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STOCHASTIC STAR FORMATION AND SPIRAL STRUCTURE OF GALAXIES

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ABSTRACT

Self-propagating star formation in a differentially rotating disk is capable of producing persistent large-scale spiral features. The basic idea of the process is that aggregates of stars are created by a chain reaction mechanism in which shock waves from high-mass stars induce the formation of more high-mass stars which then in turn cause further shock waves, etc. The differential rotation of the galaxy then stretches these aggregates into spiral features.

We have extended the original deterministic model in two crucial ways. First, we implement a stochastic description of the star formation process; second, we increase the size of the array to avoid the dominance of boundary effects. Our results show that this model can generate spiral features that appear very similar to those of real galaxies, in particular with respect to the density and pitch angle of these features. The spiral features appear to rotate quasi-rigidly, and, although an individual feature may only have a lifetime of the order of a galactic rotation period, they continually regenerate such that the model galaxy exhibits well-defined spiral structure for very many rotation periods. The morphological type of the galaxy is found to be a result of the form of the galactic rotation curve alone.

Subject headings: galaxies: structure — stars: formation

I. INTRODUCTION

Spiral structure in galaxies is known to be closely associated with groupings of bright young stars, gas, and dust. Because of differential rotation, aggregates of these bright young stars tend to be drawn into spiral segments. Oort (1962) pointed out, however, that the problem of the existence of spiral arms is not that of spiral segments that last for the lifetime of bright O and B stars, but rather that of persistent features which in most cases extend all the way from the galactic nucleus to the edge of the galaxy. This is normally taken as suggesting that the spiral structure is not produced by local processes but is due to a global property of the galaxy. The gravitational theory of spiral density waves of Lin and Shu (1964) reflecting this point of view has been very successful in providing an explanation of the large-scale coherence of the spiral structure. However, as pointed out by Toomre (1977), difficulties remain in explaining the persistence of such features for more than a few rotations of the galaxy.

In this paper, we explore precisely the opposite point of view, namely, that the large-scale ordering is induced by purely local processes. Recent results (e.g., Berkhuijsen 1974; Elmegreen and Lada 1977; Herbst and Assousa 1977; Blitz 1977) seem to suggest that star formation proceeds as a spatially ordered chain reaction, as first suggested by Öpik (1953) and later by Blaauw (1964). It is not yet clear exactly which basic physical mechanisms would produce such triggering of star formation activity, and several have been proposed by these authors. Most of them rely on compressing interstellar clouds by supernova explosions or ionization-front shocks and stellar winds from bright massive stars. Taking the present observational evidence at face value, the possibility exists that the spiral segments delineated by bright O and B stars are extended self-reproducing structures. The question that must be asked is the following: Can they be sufficiently extended so as to completely delineate the spiral arms; and if so, how long do they persist?

In a recent paper Mueller and Arnett (1976) (hereafter MA) studied the structures produced by a model in which stars were produced by a chain reaction mechanism in a differentially rotating galaxy. The chain reaction is produced by high-mass stars which become supernovae. The expanding shells of gas from these supernovae trigger nucleation of stars in nearby regions. Some of these new stars will also be massive stars which will in turn become supernovae repeating the process. MA dubbed the process "self-propagating star formation" (SPSF). Notwithstanding the extreme simplicity of the model, some rather tantalizing results were obtained. MA observed large irregular spiral structures which, although they did not persist for long periods of time, did regenerate from time to time as the galaxy evolved.

In this paper we want to pursue the SPSF model more fully in order to determine, first, its suitability as a model for real galactic evolution, and second, if it is suitable, the important parameters for ascertaining the features of known galactic structure. We extend the model in two important ways. First, we make the model stochastic instead of completely deterministic by introducing a finite probability for star formation. The reason for this is that physically we have no assurance that each massive star will indeed induce creation of another massive star. The possibility of triggering star formation will depend on the details of the local matter distribution and its dynamic properties. Furthermore, the number of massive stars created is only a small percentage of the total stars created (e.g., Salpeter 1965). Second, we have extended the discrete array chosen to represent the galaxy to a size large enough to ensure that finite size effects do not dominate the results.

Our results indicate that stochastic self-propagating star formation (SSPSF) produces large-scale spiral features that are stable over long times of the order of the lifetime of the galaxy. These features, given by the loci of star formation, rotate "quasi-rigidly." The spiral structures obtained using the observed rotation curves for Sb and Sc galaxies correspond very well with the appearance of the arms in these galaxies (e.g., pitch angle and arm density). Each morphological type appears to be due simply to the rotation curve that exists in the galaxy. That is to say, in this model there is no evolution between morphological types.

