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A COMPREHENSIVE STUDY OF 12 VERY RICH CLUSTERS OF GALAXIES. I. PHOTOMETRIC TECHNIQUE AND ANALYSIS OF THE LUMINOSITY FUNCTION*

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ABSTRACT

A study of 12 very rich clusters of galaxies based on photographic photometry has been completed at Lick Observatory. A detailed analysis of the luminosity functions indicates that, although they are similar in general appearance, variations exist which are significantly greater than would be expected for statistical fluctuations of a universal function. The differences are discussed in terms of variation of the Schechter parameters M^*, the bright-end turnover, and α, the faint-end slope. Two clusters have deviations from "$M^*_{\text{universal}}$" in the 3–4 σ range, A274 showing an excess and A2029 a deficiency of bright galaxies for their respective richnesses. Some attention is also given to evidence for different forms of the luminosity function, as may be exemplified by A168.

The nonuniversality of the luminosity function implies that M^* may be unreliable as a standard candle. It is unclear from available data, however, whether the fluctuations in M^* are correlated with cluster characteristics in such a way that the distance of a cluster could systematically affect estimation of its distance modulus. Thus the usefulness of M^* as a standard candle is questionable until the nature and extent of variations in this parameter are better understood.

The use of the first-ranked cluster member as a standard candle is also investigated. Clusters such as A2029 indicate that the first-ranked cluster member is a special object in at least some cases, lending some support to its use as a standard candle. The small scatter in M_1 for these 12 clusters can be reduced even further if a correction for Bautz-Morgan Type is applied. The rms variation in M_1 after correction is only ± 0.33 mag.

The unusually steep bright ends of luminosity functions for clusters containing cD galaxies, as in A2029, suggest that cannibalism of massive cluster members by the cD galaxy could be responsible for evolutionary changes in the distribution. On the other hand, A665 also has a steep bright end, but has no cD galaxy. This example, together with A274 where the number of bright galaxies significantly exceeds that predicted by the universal distribution, indicates that there are cases where the differences in the distributions are more likely to be initial ones than evolutionary ones.

Subject headings: galaxies: clusters of — galaxies: photometry — luminosity function

I. INTRODUCTION

This is the first of a pair of papers discussing the results of a photometric study undertaken at Lick Observatory of 12 very rich clusters of galaxies. This paper contains a brief discussion of the techniques of data gathering and reduction and a detailed look at the luminosity functions of the clusters. Paper II discusses the dynamical aspects of the clusters, including the spatial distributions of the galaxies, the M/L ratios, and the population types of the clusters.

The primary aim in this paper is to demonstrate (1) that the photometric scheme used is a reliable one, and (2) that the resulting luminosity functions show significant variation, inconsistent with the hypothesis of a universal luminosity function.

II. THE DATA

A detailed discussion of the selection of objects and the methods of data acquisition and processing can be found in Dressler (1976). A brief summary is included here.

a) Selection of Objects

Abell's (1958) catalog of rich clusters of galaxies was surveyed to provide the richest possible clusters within reach of the 91 cm Crossley telescope, and to provide a spread in Bautz-Morgan type. All but one of the clusters were classified by Abell as richness 2 or greater (as rich as or richer than Coma). To cover the Bautz-Morgan Type I clusters, three Type I examples of giant cD galaxy clusters were selected from Morgan and Lesh (1965) and from Matthews, Morgan, and Schmidt (1964).

No attempt was made to pick unusual clusters; the luminosity function was not an issue at the time of selection. The clusters picked were those which seemed well defined, reasonably regular, and relatively isolated from other clusters.

Table 1 contains a summary of the basic cluster descriptions.

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* *Lick Observatory Bulletin*, No. 771.

TABLE 1

BASIC CLUSTER DATA

				NO. OF PLATES				
CLUSTER	ABELL RICHNESS	BAUTZ-MORGAN	Z	B	V	F	ΔV	COMMENTS
A2256..........	2	II–III	0.0594	4	4	4	1274	Rich, regular, resembling Coma
A2029..........	2	I	0.0774	2	6	3	1514 [788]	cD-type cluster
A274...........	3	III	0.1289	...	1	1	...	Spiral, rich (?), not very dense
A168...........	2	II	0.0449	4	4	4	576	Spiral, rich, not very dense
A154...........	1	I–II	0.0652	2	2	2	829	Coma-like core
A2670..........	3	I	0.0774	...	4	4	890*	cD-type cluster
A98............	3	II–III	0.1034	...	...	6	786	
A1940..........	3	II	0.1389	...	...	6	715	Contaminated field
A1413..........	3	I	0.1427	...	...	6	...	cD-type cluster, very rich
A665...........	5	III	0.180	...	...	6	...	
A2218..........	4	I	0.1641	...	...	2	...	cD in regular, very rich cluster
A401...........	2	I	0.0750	...	1	1	1390†	cD-type cluster

* Oemler 1974*b*. † Hintzen, Scott, and Tarenghi 1977.

b) *Acquisition*

The photometry used in this study came from photographic plates taken with the Crossley reflecting telescope at Lick Observatory at a scale of 38".7 mm^{-1}. All the clusters were studied in an F band with O98 emulsions and 2 mm of RG1 filter; in addition, five of the clusters were studied in the B and/or V band as well.

All plates were calibrated with spot sensitometry. The spots were then scanned by the Lick microdensitometer–PDP8/I system in order to determine the relation of incident intensity to plate opacity. The plates themselves were then scanned with an 18 μm × 20 μm slit, and the entire area (~0.2 sq. deg.) was recorded on magnetic tape.

c) *Processing*

The scale of 39" mm^{-1} of Crossley plates and the relatively small area to be studied allowed consideration of both automated and visual galaxy/star discrimination. The author determined after some experimentation that visual discrimination using the best plate of the cluster was more reliable than the simple computer discrimination schemes which could be employed. Although star/galaxy discrimination was quite difficult near the plate limit of $m_{F(23.8)} \approx 20.0$, in fact very little data analysis was done with galaxies fainter than 19.0 mag where the problem was not serious. The galaxy selection process was accomplished by having the computer drive the microphotometer and stop for visual inspection of every image on the plate whose central intensity was above a minimum threshold of approximately 10% of the sky intensity.

With a list of positions of galaxies to be studied, the computer retrieved from the magnetic tape appropriate areas containing each galaxy and sufficient surrounding "sky." The computer then determined accurate centroid positions of the galaxy, determined the intensity of the sky, and searched for any detectable eccentricity and orientation of the galaxy. Finally, the intensity in elliptical annuli as a function of semimajor axis was determined, and the total intensity within an isophote 5% of sky was calculated. All of the above information was stored on tape for future reference. Operating in a totally automated mode, the computer processed approximately 100 galaxies per hour.

d) *Analysis of Errors*

In all but three clusters, six or more plates were used to reduce the inherent scatter of the detection and measurement procedure. Figure 1 shows a comparison of two plates of average quality of Abell 98. A variety of plate-to-plate comparisons were made with plates of different depth, seeing, sky fog nonuniformity, and camera orientation in order to estimate the effects of these variables on the random and systematic errors. The only systematic errors noted were in the comparison of good and bad seeing plates, where apparently the light lost due to the seeing disk caused a substantial underestimation of the brightness of the small, faint galaxies on the poor plates. The random errors in the

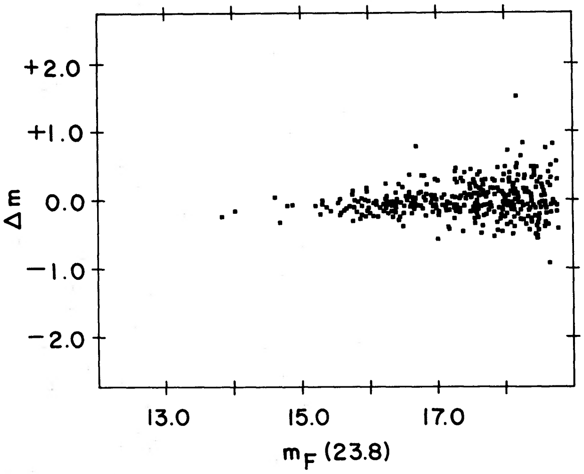

FIG. 1.—A comparison of the magnitudes of the galaxies in A98 as derived from independently reduced photographic plates.

