

PERIODIC TIMING RESIDUALS IN PULSATING BINARY X-RAY SOURCES AND
 ORBITAL PRECESSION IN HERCULES X-1*

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ABSTRACT

Intrinsic variations of pulse frequency and reflection of pulses produce systematic residuals in pulse arrival times from pulsating binary X-ray sources. We use an iterative technique to calculate higher-order terms in the arrival time curve of a variable-frequency binary source. We also formulate a general framework for studying time delays caused by reflection of pulses. By applying these methods to Her X-1, we find that reflection effects and variations in pulse frequency due to accretion torques, stellar wobble, and changes in pulse shape can mimic the effects of orbital precession with the 35 d period. We suggest observational tests to resolve this ambiguity in the interpretation of the timing data.

Subject headings: stars: binaries — stars: pulsation — X-rays: binaries — X-rays: sources

I. INTRODUCTION

Arrival times of X-ray pulses from binary X-ray sources are affected by variations in pulse frequency and by reflection of pulses (Lamb *et al.* 1975; Fabian and Pringle 1976; Milgrom and Avni 1976). The resulting arrival time variations provide information on the structure of neutron stars and on the accretion process. However, they also interfere with determination of the orbit. Thus the problem of separating these variations from those due to orbital motion is fundamental for interpreting the data.

An important example of this problem occurs in the Her X-1 system. The results of timing observations have been interpreted as evidence for counter-precession of the orbital plane with a 35 d period (Deeter and Boynton 1976, hereafter DB; Fechner and Joss 1977, hereafter FJ). Such a precession is predicted by models of the 35 d cycle (Roberts 1974; Petterson 1975) whose essential *phenomenological* ingredient is a precessing accretion disk (Katz 1973; Gerend and Boynton 1976), but which postulate forced precession of HZ Her as a mechanism *leading to* disk precession. Such a misalignment between the spin axis of HZ Her and the normal to the binary plane would be contrary to published calculations of the tidal dissipation rate (Chevalier 1976) and would therefore have important implications for binary evolution. Finally, measurement of orbital precession could provide an independent determination of the inclination angle of the binary system; this would help in determining the mass of Her X-1.

It is therefore important not only to establish the *statistical significance* of the observed residuals but also to examine their *character* in order to determine (1) whether the interpretation of the data in terms of a precessing orbit is unique; and (2) whether the inclination angle and precession amplitude can be determined.

In this *Letter* we study sources of arrival time variations other than orbital motion and use our results to evaluate the Her X-1 observations. In § II we discuss the Her X-1 data. In § III we study the effects of variations in the intrinsic pulse frequency by using an iterative technique to calculate higher-order terms in the arrival time curve. In § IV we develop a formalism for studying arrival time variations caused by reflection of pulses. We summarize our conclusions in § V.

II. THE HERCULES X-1 OBSERVATIONS

The time-delay curve for a constant frequency source in a counter-precessing, circular orbit is given by

$$\begin{aligned} \Delta t(t) = \tau_0 \cos^2 \frac{\beta}{2} \left\{ \cos [\omega(t - t_0)] + 2 \tan \frac{\beta}{2} \tan \bar{i}_0 \cos [\omega(t - t_0) + \omega'(t - t_p)] \right. \\ \left. - \tan^2 \frac{\beta}{2} \cos [\omega(t - t_0) + 2\omega'(t - t_p)] \right\}, \end{aligned} \quad (1)$$

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where $c\tau_0 = a_x \sin i_0$, a_x is the radius of the orbit, i_0 is the inclination of the invariant plane to our line of sight, β is the precession amplitude, ω is the orbital frequency (period 1.7 d), t_0 is the orbital epoch, $\tilde{i}_0 = (\pi/2) - i_0$, ω' is the precession frequency (period 35 d), and t_p is the precession epoch. The second and third terms in equation (1) are the "precession residuals." They are periodic at the sideband frequencies $\omega + \omega'$ (period 1.62 d) and $\omega + 2\omega'$ (period 1.55 d).