II. THE MODEL

The model we choose is a two-dimensional disk which we divide into N rings. Each ring is further divided azimuthally into cells such that each cell in the array has the same area. The rings are allowed to rotate following an imposed rotation curve. The nearest neighbors of any cell are defined as those cells having contiguous borders. Due to differential rotation the nearest neighbors continuously change and can vary in number between five and seven depending on exactly how adjacent rings line up at any given time step. The model is also discrete in time. All new stars are created and all existing stars age simultaneously every time step. Although we keep account of stars for many time steps, all supernovae occur at the time step immediately following the star's creation.

In the context of the SSPSF model the terms "star" and "supernova" do not necessarily refer to the single objects normally denoted by these terms. For simplicity of expression, a cell in which SSPSF induces star formation is spoken of as containing a "star," and a cell which induces star formation in an adjoining cell is spoken of as having become a "supernova." Any cell is allowed to contain only one "star" at any given time. In actuality, the term "star" corresponds to groups of young stars, open clusters, and associations with their H II regions. By the same token "supernova" corresponds to a star or group of stars capable of triggering star formation.

For the initial state of the galaxy we randomly populate about 1% of the cells with bright young stars. These stars then have a finite probability (P_{st}) of creating another bright massive star in an adjoining cell at the next time step. Thus a chain reaction can be initiated. Once a star has been created in a given cell, it becomes harder to create a new star in that cell for a refractory period of length τ_r (τ always refers to time in units of time steps). In addition to this stimulated star formation, we let a small number of stars be spontaneously created randomly throughout the galaxy (P_{sp}). These act as new centers of nucleation and help stabilize the model.

The difference between our model and the MA model is threefold. First, our model is stochastic: if a cell has a supernova as a neighbor, there will not necessarily be a massive star created in it at the next time step, but the probability of creating such a star will be just P_{st}. Second, we use a larger number of rings and cells. As we will discuss later, if the number of rings chosen is too small, edge effects will dominate destroying the spiral structure quite early. Third, the probability of nucleation of a star in a cell in which a star has recently been created is not zero for τ_r time steps. Rather, the probability of nucleation changes smoothly over that length of time. A number of forms have been tried for this function, and, although it is important that the probability is of reasonable magnitude, the results are not very sensitive to the exact form. The function usually used is

$$P = P_{\mathrm{st}}\tau_a/\tau_r \tag{1}$$

for $\tau_a \leq \tau_r$, where P is the probability of nucleating a star in the cell in question and τ_a is the age of the star last created in that cell. For $\tau_a \geq \tau_r$, $P = P_{\mathrm{st}}$.

The model parameters may be given some physical significance in the following manner. For example, if we choose a galactic radius G_R and N rings, then the size of a cell is G_R/N, and the time step can be taken as the time it takes a supernova remnant (or an ionization front produced by a bright massive star as suggested by Elmegreen and Lada 1977) to reach an adjacent cell. That is, the time step is given by

$$\delta t = \frac{G_R}{NV_I}\,, \tag{2}$$

where V_I is taken as the velocity of propagation of the supernova remnant (or the ionization-front propagation velocity). Finally, τ_r can be considered as the length of time it takes for gas to return to a cell which has experienced a supernova. The return can come about in two ways, either due to the propagation of a molecular cloud into the cell under consideration, or due to the diffusion of gas from other high-density regions. We choose the latter since it must be remembered that, although we have not explicitly taken the gas into account, the model implicitly assumes a homogeneous disk of gas modified only by the existence of a finite τ_r. Furthermore, our model is static in that, except for the imposed rotation curve, no explicit stellar or gas dynamics (such as molecular clouds) are considered. Therefore, for the case of supernova-triggered SSPSF, the gas return proceeds by diffusion at the velocity of sound (V_s) of the interstellar gas so that

$$\tau_r = V_I/V_s\,. \tag{3}$$

As will be discussed below, within limits, the results are not very sensitive to the value of τ_r.

In order to make more quantitative statements about the density and ordering of the star distributions, we use the concept of normalized geometrical entropy introduced by Schulman and Seiden (1978). The array is coarse grained into 3 × 3 blocks of cells. The entropy of such an array is then

$$S = \frac{1}{9J} \log \prod_{i=1}^{J} \frac{9!}{j_i!(9 - j_i)!}, \tag{4}$$

where J is the number of blocks in the array and j_i is the number of occupied cells in block i. This entropy reflects both changes in ordering and density. Since we are primarily interested in the order induced by SSPSF, we divide equation (4) by the entropy of a random distribution of the same density. The normalized entropy is then (see Schulman and Seiden 1978)

$$\bar{S} = S/\langle S \rangle, \tag{5}$$

where

$$\langle S \rangle = \frac{1}{9} \sum_{k=1}^{9} \rho^k (1 - \rho)^{9-k} \frac{9!}{j_i!(9 - j_i)!} \times \log \left(\frac{9!}{j_i!(9 - j_i)!} \right). \tag{6}$$

A value $\bar{S} = 1$ (logarithms are taken to base 2) signifies a completely random state; deviations below unity provide a measure of the order of the system.

