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CONSTRAINTS ON THE PROPERTIES OF CIRCUMSTELLAR SHELLS FROM OBSERVATIONS OF THERMAL CO AND SiO MILLIMETER LINE EMISSION

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ABSTRACT

An attempt to detect CO and SiO microwave emission from Betelgeuse and long-period variables (LPVs) is described. The absence of SiO 86 GHz emission from the Betelgeuse circumstellar shell is shown to require that either all Si is associated into silicate dust grains or SiO molecule formation is inhibited. The former explanation is consistent with published estimates of the column density of silicate grains. The latter explanation is supported by the absence of emission in the CO 115 GHz line. Circumstellar absorption lines are not present in the TiO $\gamma(0, 0)$ band near 7100 Å. TiO, like SiO and CO, appears to be underabundant in the shell.

The new detections of circumstellar microwave emission include an observation of the CO 115 GHz line from R Cas (the first LPV for which both SiO 86 GHz emission and CO 115 GHz emission are seen) and of the SiO 86 GHz line from the S star χ Cyg. The absence of the isotopic line ^{30}SiO at 84 GHz shows that the shells around R Cas, R Leo, and χ Cyg have an optical depth in the 86 GHz line of $\tau < 10$. This suggests that the SiO molecules contain a minor fraction of the total Si and that a substantial fraction of the Si is in the silicate grains. The shell around R Leo has been detected through the K I fluorescent emission. This confirms that the shells are large.

Subject headings: radio sources: lines — stars: abundances — stars: circumstellar shells — stars: long-period variables

I. INTRODUCTION

Red supergiants are immersed in extensive, expanding shells of gas and dust. Traditional signatures of these circumstellar shells include both Doppler-shifted absorption cores in resonance and low-excitation lines of neutral and singly ionized atoms, and infrared emission by dust grains. A new diagnostic is now available: molecular emission lines at millimeter and centimeter wavelengths. This paper discusses observations of thermal CO and SiO emission from the circumstellar shells of long-period variables (LPVs) and the M2 supergiant Betelgeuse.

Thermal emission from LPVs in the SiO 86 GHz $J = 2 \rightarrow 1$ transition in the ground vibrational level was discovered by Buhl *et al.* (1975). In contrast to the narrow SiO maser lines from excited vibrational levels, the 86 GHz line was found to be broad, with a width consistent with the expansion velocity (~ 10 km s^{-1}) of the shell. Reid and Dickinson (1976) analyzed the Buhl *et al.* observations of R Cas, W Hya, R Leo, and VY CMa to derive the stellar radial velocity from the centroid of the 86 GHz line with the assumption of a spherically symmetric shell. This important datum is difficult to extract from the absorption and emission lines in the optical spectrum. Zuckerman *et al.* (1977) conducted a search for CO 115 GHz $J = 1 \rightarrow 0$ emission from circumstellar shells. A broad CO line was found in seven carbon-rich and two oxygen-rich stars. The former included the LPVs R Scl, V Hya, and V Cyg. (Subsequent to the observations reported here, Lo and Bechis 1977 completed a search for CO emission in evolved stars and reported positive detections in seven stars including o Cet and χ Cyg.)

The observations described here were intended to answer two questions.

i) Can thermal SiO or CO emission be detected from the circumstellar shell around the bright M2 supergiant Betelgeuse?

ii) Do the shells around oxygen-rich LPVs provide a detectable CO 115 GHz line to complement the SiO 86 GHz line?

II. OBSERVATIONS

The observations were obtained with the 11 m telescope of the National Radio Astronomy Observatory.[1] A cooled 80–120 GHz mixer-receiver was used with a single feed at both the CO (115.2712 GHz) and SiO (86.8469 GHz) frequencies. Filter banks of 256 channels, with channels of width 256 kHz and 500 kHz each, were simultaneously fed by the single feed. Position switching was done by alternately observing for 30 s each the stars and reference positions located about 2′ away. Interstellar CO appears in some spectra; the interstellar nature of these narrow lines

[1] The National Radio Astronomy Observatory is operated by Associated Universities, Inc., under contract with the National Science Foundation.

