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OBSERVATIONAL LIMITS ON THE LOCATION OF PULSAR EMISSION REGIONS

J. M. CORDES
Department of Physics and Astronomy, University of Massachusetts, Amherst
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ABSTRACT

We assume, following Radhakrishnan and Cooke, that a pulsar's beam is associated with the polar region of a magnetic dipole. Under the assumption that high radio frequencies originate closer to the star than do low frequencies, we place limits on the location of the emission region from the apparent absence of differential retardation and aberration in pulse timing measurements. The lowest observed frequencies must originate at less than a few percent of the velocity-of-light radius for PSR 0525+21, PSR 0950+08, and PSR 1133+16. High-frequency radiation originates at distances not greater than 5 to 10 times the neutron star radius (10^6 cm), or less than 1% of the light-cylinder radius.

Subject heading: pulsars

I. INTRODUCTION

A pulsar's radiation is evidently due to some radiation beam attached to a rotating neutron star. Radhakrishnan and Cooke (1969) first proposed that the axis of the beam coincides with the magnetic pole of an approximately dipolar field. Impetus behind their model included polarization observations of the Vela pulsar (PSR 0833−45) which showed rotation of the polarization position angle through the observed pulse. The frequency independence of the angle rotation suggested a geometric origin and, indeed, the changing orientation—due to stellar rotation—of some vector attached to the star appears to closely mimic the observed angle rotations of many pulsars (Backer, Rankin, and Campbell 1976; Manchester and Taylor 1977). Also consistent with a cylindrically symmetric beam are the observations of both single-lobed and double-lobed average pulse profiles and the tendency for pulse-to-pulse intensity fluctuations to exhibit approximate mirror symmetry about the pulse centroids (Backer 1976; Taylor, Manchester, and Huguenin 1975).

The purpose of this paper is to discuss the radial locations of the pulsar emission region. Observations strongly support the view that emission occurs in an annular region centered on the magnetic pole. However, only theory to date has guided discussions of the actual radius or radial range where radiation is produced. Different expositions predict the quite different emission radii of one stellar radius (Sturrock 1970) and 10–100 stellar radii (Ruderman and Sutherland 1975). Our discussion will be within the context of the Radhakrishnan and Cooke model, also called the polar cap model (after Sturrock 1970), but which we call the *plasma flow model* because the essential feature of the model is that the primary relativistic motion of plasma is approximately one-dimensional in the corotating frame of reference. Corotational velocities are assumed to be secondary, and in this paper we put upper limits on aberration associated with corotation. First we review evidence that suggests that different radio frequencies are radiated at different radii. We then discuss observable consequences of such spatially differentiated emission, after which we use observations to place constraints on the radii of emission. Finally, we discuss additional implications of the assumptions underlying our treatment of the problem.

II. EVIDENCE FOR A RADIUS-TO-FREQUENCY MAPPING

A sizable fraction of pulsars show average pulse intensity profiles that comprise two lobes or components of emission, as shown schematically in Figure 1. The separation of components $\Delta\theta_p$ (in degrees longitude where 360° equals one pulse period) typically decreases with increasing frequency according to ν^{-x}, as shown in Figure 1. The average value of x is 0.25 for frequencies smaller than some critical frequency ν_c for 12 pulsars examined by Manchester and Taylor (1977). At frequencies higher than $\nu_c \approx 1$ GHz the exponent x becomes smaller and may actually become slightly negative (Sieber, Reinecke, and Wielebinski 1975).

Komesaroff (1970) and Tademaru (1971) derived $\nu^{-1/4}$ dependences for component separations by analyzing curvature radiation from groups of particles in the plasma flow model. Implicit in their models is the fact that, owing to the beaming process, the spectrum of curvature radiation, and the changing size of the first Fresnel zone, different frequencies receive their greatest contributions from different radii. A radius-to-frequency mapping is explicit in the model of Ruderman and Sutherland (1975) who propose that radiation is related to the local plasma frequency and therefore to the particle density. The particle density decreases as r^{-3} for plasma near the magnetic pole of a dipolar field. The derived component separation varies as $\nu^{-1/3}$ in the Ruderman and Sutherland model.