The number of parameters entering the model itself is only five. These are the number of rings (N), the refractory period (τ_r), the stimulated probability (P_{st}), the spontaneous probability (P_{sp}), and the relative angular rotation curve (i.e., in units of radians per time step). We have considered a number of variations of these parameters and will describe below the effects of these variations. However, a majority of the computer runs have been carried out with the following set of parameters. The number of rings used is 49, which results in 7350 cells. $P_{st} = 0.28$, $P_{sp} = 0.0002$, $\tau_r = 11$, and the rotation curve chosen is that for either M101 or M81 as given by Roberts and Rots (1973). Unless explicitly stated to the contrary, the results described below are valid for both larger and smaller arrays. As noted above, these parameters may be further related to other physical parameters of a real galaxy. For example, choosing $G_r = 15$ kpc and $V_I = 10$ km s^{-1} gives a cell of 306 pc and a time step of 15 million years. It should be remembered, however, that although parameters such as G_r and V_I are certainly necessary to compare the model to real galaxies, they do not enter directly into the model itself (eqs. [2]–[3]).

III. RESULTS

A set of typical results is shown in Figure 1, where stars up to age 10 time steps old are plotted. The older stars can be considered to represent the remains of the supernova and the smaller stars which were created in a given cell along with the massive star. The symbol size is inversely proportional to the age of the star; thus the smaller symbols represent older stars. Including older stars produces much clearer spiral structure since it connects the regions of active star formation which are genetically related. For the particular choice of parameters given above, 10 time steps equals a lifetime of 150 million years which is long for representing the bright stars that delineate the arms in real galaxies. However, from our experimentation, we find that the results are essentially the same for twice the number of rings and five time steps. This case gives a lifetime of 37 million years for the oldest stars represented. The central eight rings have been kept empty of stars; that is, we do not consider the problem of the evolution of the galactic

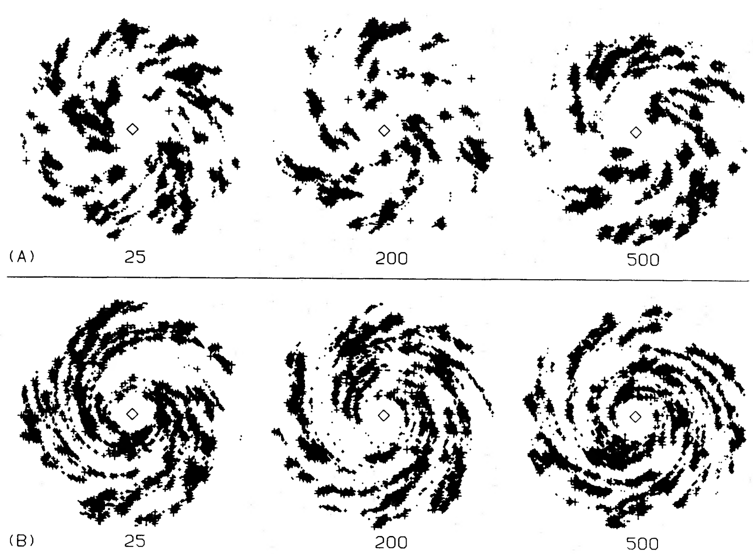

FIG. 1.—Evolution of two model galaxies. The top three examples (A) are for the rotation curve of M101, and the bottom three (B) are for M81. The number below each figure is its age in time steps. Each time step equals 15 million years. The outside rim completes one revolution in about 32 time steps.

nucleus. The diamond indicates the center of the array.

Figure 1*a* shows results for the rotation curve of the Sc type galaxy M101. It has the ragged sparse form which is actually observed for M101. Also the pitch angle of the model arms agrees very well with the observed angle of the arms in M101. Well developed spiral structure starts up almost immediately, and, although the number and length of the arms continually change, clear spiral structure can be seen for very long times. The 500 time steps in Figure 1 correspond to 7.5×10^9 years, or about 13 galactic rotations. The spirals continue to persist for as long as the calculation is carried on (our present longest run is 2000 time steps). Figure 1*b* shows results for a run having the same model parameters as Figure 1*a* except for the rotation curve, which in this case is for the Sb type galaxy M81. As can be clearly seen, the spiral structure is much tighter in this case, the density of stars is greater and the arms are less ragged. These characteristics are typical of those observed in Sb galaxies (Lequeux 1969).

