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INFRARED PHOTOMETRY, BOLOMETRIC MAGNITUDES, AND EFFECTIVE TEMPERATURES FOR GIANTS IN M3, M13, M92, AND M67

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ABSTRACT

Broad-band infrared *J*-, *H*-, and *K*-magnitudes and narrow-band CO and H_2O indices are presented for a selection of giants reaching 3 mag below the red-giant tip in the globular clusters M3, M13, and M92, and in the old open cluster M67. Comparison of these data with a calculated grid of model atmospheres gives the following results: (1) the models satisfactorily predict the broad-band colors; (2) $V - K$ measurements accurate to ± 0.1 mag can be used to give effective temperatures to ± 100 K independent of surface gravity or metal abundance for metal-poor stars; (3) there is disagreement with a previous (T_{eff}, spectral type)-calibration based on DDO photometry.

Empirical bolometric magnitudes are derived by integrating the observed energy distributions out to 2.2 μm. The derived luminosities and effective temperatures are plotted in a H-R diagram and are compared with a set of evolutionary tracks for metal-poor stars due to Rood. The agreement is good.

The CO index, which is sensitive primarily to luminosity and effective temperature for Population I giants, becomes sensitive to metal abundance for very metal-poor stars. The relative metal abundance of M3 and M13 derived from CO is reversed from that derived by some other methods. Some possible explanations are considered.

Because of the importance of Rayleigh scattering in these cluster stars, $B - V$ depends on surface gravity. Gravities determined from $B - V$ colors are used to derive crude masses for the stars. These masses ($\sim 0.6\ M_{\odot}$) are roughly consistent with estimates of the turnoff mass.

The CNO abundances are suggested as the second parameter affecting the color-magnitude diagrams of globular clusters.

Subject headings: clusters: globular — clusters: open — infrared: sources — stars: atmospheres — stars: late-type — stars: luminosities

I. INTRODUCTION

This paper presents the initial results of a program directed toward establishing a library of infrared magnitudes and narrow-band indices for globular and open cluster giants covering a wide range in metal abundance. Ultimately, these data will be used in the construction of stellar synthesis models for systems covering a range in total mass and metal abundance. Recent observational work has led to the idea that metallicity differences can account for at least some of the variations in color and band strengths within individual galaxies and between galaxies of differing absolute luminosity (e.g., Sandage 1972; Faber 1973; Strom *et al.* 1976; Frogel *et al.* 1978 [hereafter Paper I]; Aaronson, Frogel, and Persson 1978 [hereafter Paper II]; Persson, Frogel, and Aaronson 1978 [hereafter Paper III]). Metallicity differences act directly on the broad-band colors and various narrow-band indices of individual stars and indirectly through changes in the stellar luminosity function. In order to investigate these effects on composite stellar populations, it is important to calibrate the infrared indices as functions of metallicity and population mix for individual stars and for star clusters with known values of Fe/H. To begin this program we have measured the *J*-, *H*-, and *K*-magnitudes and CO and H_2O band strengths for a selection of giants in the globular clusters M3, M13, and M92, and in the old open cluster M67.

In addition to their eventual use in stellar synthesis models, the data (§ II) have three important immediate applications to which the bulk of this paper is devoted.

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In §§ III and IV a new set of model atmosphere calculations is described, and the predicted colors are compared with the broad-band data. A basic result of the calculations is that reliable effective temperatures can be derived from $V - K$ colors independent of metal abundance or surface gravity. The infrared data further allow empirical bolometric magnitudes to be derived, and thus the theoretical H-R diagram, $\log L/L_{\odot}$ versus $\log T_{\rm eff}$, can be constructed from observational parameters. In § V this H-R diagram is compared to a set of evolutionary tracks for metal-poor stars computed by Rood (1972). In § VI the narrow-band CO indices are shown to be metal-sensitive for metal-poor giants. The relative metallicities of M3 and M13, as derived from the CO indices of their stars, are reversed from those derived from lines of the heavier elements. A summary is given in § VII.

II. OBSERVATIONS

Most of the globular cluster data were obtained with the 5 m Hale telescope during 1976 and 1977. Some of the M67 observations were made with the 2.5 m and 1.5 m telescopes on Mount Wilson. The InSb detecting system and filters are described fully in Papers I and II. Briefly, three broad-band filters centered at 1.25, 1.65, and 2.20 μm are used to determine *J*-, *H*-, and *K*-magnitudes. The bandwidths of these filters are 0.28, 0.30, and 0.40 μm, respectively. The H_2O and CO absorption strengths (indices) are measured with intermediate-band filters at 2.00 and 2.36 μm, respectively, and with a continuum filter at 2.20 μm. The bandwidths of these filters are 0.08, 0.08, and 0.11 μm, respectively. All observations are ultimately referred to α Lyr which is assumed to be 0.00 mag at all wavelengths. The standard stars which define the photometric system are given in Papers I and II. A detailed description of the transformation between the Johnson system (Johnson 1966*a*) and the present system is given in the Appendix of Paper I. The centering of the focal-plane aperture was checked for each star observed, to minimize the effects of differential refraction. Sky chopping was always in the north-south direction, and care was taken to ensure that no stars were present in the reference beam. Several instrumental corrections which were applied to the galaxy measurements of Papers I and II are minimized when stars are measured. The only instrumental corrections to the data of this paper arose from the occasional measurement of the standard and cluster stars with apertures of different diameters.

Table 1 lists the clusters observed. The individual stars were selected on the basis of brightness and location in the clusters. No attempt was made to obtain any type of complete sample, although the brightest giants were almost always observed. Between four and six stars could be observed per hour with the 5 m reflector to an accuracy of better than 0.02 mag in the colors and magnitudes. Measurements of the CO and H_2O indices required about twice as much time as the *JHK* measurements. Inspection of the data after the

TABLE 1

INTEGRATED PARAMETERS FOR CLUSTERS

Cluster	E_{B-V}	$(m-M)_0$	Source
M3	0.0	14.83	1
M13	0.03	14.33	1
M92	0.02	14.57	1
M67	0.06	9.38	2

Sources: 1. E_{B-V} and $(m-M)_0$ from Sandage (1970).
2. E_{B-V} and $(m-M)_0$ from Eggen and Sandage (1964).

TABLE 2A

M3 PHOTOMETRY*

Star	Observed							
	K	$U-V$	$B-V$	$V-K$	$J-H$	$H-K$	CO	H_2O
I-21	9.95	2.47	1.37	3.10	0.71	0.09	0.085	0.04
II-18	11.73	1.43	0.94	2.36	0.56	0.09	0.00	-
II-46	9.25	-	-	3.47†	0.77	0.11	0.07	0.04
III-28	9.60	2.63	1.37	3.21	0.72	0.10	0.055	0.045
III-77	10.33	2.31	1.15	2.97	0.68	0.12	0.08	-
IV-25	10.78	2.00	1.13	2.82	0.64	0.13	0.05	-
193	12.42	1.25	0.90	2.38	0.58	0.06	-	-
216	11.46	1.68	1.00	2.58	0.60	0.08	0.04	-
1397	9.16	3.19	1.56	3.49	0.79	0.13	0.08	0.03
AA	9.26	3.13	1.57	3.43	0.80	0.12	0.05	0.045
BI	11.50	1.61	1.03	2.61	0.59	0.08	0.075	0.02

**UBV* photometry from Johnson and Sandage (1956).

†*V*-magnitude from Sandage (1953).

Typical 1 σ_m errors in the infrared photometry are ±0.02 mag for K, $J-H$, and $H-K$, ±0.03 mag for $V-K$, and ±0.015 mag for CO and H_2O for all stars in Table 2, unless noted otherwise.

TABLE 2B

M13 PHOTOMETRY*

Star	Observed K	Observed $J-H$	Observed $H-K$	Reddening Corrected K	$U-V$	$B-V$	$V-K$	$J-H$	$H-K$	CO	H_2O
I-2	11.91	0.50	0.11	11.90	1.33	0.84	2.27	0.49	0.11	-	-
I-18	11.42	0.54	0.11	11.41	1.41	0.89	2.45	0.53	0.11	-0.01	-
I-23	10.57	0.54	0.12	10.56	1.58	0.97	2.56	0.53	0.12	-0.05	-
I-24	10.06	0.62	0.11	10.05	-	1.10	2.72	0.61	0.11	-0.01	-
I-48	8.49	0.73	0.11	8.48	-	1.55	3.47	0.72	0.11	0.00	0.035
II-67	8.50	0.77	0.14	8.49	3.23	1.56	3.54	0.76	0.14	-0.005	0.03
II-76	9.49	0.66	0.11	9.48	2.31	1.24	2.95	0.65	0.11	0.03	-
II-90	8.66	0.78	0.13	8.65	-	1.57	3.49	0.77	0.13	0.02	0.05
III-18	9.80	0.62	0.14	9.79	2.18	1.17	2.89	0.61	0.14	-0.005	-
III-56	8.75	0.76	0.13	8.74	-	1.44	3.31	0.75	0.13	0.04	0.035
III-63	8.96	0.72	0.12	8.95	2.56	1.36	3.16	0.71	0.12	0.03	0.03
III-72	13.05	0.51	0.00	13.04	1.03	0.78	2.00	0.50	0.00	-	-
III-73	9.23	0.64	0.12	9.22	2.41	1.27	3.01	0.63	0.12	0.04	0.03
IV-25	8.58	0.77	0.14	8.57	3.13	1.51	3.43	0.76	0.14	0.01	0.04

**UBV* photometry from Kadla (1968) and Cathey (1974).

first observing run showed that the H_2O index was not providing much information for stars as hot as those in this program, so observations of it were discontinued. Tables 2A–2D present the observed and reddening-corrected infrared photometry. The optical data are from the sources cited with reddening corrections applied from Table 1.[1]

The reddening-corrected observations (hereafter denoted by a subscript zero) and plotted in Figures 1–3. The mean relationships for field giants and dwarfs are as given in Papers I and II. Before discussing the data, we shall present a grid of model atmosphere colors. This allows a quantitative comparison of theory and observations to be made.

III. CALCULATED MODELS

a) *Description*

A grid of atmospheric models for metal-poor K giants was constructed using the ATLAS6 code. This program has been described fully by Kurucz (1970, 1975, 1978*b*). The models assume plane-parallel atmospheres in radiative equilibrium under LTE conditions. Convection, which is not important for the values of surface gravity g and effective temperature T_{eff} considered here, is treated with a mixing-length approximation with the ratio of mixing length to pressure scale-height equal to 2. The opacity arising

[1] The results of Sandage and Walker (1966) for M92 were used for consistency even though Cathey (1974), who observed several of our M92 stars, has found a problem with the Sandage and Walker photometry. For the present purposes, the errors in the Sandage and Walker (1966) photometry for stars on the giant branch are not serious.