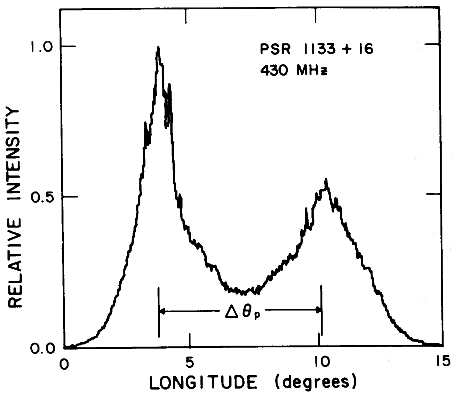

FIG. 1*a*

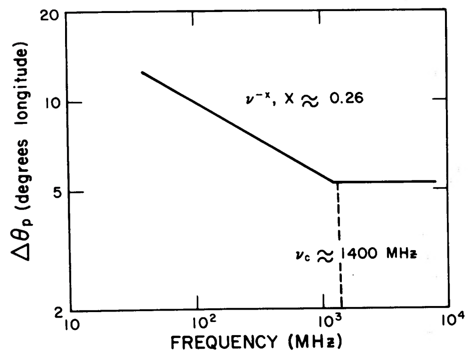

FIG. 1*b*

FIG. 1.—(*a*) An average pulse profile of total intensity formed by summing several hundred single pulses obtained at 430 MHz. The lobe separation is $\Delta\theta_p \approx 6\overset{\circ}{.}4$. Noisiness of the profile is due to intensity fluctuations in pulses used to form the profile. (*b*) Schematic frequency dependence, after Sieber (1973), of the lobe separation $\Delta\theta_p$ for PSR 1133+16. At low frequencies the lobe separation varies according to a power law ν^{-x} where $x = 0.26$ for PSR 1133+16; for 11 other pulsars examined by Manchester and Taylor (1977), $0.08 \leq x \leq 0.5$ at frequencies below ν_c. Above a break frequency ν_c, the exponent of the power law is zero although for some pulsars it may become positive.

An attendant feature of any radius-to-frequency mapping is that, relative to the total spectral extent of a pulsar's radiation, the radiation at a given radius is narrow-band. The underlying kinematic cause of radiation is most likely the motion of particles along curved magnetic field lines, the radiation of which is essentially broad-band. However, it is well known that pulsar radiation must involve coherence among the radiating particles, and it is likely that coherence will drastically modify the single-particle curvature radiation spectrum. Benford and Buschauer (1976) demonstrate that radiation is narrow-band if a plasma wave—via its particle density fluctuations—causes the coherent amplification of curvature radiation.

In the following we assume the existence of a radius-to-frequency mapping. We can determine the mapping from the observed dependence on frequency of the average profile width and by assuming a structural form for the magnetic field near the magnetic pole.

III. DETERMINANTS OF AVERAGE PROFILE WIDTHS

a) Polar Cap Size and Average Profile Width

We assume that radiation at frequency ν arises from an average radius $\bar{r}(\nu)$ and that the angular width of the average profile is related to the width of the zone of radiation at radius $\bar{r}(\nu)$. The width of the radiation zone is determined by that of the polar cap which itself is defined by those field lines that extend through a cylinder (coaxial with the rotation axis) where rigid corotation ceases. The radius of the velocity-of-light cylinder—where corotational velocities would be the speed of light—is clearly an upper limit on the actual radius. The polar cap is the region on the stellar surface where particles emanate from the star and are accelerated to relativistic energies (Goldreich and Julian 1969).

Let $r_{\rm LC} = cP/2\pi$ be the velocity-of-light radius, where P is the pulse period, and let R be the stellar radius. The polar cap size for a purely dipolar field is

$$\Delta\theta_{\rm PC} = 2(R/r_{\rm LC})^{1/2} . \qquad (1)$$

If the radiation zone is defined by those field lines which also bound the polar cap, then the radiation zone width at radius $\bar{r}(\nu)$ is

$$\Delta\theta_{\rm RZ} = (\bar{r}(\nu)/R)^{1/2}\Delta\theta_{\rm PC} . \qquad (2)$$

Curvature radiation from a single particle is beamed in the local magnetic field direction. It is probable that the beamwidths of curvature-radiating particles are negligibly small compared to the angular width of the radiation zone (Cordes 1976). Therefore the field-line tangents of those field lines that border the radiation zone define the observed average profile width. At an angle θ from the magnetic pole, the local tangent vector makes an angle $3\theta/2$ with the pole. Therefore the average profile width is

$$\Delta\theta_p = 3\Delta\theta_{\rm RZ}/2 \qquad (3)$$

as long as aberration due to stellar rotation is not important.