The observed residuals have the following properties: (1) Their amplitude is $\epsilon\tau_0$, where $\epsilon \sim 10^{-3}$, with a 2σ uncertainty $\Delta\epsilon \sim \epsilon$. (2) Their precise time dependence is unknown. It was not established that this time dependence is described by the above two frequencies and those two frequencies alone. Indeed, the residuals may have a spectrum of periods from ~ 1.55 d to ~ 1.65 d. This conclusion follows from a detailed study of the range of parameters given by FJ, who varied all the precessional parameters in their fit, and is consistent with the findings of DB. The uncertainties in the periodicities of the residuals result partly from the fact that only ~ 8 days of data were analyzed in each of the four 35 d cycles studied.

III. VARIATIONS IN INTRINSIC PULSE FREQUENCY

Consider a variable-frequency source in a binary system. Let $\Omega(t) = \dot{\phi}(t)$ be the intrinsic pulse frequency and $\Delta_1(t)$ the time delay caused by the orbital motion alone (neglecting effects $\sim v^2/c^2$). The time-delay curve is commonly defined as the difference between the actual arrival time of a certain pulse phase $\phi(t)$, minus the expected arrival time calculated for a constant-frequency ($\bar{\Omega}$) stationary source, the difference being evaluated at the expected arrival time. The time-delay curve is thus given by the functional equation

$$\Delta t \left\{ \bar{\Omega} + \Delta_1(\bar{t}) + \frac{1}{\bar{\Omega}} [\phi(t) - \phi(\bar{t})] \right\} = [t + \Delta_1(t)] - \left\{ \bar{t} + \Delta_1(\bar{t}) + \frac{1}{\bar{\Omega}} [\phi(t) - \phi(\bar{t})] \right\}, \quad (2)$$

where \bar{t} is an arbitrary reference time and $\bar{\Omega}$ is the average observed pulse frequency during the observing interval [chosen so that there will be no linear term in t in the expression for $\Delta t(t)$]. When the fractional variation in pulse frequency is small, $\Delta t(t)$ can be calculated from equation (2) by iteration, with each iteration yielding terms of higher order in small parameters.

a) First-Order Effects

To model periodic variations in pulse frequency with a period shorter than the observing interval, we approximate the intrinsic pulse phase by $\phi(t) = \Omega_0 t + A \cos(\omega_1 t + \theta_1)$ and assume a circular orbit, so that $\Delta_1(t) = \tau_0 \cos[\omega(t - t_0)]$. Then the first iteration of equation (2) yields the time-delay curve to lowest order in $\omega A/\Omega_0$ and in $\omega_1 A/\Omega_0$, namely,

$$\Delta t(t) = \tau_0 \left\{ \cos[\omega(t - t_0)] - \frac{A}{\tau_0 \Omega_0} \cos(\omega_1 t + \theta_1) \right\}. \quad (3)$$

Specializing to Her X-1, the observed residuals could be due to intrinsic frequency variations if their period is ~ 1.62 d (i.e., $\omega_1 \sim \omega + \omega'$) and if $A = \epsilon\tau_0\Omega_0 \approx 7 \times 10^{-2}$. In §§ IIIb and IIIc we consider specific mechanisms that can cause such variations.

b) Accretion Torques

Mass transfer into the accretion disk and the flow within it are modulated with a ~ 1.62 d period in several models of Her X-1 (Roberts 1974; Gerend and Boynton 1976; Mazeh and Shaham 1977). If this modulation persists at the Alfvén surface, the accretion torque could induce a ~ 1.62 d variation in the rotation frequency. We use the relation between accretion torques and spin-up rates (Elsner and Lamb 1976; Rappaport and Joss 1977; Schreier 1977) to show that the torque in Her X-1 could be sufficiently large to produce the observed residuals. The observed long-term spin-up rate of Her X-1 is $3 \times 10^{-6} \text{ y}^{-1}$, much smaller than the maximum long-term rate allowed by theory, which is $1.2 \times 10^{-4} \text{ y}^{-1}$. This indicates a near balance between spin-up and braking stresses at the Alfvén surface (see Davidson and Ostriker 1973; Ghosh and Lamb 1978). Short-term variations of the period could be even larger if the crust of the neutron star is only weakly coupled to the liquid interior (Fabian and Pringle 1976; Lamb, Pines, and Shaham 1976). Considering the uncertainties in the mass of Her X-1 and in the moment of inertia of the crust (Pandharipande, Pines, and Smith 1976), the theoretical short-term spin-up rate could be as large as $2 \times 10^{-5} \text{ d}^{-1}$, 10 times larger than the $A\omega_1^2/\Omega_0$ of $2 \times 10^{-6} \text{ d}^{-1}$ required. Thus the observed residuals may be due to a ~ 1.62 d modulation in the balance between the spin-up and braking stresses. The associated modulation of X-ray luminosity need not be larger than $\sim 1\%$ (Ghosh and Lamb 1978). It is interesting to examine the data in order to observe or set limits on any ~ 1.62 d modulation of X-ray intensity.