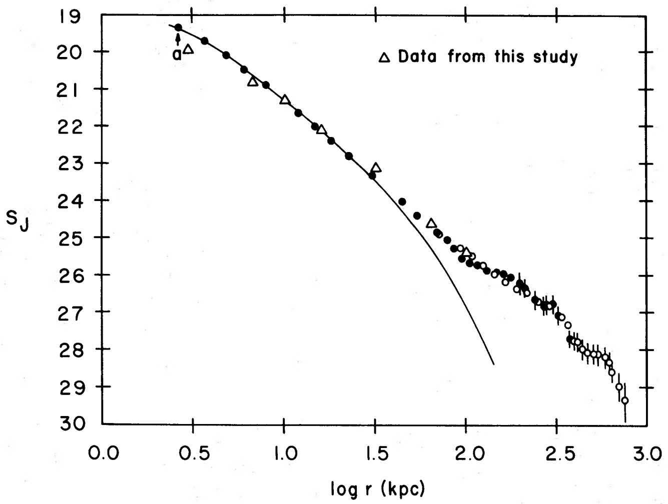

FIG. 2.—Reproduction of Oemler's (1973) Fig. 4, showing the surface brightness profile of the cD in A2670. The triangles indicate values found in the present study.

magnitudes of the galaxies were assessed to be on the order of 0.1 mag for the comparison of the best plates and 0.2 mag for the worst.

As another test of the size of possible systematic errors, the profile of the cD galaxy in A2670 was compared with Oemler's (1973) determination. Figure 2 shows good agreement between these two studies, with the exception of the bright core which was apparently affected by the seeing disk and guiding errors in this study. The core contains a very small fraction of the total luminosity, however, and therefore is unimportant to the determination of the total magnitude.

The absolute photometric calibration was accomplished by measuring the sky brightness during the exposures by means of a 50 mm telescope with a 1P21 photomultiplier. The calibration was made with bright stars as photoelectric standards; and once the brightness of the night sky was known, the brightness of any object on the plate could be determined. Several such calibrations were available for each cluster.

In order to check the accuracy of this system and as a further check of systematic departures of the photometry, the magnitudes of the brightest galaxies in the clusters were compared with the results of Sandage (1973), Oemler (1973), Gunn and Oke (1975), and Gunn (1976) (Fig. 3). Only in the case of A2029 is there a serious discrepancy, the result of this study agreeing with Gunn's measurement but ~0.5 mag fainter than Sandage's result.

e) *Corrections Applied to Data*

The usual cosmological corrections were applied to the data, the *K*-dimming taken from Sandage (1973) and the dependence of surface brightness with redshift being used to correct the isophotal magnitudes of the distant clusters. Color transformations were taken from Gunn and Oke (1975) and Oemler (1974*a*), and galactic extinction was taken from Sandage (1973). The values $H_0 = 50$ km s^{-1} Mpc^{-1} and $q_0 = +1$ have been adopted for this study.

The final isophotal magnitudes and finding charts for the 12 clusters can be found in Dressler (1976).

The background correction used in this study was taken from Figure 2 of Oemler's 1974*a* work. In order to establish that the magnitude system in the present study was comparable with Oemler's, and in order to check Oemler's claim that there is no advantage in

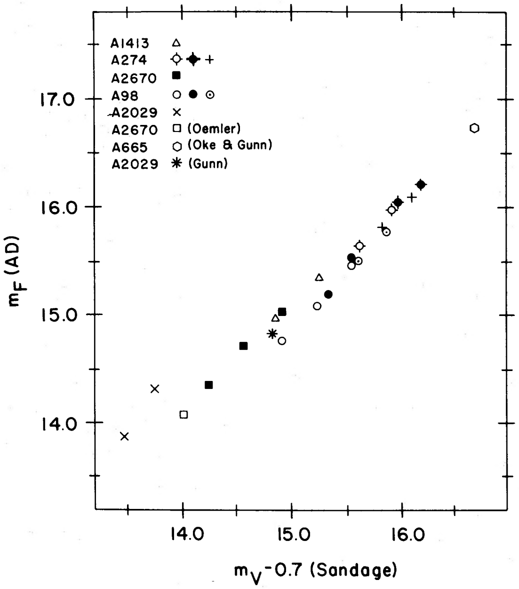

FIG. 3.—Comparison of the photometry of this study with that of other authors. The data for five clusters is compared with data from Sandage (1973) and for three other clusters comparison with other studies is included. Multiple symbols for the same cluster indicate the comparison of more than one galaxy in that cluster. The appearance of the same symbol more than once in the diagram indicates the comparison of different aperture measurements of the same galaxy.

determining the background correction in the immediate vicinity of each cluster, background counts were independently determined for the clusters in this study. The Shane-Wirtanen counts in 10′ × 10′ bins (unpublished) and the Crossley data were used to estimate the background in the following way. The Crossley data were examined to determine at what magnitude m_{SW} the number of galaxies in the area covered by the Crossley plate was equivalent to the Shane-Wirtanen count in the same area. The Shane-Wirtanen background was then found by averaging over many square degrees at a distance of several degrees from the cluster. The Crossley background at $m_F = 18.6$ was then calculated as the Shane-Wirtanen background multiplied by dex $[0.6(18.6 - m_{\mathrm{SW}})]$, 0.6 being the average value of $d \log N/dm$ for the magnitude range in question (see Fig. 2 of Oemler 1974). The backgrounds calculated in this way agreed very well with Oemler's data, the average value being 345 galaxies per square degree with a standard deviation from field to field of 25%. This latter figure is in very good agreement with the expected fluctuation in the angular covariance function for galaxies in this magnitude range over an angular size of 1/2°, and is considerably larger than a $\sqrt{N}$ fluctuation.[1]

Oemler's relation was thus adopted as given in all cases except A274 and A1940. In A274 the background was reduced by 20% in response to low Shane-Wirtanen counts and to prevent the cluster spatial profile from being a strong function of magnitude (see Paper II). (This correction tends to *moderate* the unusual luminosity function discussed in § III.) In A1940 the background was increased by 20% in order to offset the contamination of the cluster by foreground and background clusters (see Faber and Dressler 1977).

The number of background galaxies subtracted in each cluster can be found in Table 2. Incompleteness corrections were made by determining the distribution $I_{\mathrm{tot}} = f(I_0)$ (where I_{tot} is the total intensity of the galaxy out to some isophote, and I_0 is the measured central intensity) from the data where the sample was complete. This distribution function was then used to predict what fraction of galaxies with a given I_{tot} would be missed because their central intensities were below the detection threshold. (Though I_0/I_{tot} is not actually independent of absolute magnitude, this assumption was adequate for this determination.) On the average this incompleteness correction was well described by the integral of a Gaussian curve centered on $m_F = 19.0$ with a standard deviation of 0.40 mag. The incompleteness correction for each cluster can be found in Table 2.