TABLE 2C

M92 PHOTOMETRY*

Star	Observed K	Observed $J-H$	Observed $H-K$	Reddening Corrected K	$U-V$	$B-V$	$V-K$	$J-H$	$H-K$	CO	H_2O
II-12	12.41	0.42	0.09	12.40	0.95	0.75	2.11	0.41	0.09	-	-
II-70	10.44	0.56	0.11	10.43	1.57	0.96	2.52	0.55	0.11	-0.005	0.03
III-4	12.00	0.48	0.07	11.99	0.82	0.69	2.06	0.47	0.07	-	-
III-13	8.93	0.64	0.12	8.92	2.41	1.31	3.03	0.63	0.12	0.01	0.035
III-65	9.57	0.62	0.11	9.56	2.06	1.16	2.72	0.61	0.11	-0.01	0.03
III-82	10.79	0.54	0.13	10.78	1.53	0.92	2.47	0.53	0.13	0.01	-
IV-2	11.03	0.55	0.09	11.02	1.30	0.88	2.42	0.54	0.09	-0.005	0.045
IV-10	10.88	0.53	0.10	10.87	1.37	0.93	2.48	0.52	0.10	0.02	-
IV-114	11.42	0.52	0.08	11.41	1.20	0.85	2.35	0.51	0.08	0.03	-
VII-18	9.12	0.63	0.14	9.11	2.30	1.29	2.93	0.62	0.14	-0.02	0.03
VIII-43	12.42	0.48	0.02	12.41	0.84	0.73	2.14	0.47	0.02	-	-
X-49	9.24	0.64	0.12	9.23	2.26	1.17	2.85	0.63	0.12	-0.02	0.025
XI-19	10.16	0.60	0.10	10.15	1.76	1.03	2.64	0.59	0.10	-0.03	0.04
XII-8	10.07	0.56	0.11	10.06	1.60	1.04	2.63	0.55	0.11	0.02	-

**UBV* photometry from Sandage and Walker (1966).

TABLE 2D

M67 Photometry*

Star	Observed K	Observed $J-H$	Observed $H-K$	Reddening Corrected K	$U-V$	$B-V$	$V-K$	$J-H$	$H-K$	CO	H_2O	Notes
84	8.10	0.54	0.09	8.08	1.97	1.06	2.33	0.52	0.08	0.08	-	1
94	11.44	0.28	0.06	11.42	0.53	0.50	1.23	0.26	0.05	0.03	-	
105	7.47	0.65	0.10	7.45	2.48	1.20	2.67	0.63	0.09	0.13	0.085	
108	6.57	0.69	0.11	6.55	2.83	1.32	2.99	0.67	0.10	0.11	0.05	
115	11.17	0.31	0.05	11.15	0.65	0.57	1.32	0.29	0.04	0.015	-	
117	10.58	0.47	0.08	10.56	0.95	0.71	1.87	0.45	0.07	0.055	-	
141	7.99	0.54	0.09	7.97	1.99	1.05	2.33	0.52	0.09	0.07	-	1
151	8.06	0.53	0.09	8.04	1.96	1.04	2.27	0.51	0.08	0.07	-	1
164	8.01	0.54	0.09	7.99	2.04	1.07	2.38	0.52	0.08	0.06	0.01	
170	6.60	0.67	0.11	6.58	2.74	1.30	2.93	0.65	0.10	0.13	0.04	
193	9.96	0.51	0.07	9.94	1.72	0.96	2.14	0.49	0.06	-	-	1
223	8.04	0.55	0.11	8.02	1.98	1.04	2.38	0.53	0.10	0.035	-	1
224	8.23	0.54	0.08	8.21	2.12	1.07	2.37	0.52	0.07	0.07	0.04	
227	10.82	0.45	0.14	10.80	1.39	0.86	1.89	0.43	0.13	-	-	2
231	9.11	0.56	0.08	9.09	1.83	0.99	2.23	0.54	0.07	-	-	1
244	8.60	0.46	0.10	8.58	1.51	0.88	2.02	0.44	0.09	0.04	-	1
I-17	10.09	0.46	0.10	10.07	1.61	0.93	2.15	0.44	0.09	0.04	-	
II-22	10.76	0.43	0.07	10.74	1.42	0.87	2.01	0.41	0.06	0.05	-	
III-34	8.77	0.53	0.07	8.75	1.90	1.03	2.39	0.51	0.06	0.075	-	
IV-20	8.60	0.55	0.08	8.58	1.86	1.01	2.45	0.53	0.07	0.065	-	
IV-77	10.66	0.46	0.07	10.64	1.53	0.93	2.07	0.44	0.06	0.05	-	
IV-68	10.84	0.43	0.12	10.82	1.25	0.81	1.95	0.41	0.11	0.025	-	
IV-81	10.89	0.38	0.06	10.87	1.10	0.76	1.74	0.36	0.05	0.03	-	

**UBV* photometry from Eggen and Sandage (1964).

NOTES: 1. Due to poor seeing, the errors in the infrared broad-band colors and magnitudes for these stars are ±0.04 mag.
2. The errors for this star are ±0.05 mag.

from atomic lines is evaluated using the opacity distribution function (ODF) technique described in Kurucz, Peytremann, and Avrett (1974) and Kurucz (1975) based on the line list of Kurucz and Peytremann (1975) with a microturbulent velocity of 2 km s^{-1}. The damping constants are described in Peytremann (1975) and Kurucz, Peytremann, and Avrett (1974). The solar abundances are those of Kurucz (1970) as modified by improved determinations between 1970 and 1975. The values of $T_{\rm eff}$ in the grid range from 3800 to 5250 K, and the values of $\log g$ from 0.5 for the more metal-deficient models to $\log g = 3.0$ for the least metal-deficient models. The metal abundances with respect to the Sun (denoted Fe/H) are 0.01, 0.03, and 1.0. Even at Fe/H = 0.01 the metals (particularly Fe and Mg) were the dominant source of electrons over most of the atmosphere for the lower $T_{\rm eff}$ models. The code contains opacities from atomic lines but does not include the effects of molecular bands. This should not be a serious problem, since high-resolution echelle spectra of several of the coolest stars in the sample (Cohen 1978) show only very weak molecular bands. For example, the MgH band at 5200 Å is not detectable in several stars in M3 and M13 (the G-band of CH is seen). We thus expect only CO to be of any importance as a source of molecular opacity.

Because of the broad-band nature of the observations, the ODF spectra were used to compute *UBVRIJHK* for each model. The effective wavelengths are those given by Johnson (1966*a*) for *UBVRI*, while they are from Wilson *et al.* (1972) and Paper I for *JHKL*. The theoretical *JHK* colors were calculated by convolving the filter-response curves (which are nearly square waves) with the predicted F_ν distribution. Colors from B to I were predicted using the ODF spectra over the 600 Å of maximum filter response, with the response functions of Golay (1974) and Johnson and Mitchell (1962). U was integrated over the entire response function given by Golay (1974). Differences between the predicted and observed $U - V$ colors are due to the high density and possible omission of some atomic lines in the bandpass in these cool stars. The neglect of molecular opacities may also affect the predicted $U - V$ colors for the more metal-rich models. These problems with U do not affect the subsequent discussion, however. There is also an uncertainty of about ± 0.03 mag in the predicted H-magnitude because of its location near the minimum in the H^- opacity and the maximum of the Planck curve (Gingerich and Kumar 1964), the neglect of molecular absorption, and the use of ODF spectra in a region of rapidly varying flux. To normalize the colors, a model for Vega was calculated assuming $T_{\rm eff} = 9500$ K, $\log g = 4.0$, and Fe/H = 1.0 (Kurucz 1978*a*). The resulting colors were 0.00 ± 0.01 mag from U to I; but the $V - J$, $V - H$, and $V - K$

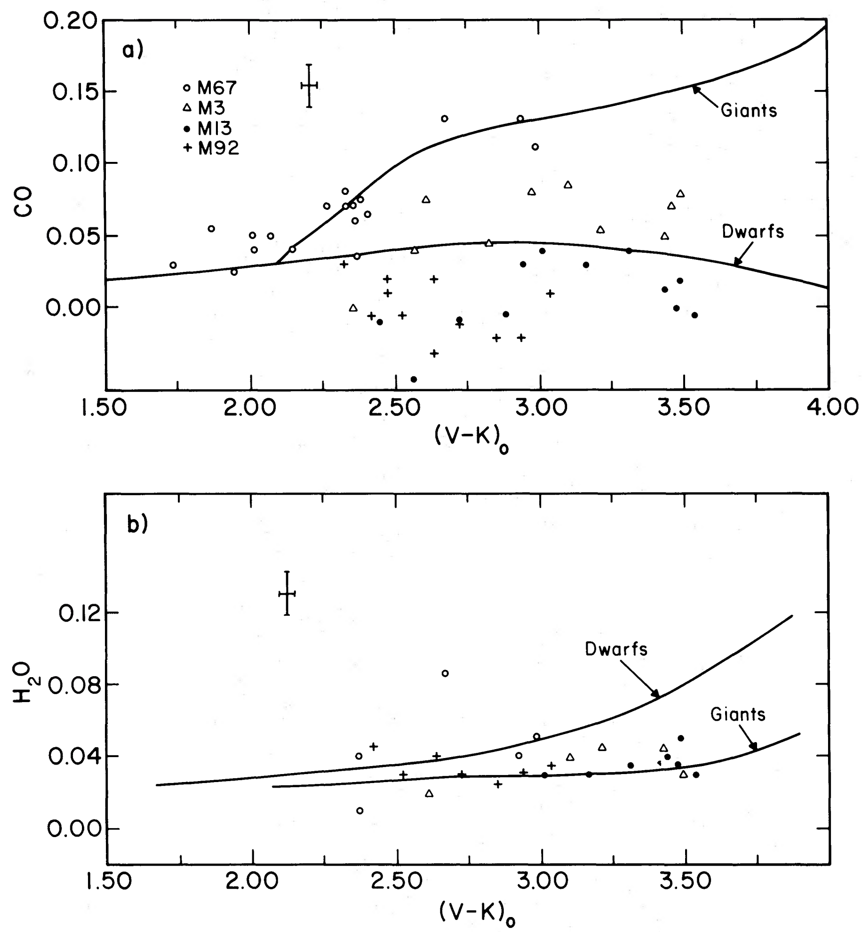


FIG. 1.—The observed narrow-band indices are plotted as a function of $V - K$ (corrected for reddening). The mean dwarf and giant lines are from Papers I and II. The error bar is a nominal ± 0.015 mag error in CO and ± 0.03 mag in $V - K$ ($1\ \sigma_m$).