c) Stellar Wobble

Stellar wobble can cause an apparent change in pulse frequency. We follow the assumptions and notation of Ruderman (1970),¹ who calculated the pulse phase from the motion of a fixed point on the stellar surface. (The results may be different for special beam geometries.) Let ω_w be the wobble frequency, α_w the wobble amplitude, and β_w the

¹ In equation (3) of Ruderman (1970), α and β should be interchanged.

angle between the magnetic axis and symmetry axis of the star. Ruderman found that when $\beta_w < \alpha_w$ and $\beta_w \ll 1$ the apparent pulse phase $\phi_{\text{app}}(t)$ has a sinusoidal time variation with frequency ω_w and amplitude $\beta_w/\sin \alpha_w$. We find in addition that when $\alpha_w < \beta_w$ and $\alpha_w \ll 1$, $\phi_{\text{app}}(t)$ has a sinusoidal time variation with frequency ω_w and amplitude $\alpha_w/\tan \beta_w$.

Specializing to Her X-1, the observed residuals could be due to wobble if the wobble period is ~ 1.62 d and if either $\beta_w/\sin \alpha_w \approx 0.07$ or $\alpha_w/\tan \beta_w \approx 0.07$. There is a wide range of wobble angles that satisfy these requirements; in particular, α_w and β_w can be as small as desired. Note, however, that a ~ 1.62 d wobble is apparently disjoint, by current thinking, from the rest of the 35 d phenomenology.

d) Variations in Pulse Shape

Periodic 1.62 d variations in intrinsic pulse *shape* will induce a 1.62 d periodicity in the pulse arrival times, if the latter are determined in the usual way by cross-correlating the pulses with an average pulse template. Pulse-shape variations can be caused by changes in the accretion flow pattern, or by stellar wobble.

e) Higher-Order Terms

A 35 d variation in the pulse frequency of Her X-1 could be caused by stellar wobble or by varying accretion torques (Lamb *et al.* 1975). From equations (1) and (3) it follows that such a variation ($\omega_1 = \omega'$) cannot explain the observed residuals in lowest order, as the residuals would have the wrong period. We now consider whether such a variation could explain the residuals through higher-order effects.

To calculate the higher-order terms in the time-delay curve, we generalize our treatment of § IIIa and write $\phi(t) = \Omega_0 t + A g(t)$, where $g(t)$ has a 35 d period, an average value over this period of zero, a magnitude ~ 1 , and no variation on time scales shorter than 35 d. Since the timing data are from ~ 8 days in each HIGH state, we choose \bar{t} of equation (2) as the center of the HIGH state, and expand $g(t)$ to second order in t around \bar{t} . We assume a circular orbit. The term in Δt linear in $(t - \bar{t})$ vanishes if we choose $\bar{\Omega}$ to be $\Omega_0 + A \dot{g}(\bar{t})$. In the next higher order we obtain

$$\Delta t(t) = \tau_0 \cos(\omega t - \omega \eta + \theta_0) - \eta - \frac{1}{2} \delta (t - \eta - \bar{t})^2 + \frac{1}{2} \tau_0 \omega \delta (t - \eta - \bar{t})^2 \sin(\omega t - \omega \eta + \theta_0), \quad (4)$$

where

$$\delta = A \ddot{g}(\bar{t}) / [\Omega_0 + A \dot{g}(\bar{t})], \quad \eta = \tau_0 \cos(\omega \bar{t} + \theta_0),$$

θ_0 is a reference phase, and we have kept only the dominant terms.