The sensitivity to P_{st} is quite strong. In Figure 2 we plot the total star density as a function of P_{st}. Good spiral formation is obtained in the region $0.24 \leq P_{st} \leq 0.3$. For values below 0.24 the spiral segments are very short and sparse; in fact, for $P_{st} < 0.2$ the galaxy never stabilizes, and the star population remains quite low and fluctuates wildly. The lifetime of any star-forming region is quite short, and the galaxy is kept alive only by P_{sp}. For $P_{st} > 0.3$, the galaxy fills up quite rapidly (at $P_{st} = 0.4$, 70% of the cells are occupied by stars of all ages); and, although some spiral structure can still be seen, the density of stars is quite high, giving the resulting galaxies a much more homogeneous appearance.

It is clear that these values of P_{st} are determined by the requirement that a propagating structure have a probability of unity to survive to the next time step. With the minimum number of neighbors being five, the minimum P_{st} must be 0.2. For values of P_{st} greater than 0.3, too many descendants are produced, and a "population explosion" occurs.

Figure 2 also shows the average geometrical entropy as a function of P_{st}. The actual value of the entropy is sensitive to how many star ages are included in its definition. As mentioned above, we must plot more than just the new stars to see good spiral structure. The same is true with entropy. Lower values for the entropy are obtained if enough older stars are included to connect the genetically related star-forming areas, since these are just the areas where ordered structures are induced by the SSPSF mechanism. The data of Figure 2 show that although the number of new stars created per time step changes by a factor of 50, the entropy changes by only 10% and shows a small minimum at $P_{st} \approx 0.28$. Since the entropy is a measure of the order induced by the SSPSF process, its relative constancy shows that the order itself is not a strong function of P_{st}.

The dependence of the model on P_{sp} is not very critical. As MA observed, a finite P_{sp} is needed to prevent the galaxy from dying out. However, unless P_{sp} becomes large enough to compete with P_{st} in the number of stars it creates per time step ($P_{sp} \geq 0.002$), its exact magnitude is not important. At values less than about 1×10^{-5} the galaxy either dies or undergoes very large fluctuations in star density, but for values greater than this the galaxy evolves in approximately the same way independent of the exact value of P_{sp}.

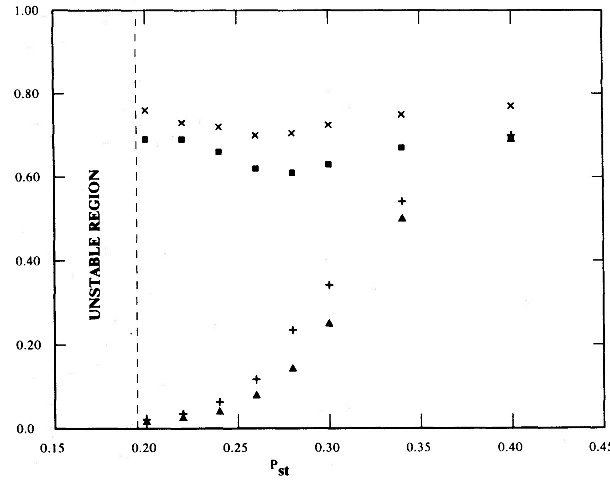

FIG. 2.—The total star density (*triangles*, M101; +'s, M81) and geometrical entropy (*squares*, M101; ×'s, M81) as a function of P_{st}.

To verify the stability of the presence of the pattern, we tested a number of different initial random distributions of stars, including the case of no stars at all. For this latter case, stars start to appear due to P_{sp}. A number of runs of the same system were also made with different random number sequences. The details of the structure changed from run to run, but the spiral features in all cases appeared with identical overall characteristics such as pitch angle, number of arms, density, etc.

As would be expected, the evolution time in time steps is proportional to the number of rings chosen. However, since by equation (2) the size of the time step is inversely proportional to N, the actual time of evolution is independent of N. Therefore, as long as N is large enough to avoid the dominance of size and boundary effects, the number does not directly alter the course of the evolution. This will be discussed in more detail later on.

The dependence of the evolution on τ_r is quite weak, so that good spiral features appear at all τ_r (at least up to 40, the largest we have tried). However, at large τ_r one can clearly see the growth of the arm marked by a progression of older to younger stars. For small τ_r this pattern is washed out.

This brings us to the last parameter of importance, the rotation curve. Figure 1 shows that the form of the galaxy is clearly a function of the rotation curve. The two rotation curves yield model galaxies with different morphological types, which are similar to the morphological types of the galaxies from which the rotation curves were taken. An important point is that the morphological type does not change during the evolution of the model galaxies. Although the number and raggedness of the arms change with time, their pitch angles remain pretty constant. Therefore, in this model the galactic type is simply characteristic of the angular rotation curve. It is interesting to note that the spiral density wave theory (Roberts, Roberts, and Shu 1975) also connects the morphological type with physical parameters of the galaxy. For this theory, where the dynamics of the gas and stars are explicitly considered, the total mass of the galaxy and its distribution are the critical parameters. These parameters are just those that determine the rotation curve.