The data were analyzed to as faint a limit as possible, until the background became equal to the number of true cluster members. This resulted in a faintness limit of $m_F \approx 18.6$–19.2, depending on cluster richness. In no case within the adopted limits did the incompleteness correction at a given magnitude exceed 30%.

TABLE 2

BACKGROUND AND INCOMPLETENESS CORRECTIONS

Cluster	Total No. of Galaxies $m_F < 18.6$	% of Total No. Subtracted as Background Galaxies*	% of Total No. Added to Compensate Incompleteness*
A2256......	372	20	4
A2029......	328	23	3
A274.......	93†	38	5
A168.......	173	43	6
A154.......	191	39	7
A2670......	213	35	7
A98........	279	27	4
A1940......	173	50	6
A1413......	222	32	6
A665.......	129	57	9
A2218......	239	31	4
A401.......	231	22	3

* See § II*e* for distribution in magnitude.

† Restricted area due to contamination by foreground cluster.

III. THE LUMINOSITY FUNCTIONS

It is clear from even a casual inspection of the Palomar Sky Survey that the luminosity functions of clusters of galaxies are quite similar, consisting of a more or less flat faint end terminated by a steep cutoff for high-luminosity galaxies. Though the exact form of the luminosity function may be uncertain, it is apparent that this cutoff occurs at roughly the same absolute magnitude in different clusters.

The lack of variety among cluster luminosity functions can be understood if one supposes that galaxies formed at an earlier epoch than clusters and under relatively uniform conditions over large volumes of space. In this case the luminosity function of a cluster would reflect only statistical selection from a pool of galaxies whose luminosity distribution is universal.

On the other hand, if there are variations from cluster to cluster which are larger than what would be expected from sampling statistics, this could imply additional possibilities. (1) Large-scale (cluster size) inhomogeneities of density, temperature, or angular momentum might have existed in the early universe. (2) Clusters may undergo evolutionary changes which result in alteration of the original luminosity function. For example, a cD galaxy might cannibalize other massive cluster members. Collisions and stripping might also alter galaxy characteristics. (3) Clusters themselves might have formed at an earlier epoch than individual galaxies.

No significant variation in cluster luminosity functions has been claimed by previous authors. Indeed, Abell (1962) has suggested that the luminosity function is truly universal, that is, both the form *and* parametrization are the same in every cluster. In this study we undertake a critical evaluation of this notion of universality, concentrating on very rich clusters where statistical fluctuations are minimal.

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[1] The author thanks the anonymous referee for pointing this out.

a) *The Form of the Luminosity Function*

A short summary of the history of suggested forms for the luminosity function of clusters of galaxies can be found in Abell (1962). Abell suggests that the integrated luminosity function, that is, the number of galaxies brighter than a given magnitude, is well described by the expression

$$N(<m) \propto 10^{ax+b}$$
$$(a \sim 0.8 \qquad m < m^* ,$$
$$a \sim 0.25 \qquad m > m^*) . \qquad (1)$$

In the log N versus magnitude diagram this function appears as a pair of straight lines intersecting at m^*. Data from Oemler's study (1974*a*) and the present work indicate that a very sharp break at m^* in the luminosity function may be uncommon, so the Abell form might better be thought of as the asymptotic limits of the bright and faint ends with some smooth transition around the break.

A second analytic expression has been suggested for the differential luminosity function by Schechter (1976). His expression is

$$n(L) = n^*(L/L^*)^{\alpha} \exp(-L/L^*) dL , \qquad (2)$$

which can be interpreted as a power law at faint luminosities ($L \ll L^*$) terminated by an exponential for $L > L^*$. The characteristic magnitude M^*, represented in this expression by L^*, again parametrizes the transition between the two regions. The differential luminosity function proposed by Schechter increases monotonically and is continuous, in contrast to the differential function implied by Abell's integral form, which has a discontinuity and a local maximum at m^*.

Schechter compared his form of the luminosity function with the data collected by Oemler and found fairly acceptable fits in all but two cases, these being apparently too flat in slope at the faint end (A665 and A2670). Schechter also compared his expression with the field galaxy distribution and found, again, reasonable agreement with the form, and perhaps more interestingly, that the parameters obtained in the fit for the field galaxy distribution agreed well with the values for the cluster fitting.

A further investigation by Turner and Gott (1976) of small groups of galaxies (~ 10 members) indicates a good fit to the Schechter form with a similar M^*, but a value of the faint-end slope α which is somewhat flatter, $\alpha \sim -1$. They suggest that the value Schechter found by rich clusters (also in agreement with Abell's results) of $\alpha \sim -1.25$ could be peculiar to rich clusters. This paper will deal with this question in more detail subsequently.

To summarize, then, there have been two widely used suggestions for the form of the luminosity function, Abell's and Schechter's; there has been little or no evidence of variation in form or in the value of M^*, and only marginal evidence for a variation of the faint-end slope.

The luminosity functions for the clusters in this study are shown in Figure 4. In the discussion to follow, three types of variation are discussed. First, using the concept of the "break" introduced by Abell, the luminosity functions are investigated for variations of M^* too large to be purely statistical fluctuations. A2029 and A274 are found to have the largest deviations from a universal M^*, though others are possibly significant. Second, the evidence for variation of the faint-end slope is considered. A2670 is confirmed to have a very flat faint end as reported by Oemler, and A401 is presented as a further example of this type of variation. Third, after discussing the variation in these parameters, the evidence for variation in the form itself is considered. A168 is suggested as an example of a luminosity function apparently inconsistent with either the Abell or Schechter form. Finally, the ratios of "bright" to "faint" galaxies in the 12 clusters are examined for further evidence of variations in the distributions.

b) *The Variance of M^**

This parameter has received much attention as a potential standard candle (Bautz and Abell 1973; Austin, Godwin, and Peach 1975) and has been highly recommended by virtue of the small scatter in the [m^*, log (cz)] diagram for 12 clusters studied by these authors. The technique, of course, rests on the assumption of a universal luminosity function.

In order to investigate the variations of M^* in the 12 clusters in this study, it was necessary to estimate the expected variation of M^* due to a finite sample size and then to compare this with the observed fluctuations.

A standard procedure for defining M^* had to be adopted. It was decided that the Abell formulation was not suitable for this purpose for several reasons. Since the value of M^* was to be determined by the computer to eliminate any biases of the observer, some sort of fitting routine, rather than eye-estimates, had to be adopted. A χ^2 minimization routine using the Abell function as a model would not be very reliable, since the errors in the bins would not be uncorrelated in the integrated distribution, and because the non-physical cusp of the Abell break gives a poor fit to many clusters. (Thus, without providing some arbitrary smoothing around the break, even the best χ^2 fit would not have been very good.)

Schechter's expression for the differential function avoided both these problems and was adopted to study the variation of M^*. Further discussion of the Abell function follows below.

The procedure amounted to the following statement: Suppose all clusters exhibited only a statistical fluctuation of a universal Schechter function. How large a variation in M^* would be expected due to finite sample size?