The last term in equation (4) represents residuals which, during the HIGH state, have periodicities close to 1.7 d. For these to be comparable in size to the size, $\epsilon \tau_0$, of the observed residuals, δ would have to be $\sim 4 \times 10^{-5} \text{ d}^{-1}$. From the third term in equation (4), the data during the same HIGH state would then necessarily show an average spin-up rate of the same size. This is ruled out by FJ, who showed that $\dot{\Omega}/\Omega < 10^{-7} \text{ d}^{-1}$. Expanding equation (1) to second order in ωt around $\omega \bar{t}$, we also find that equation (4) cannot be made formally equivalent to (1) for any values of the precession parameters. Thus we can state quite generally that the observed residuals are not due to higher-order terms.

IV. REFLECTION OF PULSES

Consider a primary pulse emitted by a constant-frequency (Ω_0) source in a binary system. Let $\Delta_1(t)$ be the time delay of the primary pulse caused by the orbital motion of the source. A reflected pulse is produced by scattering of the primary pulse in the binary system. Let $a_n(t)$ be the ratio of the amplitude of the n 'th harmonic of the reflected pulse to the corresponding amplitude of the primary pulse in the direction of our line of sight, and $\Delta_{2,n}(t)$ be the additional time delay of the n 'th harmonic of the reflected pulse relative to the primary pulse. The quantities $a_n(t)$ and $\Delta_{2,n}(t)$ are determined by the effects of light travel time, photon diffusion, albedo, and the angular dependence of the reflected pulses (see Avni and Bahcall 1974; Basko, Sunyaev, and Titarchuk 1974; Middleditch and Nelson 1976; Milgrom and Avni 1976). Combining the primary and reflected pulses to first order in $a_n(t)$, we find that the time delay curve of the n 'th harmonic of the composite pulse is

$$\Delta t_n(t) = \Delta_1(t) + \frac{1}{(n+1)\Omega_0} a_n(t) \sin[(n+1)\Omega_0 \Delta_{2,n}(t)]. \quad (5)$$

Thus a general method to distinguish between orbital time delays and reflection effects is to determine separately the time-delay curves of the Fourier components of the pulse. This follows from the explicit appearance of $(n+1)\Omega_0$ in equation (5), and from a possible dependence of $a_n(t)$ and $\Delta_{2,n}(t)$ on n , e.g., if the reflecting region is larger than the pulse wavelength. The relative shift of Fourier components causes a variation in pulse shape. A search for such a variation may be more efficient than a Fourier decomposition if the pulse shape contains narrow features. Also $\Delta t_n(t)$ may depend on X-ray energy. If the observed residuals are energy-dependent, they cannot have a purely orbital origin.

Specializing to Her X-1, and considering the fundamental frequency ($n = 0$) which contains most of the pulse

power, we find that if the observed residuals are due to reflection, then $a_0(t) \gtrsim \epsilon\tau_0\Omega_0 \approx 0.07$. We consider below reflection from several possible regions in the binary system.

a) Reflection off a "Precessing" Disk

Consider pulses reflected from an accretion disk that has a stationary form which counterrotates with a 35 d period. Expanding $a(t)$ and $\Delta_2(t)$ (omitting the subscript 0) around \bar{i} , the center of the HIGH state, we find

$$\Delta l(t) = \tau_0 \left\{ \cos [\omega(t - t_0)] + \frac{1}{\Omega_0\tau_0} [a(\bar{i}) + \dot{a}(\bar{i})(t - \bar{i})] \sin [\Omega_0\dot{\Delta}_2(\bar{i}) + \Omega_0\Delta_2(\bar{i})(t - \bar{i})] \right\}. \quad (6)$$

If $\Delta_2(t)$ has amplitude T , then $\dot{\Delta}_2 \approx \omega'T$, and the residuals of equation (6) have an approximate frequency $\Omega_0\omega'T$. The observed residuals, in a single HIGH state, could be due to this effect if $T \approx (\omega + \omega')/\omega'\Omega_0 = 4$ s and $a \gtrsim 0.07$. These conditions can be marginally satisfied in particular disk geometries: for example, the pulses might be reflected from a "hump" or "twist" in the disk, at a distance ~ 4 lt-s from the source, and extending to a height ~ 2.5 lt-s above the orbital plane. The corresponding angular height is similar to that discussed by Jones and Forman (1976). The reflected pulses must then be approximately phase-coherent, so that travel-time-smearing effects do not significantly reduce the amplitude of the reflected pulse.