b) Retardation and Aberration

If radiation at two frequencies ν_1 and ν_2 is produced at radii $r(\nu_1)$ and $r(\nu_2)$, then differential aberration and retardation may be observable in the relative arrival times of pulses at the two frequencies. To first order (ignoring field-line curvature), retardation causes pulses at frequency $\nu_1 > \nu_2$ to arrive later than those at frequency ν_2 by an interval

$$\Delta t_{\rm ret}(\nu_1, \nu_2) \equiv t(\nu_1) - t(\nu_2) \approx [r(\nu_2) - r(\nu_1)]/c . \qquad (4)$$

Aberration due to corotation causes radiation beams to be bent into the azimuthal direction such that emission is received earlier than if there were no rotation. Clearly, aberration is greater for emission arising at larger radii. Therefore, to first order, aberration causes pulses at frequency $\nu_1 > \nu_2$ to arrive later than those at frequency ν_2 by an interval

$$\Delta t_{\mathrm{ab}}(\nu_1, \nu_2) \approx \sin \alpha \Delta t_{\mathrm{ret}}(\nu_1, \nu_2)\,, \quad (5)$$

where α is the angle between the rotation axis and the magnetic pole. Equation (5) assumes that corotational velocities are much less than the speed of light, or that $\sin \alpha r(\nu) \ll r_{\mathrm{LC}}$.

IV. LIMITS ON EMISSION RADII

We presume that radiation at frequency ν arises from a radius $r(\nu)$ which may vary from pulse to pulse or on a variety of time scales over a range $\Delta r(\nu)$ centered on a mean value $\bar{r}(\nu)$. The total range in which all radio frequencies are emitted is $\Delta r_T = r(\nu)_{\max} - r(\nu)_{\min}$. We can put limits on both $\Delta r(\nu)$ and Δr_T on the basis of observed average profile properties.

a) *Average Profile Widths*

We associate average-profile broadening with the radial change of the radiation zone width. It is clear that $\Delta r(\nu)$, the radial range for a given frequency ν, must be much smaller than the radial change required to account for the frequency-dependence of profile widths. That is, radiation at a given frequency must arise from a radial range that is much smaller than the total radial range for all frequencies; otherwise profile widths would probably not be observably frequency dependent.

The ratio of maximum profile width (at a low frequency) to minimum profile width (at a high frequency) is related both to the ratio of frequencies (by observation) and to the ratio of radii (knowing the field line topology). We have

$$\Delta\theta_{p_{\max}}/\Delta\theta_{p_{\max}} = (\nu_{\max}/\nu_{\min})^{-x}$$
$$= (r_{\max}/r_{\min})^{1/2} \equiv (1 + \Delta r_T/r_{\min})^{1/2} \quad (6)$$

where the second equality holds for a dipolar field. We solve for

$$\Delta r_T/r_{\min} = (\Delta\theta_{p_{\max}}/\Delta\theta_{p_{\min}})^2 - 1 \quad (7)$$

for three pulsars that have bimodal average pulse profiles. For PSR 1133+16, $x = 0.26$ between 40 and 1400 MHz (Sieber, Reinecke, and Wielebinski 1975) and $\Delta\theta_{p_{\max}}/\Delta\theta_{p_{\min}} \approx 2.4$ and thus $\Delta r_T/r_{\min} \approx 5$. PSR 0525+21, which has a pulse period 3 times that of PSR 1133+16, yields $\Delta r_T/r_{\min} \approx 2$ while PSR 0950+08 with a period that is one-fifth that of PSR 1133+16, yields $\Delta r_T/r_{\min} \approx 32$. Table 1 summarizes parameters of the average profiles and derived quantities. Note that derived values of $\Delta r_T/r_{\min}$ are actually lower limits because observations at frequencies lower than those referred to may yield larger values of $\Delta\theta_{p_{\max}}$. Thus the derived depth of the radiation zone Δr_T depends on the smallest radiated frequency. A few pulsars have been observed at 40 MHz (Craft 1970). Some show spectra with low-frequency turnovers at a few hundred megahertz that are evidently intrinsic to the pulsar, but some show no evidence of turnovers even at 40 MHz (Sieber 1973).

b) *Dispersion Measurements*

The dispersion measures of pulsars are determined by measuring the difference in arrival time of pulses between two frequencies. Most published measurements to date involved the summing of many pulses at two or more frequencies to form average pulse profiles whose relative positioning in time yielded the dispersion measurement. Observations by Craft (1970) presently allow us to put limits on Δr_T because differential aberration and retardation over this radial range are not evident.