In order to estimate the expected variation in M^* the following scheme was used. A series of Monte Carlo models was generated, the computer providing N random numbers from 0 to 1, and the luminosities were assigned in proportion to the probabilities predicted by the Schechter function with $M_F^* = -22.6$

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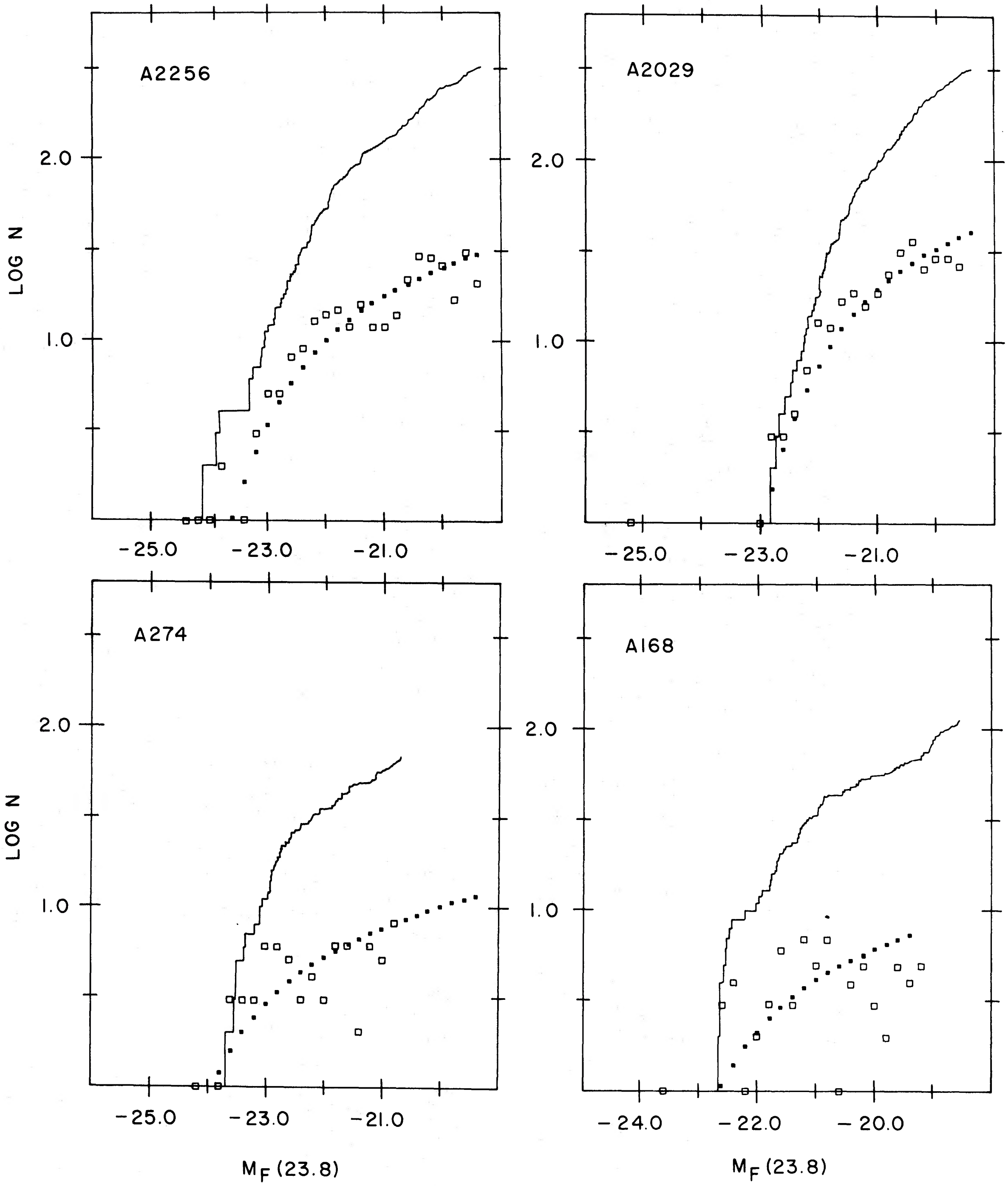

FIG. 4.—The luminosity functions of the clusters. *Solid line*: actual integrated distribution (corrected for background and incompleteness); *open squares*: differential distribution in 0.2 mag bins; *solid squares*: best-fitting Schechter function.

and $\alpha = -1.25$. For example, if the ith random number was 0.45, the ith galaxy would be assigned a magnitude M_i such that in a perfect Schechter function 45% of the galaxies would have $M < M_i$. Thus each model cluster of N galaxies would be statistical fluctuation on the Schechter model. The computer would then treat the simulated cluster as real data and find by χ^2 minimization a Schechter function that best described the data, allowing M^* to vary as a free parameter. (Unlike the Schechter technique, N^*, the richness normalization, is not taken as a free parameter—here the Schechter function is normalized so that the total number of galaxies predicted equals the observed number in the data.) By generating several