Another manifestation of the effect of the rotation curve can be seen in Figure 2; i.e., there is an entropy difference between the M81 and M101 models. Because the geometrical entropy is a measure of the order of the system, we would expect it to depend on the rotation curve since a change in rotation curve would change the way the stars were distributed in the boxes chosen for the coarse graining. M81 has a greater differential rotation than M101 so that the resultant genetic star tracks are stretched out over greater areas, raising the entropy. Since the geometrical entropy that we have defined appears to be a function of galactic type, it might therefore be a useful concept for classifying observed galaxies.

This sensitivity to the form of the rotation curve is in direct contrast to that reported by MA. We believe the main difference lies in the size of the array that they chose. They used an array of only 30 rings (2500 cells), which gave pronounced spiral structures when $6 \leq \tau_r \leq 10$. Since their model was deterministic ($P_{st} = 1$), it led to structures of the order of $2\tau_r$ wide; therefore, their array had room for only two to three such structures, which interfered with each other. Another way to look at it is that it takes on the average only about 20–25 time steps to reach from center to edge of the galaxy, so that after times that are three to four times greater than this the galaxy is filled with interfering structures. This corresponds well with the time scale MA observed for good spiral structure. In our case we have 49 rings, and, since $P_{st} = 0.28$, the average time it takes to reach the edge is about 150 time steps. Furthermore, since we do allow some star formation during the refractory period, our structures are not as rigid and their interference with each other is not as severe. Finally, for our model the lifetime of the large spiral features is of the order of a few hundred time steps, so they do not live long enough to strongly interfere. Therefore, the galaxy does not fill up for this model, and the spiral arms continue to regenerate for quite long times.

In choosing any sort of finite model such as the one described here, one must pay particular attention to the finite size effects so as to be sure that they do not overwhelm the effects one is looking for. For our model we have found that the minimum usable array contains about 40 rings (we have carried out runs over the range $25 \leq N \leq 149$). At least this size is necessary to see clearly defined long-lived spiral structure throughout the body of the disk. However, the effects of finite size also show up in the failure of the model to always scale with N. For example, the density of stars created per time step is not completely independent of N even for $N > 40$. This density is 50% smaller for $N = 149$ than it is for $N = 49$. In order to get the densities found for $N = 49$, P_{st} must be increased from 0.28 to 0.30 for $N = 149$. The question of finite size effects is still not completely understood and needs to be further explored.

In the Introduction, we mentioned that the spiral arms rotate quasi-rigidly. What we mean by this can be seen in Figure 3, where we show six pictures separated by only five time steps between each picture. (The diamonds on the outer edge are fiducial marks that indicate the rotational motion of the edge of the galaxy.) We have indicated two representative features by the solid lines drawn on the figures. Consider first the figure shown at the top of $\tau = 30$. It is about 120° in length and extends radially over about the outer 25% of the disk. As time progresses, the feature lengthens since the leading edge (at smaller radius) rotates faster than the trailing edge. At $\tau = 55$ the feature has stretched to over 290°, the leading edge having made one complete revolution. Note also that at some stages the feature lines up with other genetically distinct features, giving the appearance of a much larger spiral. This is a common result in this model, especially at later times when the total star density is larger. Although it is possible to have single spirals which are quite large, it is very common to see a long arm made up of a number of spiral pieces. The second feature starts as a small new patch at the bottom of $\tau = 30$. As time progresses, it lengthens and catches up to the trailing edge of the first feature since it is rotating faster. It catches the first feature at $\tau = 50$, after which it "slows down," apparently because it bumps into the refractory region surrounding the trailing edge of the first feature. In any event it can be seen that the spiral features do preserve their shape for times as long as a rotation period. That is to say, although the persistence of spiral appearance over long times is due to the continuous generation of new spiral features, a well defined feature can last at least for times of the order of a galactic rotation period.

A further point worth noting about the character of the patterns is that they consist of a number of spirals; the classic "two-armed" spiral is not generally a result of this model. The multiarmed configuration is, however, just what is observed for the majority of galaxies. In addition, as will be seen below (Fig. 5*b*), the model also gives a good representation of an Sb galaxy such as M81, a representative of the two-armed class.