colors were all $+0.09$ mag. The corrections required to force the colors of the Vega model to be 0.00 at all wavelengths were made to all the predicted colors of the K giant models.[2]

We estimate that the effects of the CO and H_2O absorption bands on $J - K$ and $V - K$ are less than 0.05 mag based on the observed CO indices. In comparing the observed and predicted colors (§§ IV and V), the observed K-magnitudes were made brighter by $0.20 \times$ (CO absorption index) as the CO bands occur over about 20% of the K bandpass. The maximum correction to K for the stars discussed in this work was 0.04 mag. A correction to the calculated $B - V$ colors for the CH band can be obtained using the photometric and spectroscopic observations of Norris and Zinn (1977) in the region 4290–4320 Å for globular cluster stars and Griffin (1968) for higher metallicity stars. An estimate of the ratio of the total absorption by CH in the B bandpass to that of CH within this 30 Å region can be obtained from Moore, Minnaert, and Houtgast (1966). The resultant approximate correction for CH in $B - V$ is redder by 0.02 mag for M92, 0.03 mag for M13, and 0.09 mag for M67. The correction for normal metallicity stars is uncertain by $(+0.05, -0.03)$ mag, while that for the globulars is uncertain by ± 0.01 mag.

The theoretical bolometric corrections (denoted BC) were obtained by integrating the calculated F_ν over frequency for each model and dividing by the predicted V flux. The bolometric correction scale was normalized by using the solar fluxes given in Allen (1963). Table 3 lists the predicted colors and bolometric corrections for each model calculated, and Figure 4 shows the

[2] The $U - V$ colors for the Sun (0.64 mag) calculated in the manner described above from a [5770, 4.43, 1.0] model is too blue by 0.14 mag, although the rest of the colors out to I are reproduced fairly well. We assume this to arise mostly from the effect of missing opacity sources, both atomic and molecular, not important in Vega but important in the Sun. We thus add 0.14 mag to the calculated $U - V$ colors to give the colors listed in Table 3. Based on a comparison with the observations (Fig. 2), we still appear to be missing some opacity at U for solar metal-content models as our computed $U - V$ colors are too blue to match the mean giant line at a reasonable surface gravity.

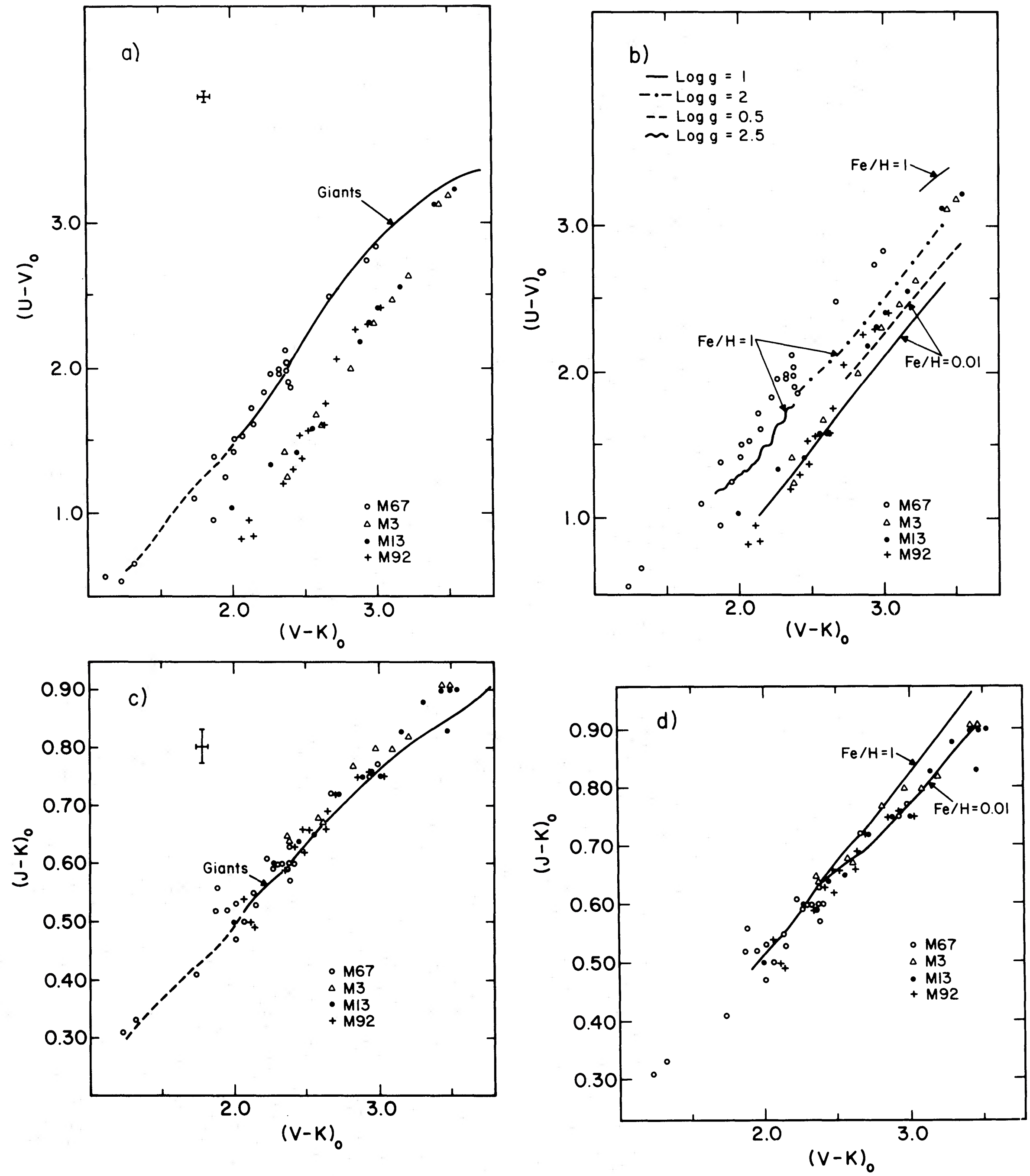


FIG. 2.—In (*a*) and (*c*) the reddening-corrected colors are shown with the mean field-giant line from Paper I. The dashed lines are from Johnson (1966*a*). In (*b*) and (*d*) the colors are replotted together with the model results from Table 3. The error bar is a nominal ±0.03 mag error (1 σ_m).

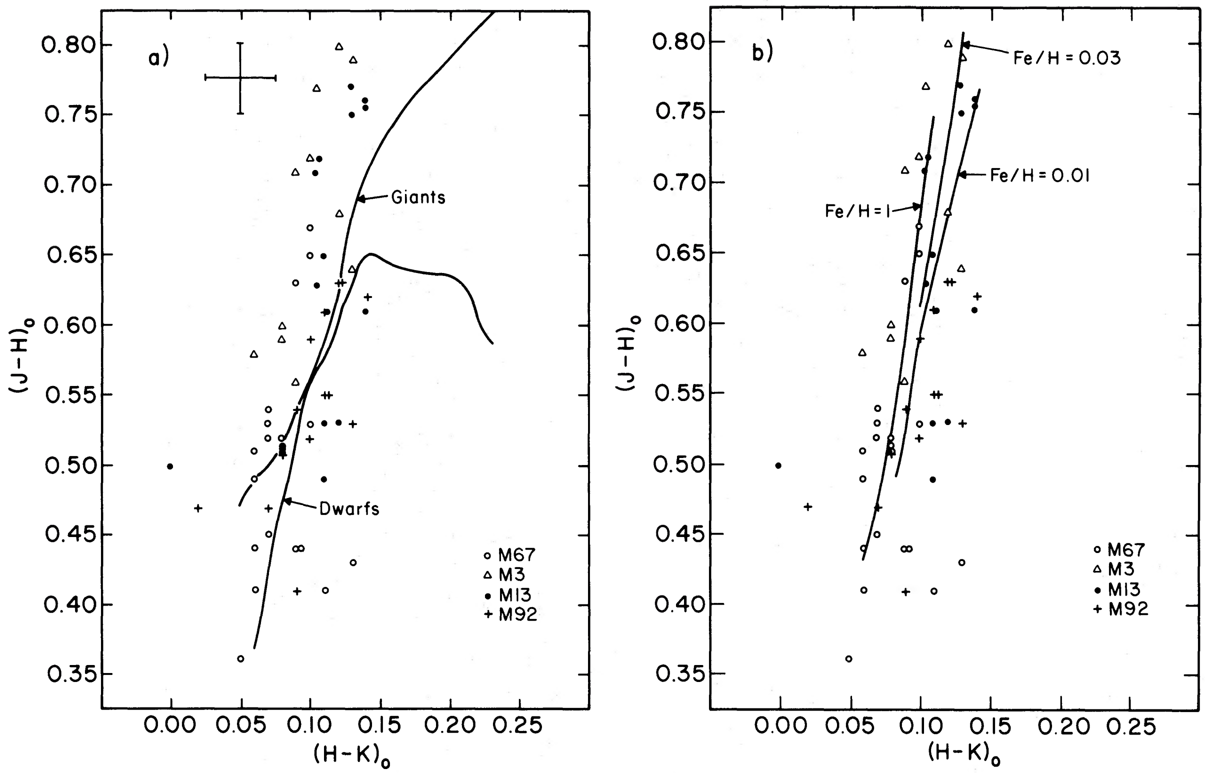
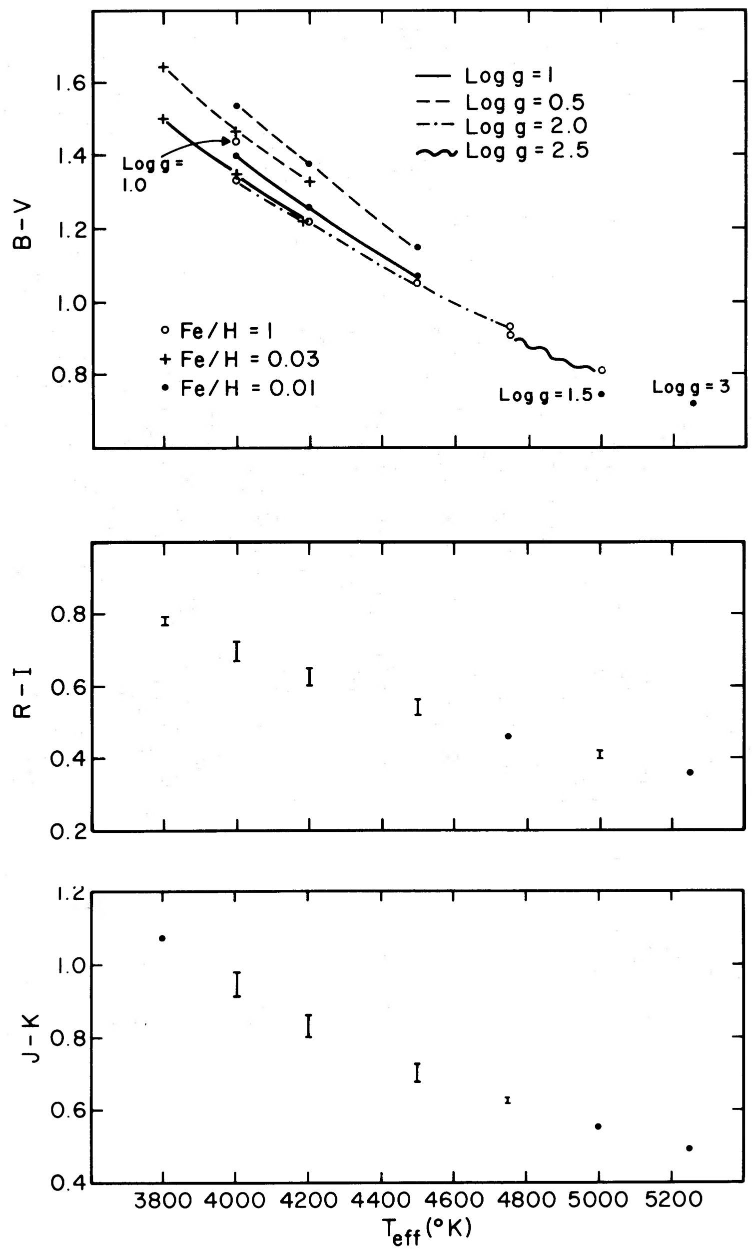