TABLE 1

CO AND SiO OBSERVATIONS

Star	Line	T_A^* (K)	v_{LSR} (km s^{-1})	Δv_{base} (km s^{-1})
α Ori........	CO	< 0.10	...	
	^{28}SiO	< 0.06	...	
R Cas........	CO	0.12 ± 0.06	$+24 \pm 2$	21 ± 4
	^{28}SiO	0.17 ± 0.03	$+22 \pm 2$	26 ± 3
	^{30}SiO	< 0.06	...	
R Leo........	CO	< 0.10	...	
	^{28}SiO	0.16 ± 0.06	0 ± 2	16 ± 5
	^{30}SiO	< 0.10	...	
χ Cyg........	^{28}SiO	0.14 ± 0.04	$+9 \pm 2$	14 ± 4
	^{30}SiO	< 0.06	...	
S Per........	^{28}SiO	< 0.15	...	
o Cet........	^{28}SiO	< 0.010	...	

was confirmed by short observations of positions close to but removed from the stars.

Table 1 lists the results of the observations. The intensity is given as T_A^*, where this quantity is defined and calibrated according to Ulich and Haas (1976). In Table 1, v_{LSR} is the velocity of the line with respect to the local standard of rest and Δv is the full width at half-maximum (FWHM) with no correction for the broadening produced by the filters. Figure 1 shows the CO profile for R Cas. The profile shown was obtained with the 500 kHz filters and has not been smoothed in any way.

The new detections of circumstellar emission include the SiO 86 GHz line from the S star χ Cyg and the CO $J = 1$–0 115 GHz line from R Cas. Thermal SiO emission from R Cas and R Leo was first detected by Buhl *et al.* (1975). Our confirming observations of these stars give similar profiles; the only discrepancy occurs for R Leo, where we obtain $\Delta v = 16 \pm 5$ km s^{-1} (Table 1) and the earlier profile gives $\Delta v = 8 \pm 1$ km s^{-1} (Reid and Dickinson 1976).

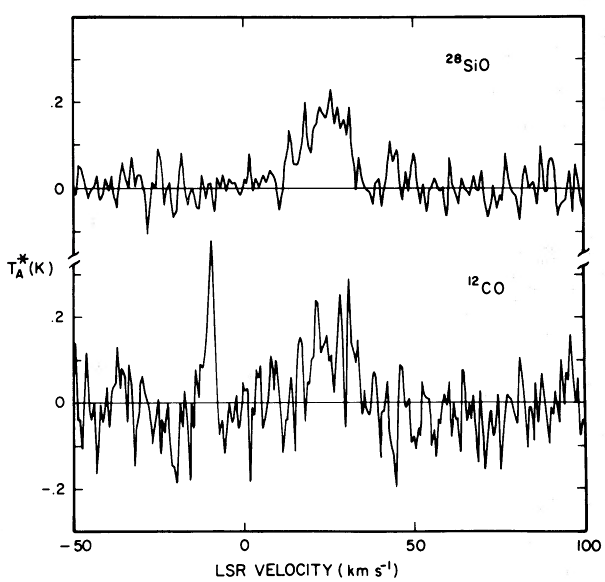

FIG. 1.—The SiO 86 GHz and CO 115 GHz line profiles for R Cas. The narrow line in the lower spectrum is an interstellar CO line from a diffuse cloud.

III. THE BETELGEUSE SHELL

a) The Predicted SiO *Antenna Temperature*

The absence of CO and SiO line emission from the Betelgeuse shell is interpreted using a recent model for the circumstellar gas shell (Bernat 1976, 1977). The radial expansion is assumed to begin at a radius $r_i \approx 50\, r_*$ (r_* is the stellar radius) where the hydrogen density n(H) $\approx 2 \times 10^6$ cm^{-3} and the electron density $n_e \approx 700$ cm^{-3}. The velocity of expansion is assumed to be constant above r_i, and the gas density to decrease as the square of the radius; Bernat *et al.* (1978) provide some observational evidence that this density decrease extends to a large distance from the star ($r \approx 1200\, r_*$). The total hydrogen column density N(H) $\approx 1.3 \times 10^{22}$ cm^{-2} for the line of sight to the star. This estimate is similar to earlier estimates; e.g., Weymann (1962) obtained N(H) $\approx 2 \times 10^{22}$ cm^{-2}. The adopted model is based upon an analysis of the distinctive, sharp, blueshifted lines produced in the cores of photospheric resonance and low-excitation lines by the smoothly flowing expansion above r_i.