As was previously mentioned, the variable P_{sp} was necessary to ensure stability of the galaxy although its exact value was not critical. We have been able to eliminate P_{sp} completely by choosing a slightly different model. Instead of randomly populating the galaxy by a few stars, we let its center (the first few rings) be continuously occupied by bright young stars. That is, the original excitation for all star production

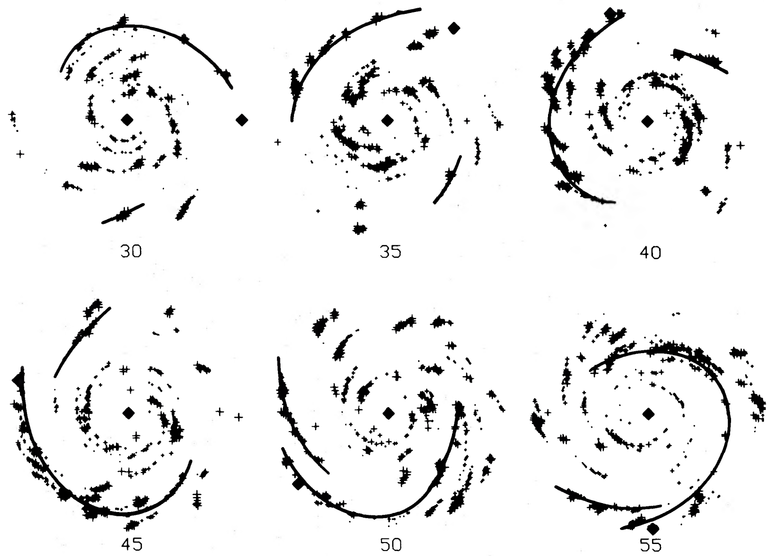

FIG. 3.—Evolution of an M81 type galaxy over a period of the order of one complete rotation. The figures shown are five time steps apart and illustrate the "quasi-rigid" rotation of the spiral features. The square on the edge is a fiducial mark showing the rotation of the edge of the galaxy.

then comes from the galactic nucleus. In Figure 4 we show some results for this type of galactic evolution, in this case for an M81 rotation curve. The star population slowly grows out from the nucleus, irregularly at first, but then the spiral arms become well formed and the structure becomes indistinguishable from the structures of Figure 1. At times greater than approximately 200 time steps the values obtained for star density and entropy are also the same as for the case of finite P_{sp}. This time (order of N/P_{st}) is just what we would expect for propagation from the center to the outer edge. Therefore, if we choose to let all excitation originate from the galactic nucleus, we can completely eliminate the parameter P_{sp}.

IV. DISCUSSION

The results of the previous section show that the process of stochastic self-propagating star formation is able to produce large-scale spiral structure over a period of many galactic rotations. The morphological type depends upon the particular rotation curve; and as the model evolves, this type remains constant. This appears to be a basic property of the SSPSF model

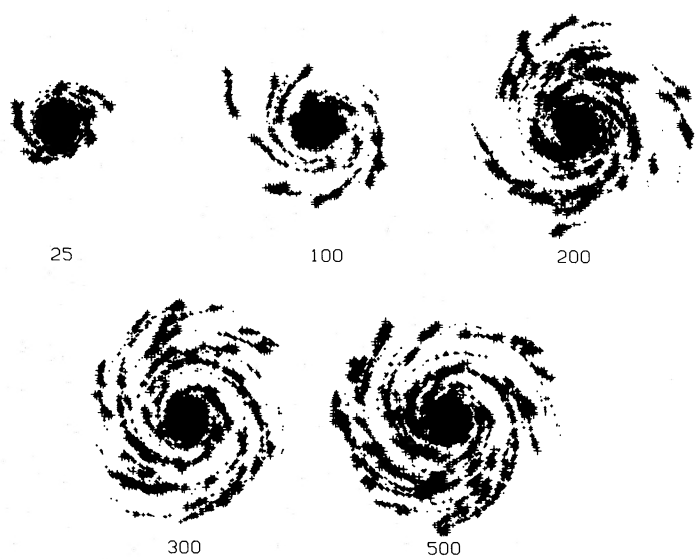

FIG. 4.—Evolution of an M81 type galaxy with no spontaneous nucleation of stars ($P_{sp} = 0$). SSPSF is initiated only at the nucleus.