In this case, $a(t)$ and $\Delta_2(t)$ have a 35 d period and the reflection residuals are therefore the same at the same 35 d phase of every HIGH state. On the other hand, in Her X-1 the frequencies corresponding to the orbital period and the 35 d cycle satisfy $\omega \approx 20.5 \omega'$. Therefore the precession residuals change sign between consecutive 35 d cycles (see eq. [1]). Thus it is possible to distinguish observationally between these two sources of residuals. This has not yet been done, since the data analyzed by FJ contain two "even" 35 d cycles, and since DB (whose data contain two "odd" and two "even" 35 d cycles) did not treat all the precession parameters as free parameters.

If the precession period is 70 d (Chester 1977), the 1.66 d component of the precession residuals keeps the same sign in alternate 35 d HIGH states, while the 1.62 d component flips sign. Thus accurate timing observations can test this hypothesis. With a 70 d period, the reflection residuals retain the same sign in consecutive HIGH states if the accretion disk is highly symmetric, but no such simple relation is necessary in general.

b) Reflection off a Corotating Feature

If the accretion disk counterrotates with a 35 d period, then the intensity of X-ray pulses incident on any reflecting material that corotates with the binary system is modulated with a 1.62 d period. In this case the function $a(t)$ of equation (5) will contain a periodic component at 1.62 d, which can mimic orbital precession.

If, however, the disk in Her X-1 is larger than a few light seconds (as is possibly indicated by the optical light curve), then the factor $\sin [\Omega_0\Delta_2(t)]$ oscillates several times in one orbital period, and the 1.62 d periodicity will not show up in the data.² {The amplitude of $\Delta_2(t)$ must be larger than the disk radius; therefore $\sin [\Omega_0\Delta_2(t)]$ oscillates at least 4 times the ratio of the disk radius to the pulse wavelength, in each orbital period.} Also, if the disk is so large, then the condition $a \gtrsim 0.07$ requires that the size of the reflecting surface be a few light seconds and that the reflected pulses be approximately phase-coherent during a large fraction of the orbital period. These requirements cannot be satisfied by a corotating reflecting surface. Therefore the observed residuals can be due to this effect only if the disk is rather small.

c) Nonstationary Features

So far we have considered reflection from features that counterrotate with the disk or corotate with the binary system. In Her X-1 there may also be 1.62 d variations of the reflecting material itself, which could result from preferential accretion at particular 1.62 d phases (Crosa and Boynton 1977; Mazeh and Shaham 1977), the periodic appearance of new reflecting elements, or periodic variations in the disk shape. Considering that the reflected pulse must make up at least $\sim 7\%$ of the total pulse, this mechanism does not seem to provide a feasible explanation for the observed residuals unless the significance of the residuals depends strongly on data taken close to the end of the HIGH state, in which case the unscattered pulse intensity in our direction could be appreciably smaller than in the direction of the reflecting material.

V. CONCLUSIONS

Our results illustrate that whenever timing data are used to establish a small-amplitude orbital effect, or to extract orbital elements from such an effect, other sources of systematic residuals must be considered for their possible influence. We have shown that frequency variations and reflection of pulses in pulsating binary X-ray sources can produce measurable timing residuals with a variety of periodicities. When high-quality timing data become available, they should be searched for all such periodicities, not simply analyzed in terms of predetermined periods.

We find that (1) timing residuals caused by intrinsic variations of pulse frequency cannot be observationally distinguished from those caused by orbital motion; the basic period of the residuals is equal to the period of the intrinsic variations, but higher-order terms in the time-delay curve contain additional periodicities. (2) Timing residuals caused by reflection of pulses can be identified observationally by studying the arrival times of their Fourier

² Except in the unlikely event that the data sampling happens to coincide with a particular phase of this oscillation.

components, and by studying their energy dependence; the periodicities of these residuals are determined by the periods of variations of the properties and locations of reflecting material, and by the ratio of the system dimensions to the pulse wavelength. (3) The timing residuals observed so far from Her X-1 could be due to frequency variations or reflection of pulses, and need not be due to orbital precession.