FIG. 3.—Same as Fig. 2

TABLE 3

COLORS FROM MODEL ATMOSPHERES

T_{eff}	Model log g	Model Fe/H	*BC*	*U−V*	*B−V*	*V−R*	*V−I*	*V−K*	*J−H*	*H−K*
5250	3.0	1.00	0.11	1.15	0.72	0.59	0.95	1.91	0.43	0.06
5000	2.5	1.00	0.18	1.40	0.81	0.65	1.05	2.13	0.48	0.07
5000	1.5	0.01	0.21	1.00	0.75	0.64	1.06	2.13	0.47	0.08
4750	2.5	1.00	0.29	1.69	0.91	0.71	1.17	2.39	0.55	0.09
4750	2.0	1.00	0.29	1.74	0.93	0.72	1.17	2.38	0.55	0.08
4500	2.0	1.00	0.42	2.11	1.05	0.80	1.31	2.69	0.63	0.09
4500	1.0	0.03	0.43	1.76	1.05	0.80	1.34	2.68	0.61	0.10
4500	1.0	0.01	0.45	1.73	1.07	0.82	1.37	2.69	0.59	0.11
4500	0.5	0.01	0.48	1.94	1.15	0.86	1.42	2.73	0.59	0.10
4200	2.0	1.00	0.65	2.65	1.22	0.90	1.51	3.11	0.75	0.11
4200	1.0	0.03	0.65	2.24	1.22	0.91	1.53	3.10	0.72	0.12
4200	0.5	0.03	0.69	2.50	1.33	0.96	1.59	3.16	0.71	0.13
4200	1.0	0.01	0.67	2.23	1.26	0.94	1.57	3.10	0.69	0.12
4200	0.5	0.01	0.73	2.49	1.38	1.00	1.65	3.18	0.68	0.12
4000	2.0	1.00	0.84	3.05	1.36	0.99	1.66	3.45	0.85	0.12
4000	1.0	1.00	0.86	3.42	1.44	1.02	1.70	3.47	0.84	0.12
4000	1.0	0.03	0.84	2.63	1.35	0.99	1.68	3.44	0.81	0.13
4000	0.5	0.03	0.90	2.91	1.47	1.05	1.75	3.50	0.81	0.13
4000	1.0	0.01	0.87	2.62	1.40	1.03	1.73	3.44	0.77	0.14
4000	0.5	0.01	0.95	2.90	1.54	1.10	1.83	3.54	0.77	0.14
3800	1.0	0.03	1.09	3.04	1.50	1.09	1.86	3.83	0.92	0.16
3800	0.5	0.03	1.16	3.37	1.64	1.15	1.94	3.90	0.92	0.15



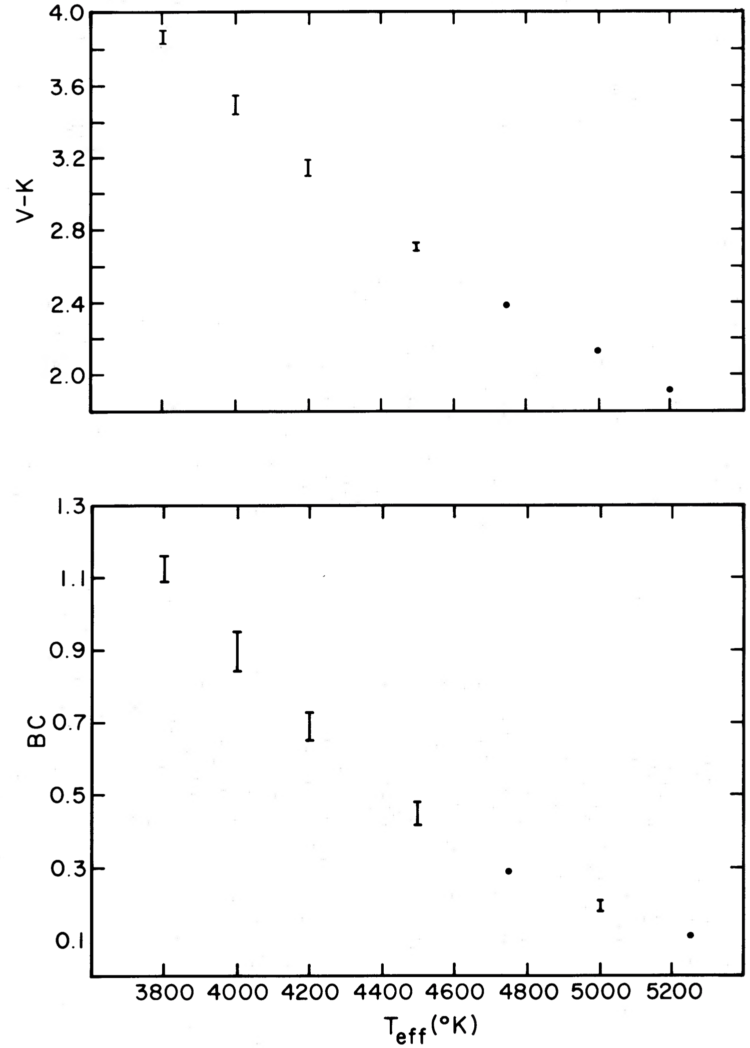


FIG. 4.—Parameters from the model atmospheres in Table 3 are plotted as a function of T_{eff}. The metallicity and surface gravity dependences are shown explicitly for $B - V$ only. For the other quantities, the total range for the models calculated is shown as vertical bars.

variations in these quantities as functions of $T_{\rm eff}$, log g, and Fe/H.[3]

As an indication of the accuracy of the model predictions, a comparison has been made with the preliminary predicted color indices and bolometric corrections (Bell and Gustafsson 1977) calculated from the Gustafsson *et al.* (1975), Bell *et al.* (1976*a*) grid which includes the effect of some molecular opacities on the colors shortward of R. A detailed comparison between the atmospheric structure of the Bell and Gustafsson grid and the ATLAS6 grid will be given by Kurucz (1978*b*). For $V - R$ and $R - I$, the differences in predicted color between the two grids for various models are ± 0.02 mag and -0.03 ± 0.02 mag, respectively, and presumably relate to the different normalization procedures. For $B - V$ with the CH correction described above, the range of the differences is 0.03 to 0.11 mag, and for $U - V$ the models' results differ by up to 0.2 mag. The slightly larger scatter in the $B - V$ differences presumably reflects the inaccuracy of our CH correction for solar metallicity stars while for $U - V$ the dominant problem is missing atomic opacity at U. Nevertheless, this agreement for such cool stars from two independently written, massive computer codes for model atmosphere is encouraging. In summary, we feel that the *relative trends* of colors in Figures 2 and 3 are correct to ± 0.03 mag, and that the colors for any given model are correct to within ± 0.05 mag except for U and for B in stars of normal metallicity where molecular opacity sources become important.

b) *Temperature Scale*

Figure 4 shows that $V - K$ is predicted to be an excellent indicator of $T_{\rm eff}$; for an observed $V - K$ with an error of ± 0.05 mag, the resulting uncertainty in $T_{\rm eff}$ is only ± 70 K, *independent of surface gravity and* Fe/H *in the range considered here*. The reason is that H^- completely dominates the opacity at both wavelengths. The $J - K$ color index, while not as good as $V - K$, is still quite adequate for determining $T_{\rm eff}$. The uncertainty in $T_{\rm eff}$ resulting from variations in g and Fe/H at constant $J - K$ is ± 100 K, while the change in $J - K$ for a change in $T_{\rm eff}$ of 100 K is 0.05 mag. Since J and K are measured with the same equipment on the same night, this observational accuracy is easily achievable. Furthermore, $J - K$ is less affected by reddening than is $R - I$ or $V - K$. The bolometric correction (which requires photometry at all wavelength from U to K) is also a very sensitive function of $T_{\rm eff}$ (Fig. 4), but obviously it is more difficult to obtain than a simple $V - K$ color. An error in BC of ± 0.10 mag yields an error in $T_{\rm eff}$ of ± 100 K, again independent of surface gravity and Fe/H in the range considered here. For a given $R - I$, on the other hand, variations in g and Fe/H over ranges suitable for globular cluster giants correspond to an uncertainty in $T_{\rm eff}$ of ± 200 K. Furthermore, for a difference in $T_{\rm eff}$ of 100 K, the $R - I$ colors differ by only 0.03 mag. So use of $R - I$ colors to determine $T_{\rm eff}$ properly requires an observational error of less than 0.02 mag. Uncertainties in color transformations from another system such as Eggen's (102, 65) system (Eggen 1972) to $R - I$ are too large to permit their use in determining $T_{\rm eff}$. Even with no observational errors, an error in $T_{\rm eff}$ of ± 200 K will result unless the surface gravity and metallicity are already known.