Craft's measurements over a wide range of frequencies (40–430 MHz) involved a comparison of the arrival times of average profiles at the different frequencies. The frequency-dependent shapes of average profiles—notably the separation of components—offers an ambiguity as to which longitude should be taken as a fiducial point. For double-lobe average profiles the midpoint between lobes is the fiducial point which yields measurements that are consistent with the cold-plasma dispersion law. This dispersion law, applicable to a tenuous medium with no magnetic field, predicts that pulses at frequency $\nu_1 > \nu_2$ arrive later than those at ν_2 by an interval (to first order)

$$\Delta t_d(\nu_1, \nu_2) \equiv t(\nu_1) - t(\nu_2)$$
$$\approx (e^2/2\pi mc) \int dl n_e(l)(\nu_1^{-2} - \nu_2^{-2})\,. \quad (8)$$

TABLE 1

PROFILE WIDTHS, DISPERSION ERRORS, AND EMISSION RADII

PSR	ν_1 (MHz)	ν_2 (MHz)	$\Delta\theta_p(\nu_1)$ (degrees)	$\Delta\theta_p(\nu_2)$ (degrees)	x (in ν^{-x})	$\Delta\phi$ (rad)	$\Delta r_T/r_{\min}$	$r_{\min}$ (cm) (upper limit)	$r_{\max}$ (cm) (upper limit)	$r_{\max}/r_{\mathrm{LC}}$
0525+21[a]......	112	613	19.0	11.0	0.21	0.034	2.0	$\lesssim 10^{8.2}$	$\lesssim 10^{8.7}$	$\lesssim 0.026$
0950+08[a,b].....	40	430	23.0	4.0–5.0	0.55	0.160	32.0–20.2	$\lesssim 10^{6.5-6.7}$	$\lesssim 10^{8}$	$\lesssim 0.08$
1133+16[a,c].....	40	1400	12.5	5.3	0.26	0.018	4.6	$\lesssim 10^{7.0}$	$\lesssim 10^{7.8}$	$\lesssim 0.01$

NOTE.—Errors in $\Delta\theta_p$ of $\lesssim 10\%$ imply errors in $\Delta r_T/r_{\min}$ of $\lesssim 30\%$. The quantity $\Delta\phi$ is itself an error interval for the fiducial timing points of pulse profiles and is probably accurate to 50%. Hence upper limits on $r_{\max}$ and $r_{\min}$ are accurate to ~50%.

REFERENCES.—[a]Craft 1970. [b] Cordes and Hankins 1977. [c] Sieber *et al.* 1975.

Additional terms in Δt_d are smaller by a factor of at least $10^{-3.5}$ by observation (Tanenbaum, Zeissig, and Drake 1968) and are probably several more orders of magnitude smaller for particle densities and magnetic fields thought to be present in the interstellar medium.

Consistency of observations with equation (8) requires either that all frequencies are incident simultaneously on the dispersive medium (the interstellar medium) or that differential emission of frequency components also obeys a ν^{-2} law. Retardation and aberration contribute time delays according to equations (4) and (5). The total time difference between the arrival of fiducial points in the average profiles at two frequencies is then

$$\Delta t_T = \Delta t_d + \Delta t_{\text{ret}} + \Delta t_{\text{ab}}$$
$$\approx \Delta t_d + (1 + \sin \alpha)\Delta t_{\text{ret}} . \qquad (9)$$

By hypothesis we assume that frequency-dependent radii produce the frequency dependence of average profile widths so that (for a dipolar field) $r(\nu) \propto \nu^{-2x} \approx \nu^{-0.5 \rightarrow -1.1}$ and hence that the retardation and aberration terms, which do not obey a ν^{-2} law, are smaller than the measurement errors of the dispersion term in equation (9).