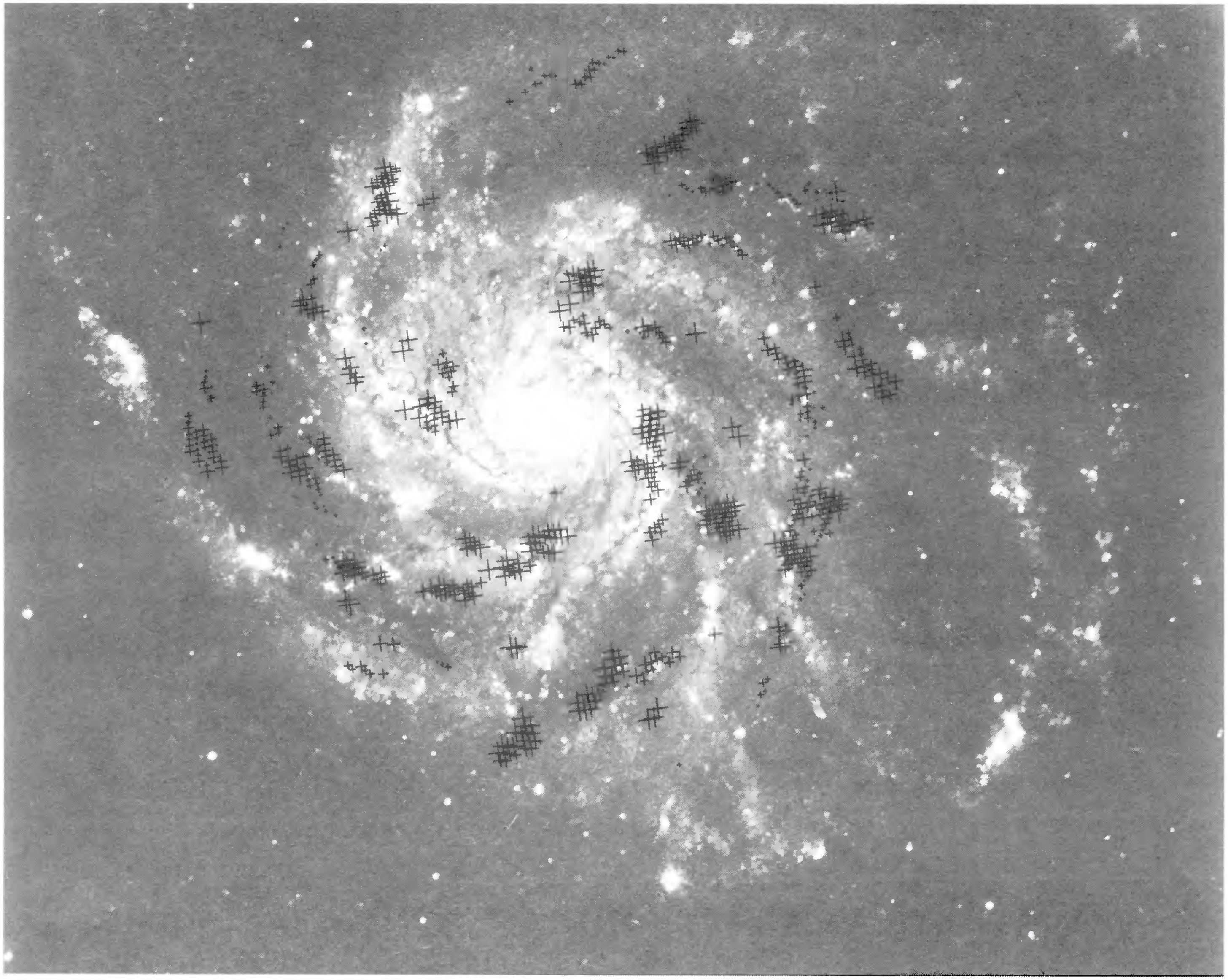

FIG. 5a

FIG. 5.—Superposition of a model galaxy upon a photograph of the galaxy from which the rotation curve was taken. (*a*) M101, (*b*) M81. In the latter case the model was projected at the 58° inclination angle of M81. (Photographs are from Sandage 1961.)