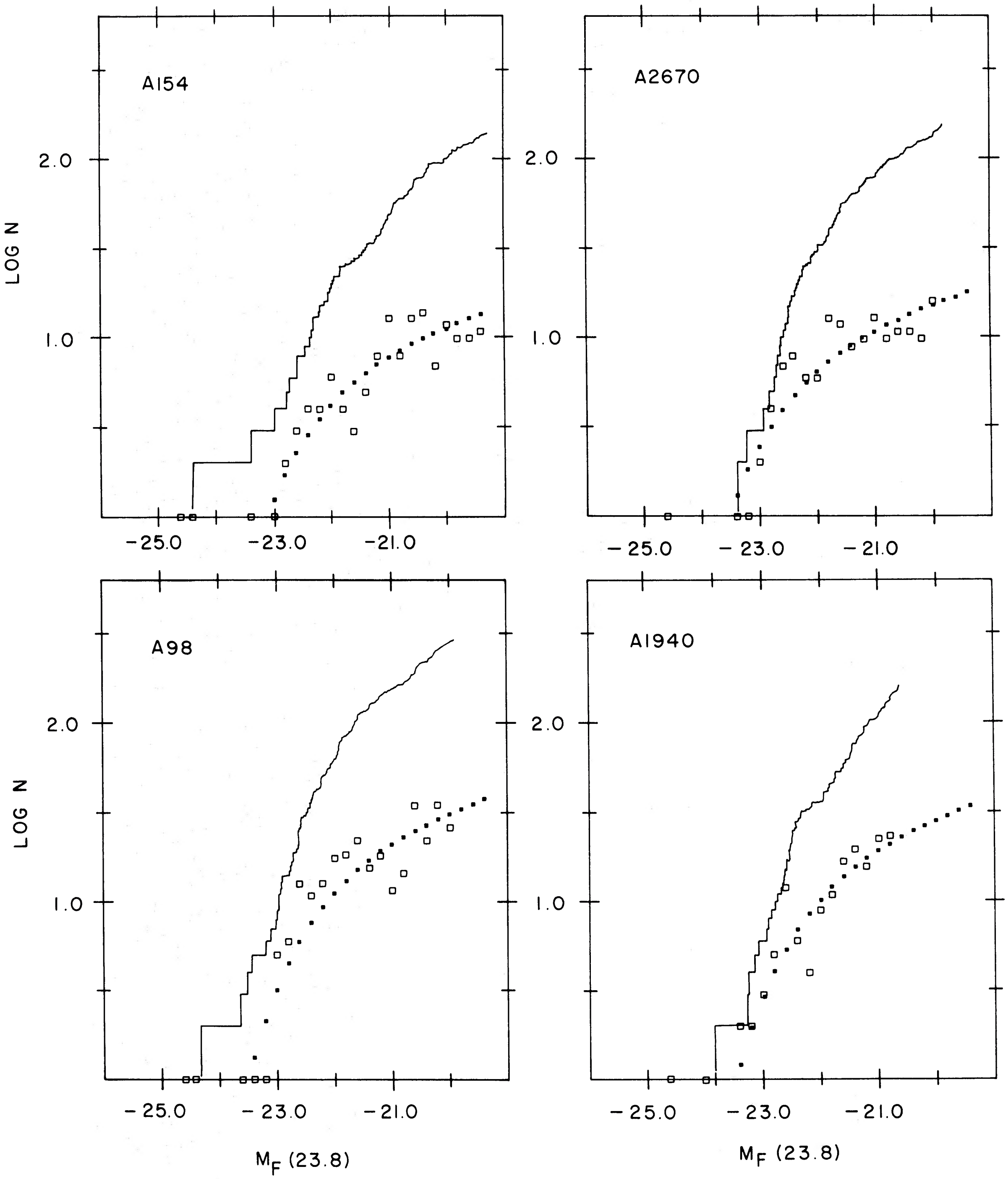

FIG. 4.—*Continued*

thousand such models the dispersion $\sigma(M^*)$ for the N-size cluster could be determined.

Because the 12 clusters studied had different numbers of galaxies and different faintness limits (due to background limitations), $\sigma(M^*)$ was determined for a grid as $\sigma(M^*) = f(N, M)$, where M is the faintness limit of the data samples. Table 3 gives a summary of the results.

The value of $\sigma(M^*)$ for a given value of M was found to vary as $\sqrt{N}$ to the accuracy of the determination; therefore, values in the table reflect the best fit of $\sigma \propto \sigma_0/\sqrt{N}$. Interpretation of the table leads to some obvious conclusions. It would be difficult to detect a variation of M^* in any Abell richness 0 to 1 class cluster (the upper left corner of the table), since even a variation of M^* of 0.5–1.0 mag would not be

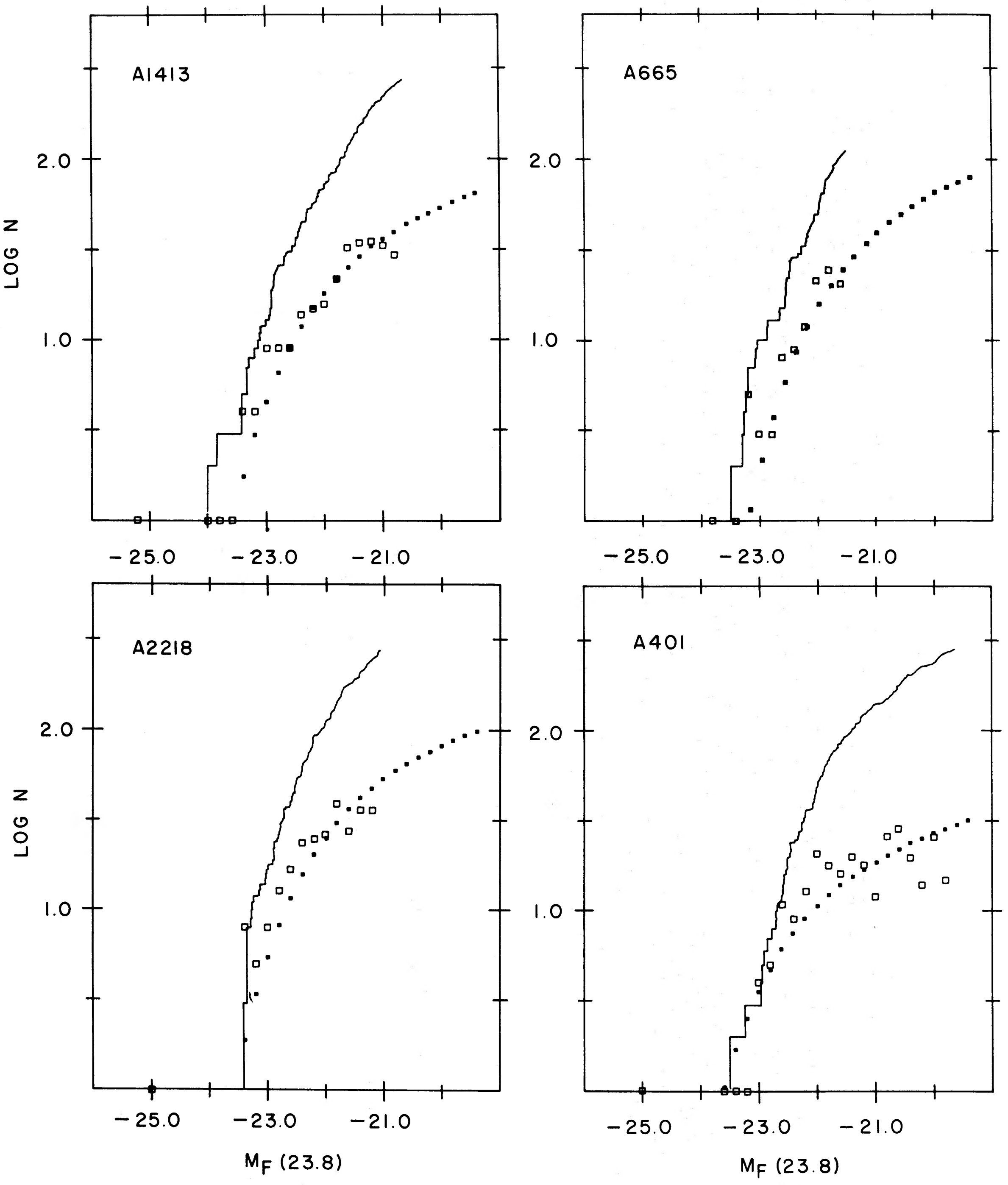

FIG. 4.—*Continued*

statistically significant. However, a richer cluster like Coma (300 galaxies brighter than $M_F = -20$) should show a deviation of only about 0.2 mag if the luminosity function is truly universal. Furthermore, if the cluster is very rich, the accuracy of the determination of M^* does not suffer severely even if the total number of galaxies observed is lower (due to a brighter M). This is because the lack of galaxies on the fainter, flat part of the curve does not raise the uncertainty of M^* as much as the deficiency of galaxies on the bright end in a poorer cluster. In other words, because of the steepness of the curve at the bright end, it is easier to detect a deviation of M^* in a rich cluster with a bright M than in a poorer cluster with a much fainter M.

Before comparing the data for the program clusters to the results of the Monte Carlo modeling, one

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TABLE 3

σ_{M^*} (No., M)

Sample No.	< -19	< -20	< -21	< -22
50......	0.46	0.42	0.33	0.24
100......	0.33	0.30	0.24	0.18
200......	0.23	0.21	0.17	0.13
300......	0.19	0.17	0.14	0.10

additional issue should be raised. Since the aim here is to test the hypothesis that the variations in luminosity functions are purely statistical, it is important to consider any selection effects in the choice of the 12 clusters which might have increased the probability of finding an abnormal M^*. In this sample the atypical distribution in Bautz-Morgan (BM) type (the high percentage of BM Type I clusters) is the only point of concern. One might expect that the variance in M^* could be correlated with Bautz-Morgan type if, for example, BM Type I clusters are systematically those with a fainter than usual M^*, thus resulting in increased contrast between the brightest galaxy and the fainter cluster members. For the rich clusters in this study, however, the Bautz-Morgan type is determined by the brightest few galaxies, while M^* is determined by the brightest 20 or more. Therefore, within the context of the statistical model, the validity of which is being tested here, these two quantities are virtually uncorrelated, so selecting for Bautz-Morgan type should not significantly affect M^*.