The precision of a dispersion measurement, typically one part in 10^4 (Craft 1970), is limited by the accuracy to which a fiducial point in pulse longitude can be established given a finite signal-to-noise ratio and given estimation errors (due to pulse fluctuations) in the average pulse shape. Therefore a *range* of longitudes qualifies as the fiducial marker which will render dispersion measurements consistent with the cold-plasma dispersion law. The midpoint between pulse profile lobes appears to be the center of this range; therefore retardation and aberration, which would take the midpoint out of the range of fiducial points, must yield longitude increments that are smaller than this range. If $\Delta\phi$ is the longitude range in radians, then we must have $(1 + \sin \alpha)\Delta t_{\text{ret}} < P\Delta\phi/2\pi$ and hence an upper limit on Δr_T is

$$\Delta r_T \leqslant \Delta\phi r_{\text{LC}}/(1 + \sin \alpha) . \qquad (10)$$

Combining equation (10) with equation (7) yields an upper limit on $r_{\min}$:

$$r_{\min} \leqslant \Delta\phi r_{\text{LC}}/\{(1 + \sin \alpha)[(\Delta\theta_{p_{\max}}/\Delta\theta_{p_{\min}})^2 - 1]\} . \qquad (11)$$

In Table 1 we show the relevant data for PSR 0525+21, PSR 0950+08, and PSR 1133+16 and the resulting upper limits on the maximum and minimum radii. The minimum radius corresponds to the high-frequency range of the pulsar spectrum where the profile lobe separation has reached a comparatively stable value. Derived values assume $\sin \alpha \approx 1$ because if $\sin \alpha$ were significantly smaller than unity, the observed average profile widths would be larger fractions of the pulse period than they are. Observation of double-lobed average profiles probably implies that the center of the pulsar's beam of radiation passes close to the line of sight (Komesaroff 1970; Backer 1976; Ruderman and Sutherland 1975). Therefore, measured profile widths are related to the true width of the pulsar beam.

c) *Comments*

The results summarized in Table 1 suggest that for PSR 0525+21 and PSR 1133+16 ranges in radius of 2:1 and 5:1 are respectively required to explain the frequency dependence of average profiles. The variation with frequency of the average profiles of five other pulsars examined by Sieber, Reinecke, and Wielebinski (1973) indicates similar radial ranges.

PSR 0950+08 apparently requires the largest radial range, but some qualifying remarks on our use of the observations is in order here. Most observations by Craft (1970) of PSR 0950+08 made use of linearly polarized feeds; 430 MHz observations used a circularly polarized feed. Linearly polarized power has a double-component average profile whose component separation is larger than that of the average profile of the total intensity. The small amount of circularly polarized power at 430 MHz (Cordes and Hankins 1977) implies that measurements with a circularly polarized feed are good approximations to measurements of the total intensity. If the component separation of the linear polarization profile is used at 430 MHz instead of that of the total intensity, then the radial range is reduced by a factor of $\frac{2}{3}$.

The derived high-frequency radius for PSR 0950+08 is within a factor of a few (3 to 5) times the typical radius attributed to a neutron star. All "maximum" radii are well within the velocity-of-light cylinder, although the upper limit for PSR 0950+08 is near a value where retardational and aberrational effects might be observable if more accurate dispersion measurements are made.

d) *Nondipolar Fields*

Another component of our discussion involves the relationship of the polar cap size to the measured width of the average profile. Given a polar cap size, the observed profile width depends on the radius of emission (as well as on how close the magnetic pole comes to the line of sight). Sturrock's (1970) model constrained the emission radius to be within a stellar radius of the surface. Polar cap sizes predicted by equation (1) produce average profiles that are then too narrow by nearly a factor of 10. Roberts and Sturrock (1972) argued that polar caps are actually larger than those predicted from equation (1) because rigid corotation ceases at a radius where centrifugal and gravitational forces balance. They represent this radius as

$$R_Y = R^{1-\eta} r_{\text{LC}}^{\eta} , \qquad (12)$$

where $0.5 \leqslant \eta \leqslant 0.7$. The polar cap size then becomes

$$\Delta\theta_{\text{PC}} \approx 2(R/R_Y)^{1/2} = 2(R/r_{\text{LC}})^{\eta/2} . \qquad (13)$$

The polar cap is actually ellipsoidal, with major axis aligned in the longitudinal direction of the pulsar. The ratio of minor axis to major axis is not, however, much different from unity (Roberts and Sturrock 1972).