Thus an important conclusion of this paper is that $V - K$ and $J - K$ are expected to be the best, easily observable photometric indices for determining $T_{\rm eff}$ in the range 3800 to 5250 K. A comparison of the observed colors with those predicted from the models should yield values of $T_{\rm eff}$ accurate to ± 100 K (allowing for the estimated uncertainty in the predicted colors) independent of Fe/H or g.

c) *Comparison with Previous $T_{\rm eff}$ and Bolometric Correction Scales*

Here we compare our predicted $V - K$ colors and bolometric corrections as functions of effective temperature with the work of Johnson (1966*a*), Schlesinger (1969), and Osborn (1971). First we point out that our $V - K$ colors are on the same system as that of Johnson (with an 0.02 mag zero-point shift) and that our ($V - K$, spectral type)-relationship is based ultimately on his (Paper I). Johnson's (1966*a*) ($V - K$, bolometric correction)-relationship is also in good agreement with ours (see § IV below).

For a given value of $V - K$ our predicted value of $T_{\rm eff}$ (Fig. 4 and Table 3 including a CO correction appropriate for solar metallicity) is about 100 K hotter than that of Johnson (1966*a*). Although Johnson's scale is based on a combination of observed stellar radii and spectral energy distributions and on theoretical model atmospheres, he had almost no data available for stars with $T_{\rm eff}$ in the range 3800–5000 K (see his Tables 5 and 7). In order for Johnson to establish a ($T_{\rm eff}$, spectral type)-calibration in this range, he had to interpolate using data primarily on cool supergiants and very cool Mira stars. Johnson's (1966*a*) results were given strong weight by Schlesinger (1969); however, Schlesinger's temperature scale is in places more than 150 K cooler than Johnson's and hence more than 250 K cooler than ours. If Schlesinger's spectral types are transformed to $V - K$ values via Johnson's calibration, we find that for a given $V - K$ Schlesinger's BC values are as much as 0.2 mag too large. This difference is puzzling in view of the statement by Schlesinger that his scale is based in part on Johnson's.

Osborn's (1971) temperature and bolometric correction scales are identical to those of Schlesinger

[3] The interesting metallicity dependence of $B - V$ in these cool stars was first pointed out by Böhm-Vitense and Szkody (1974). As the metallicity decreases below the solar value, the $B - V$ color becomes bluer, as is expected from a decrease in line blanketing largely in the B bandpass. Eventually, the opacities become so low that Rayleigh scattering which is proportional to λ^{-4} becomes an important opacity source relative to H^-. Thus any further decrease in the metallicity causes $B - V$ to become somewhat *redder*. A decrease in surface gravity also causes Rayleigh scattering to increase in importance relative to H^- at V. This has the effect of making $V - K$ slightly redder as well.

(1969). This fact is reflected in the temperature of the three coolest M67 giants (see § V) which we find to be about 250 K hotter than did Osborn (1974). This difference translates into a change in $V - K$ of 0.4 mag for these three stars. Since the dominant opacity sources longward of B are H^- and Rayleigh scattering, both of which have well-studied frequency dependences, our $V - K$ colors as a function of $T_{\rm eff}$ must be essentially correct. Osborn's neglect of the effect of Rayleigh scattering on the DDO temperature index, $C(4548)$, as well as his temperature scale problem, also seriously affect his globular cluster results (Osborn 1973), as here he depends on the Fe/H correction to the DDO indices. The complicated propagation of these errors through the Fe/H correction procedure and into his results is shown by a comparison of our results for the M3 giants to his. Our temperature for M3 III-28 (near the top of the giant branch in M3) is 4100 K; Johnson's (1966*b*) is 4000 K, Osborn's (1973) is 4400 K. His high temperature for the giants in M3 led Osborn to infer a small metal deficiency for this cluster; yet his result is in itself inconsistent, since clusters which are less metal deficient have cooler giants, so the M3 red-giant tip should be considerably cooler than 4400 K.

In summary, although our $[T_{\rm eff}, (V - K)_0]$-scale is derived theoretically rather than observationally, we feel that it is the best such scale available until more stellar diameters are measured and until model atmosphere codes are written which include all molecular opacity sources. As will be seen below comparisons of the observed values of $(V - K)_0$, $(J - K)_0$, and BC with model predictions give values of $T_{\rm eff}$ which are consistent to within 100 K. Our temperature scale is close to that of Johnson (1966*a*) which was derived from the best data available at the time.

IV. DISCUSSION OF THE DATA

a) CO *and* H_2O *Absorption Indices*

Figure 1 displays the CO absorption index of the stars plotted against $(V - K)_0$. Since $(V - K)_0$ depends to a good approximation only on $T_{\rm eff}$, the differences between the cluster stars at a constant value of $(V - K)_0$ reflect differences in the CO abundance. While the CO strengths of the M67 stars are generally normal, the globular cluster stars exhibit CO absorption strengths considerably less than normal for a given effective temperature. Since the CO bands are less saturated for these metal-poor giants, a relatively small change in CO abundance will produce a measurable change in the CO index. If the relationship between CO strength and metal abundance is monotonic, then the relative amount of CO deficiency is consistent with a sequence in order of decreasing metallicity of M3, M13, and M92. The mean CO indices for the stars in each globular cluster (at $[V - K]_0 = 3.0$) are $+0.06$, $+0.02$, and -0.01 mag, corresponding to shifts (hereafter referred to as ΔCO) from the mean field giant line of -0.07, -0.11, and -0.14 mag for M3, M13, and M92, respectively. These differences, though small, are significant, since for each cluster the dispersion in CO index about the mean is small.[4] The increase of the CO index toward higher luminosity in Population I stars noted by Baldwin, Frogel, and Persson (1973) acts to strengthen the CO ordering, since M92 has the highest luminosity giants and M3 the lowest (see Fig. 5). We will return to a discussion of this ordering in § VI.

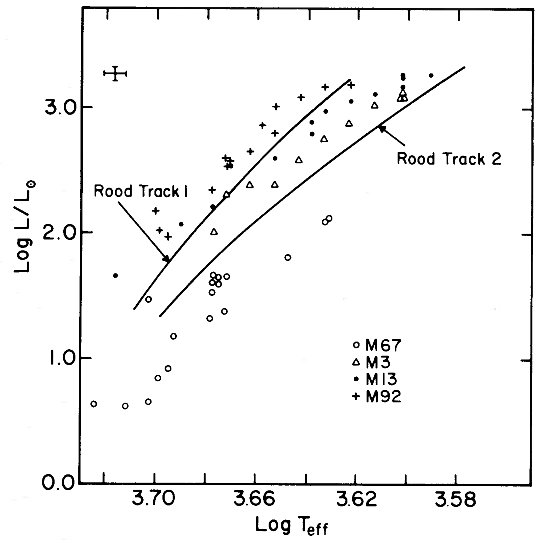


FIG. 5.—The $M_{\rm bol}$ and $T_{\rm eff}$ are plotted in a theoretical H-R diagram. The evolutionary tracks from Rood (1972) correspond to $M = 0.8\ M_\odot$, $X = 0.7$, $Y = 0.3$, $Z/Z_\odot = 0.005$ for track 1, and $Z/Z_\odot = 0.05$ for track 2.

Figure 1*b* shows the dependence of the H_2O absorption on $(V - K)_0$ (or $T_{\rm eff}$). Field giants with $(V - K)_0$ values appropriate for the stars in this paper are not expected to show any H_2O absorption (Paper II)—the reason the mean relationship in Figure 1*b* does not give 0.0 for the value of H_2O is that the continuum of late-type giants is flatter than that of α Lyr. The location of the globular cluster stars is consistent with the mean relationship for field giants.

[4] This result could depend upon the particular selection of stars observed. One effect which can influence the result concerns the numbers of weak G-band (carbon-poor) stars included in the sample. (For discussions of the properties of these stars see Zinn 1973 and Norris and Zinn 1977.) Weak G-band stars in the field display CO indices near 0.00 mag for $T_{\rm eff}$ in the range of those of the globular cluster stars (Hartoog, Persson, and Aaronson 1977). Weak G-band stars in globular clusters are not numerous on the upper giant branch, but are found mostly on the lower asymptotic branch. We would thus expect them *a priori* to be rare in our sample. The lists of Norris and Zinn (1977) contain three stars in M13 and nine stars in M92 in common with this paper. (M3 is not in common.) In M92 there is no correlation between the CO index and the presence of a G-band (six stars) or its absence (three stars). For M13 the three stars in common are all *strong* G-band stars, and the average CO index for them is 0.02 mag greater than the average for all the M13 stars. Even if all the M3 stars we observed happened to be *strong* G-band stars, it is unlikely that the mean CO index of M3 would be reduced significantly by the inclusion of a few *weak* G-band members. We believe, therefore, that the greater mean CO index of the M3 stars over the M13 stars will not be eliminated by any reasonable corrections for the presence of weak G-band stars in the two clusters.

b) *Color-Color Relationships*

Figure 2 shows the relationships between $(V-K)_0$, $(J-K)_0$, and $(U-V)_0$ for the stars that were observed. Also indicated are the mean values for field giants from Paper I and from Johnson (1966*a*). As discussed in § III, both $(V-K)_0$ and $(J-K)_0$ are equivalent to $T_{\rm eff}$ scales. Figure 2*c* demonstrates that $(V-K)_0$ uniquely defines $(J-K)_0$ over the entire range of Fe/H considered, and that this mean relationship lies close to that for field giants. The slight systematic departure of the stars from this latter relationship is in the sense expected if for the same effective temperature the fluxes in the K filter were somewhat higher in metal-poor stars due to weakened CO absorption.