This hypothesis has been verified by generating Monte Carlo models of luminosity functions and analyzing the variation of M^* as a function of the relative luminosities of the three brightest galaxies, representative of different Bautz-Morgan types. This calculation was done for simulated clusters at Abell richness 1–3 and for both the Schechter and Abell forms of the luminosity function. As was expected in light of the previous discussion, the total variation of M^* caused by selection for Bautz-Morgan type was less than 0.05 mag.

With the table of expected variations in M^* (from the Monte Carlo simulation) in hand, M^* was determined for the 12 program clusters. Again, a Schechter function with $\alpha = -1.25$ was allowed to vary in M^*, and the best fit was chosen. Since these data were treated in exactly the same way as the model clusters, whether or not a given set of data was well fitted by the curve is not important. Any consistent parameterization could be applied to both the models and data; however, a very poor approximation to the data would tend to mask any real deviations if they did exist. An erroneous positive result, however, should not be produced by a poorly chosen representation.

A168 was such a poor fit to the Schechter form that its value of M^* of -22.43 ± 0.45 was considered indeterminate. It is therefore not included in any of the following discussion of variations in M^* and has not been plotted in the relevant diagrams. Further discussion of this cluster is found in part (*d*) of this section.

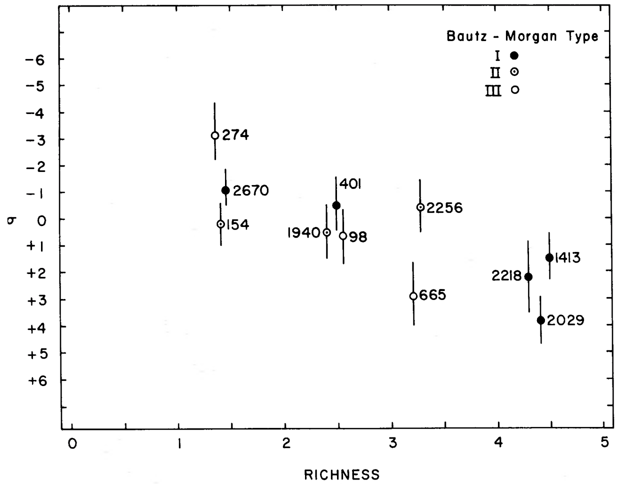

FIG. 5.—The deviation of M^* from a "universal" value of $M_F = -22.6$, given in standard deviations and plotted against cluster richness (see Paper II). A positive deviation is in the sense that M^* is too faint.

The results of the analysis of variations of M^* are given in Table 4 and Figure 5. Of the 12 clusters in this study, A2029 and A274 show the most significant variation of M^* from the universal value M_{un}^*; A665 is also probably significant, and A2218 is marginally deviant. The error bars in Figure 5 represent the combined uncertainty of the photometric zero-point and the uncertainty of the χ^2 fitting, both of which are in the 0.1–0.2 mag range. Since the faint end of the luminosity function is relatively flat compared to the bright end, the fitting procedure is insensitive to variations in the background. For example, a $\pm 1\,\sigma$ variation in the background ($\pm 25\%$) changes the best-fit value of m^* by less than 0.05 mag in A2029 and A274, and thus this uncertainty is a small one compared with the two previously mentioned.

In Figure 5 the value of M^* has been plotted versus a richness measure described in Paper II. A trend of fainter M^* with increasing richness seems present in the diagram; however, A274 should probably be excluded in the search for a trend, since the work of other authors (e.g., Oemler) indicates that the mean value of M^* for poorer clusters is close to the "universal" value. The correlation coefficient for the data excluding A274 is 0.66, indicating a $2.1\,\sigma$ result—interesting, but certainly not convincing.

If the observed scatter from the universal value of M^* was merely statistical variation, then the distribution of the observed/expected variations should be normal with a χ^2 per degree of freedom of order 1. The χ^2 of 4.0 per degree of freedom for the data in Table 4 further indicates the substantial departure from a universal M^*.

A graphic comparison of the variations in the luminosity functions is also included. Figure 6 compares the integrated luminosity functions of A2029, A2256, and A274, all arbitrarily normalized to have the same number of galaxies at $M_F = -21.0$.

To the reader who may be skeptical of photographic

TABLE 4

Deviation of M^*

Cluster	M^*	$\Delta M \equiv$ $M^* - M_{\mathrm{universal}}$	σ (Monte Carlo)	Δ/σ M.C.
A2256..........	−22.71	−0.11	0.20	−0.5
A2029..........	−21.85	+0.75	0.20	+3.8
A274...........	−23.60	−1.00	0.29	−3.4
A168...........	−22.43(?)	+0.17	0.45	...
A154...........	−22.55	+0.05	0.27	+0.2
A2670..........	−22.87	−0.27	0.24	−1.1
A98............	−22.50	+0.10	0.17	+0.6
A1940..........	−22.50	+0.10	0.20	+0.5
A1413..........	−22.38	+0.22	0.16	+1.4
A665...........	−21.94	+0.66	0.23	+2.9
A2218..........	−22.25	+0.35	0.16	+2.2
A401...........	−22.69	−0.09	0.20	−0.5

or galaxian photometry in general, a set of photographs is included which illustrates the abnormality of the luminosity functions of A2029 and A274 by comparing these two clusters with two others having "normal" luminosity functions, A2670 and A1413. A2029, containing the giant type cD galaxy (Matthews, Morgan, and Schmidt 1964) can be compared directly with A2670 since they have the same redshift of $z \sim 0.077$ (Fig. 7). Out to the 23.8 mag per square arcsec isophote the cD galaxy in A2029 is about 0.7 mag brighter than the cD in galaxy A2670, basically by virtue of its size, since the central surface brightness is comparable in the two galaxies. One can easily see, however, that the next three or four brightest galaxies in A2670 have no counterparts in A2029, and the next 20 or so are represented by only a few in A2029. The value of M^* in A2029 is about 1.0 mag below M^*_{un}, a 4 σ deviation, and it is particularly interesting that the luminosity function seems to fit a Schechter function with this very low M^* quite well.

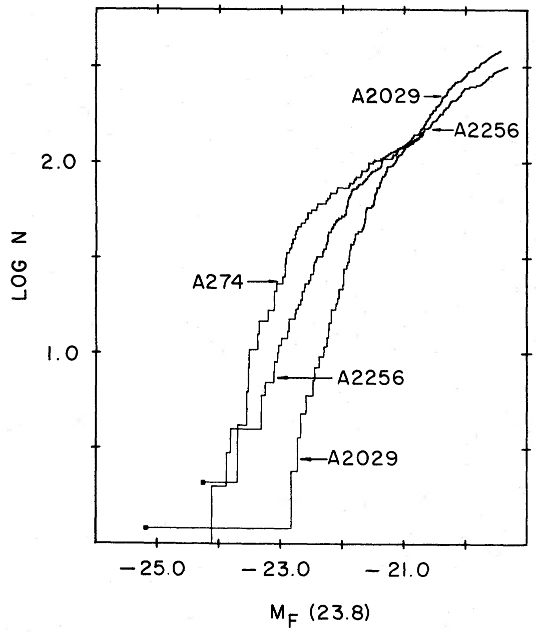

Fig. 6.—Comparison of the integrated luminosity functions of A2029, A274, and A2256, again normalized to $M_F = -21$. The log N scale is correct as given for A2256. The zero points for A2029 and A274 are indicated by the solid squares.