Roberts and Sturrock assumed $\bar{r}(\nu) \approx R$ and therefore required $\eta \approx \frac{2}{3}$ in order to get profile widths that match observed widths. If the stellar magnetic field is dipolar, however, then $\eta = 1$, and radiation at radii $\bar{r}(\nu) \approx 0.034 r_{LC}$ is required to produce 10° profile widths. The value of η determines the period–pulse-width relation. With $\eta \approx \frac{2}{3}$ the average profile width in time goes as $\Delta t \propto P^{2/3}$ while a dipolar field gives $\Delta t \propto P^{1/2}$ if radiation occurs at a fixed radius for all pulsars (e.g., at the polar cap). If radiation occurs at a fixed fraction of the light cylinder radius, then $\Delta t \propto P$. While $\Delta t \propto P^{1/2}$ is apparently inconsistent with the data (Gunn and Ostriker 1970), the $\Delta t \propto P^{2/3}$ and $\Delta t \propto P$ relations are not readily distinguished by the data.

Our results are consistent with dipolar fields persisting out to a substantial fraction (0.03–0.1) of the light cylinder radius if actual radii of emission are about equal to their derived upper limits. Alternatively, the results are also consistent with high-frequency emission occurring at or near the stellar surface and low-frequency emission originating within 10 stellar radii of the surface. The required polar cap size is then inconsistent with that expected for a dipolar magnetic field.

V. DISCUSSION

Although our results readily follow from our assumptions, it seems wise at this point to clarify and discuss some of our assumptions. Our central assertion is that different radio frequencies are emitted at different radii, a concept which has both observational and theoretical basis. A radial variation of the frequency of radiation is suggested by the contraction of average profiles with increasing frequency. An ad hoc counterproposal, however, is that all frequencies are radiated from the same narrow radial range by a broad-band mechanism but that the radiated spectrum varies systematically with distance from the magnetic pole. To reproduce the average-pulse-profile observations, the frequency of the spectral peak would necessarily decrease with distance from the magnetic pole. Neither theory nor observation can presently reject this counterproposal. However, future observations of single pulses at several frequencies simultaneously may be able to do so. In a separate paper (Cordes and Rickett, in preparation) we analyze the observable consequences of there being differential emission of frequency components as opposed to there being simultaneous emission. In particular, the frequency structure of pulse microstructure may be markedly different for the two cases. For simultaneous emission of frequency components, we expect the spectrum to fluctuate such that the local rms value divided by the local mean is approximately unity (Rickett 1975). If differential emission occurs, the spectrum may be more highly modulated.

The concept of emission in a pulsar magnetosphere itself needs some clarification. In order for a rotating beam to produce pulselike emission in a nonrotating frame of reference, the radiation must become decoupled from the pulsar's magnetic field and from the plasma that is tied to it. Decoupling may occur after a sequence of events by which various plasma waves are excited, propagate parallel to the magnetic field, and then transfer energy to quasi-free space electromagnetic waves (Hardee and Rose 1975). Our discussion thus assumes that different radio frequencies result from decoupling at different radii, and the observations suggest that decoupling takes place well within the light-cylinder radius if not also within a few stellar radii of the neutron star.

It is questionable whether or not future measurements can place stronger upper limits on the radii of emission. The observational procedure involves determination of arrival times at many frequencies. Arrival times are referenced to some fiducial point in pulse longitude which is chosen so as to make the measurements consistent with a ν^{-2} dispersion law. Specification of the fiducial point is most precise if short-time-scale structure exists in the pulses. Unfortunately the strongest upper limits are obtained from low-frequency (e.g., 40 MHz) measurements where dispersion distortion will smear out short-time-scale structure and where dispersion-removal techniques may be tedious to apply. Simultaneous multi-frequency observations at higher frequencies (e.g., 100–430 MHz) may be successful because cross-correlations of micropulses between frequency pairs can yield dispersion measurements accurate to one part in 10^6 (Rickett, Hankins, and Cordes 1975). A problem may still remain in defining a fiducial point, however. If separations of micropulses are frequency dependent in the way average profile components are, then cross-correlation functions must be computed for different longitude ranges. The fiducial point will be the longtitude from which micropulses expand away as the radio frequency gets lower. This point can be determined from cross-correlation functions as the longitude where the rate of change of the lag of peak correlation is a minimum. To determine this longitude, microstructure must be broad-band and it must be observable over a broad range of longitude. Microstructure from PSR 0950+08 is highly correlated over a 3:1 frequency range (Rickett, Hankins, and Cordes 1974), but microstructure is strong only over narrow longitude ranges for PSR 0950+08 and PSR 1133+16 (Cordes and Hankins 1977).

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J. M. Cordes: Department of Physics and Astronomy, 619I GRC Tower B, University of Massachusetts, Amherst, MA 01003