Now consider the dependence of $(U-V)_0$ on $(V-K)_0$ (or equivalently on $T_{\rm eff}$) (Fig. 2*a*). From Table 3 we see that the $U-V$ color for a given $T_{\rm eff}$ becomes bluer as the surface gravity increases due to the interplay between Rayleigh scattering and H^- opacity (recall that higher-metallicity clusters have red giants which are less luminous at a given $T_{\rm eff}$ and hence have larger surface gravities). However, in comparing giants in different clusters, the increase in surface gravity at a fixed $T_{\rm eff}$ is far outweighted by the reddening of $U-V$ produced by line blanketing as Fe/H increases. This change with Fe/H is small for Fe/H < 0.03 but becomes large between 0.03 and 1. Therefore, since the globular clusters we have observed have Fe/H ≤ 0.03, one would expect $U-V$ as a function of $V-K$ to be approximately the same for all the giants. The M67 stars show the effects of the pronounced reddening of $U-V$ at a fixed $T_{\rm eff}$ or $V-K$ due to atomic line opacities.[5] When $(V-K)_0 > 3.4$, the stars are so cool that line blanketing in the U bandpass is strong even in metal-poor stars, and the metallicity discrimination is almost lost, except perhaps for metal deficiencies more extreme than those in M3 or M13. Furthermore, the increasing importance of Rayleigh scattering in the cooler stars acts to make $U-V$ redder as the metallicity decreases (Böhm-Vitense and Szkody 1974).

Figure 3 shows that the redder globular cluster stars are significantly displaced from the $[(J-H)_0, (H-K)_0]$-relationship for field giants which is itself displaced from the blackbody line (cf. Catchpole and Glass 1974). This latter displacement is due to the H^- opacity minimum predicted by Gingerich and Kumar (1964) and noted in § III above. The location of the redder globular cluster stars in Figure 4 would at first imply that they have an even more pronounced flux peak in the H-band. Evidence for this was first noted by Glass and Feast (1973), who suggested that the origin of the effect could be a reduced importance of other opacity sources relative to H^-. The predicted $J-H$ and $H-K$ colors from Table 3 (which do not include CO, CN, or other molecular opacities) indicate, however, that decreasing metallicity moves the stars toward the lower right in Figure 3. A similar metallicity effect was noted by Mould and Hyland (1976) for M dwarfs. This is due to the increased transparency and hence higher pressures in the more metal-poor atmospheres, and to the increased relative importance of Rayleigh scattering which acts to smooth out the 1.6 μm transparency peak. The results of Table 3 reproduce approximately the position of the globular cluster giants in Figure 4, but incorrectly position the mean field-giant line to the left of the globulars and predict a 1.6 μm bump which is too large. In reality this is probably smoothed out by the neglected molecular opacities of CN, CO, and H_2O (see Bell *et al.* 1976*b*), so that for the field giants the effect of the additional molecular opacities overpowers the effect of lower gas pressures and less important Rayleigh scattering. Thus distance from the mean field-giant line in a $(J-H, H-K)$-plot is not a simple function of metallicity, and no ordering of the globular clusters in terms of metallicity can be obtained from Figure 4.

c) *Empirical Bolometric Magnitudes*

Empirical bolometric corrections can be obtained from the observed broad-band photometric colors. The data of Table 3 were supplemented with R-magnitudes from Cathey (1974) and with transformed $R-I$ colors from Eggen (1972) when available. $K-L$ colors were extrapolated from Johnson's (1966*a*) mean colors of red giants, although the final values for the bolometric corrections are not sensitive to the choice of these colors. Small blanketing corrections to the K-magnitudes based on the observed CO indices were made as discussed in § III; these corrections were never larger than 0.04 mag. The U to L colors were transformed into fluxes via the absolute calibrations cited above and integrated to yield empirical bolometric corrections. These values are listed in Table 4. Bolometric magnitudes $M_{\rm bol}$ were then obtained from the corrected V-magnitudes (references in Table 2) and the assumed distance moduli of Table 1. The principal uncertainties in these values of $M_{\rm bol}$ are the values of $(m-M)_0$. The final values of $M_{\rm bol}$ are given in Table 4.

V. LOG L, $T_{\rm eff}$ DIAGRAM

a) *Comparison with Evolutionary Tracks*

We can now compare the observed colors and the empirically determined bolometric corrections with the model predictions (§ III and Fig. 4) to derive values of $T_{\rm eff}$. As discussed in § III, the model values of $T_{\rm eff}$ which are implied by the observed values of BC, $(V-K)_0$, and $(J-K)_0$ are practically independent

[5] Eggen and Sandage (1964) estimated that blanketing values of $\Delta(U-V) = +0.09$, $\Delta V = -0.02$ were appropriate for the M67 stars. If these values are applied to the data of Fig. 2, the M67 stars will depart systematically from the mean relationship for field giants. Any blanketing correction to the K-magnitude which is of the proper sign will only increase this discrepancy. Several possible explanations are: the mean $(U-V)_0$, $(V-K)_0$ line has systematic errors, the mean line is correct but it is inappropriate for the M67 giants, and the blanketing corrections are not valid. With the data presently available, we cannot choose between these alternatives.

TABLE 4

OBSERVED AND DERIVED PHYSICAL PARAMETERS

Star	BC	M_{bol}	T_{eff}	log g (M=1 $M_\odot$)	Star	BC	M_{bol}	T_{eff}	log g (M=1 $M_\odot$)
		M92					M13		
II-12	0.26	-0.32	5000	2.2	I-2	0.22	-0.38	4900	2.1
II-70	0.37	-1.99	4600	1.3	I-18	0.27	-0.74	4750	1.9
III-4	0.23	-0.75	5000	2.0	I-23	0.32	-1.53	4700	1.6
III-13	0.63	-3.25	4200	0.7	I-24	0.42	-1.78	4500	1.4
III-65	0.54	-2.83	4500	1.0	I-48	0.90	-3.28	4000	0.6
III-82	0.39	-1.71	4700	1.5	II-67	1.15	-3.45	3900	0.5
IV-2	0.35	-1.48	4700	1.6	II-76	0.53	-2.43	4350	1.1
IV-10	0.37	-1.59	4700	1.5	II-90	0.92	-3.11	4000	0.7
IV-114	0.33	-1.14	4750	1.7	III-18	0.55	-2.20	4350	1.2
VII-18	0.61	-3.14	4300	0.8	III-56	0.79	-3.07	4100	0.7
VIII-43	0.23	-0.25	4950	2.2	III-63	0.71	-2.94	4200	0.8
X-49	0.52	-3.00	4400	0.9	III-72	0.07	+0.64	5200	2.6
XI-19	0.49	-2.27	4500	1.3	III-73	0.63	-2.73	4300	0.9
XII-8	0.50	-2.38	4550	1.2	IV-25	0.93	-3.26	4000	0.6
		M67					M3		
84	0.33	+0.70	4750	2.5	I-21	0.74	-2.52	4200	1.0
94	0.00	+3.27	5800*	3.9	II-18	0.30	-1.04	4700	1.8
105	0.52	+0.22	4450	2.2	II-46†	0.91	-3.05	4000	0.7
108	0.67	-0.51	4300	1.8	III-28	0.80	-2.82	4100	0.8
115	0.00	+3.09	5800*	3.8	III-77	0.64	-2.17	4300	1.2
117	0.06	+2.99	5300	3.6	IV-25	0.52	-1.75	4400	1.4
141	0.34	+0.58	4750	2.4	193	0.26	-0.29	4750	2.1
151	0.31	+0.62	4750	2.4	216	0.41	-1.20	4600	1.7
164	0.34	+0.61	4750	2.4	1397	0.93	-3.13	4000	0.7
170	0.63	-0.50	4300	1.8	AA	0.91	-3.05	4000	0.7
193	0.26	+2.44	4950	3.2	BI	0.49	-1.21	4500	1.6
223	0.31	+0.71	4750	2.3					
224	0.32	+0.88	4750	2.6					
227	0.04	+3.27	5200	3.7					
231	0.20	+1.74	4900	3.0					
244	0.22	+1.00	5050	2.7					
I-17	0.24	+2.60	5000	3.3					
II-22	0.21	+3.16	5150	3.6					
III-34	0.33	+1.43	4750	2.8					
IV-20	0.40	+1.25	4700	2.7					
IV-68	0.10	+3.39	5200	3.7					
IV-77	0.19	+3.14	5050	3.6					
IV-81	0.01	+3.22	5500*	3.7					

*T_{eff} uncertain; star hotter than range of model grid.

†No *UBVRI* colors; parameters based on *JHK* only.

of metallicity and surface gravity for the range considered in Table 3. It is quite satisfying that the T_{eff}'s derived via Figure 1 from the observed (but not independent) values of BC, $(V - K)_0$, and $(J - K)_0$ are consistent to better than ± 100 K for all the stars. The mean of the three values so derived for each star is given in column (4) of Table 4. We note that values of T_{eff} derived from $(R - I)_0$ colors do not agree well with the values in Table 4. This is not surprising in view of the discussion in § III. Finally, with the values of L and T_{eff} we can derive values for the surface gravities of the stars if a mass is assumed, since $g \propto (MT_{eff}{}^4/L)$. Surface gravities so computed (based on a mass of 1 $M_\odot$ for ease in later calculations) are given in column (5) of Table 4.

We can now construct a log L versus T_{eff} diagram; this plot is shown in Figure 5. Superposed on Figure 5 are the evolutionary tracks calculated by Rood (1972) for metal-deficient red giants with $M = 0.8\ M_\odot$. The qualitative agreement of the empirically determined parameters with Rood's evolutionary tracks is good.[6] Note that the ordering of the globular clusters in T_{eff} of their red-giant branches for a given bolometric magnitude is M92, M13, M3. The sensitivity of Rood's evolutionary tracks to helium content Y is sufficiently small that no useful constraint on Y can be obtained. The upper end of Rood's tracks is the theoretical location of the helium flash, and the most luminous of the observed globular giants coincide well with the

[6] The recently published Yale tracks (Ciardullo and Demarque 1977) are systematically too cool by 0.03 in log T_{eff} (300 K in T_{eff}) compared to our empirical tracks as shown in Fig. 5.

predicted luminosity limit in all three globular clusters. Furthermore, the asymptotic giant branch cannot extend in luminosity beyond the theoretical location of the helium flash.

For solar helium abundance ($Y = 0.30$) and for a globular red-giant mass of 0.8 $M_\odot$, ages deduced from the time scales of Rood (1972) are 10×10^9 yr. However, these ages depend strongly on M and Y: For $M = 0.7\ M_\odot$, $Y = 0.3$, the ages are 14×10^9 yr; and for $M = 0.8\ M_\odot$, $Y = 0.2$, they are 17×10^9 yr. Although these ages are not inconsistent with Sandage's (1970) value of 11×10^9 yr, we regard the latter value (based on the $M_{\rm bol}$ at main-sequence turnoff) as more reliable because of the strong dependence of the evolutionary tracks for red giants on the details of the convection theory and the uncertainty in the mass of the globular red giants.