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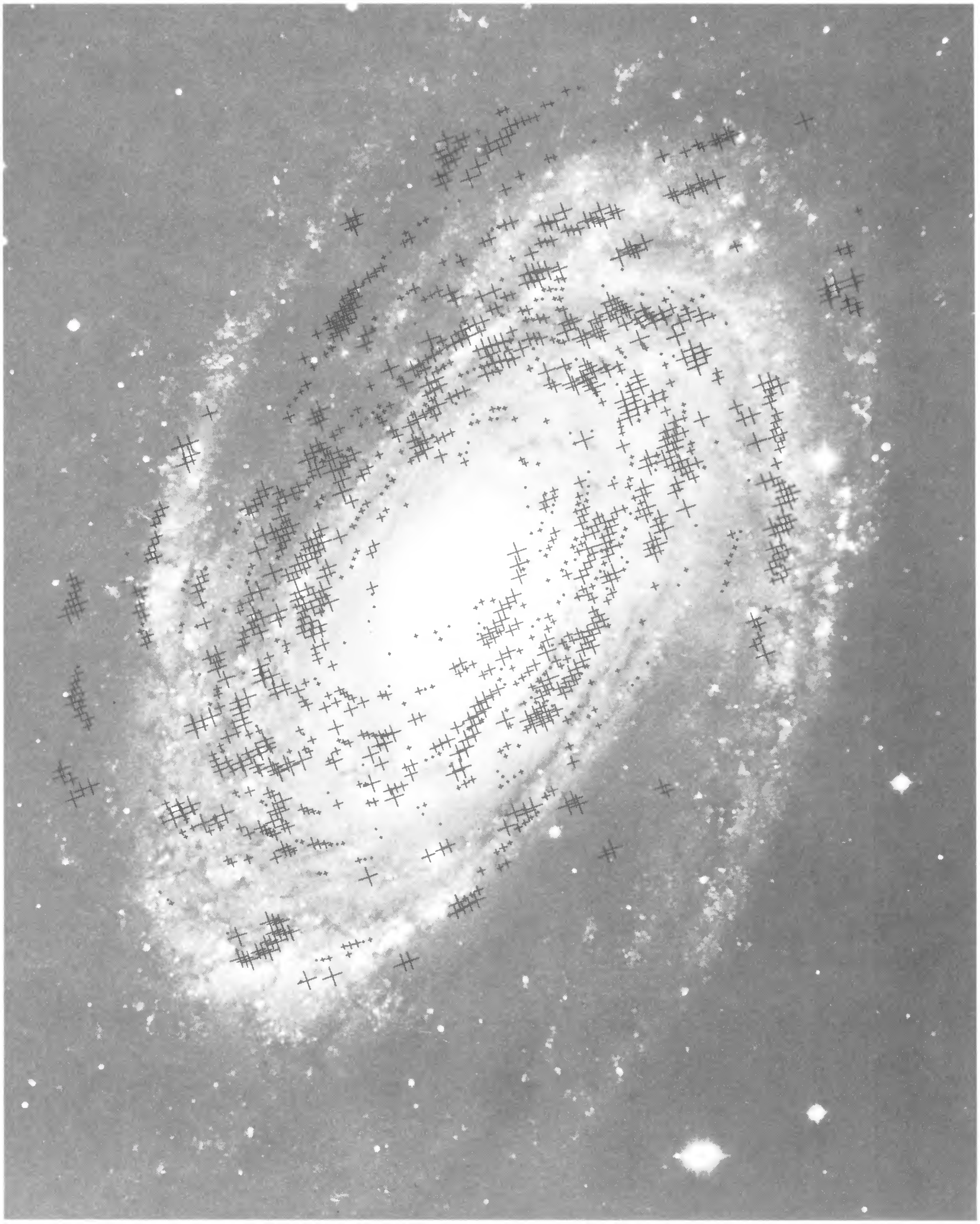

FIG. 5b

FIG. 5.—*Continued*

and one that, we believe, distinguishes this model from some of the gravitational *N*-body models (Toomre 1977).

The quality of the representative galactic forms that we have obtained can be seen by an examination of Figure 5. In this figure we have superposed appropriate model galaxies over photographs of M101 and M81. Of course, we do not find all the arms seen in the photograph, nor can we align all the arms we do find since the actual positions of the model's arms occur at random. However, we do find many arms that can be superposed over observed arms for a considerable portion of their length. Furthermore, in both cases all the model arms have the same curvature as the observed arms over the whole of their length. This reiterates the point that the arm structure is simply a direct consequence of the form of the rotation curve.

The present model is still so simple that a detailed comparison with observations would probably be premature. In particular, we make no reference to the gas component. Recent observational evidence indicates that for some galaxies the spiral structure is also displayed in the gas with the characteristics predicted by the spiral density wave theory (e.g., the innermost 9 kpc of M81 [Visser 1977]). It may be that the simplest SSPSF model corresponds to the case of M33 where the neutral hydrogen shows no spiral structure despite well defined optical spiral structure (see, e.g., the review by Burton 1973).

However, it is worthwhile, even at this stage, to estimate the supernova rate required. For the values of P_{st} that give good spiral arms, we find that about 2% of the cells undergo star formation each time step. As mentioned in § II, our "stars" do not correspond to single real stars; therefore, we must scale our results in order to compare the rate to a real galaxy. If the size of a real association due to one supernova is g, the number of volume elements g^3 in the galaxy is $\pi G_R^2 d/g^3$, where d is the thickness of the galactic disk. The time step (from eq. [2]) is g/V_I since $N = G_R/g$. Therefore, the rate of supernova formation is

$$R = 0.02\pi G_R^2 d V_I/g^4 \,. \qquad (7)$$

For $G_R = 15$ pc, $d = 200$ pc, $V_I = 20$ km s^{-1}, and $g = 50$ pc (this is a typical value for the associations found around supernovae; see, e.g., Ögelman and Maran 1976) we find a rate of about 1 supernova per 100 years. This is well within the rates observed (Tammann 1977), which shows that the value of P_{st} that gives good spiral structure is a reasonable one.

In conclusion, we have shown that large-scale spiral ordering can be produced by short-range processes as a stable configuration of the galaxy, and the values of the parameters required to generate this ordering are consistent with the astronomical evidence.

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