A274 is the only richness 3 cluster other than A2670 classified by Abell as distance class 4 (all others are supposedly more distant). The interpretation was that A2670 and A274 were two clusters of similar richness and galaxy content, the main difference being the presence of a giant cD in A2670. One can see from Figures 7 and 8 how easy it is to believe that the brightest galaxies in A274 are comparable in absolute magnitude with the brightest in A2670 (excluding the cD). However, the published redshift of A274 is actually 0.129 (Sargent 1973, confirmed by the author for several other cluster members), and this indicates an altogether different picture. With a distance some 60% greater than expected, the brightest members are actually a full magnitude more luminous than previously estimated. Furthermore, comparison of the photographs in Figure 8 reveals what detailed photometry confirmed. There are very few faint galaxies considering the number of bright galaxies, compared with a normal cluster like A1413. These conclusions are quantitatively illustrated in the luminosity function (Fig. 4*c*). In the case of A274 the best fit value of M^* is about a magnitude brighter than the "universal" value, a 3–4 σ result.

A665, classified by Abell as the richest cluster in his catalog, also seems to show a significantly lower value of M^* than expected; but because of the difficulties encountered in producing accurate photometry in a cluster of such large redshift ($z = 0.18$), this result is less certain. On the other hand, the luminosity function presented here is consistent with Oemler's result, and the zero point of the photometry agrees well with a multichannel scan by Gunn and Oke (1975), so there is reason to believe that this 3 σ result is also significant.

A2218 and perhaps A1413 are marginal examples of variation, and their possible significance will be discussed below.

In conclusion, the author feels that A2029 and A274 convincingly demonstrate the nonuniversality of the luminosity function by showing a variation of M^*

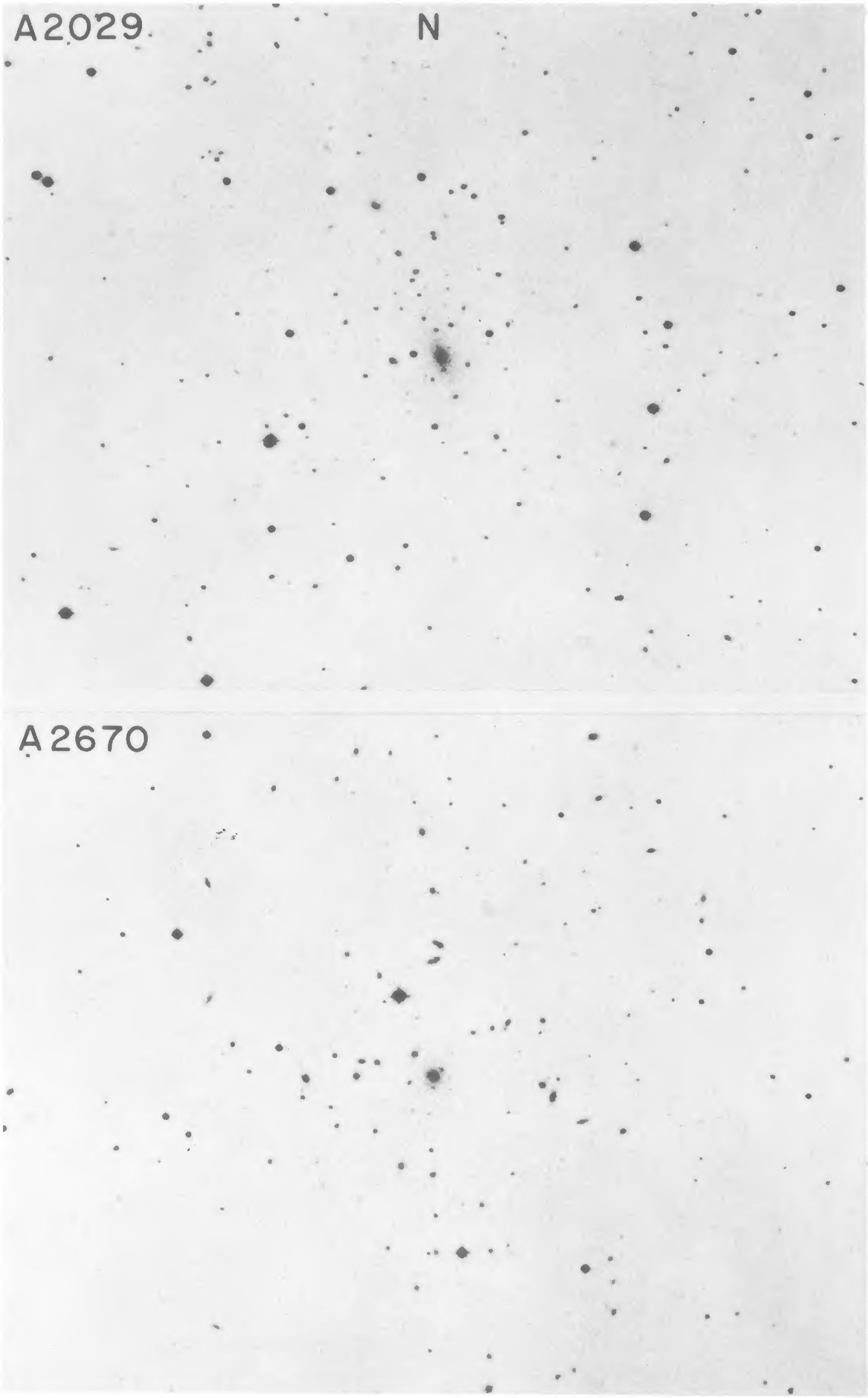

FIG. 7.—A2029 (top) and A2670. Scale ~6″.5 per mm.