The probable cause of the expansion is radiation pressure on the dust grains. Gas-grain collisions couple the gas to the expansion. Infrared interferometry (Sutton *et al.* 1977) indicates that the majority of the grains responsible for the 11 μm excess are at distances $r > 12\, r_*$ from the star. This observational limit is consistent with the size adopted for the inner boundary to the expanding shell. Gas between the star and inner boundary of the expanding shell is apparently turbulent without a net outward motion (Bernat and Lambert 1976*a*), or it is ionized so that the abundant metals are in large part doubly or more highly ionized. In either case, this gas is undetectable in the photospheric resonance line cores. Line emission by molecules in this transition zone is ignored in the following discussion.

The column density of CO and SiO in the shell is calculated on the assumption that the association of C into CO and of Si into SiO is complete throughout the shell: $N(\mathrm{SiO}) \approx 5 \times 10^{17}\ \mathrm{cm}^{-2}$ and $N(\mathrm{CO}) \approx 2 \times 10^{18}\ \mathrm{cm}^{-2}$ follow for an adopted solar Si abundance and a C abundance depleted to one-quarter of the solar value.

The antenna temperature ($T_A{}^*$) is calculated on the assumption of a uniform expansion velocity (v_{exp}), a uniform excitation temperature (T_{exc}) for SiO throughout the shell, and a SiO density with an inverse-square decrease (i.e., r^{-2}) out to a shell radius R. A treatment given by Kuiper *et al.* (1976) is adapted to the present problem.

If the shell is optically thick to 86 GHz radiation, the line profile is

$$T_A{}^*(v) = T_{\mathrm{exc}}\left[\!\left[1 - \exp\left\{-4 \ln 2\left[1 - \left(\frac{v}{v_{\mathrm{exp}}}\right)^2\right]\frac{R^2}{B^2}\right\}\right]\!\right],$$

where B is the FWHM of the antenna ($B = 78''$ is adopted) and v is the radial velocity. The profile is a maximum at the line center, with $T_A{}^*(0) \approx T_{\mathrm{exc}}$ for $R/B \gtrsim 1$, $T_A{}^*(0) \approx T_{\mathrm{exc}}(R/B)$ for $0.4 \lesssim (R/B)$, and $T_A{}^* \approx 2.7\, T_{\mathrm{exc}}\, (R/B)^2$ for $R/B \lesssim 0.4$.

The line profile in the optically thin limit is

$$T_A{}^*(v) = T_{\mathrm{exc}} \frac{\Delta v}{v_{\mathrm{exp}}} \tau_0 \frac{R^2 r_i}{B^2(R - r_i)} 16 \ln 2 \times F\left\{2(\ln 2)^{1/2}\left(\frac{R}{B}\right)[1 - (v/v_{\mathrm{exp}})^2]^{1/2}\right\},$$

where

$$F = y^{-1}\int_0^y \exp(-x^2)dx$$

and τ_0 is the optical depth along the radius vector from R to r_i with a constant velocity of expansion v_{exp} and a small-scale turbulence of Δv. The line profile is peaked at $v = \pm v_{\mathrm{exp}}$. $T_A{}^*(v)$ is expressed in terms of τ_0, because τ_0 is easily related to quantities in Bernat's model. [In considering the dependence of $T_A{}^* \approx r_i$, one should note that the product $(\tau_0 r_i)$ is a constant.]