A closer look at Figure 5 reveals an interesting systematic departure of the M13 points from the Rood track 1 in the sense that the coolest stars are significantly cooler than the track, and the hotter stars are hotter than the track. The effect is not evident in the available data for M3 or M92. The sliding temperature scale in the theoretical models will not explain this difference in slope. The effect in M13 *could* be caused by a selection effect if too many asymptotic branch stars were included at higher temperatures. This is *not* important, however, since of the four hottest stars in M13 (all of which lie to the left of the Rood track 1) only the coolest one is an asymptotic branch star, as determined from Sandage's (1970) sequences in the (V, $B - V$)-diagram. It is concluded, therefore, that the effect is real. We anticipate that if it is confirmed by observations of more stars in M13, or perhaps found in other clusters, the effect could be important from a theoretical standpoint.

b) *Mass Determinations*

The theoretical $B - V$ colors of Figure 4 and Table 3 show a strong dependence on surface gravity for a given $T_{\rm eff}$ (presumably due primarily to the varying importance of Rayleigh scattering which can be larger than the H^- opacity at 5000 Å for the cooler giants), in agreement with the calculations of Böhm-Vitense and Szkody (1974). This dependence is greatest for the largest metal deficiency and for the coolest temperatures. Surface gravities for a number of cooler stars were thus derived from Figure 4 by using the values of $T_{\rm eff}$ in Table 4 and the observed values of $B - V$ in Table 2. The corrections to $B - V$ for CH absorption given in § III were applied here. The $M_{\rm bol}$ in Table 4 were then combined with the derived surface gravities and $T_{\rm eff}$ values to obtain masses. ($U - V$ can also be used in principal to obtain masses but only for very low-metallicity stars when the dependence of $U - V$ on Fe/H is minimal [see Table 3]. Because of problems with correctly calculating $U - V$ from the models, we have not attempted this check.) Mass determinations were made only for the stars with $T_{\rm eff} < 4600$ K, as we wish to examine only the giants at the top of the giant branch where mass loss is most likely to occur. The mean mass, the number of stars included in the average, and the metallicity assumed for the globular clusters are listed in Table 5. The quoted error in the mean mass arises mostly from systematic effects, namely, uncertainty in the assumed values of Fe/H and the uncertainty of our calculated $B - V$ colors as functions of $T_{\rm eff}$ and g. The last effect contributes most of the uncertainty, since the dispersion in calculated mass is small ($\pm 0.2\ M_\odot$) within each cluster. The mean masses of Table 5 are consistent with the canonical globular-cluster red-giant mass of 0.8 $M_\odot$ (e.g., Rood 1972). This method of obtaining masses, however, will not work for the M67 giants, because our predicted $B - V$ colors with the CH correction are 0.05 mag too blue to obtain a mass of 1.0 $M_\odot$ from the observed colors. Presumably this is because the CH correction estimated in § III is too small for solar metallicity stars. The errors on the CH correction for the globular cluster giants are too small to significantly affect the values of the mean mass given in Table 5.

If mass loss were occurring along the red-giant branch in a highly individualized manner (i.e., not depending solely on a star's mass, luminosity, Fe/H, age, and $T_{\rm eff}$), the points of Figure 5 would scatter in a band about the mean line. A variable random mass loss of 0.4 $M_\odot$ would cause scatter of about ± 0.1 in $\log L/L_\odot$ for each globular in Figure 5. (McClure and Twarog 1977 claim to have detected this effect in NGC 188.) More data could place limits on such a scatter. Spectroscopic surface gravities (which should be obtainable in the near future from high-dispersion spectra) can be compared to those calculated assuming a mass of 1 $M_\odot$ to obtain actual masses for globular cluster giants which in turn will directly determine if mass loss has occurred on an extensive basis.

VI. METALLICITIES OF M3 AND M13

The metallicities of these two globular clusters have been determined in various ways, but the results have not always been consistent. We try here to separate Fe/H from C/H by studying the relative CO indices. First, we consider previous metallicity determinations.

In Table 6 are listed several Fe/H and CNO/H determinations for M3, M13, and M92. The Morgan types are based on line strengths in integrated spectra of each globular cluster (Morgan 1959). Since the metallicity also affects the $T_{\rm eff}$ and $M_{\rm bol}$ of the giant branch in a cluster, the interpretation of the Morgan types is ambiguous. The second column gives the

TABLE 5

MEAN MASSES OF GIANTS AT THE TIP OF THE GIANT BRANCH

	Mass in $M_\odot$		Assumed Fe/H
M92	0.9(+0.9,−0.4)	(6 stars)	0.01
M3	0.5(+0.4,−0.2)	(7 stars)	0.03
M13	0.5(+0.4,−0.2)	(10 stars)	0.03

TABLE 6

METALLICITY DETERMINATIONS FOR M3, M13, AND M92

	Morgan Type	Spectra	Echelle	$\delta(U-B)$	DDO	ΔS	ΔCO	M_{bol} (RG)
M13	III	−1.4	−1.6	0.15	−1.6	−1.0	−0.11	−2.9
M3	II		−1.9	0.18	−1.5	−1.6	−0.07	−2.5
M92	I	−2.1	−2.4	0.26	−2.2	−2.2	−0.14	−3.3
Parameter	[Fe/H]	[Fe/H]	[Fe/H]		[Fe/H]	[Ca/H]	CO/H	metals/H + distance modulus
Source	1	2	3	4	7	5	6	6

NOTE.—[Fe/H] ≡ log Fe/H (with respect to the Sun).

SOURCE.—(1) Morgan 1959. (2) Helfer, Wallerstein, and Greenstein 1959; Wallerstein and Helfer 1966. (3) Cohen 1978. (4) Sandage 1970. (5) Butler 1975. (6) This paper. (7) Hesser, Hartwick, and McClure 1977.

spectroscopic results for an analysis of the brightest red giant in M13 and in M92 (Helfer, Wallerstein, and Greenstein 1959; Wallerstein and Helfer 1966). The third column lists the preliminary results of the analysis of three giants in M3, five in M13, and one in M92 from a larger sample of globular cluster giants for which KPNO 4 m, high-dispersion echelle spectra have been obtained (Cohen 1978). The uncertainties in the abundance determinations based on the echelle spectra are ± 0.3 in log Fe/H on an absolute scale, but are reduced to ± 0.15 when comparing globular cluster giants in different clusters on a relative scale (Cohen 1978). The values in the fourth column are from the ultraviolet color excess at $B - V = 1.0$ (Sandage 1970) which depends largely on Fe/H and only slightly on CNO/H. Butler's (1975) study of the spectra of RR Lyr stars at a resolution of 7 Å gives the values in the fifth column. The values of ΔCO (shift of mean CO index from the field-giant line; see § IV*a*) for M92, M13, and M3 are listed in the sixth column. The errors in these means are estimated to be ± 0.01 mag. The last column lists the M_{bol} of the red giants in each cluster at a T_{eff} of 4200 K (see Fig. 5); this ordering follows that of the CO index. Evolutionary tracks for red giants with nonsolar CNO/Fe are not presently available, so that it is impossible to say whether the abundance of the heavy metals or that of CNO is the dominant factor in determining these tracks. The change in opacities will not be the only effect of varying the CNO/Fe ratio, since the energy produced by the CN cycle reactions exceeds that produced by the *p-p* chain. Iben and Rood (1970) and Rood (1972) discuss the complexity of the situation. Until evolutionary tracks are calculated, we cannot use the luminosity ranking of the globulars to determine the relative metallicities of M3 and M13.

Because of the problem discussed earlier in Osborn's (1971) metallicity calibration of the DDO indices, we disregard Osborn's (1974) metallicities, T_{eff}, and log g for globular cluster stars. Hesser, Hartwick, and McClure (1977), who also observed in the DDO system, did not get satisfactory results with Osborn's (1971) calibration and reverted to a ranking of the globular clusters based on the DDO two-color diagrams. Their CN measurements as a function of the DDO temperature parameter are not helpful, however, because the M13 and the M3 giants they observed lie essentially on the M92 line, i.e., CN was not detected in any of the three globular clusters. Metallicities which they have derived, based on a line-blanketing index, are listed in Table 6; the formal scatter of the points implies an error of ± 0.1 in the logarithm of Fe/H. However, there are some systematic uncertainties in using the DDO system for such metal-poor stars. Hesser, Hartwick, and McClure (1977) have assumed that the DDO indices behave monotonically with Fe/H. This may not be correct because of the importance of Rayleigh scattering in these very metal-poor K giants. $U - B$ (and also 38 − 42 and 42 − 45 in the DDO system) are relatively insensitive to Fe/H until Fe/H is greater than 0.03 according to our model results. Also, clearly nonmonotonic behavior in $U - B$ as a function of Fe/H is shown in Figure 8 of Böhm-Vitense and Szkody (1974). Because of these problems we must assign a larger error of ± 0.3 in log Fe/H to their results.

From Table 6 it seems probable that the heavy-metal (i.e., Fe/H) ordering of these globulars is M92, M3, M13. The CO ordering is *not* the same but is M92, M13, M3. For stars with approximately normal C/O (i.e., C < 0), the CO abundance is controlled by the carbon abundance. Clegg (1975) has shown that in presumably unevolved, metal-poor dwarfs C/Fe is approximately solar while N/Fe is decreased below solar. No information is available on O/Fe in metal-poor dwarfs, but in metal-poor giants (Conti *et al.* 1967; Lambert, Sneden, and Ries 1974) O/Fe appears to be larger than solar. One might therefore expect that C < 0 in the globular giants, which is confirmed by the scarcity of CH stars in metal-poor globulars; therefore it is the carbon abundance which controls the CO formation. Sneden (1974) found that metal-poor field giants have a C/N ratio less than one-tenth the solar value, in qualitative agreement with the theoretical predictions for mixing in red giants (Iben 1964). Therefore, if mixing has occurred in all the giants in M13 but has not occurred in those of M3, the ΔCO values for these two clusters could be explained. We do not understand enough about mixing to say why two clusters as similar as M3 and M13 should have had such a completely different mixing history for the giant branch. Furthermore, in ω Cen where there is evidence for mixing (Bessell and Norris 1976; Dickens and Bell 1976), only some of the giants show CNO anomalies. An alternative explanation is that the gas out of which M3 condensed had a larger value of C/Fe (with C/O still less than 1) than the primordial gas for M13. It is easy to imagine adding light elements locally through a supernova explosion but impossible to imagine a scenario for removing heavy elements (which could make M13 the peculiar cluster); therefore, relative to M92 (and probably relative to the Sun) M3 would be the peculiar cluster. Until more observational data are available, we prefer not to speculate further on the cause of the ΔCO anomaly in M3 and M13.