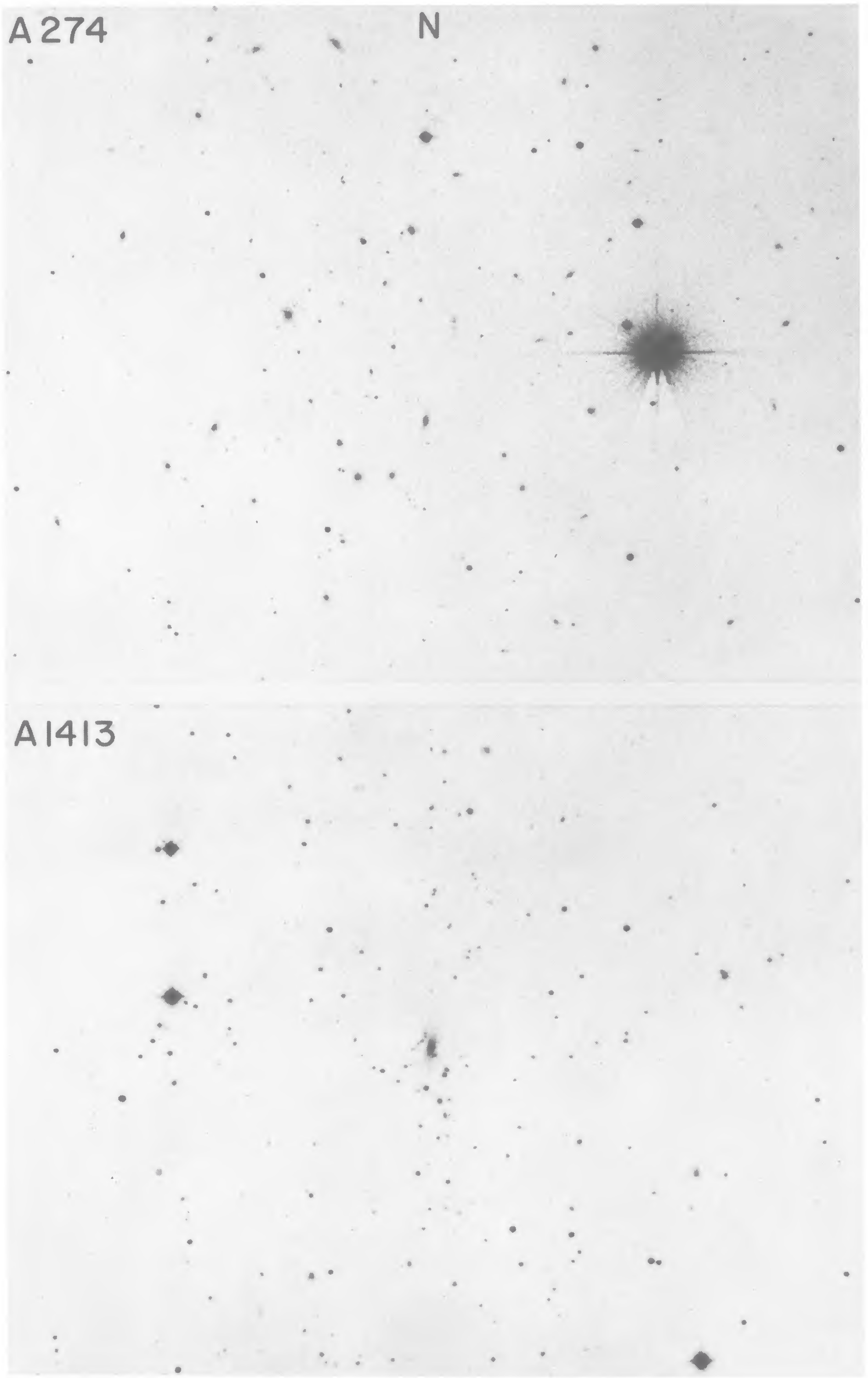

FIG. 8.—A274 (top) and A1413. Scale ~6″.5 per mm.

much greater than expected from purely statistical fluctuations.

c) Variations in the Slope of the Faint End of the Luminosity Function

Section III*b* leads naturally to another question: Can all clusters be adequately described by a Schechter function with variable M^*? In the context of Schechter's expression this question is reduced to: Are there significant variations in the faint-end slopes of cluster luminosity functions as well as variations in M^*?

As mentioned earlier, Turner and Gott (1976) have found that the composite luminosity function for small groups is better described by a faint-end slope of $\alpha \approx -1.0$ rather than the $\alpha \approx -1.25$ found by Schechter (and consistent with Abell's results). Oemler also noted that the luminosity functions of A2670 and A665 seem too flat at the faint end, and Schechter cited this as the reason for the poor fits he obtained for these clusters. The uncertainties in background and photometry are of considerable importance in A665, so that it is difficult to say anything definitive in this case. However, Schechter suggests that different treatment of A2670 by Oemler (smaller area, fainter magnitude limit) compared with his other clusters might account for the different faint-end slope. The present study, surveying a larger area of the cluster, confirms Oemler's result, although these data do not go to as faint a limit due to the greater contribution of the background. Another cluster from this sample, A401, also exhibits a flat faint end, and is an example as good as or better than A2670 (see Fig. 4*l*).

Since overestimation of the background would tend to flatten the observed and subsequently corrected luminosity function, A401 was also studied using a value of the background only half the expected value (i.e., assuming an enormous hole in the background distribution at the cluster), but the flatness of the luminosity function was not significantly changed (Fig. 9). Since this is a much larger fluctuation than should actually occur, it seems reasonable to conclude that this flat faint-end slope is real. Figure 10 illustrates the fit of A401 to a Schechter function with $\alpha = -1.0$.

Thus there are two clusters in this study with faint ends noticeably flatter than, for example, the faint end of the Coma cluster. In addition, two other clusters, A2029 and A2218, show a possible flattening at the faint end, but due to poorer statistics they cannot be considered compelling examples. A401 illustrates that it is highly unlikely that background variations could be solely responsible for these flat faint ends. However, a larger sample of clusters is probably needed to settle the question of faint-end variation. With the present data, there is no clear correlation between this faint-end flattening in clusters and any other obvious characteristics except that all are examples of very rich, regular clusters and all, with the exception of A665 as reported by Oemler, are Bautz-Morgan Type I.

Clearly more work is needed here, since it is interesting and perhaps puzzling that field galaxies and most

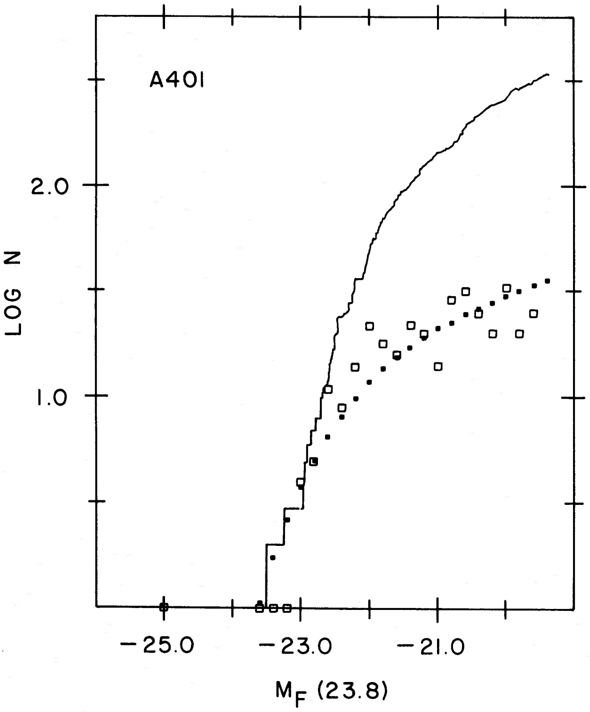

FIG. 9.—The luminosity function of A401 produced with half the background.

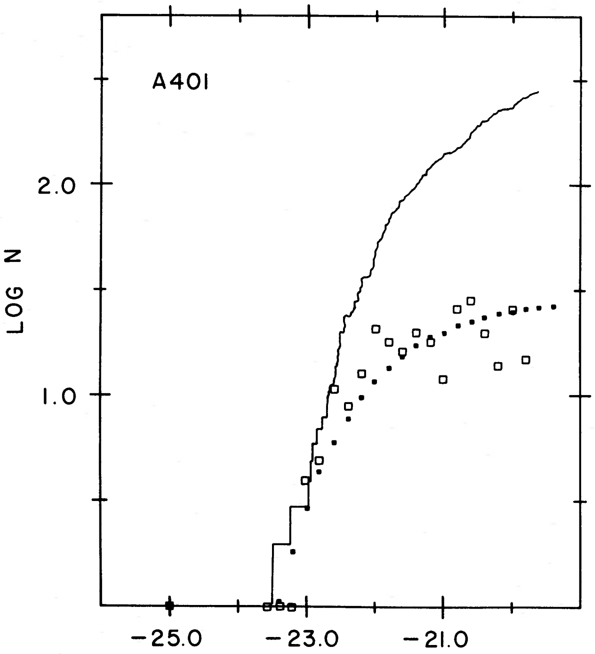

FIG. 10.—The luminosity function of A401 compared with a Schechter function with $\alpha = -1.0$.