The following observations can greatly help in determining the origin of these residuals:

a) *Accurate determination of the periodicities of the residuals.*—If the data contain components with both a 1.62 d period and a 1.55 d (or 1.66 d) period, then stellar wobble is ruled out, and frequency variations induced by accretion torques become extremely implausible, as sole explanations of the observed residuals.

b) *Measurements of the relative sign of the residuals in "odd" versus "even" 35 d cycles.*—If a sign flip is found, reflection of pulses off a precessing disk (with a precession period of 35 d) is ruled out.

c) *Searches for energy dependence of the residuals and for variation of the pulse shape with time.*—If the residuals are energy-dependent, if the pulse shape varies with a 1.62 d period, or if the time-delay curves of the various Fourier components of the pulse are not identical, then orbital precession and pure frequency variations are ruled out as the sole explanation of the observed residuals.

Finally, we note that in order to decide among possible models for interpreting the data, it is necessary to establish carefully the statistical significance of the need for introducing any additional parameters into the arrival time curve. This is particularly important when the accuracy of measuring individual arrival times is determined from the mean square residuals around the best-fit curve, and is not precisely known otherwise.

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REFERENCES

- Avni, Y., and Bahcall, J. N. 1974, *Ap. J.*, **191**, 221.
 Basko, M. M., Sunyaev, R. A., and Titarchuk, L. G. 1974, *Astr. Ap.*, **31**, 249.
 Chester, T. J. 1977, *Ap. J. (Letters)*, **213**, L9.
 Chevalier, R. A. 1976, *Ap. Letters*, **18**, 35.
 Crosa, L., and Boynton, P. E. 1977, preprint.
 Davidson, K., and Ostriker, J. P. 1973, *Ap. J.*, **179**, 585.
 Deeter, J. E., and Boynton, P. E. 1976, *Ap. J. (Letters)*, **210**, L133 (DB).
 Elsner, R. F., and Lamb, F. K. 1976, *Nature*, **262**, 356.
 Fabian, A. C., and Pringle, J. E. 1976, *M.N.R.A.S.*, **174**, 25p.
 Fechner, W. B., and Joss, P. C. 1977, *Ap. J. (Letters)*, **213**, L57 (FJ).
 Gerend, D., and Boynton, P. E. 1976, *Ap. J.*, **209**, 562.
 Ghosh, P., and Lamb, F. K. 1978, *Ap. J. (Letters)*, in press.
 Jones, C., and Forman, W. 1976, *Ap. J. (Letters)*, **209**, L131.
 Katz, J. I. 1973, *Nature Phys. Sci.*, **246**, 87.
 Lamb, D. Q., Lamb, F. K., Pines, D., and Shaham, J. 1975, *Ap. J. (Letters)*, **198**, L21.
 Lamb, F. K., Pines, D., and Shaham, J. 1976, in *X-Ray Binaries*, ed. E. Boldt and Y. Kondo (NASA SP-389), p. 141.
 Mazeh, T., and Shaham, J. 1977, *Ap. J. (Letters)*, **213**, L17.
 Middleditch, J., and Nelson, J. 1976, *Ap. J.*, **208**, 567.
 Milgrom, M., and Avni, Y. 1976, *Astr. Ap.*, **52**, 157.
 Pandharipande, V. R., Pines, D., and Smith, R. A. 1976, *Ap. J.*, **208**, 550.
 Petterson, J. A. 1975, *Ap. J. (Letters)*, **201**, L61.
 Rappaport, S. A., and Joss, P. C. 1977, *Nature*, **266**, 683.
 Roberts, W. J. 1974, *Ap. J.*, **187**, 575.
 Ruderman, M. 1970, *Nature*, **225**, 838.
 Schreier, E. 1977, *Ann. NY Acad. Sci.*, **302**, 445.

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