VII. SUMMARY AND CONCLUSIONS

Observations of the broad-band *JHK* colors and CO and H_2O indices combined with previously published *UBV* photometry for red giants in M3, M13, M92, and M67 have been interpreted with the aid of parameters computed from a grid of model stellar atmospheres in the range $3800 \leq T_{\text{eff}} \leq 5250$ K, $0.5 \leq \log g \leq 3.0$, and 0.01 Fe/H $\leq$ 1.0. The following conclusions have emerged from this work:

1. $V - K$ and $J - K$ are the best photometric indices for determining T_{eff} in this temperature range because of their insensitivity to variations in surface gravity and metallicity. Reasonable care in observational techniques and reductions should lead to T_{eff} determinations that are accurate to ± 100 K.

2. A problem has been found in Osborn's (1971) calibration of the DDO photometric system with metallicity which leads to rejection of the T_{eff}, Fe/H, and log g values for globular cluster stars which have been subsequently derived from his calibration; this reservation does not apply to the work of Hesser, Hartwick, and McClure (1977) which is based on an empirical ranking of the clusters in a method analogous to that used for the CO indices.

3. Bolometric corrections have been derived from the broad-band photometry, and a (M_{bol}, T_{eff})-diagram for the globular-cluster red giants (and those in M67) has been constructed. Rood's (1972) evolutionary tracks are in good qualitative agreement with the locations of the cluster stars.

4. For low metallicities, Rayleigh scattering becomes important, as was first found by Böhm-Vitense and Szkody (1974). The $B - V$ color then depends on surface gravity (as well as on T_{eff} and Fe/H). With assumed values for Fe/H and our T_{eff}'s based on $V - K$, approximate values of g can be obtained. We can therefore derive mean masses for the red giants in the globular clusters by using empirical values of M_{bol} and T_{eff}. The deduced mean mass of the red giants is 0.6 ($+0.6$, -0.3) $M_\odot$. The accuracy of this result depends strongly on uncertainties in the calculated colors.

5. The ordering of CO index strength at a fixed $V - K$ of 3.0 is M92 (-0.01), M13 ($+0.02$), M3 ($+0.06$). Values of M_{bol} lead to the same ordering. Other spectroscopic and photometric data indicate that the Fe/H ordering of the globular clusters is M92, M3, M13—M92 being the most metal-poor. M3 may have a ratio of CNO/Fe which is peculiar compared to the Sun and/or M92; such an excess of CNO elements could be a result of mixing in the course of evolution of the red giants or of some inhomogeneity (e.g., a supernova or supermassive star) in the primordial galactic halo.

6. Using the theoretical horizontal branch tracks of Castellani and Tornambe (1977), we note that the difference in CO abundance between M3 and M13 is in the correct sense to explain the difference in the relative populations of the red and blue horizontal branches in these two clusters. This supports the identification of the second parameter affecting the color-magnitude diagrams of globular clusters as the abundance of the CNO elements, as was suggested by Hartwick and McClure (1972).

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Note added in proof.—Eggen (1977, private communication) has suggested that $E(B - V)$ for M67 is 0.03 mag instead of 0.06 mag used here. $V - K$, $U - V$, and $J - K$ then become redder by 0.08, 0.05, and 0.01 mag, respectively, for the M67 stars, while V and K become fainter by 0.09 and 0.01 mag, respectively, than the reddening-corrected values listed in Table 2*d*. The T_{eff} for the M67 stars is decreased by approximately 50 K from the values listed in Table 4. The problem discussed in n. 5 is eliminated with the change in reddening, while the 0.05 mag discrepancy in $B - V$ between the observed colors and those predicted for giants with a mass of 1 $M_\odot$ noted in § V*d* for the M67 stars remains unaffected. For the redder stars, M_{bol} will be 0.05 mag fainter, and for the bluer stars, M_{bol} will be about 0.08 mag fainter than the values listed in Table 4.

REFERENCES

Aaronson, M., Frogel, J. A., and Persson, S. E. 1978, *Ap. J.*, in press (Paper II).
Allen, C. W. 1963, *Astrophysical Quantities* (2d ed.; London: Athlone Press).
Baldwin, J. R., Frogel, J. A., and Persson, S. E. 1973, *Ap. J.*, **184**, 427.
Bell, R. A., Eriksson, K., Gustafsson, B., and Nordlund, A. 1976*a*, *Astr. Ap. Suppl.*, **23**, 37.
Bell, R. A., and Gustafsson, B. 1977, private communication.
Bell, R. A., Gustafsson, B., Nordh, H. L., and Olofsson, S. G. 1976*b*, *Astr. Ap.*, **46**, 391.
Bessell, M. S., and Norris, J. 1976, *Ap. J.*, **208**, 369.

Böhm-Vitense, E., and Szkody, P. 1974, *Ap. J.*, **193**, 607.
Butler, D. 1975, *Ap. J.*, **200**, 68.
Castellani, V., and Tornambe, A. 1977, *Astr. Ap.*, **61**, 427.
Catchpole, R. M., and Glass, I. S. 1974, *M.N.R.A.S.*, **169**, 61P.
Cathey, L. R. 1974, *A.J.*, **79**, 1370.
Ciardullo, R. B., and Demarque, P. 1977, *Trans. Astr. Obs. Yale Univ.*, Vol. **33**.
Clegg, R. E. S. 1975, Ph.D. thesis, University of Maryland.
Cohen, J. G. 1978, in preparation.
Conti, P. S., Greenstein, J. L., Spinrad, H., Wallerstein, G., and Vardya, M. S. 1967, *Ap. J.*, **148**, 105.
Dickens, R. J., and Bell, R. A. 1976, *Ap. J.*, **207**, 506.
Eggen, O. J. 1972, *Ap. J.*, **172**, 639.
Eggen, O. J., and Sandage, A. 1964, *Ap. J.*, **140**, 130.
Faber, S. M. 1973, *Ap. J.*, **179**, 731.
Frogel, J. A., Persson, S. E., Aaronson, M., and Matthews, K. 1978, *Ap. J.*, in press (Paper I).
Gingerich, O., and Kumar, S. S. 1964, *A.J.*, **69**, 139.
Glass, I. S., and Feast, M. W. 1973, *M.N.R.A.S.*, **163**, 245.
Golay, M. 1974, *Introduction to Astronomical Photometry* (Dordrecht: Reidel), p. 89.
Griffin, R. F. 1968, *A Photometric Atlas of the Spectrum of Arcturus* (Cambridge: Cambridge Philosophical Soc.).
Gustafsson, B., Bell, R. A. Eriksson, K., and Nordlund, A. 1975, *Astr. Ap.*, **42**, 407.
Hartoog, M., Persson, S. E., and Aaronson, M. 1977, *Pub. A.S.P.*, **89**, 660.
Hartwick, F. D. A., and McClure, R. D. 1972, *Ap. J.* (*Letters*), **176**, L57.
Helfer, H. L., Wallerstein, G., and Greenstein, J. L. 1959, *Ap. J.*, **129**, 700.
Hesser, J. E., Hartwick, F. D. A., and McClure, R. D. 1977, *Ap. J. Suppl.*, **33**, 471.
Iben, I. 1964, *Ap. J.*, **140**, 1631.
Iben, I., Jr., and Rood, R. T. 1970, *Ap. J.*, **159**, 605.
Johnson, H. L. 1966*a*, *Ann. Rev. Astr. Ap.*, **4**, 193.
———. 1966*b*, *Ap. J.*, **146**, 613.
Johnson, H. L., and Mitchell, R. I. 1962, *Comm. Lunar and Planetary Lab.*, No. 14.
Johnson, H. L., and Sandage, A. 1956, *Ap. J.*, **124**, 379.
Kadla, Z. I. 1968, *Astr. Zh.*, **45**, 541.
Kurucz, R. L. 1970, *SAO Spec. Rept.*, No. 309.
———. 1975, *Dudley Obs. Rept.*, No. 9, p. 271.
———. 1978*a*, in preparation.
———. 1978*b*, in preparation.
Kurucz, R. L., and Peytremann, E. 1975, *SAO Spec. Rept.*, No. 362.
Kurucz, R. L., Peytremann, E., and Avrett, E. H. 1974, *Blanketed Model Atmospheres for Early-Type Stars* (Washington: Government Printing Office).
Lambert, D. L., Sneden, C., and Ries, L. M. 1974, *Ap. J.*, **188**, 97.
McClure, R. D., and Twarog, B. A. 1977, *Ap. J.*, **214**, 111.
Moore, C. E., Minnaert, M. G. J., and Houtgast, J. 1966, *NBS Monog.*, No. 61.
Morgan, W. W. 1959, *A.J.*, **64**, 432.
Mould, J. R., and Hyland, A. R. 1976, *Ap. J.*, **208**, 399.
Norris, J., and Zinn, R. 1977, *Ap. J.*, **215**, 74.
Osborn, W. 1971, Ph.D. thesis, Yale University.
———. 1973, *Ap. J.*, **186**, 725.
———. 1974, *M.N.R.A.S.*, **168**, 291.
Persson, S. E., Frogel, J. A., and Aaronson, M. 1978, in preparation (Paper III).
Peytremann, E. 1975, *Astr. Ap.*, **39**, 313.
Rood, R. T. 1972, *Ap. J.*, **177**, 681.
Sandage, A. 1953, *A.J.*, **58**, 61.
———. 1970, *Ap. J.*, **162**, 841.
———. 1972, *Ap. J.*, **176**, 21.
Sandage, A., and Walker, M. F. 1966, *Ap. J.*, **143**, 313.
Schlesinger, B. M. 1969, *Ap. J.*, **157**, 533.
Sneden, C. 1974, *Ap. J.*, **189**, 493.
Strom, S. E., Strom, K. M., Goad, J. W., Vrba, F. J., and Rice, W. 1976, *Ap. J.*, **204**, 684.
Wallerstein, G., and Helfer, H. L. 1966, *A.J.*, **71**, 350.
Wilson, W. J., Schwartz, P. R., Neugebauer, G., Harvey, P. M., and Becklin, E. E. 1972, *Ap. J.*, **177**, 523.
Zinn, R. 1973, *Ap. J.*, **182**, 183.

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