Limiting forms for $T_A{}^*$ are

$$T_A{}^*(0) \approx 10\tau_0 \frac{r_i R^2}{B^2(R - r_i)} T_{\mathrm{exc}} \quad \text{for } R/B \lesssim 0.5 \text{ and } R > r_i$$

$$\approx 7\tau_0 \frac{r_i}{B} T_{\mathrm{exc}} \quad \text{for } R/B \gtrsim 1 \text{ and } R > r_i\,.$$

The line-center optical depth for the SiO transition is calculated from standard expressions, e.g.,

$$\tau_0(J'') = 1.6 \times 10^{-7} \frac{A_{J'J''} b_{J''}}{B} \times \left(\frac{2J' + 1}{J'^2}\right) \frac{N(\mathrm{SiO})}{\Delta v T_{\mathrm{exc}}{}^2},$$

where N(Si) is the total column density of the molecule, $A_{J'J''}$ is the Einstein A-value for the $J' \rightarrow J''$ transition, B is the rotational constant (in cm^{-1}), Δv is the width (FWHM in $\mathrm{km\ s}^{-1}$) of the Gaussian absorption coefficient, $b_{J''} = \exp[-hcBJ''(J'' + 1)/kT_{\mathrm{exc}}]$, and T_{exc} is the excitation temperature. A similar expression gives τ_0 for the CO transition.

The SiO 86 GHz observation ($T_A{}^* < 0.06$ K) is discussed first. An immediate comparison with a zeroth-order predicted $T_A{}^*$ highlights an interesting discrepancy. The predicted radial optical depth is

$$\tau_0 \approx 30\left(\frac{N}{5 \times 10^{17}}\right)\left(\frac{100}{T_{\mathrm{exc}}}\right)^2,$$

where $\Delta v \approx 10\ \mathrm{km\ s}^{-1}$ is obtained from observations of the shell absorption lines (Bernat 1977) and of the K I fluorescent emission (Bernat and Lambert 1976*b*) and $b_2 \approx 0.8$ is adopted. The zeroth-order prediction is $\tau_0 \approx 30$. Bernat estimates the shell inner boundary to be $r_i \approx 50\, r_*$ or $r_i \approx 2''.5$. The surface $\tau \approx 1$ has an angular size $r(1) \approx \tau_0 r_i \approx 75'' \approx B$ for the zeroth-order prediction $\tau_0 \approx 30$. K I fluorescent emission has been detected to 30″ from the star, so that this angular size for $r(1)$ appears reasonable. This leads to the prediction $T_A{}^* \approx T_{\mathrm{exc}}$.

The prediction is not confirmed by the observational limit of $T_A{}^* < 0.06$ K, if the argument (see below) that $T_{\mathrm{exc}} \approx 100$ K over a large part of the shell is accepted. The assumption that the optically thick shell is small, $r(1)/B < 0.5$, requires $r(1) \lesssim 1''$ for $T_{\mathrm{exc}} \approx 100$ K. This is in conflict with the observed size of the shell. These inconsistencies demand that the shell be optically thin.

The assumption of a large optically thin shell ($R/B \gtrsim 1$) yields $\tau_0 T_{\mathrm{exc}} \lesssim 0.3$, when the limit $T_A{}^* < 0.06$ K is applied to the line profile at $v = v_{\mathrm{exp}}$ and $\Delta v/v_{\mathrm{exp}} \approx 1$. If $T_{\mathrm{exc}} \approx 100$ K, then $\tau_0 \approx 0.003$. A small shell limit, $R = 2r_i$, gives $\tau_0 T_{\mathrm{exc}} \lesssim 6$, or $\tau_0 \lesssim 0.06$. Both limits conflict with the zeroth-approximation prediction $\tau_0 \approx 30$. Unless T_{exc} is very low, the observations appear to demand a reduction in the SiO column density by a factor of at least 300 below the value obtained by assuming all Si is associated into SiO molecules.

Thanks to the smaller permanent electric dipole moment of CO, the zeroth-order approximation gives a smaller optical depth $\tau_0 \approx 0.4$. The observed limit for $T_A{}^*$ translates to $\tau_0 T_{\mathrm{exc}} \lesssim 0.5$ (large shell) and $\tau_0 T_{\mathrm{exc}} \lesssim 10$ (small shell), or $\tau_0 < 0.005$ and $\tau_0 < 0.1$, respectively. These values suggest that CO is also depleted relative to the zeroth-order approximation.

b) *The Excitation Temperature*

The excitation temperature of the lowest levels of the SiO rotational ladder depends upon the interaction of the molecule with the neighboring atoms, electrons, and dust grains as well as upon the infrared stellar radiation. An attempt is made to show that $T_{\mathrm{exc}} \approx 100$ K is applicable to SiO molecules in the inner part of the circumstellar shell.

Dilute infrared radiation from the stellar photosphere permeates the shell. The $X\,^1\Sigma^+$ ground state of SiO has its fundamental vibration-rotation transition near 8 μm. Transitions $(v'', J'') \rightarrow (v' + 1, J'' \pm 1)$ occur through the absorption of 8 μm radiation. The molecules in the shell are cold, so that all are most probably in the ground $(v'' = 0)$ vibrational state. Reemission occurs within about 0.25 s^{-1}, and the molecule is returned to the original rotational level J'' or to $J'' \pm 2$. The rotational ladders of odd and even J are not coupled by the 8 μm absorption-reemission cycles (see Morris and Alcock 1977). If the shell is optically thin to the 8 μm radiation and to the pure rotation or microwave transitions, the excitation temperature is given by

$$T_{\mathrm{exc}} = \frac{h\nu}{K} \frac{1}{\ln [(4r^2/r_*^2)(h\nu/kT_*)]},$$

where T_* is the stellar radiation temperature appropriate for the 8 μm lines. Lambert and Snell (1975) argued that the 8 μm spectrum was not contaminated by the silicate grain emission and that the 8 μm radiation was a composite of photospheric and chromospheric components. Their analysis yields $T_* \approx 4000$ K, when an approximate accounting is taken of the flux depression resulting from the photospheric SiO lines. With $T_* \approx 4000$ K, $T_{\mathrm{exc}} \approx 200$ K for $r_i \approx 50\, r_*$. T_{exc} decreases slowly outward.

Collisions may be a competing excitation mechanism near r_i, but the steep density decrease reduces their effectiveness away from r_i. Rates for collisional de-excitation are compared with the Einstein A-value for the $J = 2$ state $(A_{21} = 2.9 \times 10^{-5}\ s^{-1})$. A cross section $\sigma \approx 10^{-15}\ cm^2$ (probably an overestimate) provides a de-excitation rate via H + SiO collisions R(H, SiO) $\approx 20 A_{21}$ at r_i and an approximate r^{-2} dependence above r_i. The corresponding rate for electrons is obtained from the theoretical rate constant (Dickinson *et al.* 1977) and from $n_e \approx 700\ cm^{-3}$ at r_i: $R(e$, SiO) $\approx 100\, A_{21}$. An approximate calculation of the net rate out of $J = 2$ from the 8 μm radiative cycles is R(8, SiO) $\approx 15\, A_{21}$. Taken at face value, the rates suggest $T_{\mathrm{exc}} \approx T_e$ near r_i, where T_e is the kinetic temperature of the electrons. However, the electron density is uncertain. If $n_e \lesssim 700\ cm^{-3}$, the 8 μm radiative cycles dominate and $T_{\mathrm{exc}} \approx 200$ K. If $n_e \gg 700\ cm^{-3}$, $T_{\mathrm{exc}} \approx T_e$ over the denser inner part of the shell. An examination of the relative rates for elastic and inelastic collisions would probably show that all gas particles achieve the same kinetic temperature.

Unfortunately, this temperature is unobtainable from observations. Weymann (1962) has derived $T_e \approx 600$ K from the relative population of low-lying levels of Fe I and Ti II. However, Bernat (1976) argues that such levels are radiatively controlled and, therefore, that their populations provide no information on T_e. A theoretical study of the energy budget of the gas is unlikely to yield a definitive estimate of T_e.

In dense gas and dust mixtures, the gas and dust equilibrate; i.e., $T_e \approx T_d$, where T_d denotes the dust grain temperature. If the model silicate grains chosen by Dyck and Simon (1975) are adopted, a grain experiences a H atom collision every 1000 years when in the vicinity of r_i. This assumes that the grain is stationary with respect to the H atoms moving at their thermal velocity for $T_e \approx 100$ K. The shell gas expands outward at 10 km s^{-1}. Between collisions the dust grain moves about 10 r_i and experiences a temperature (T roughly proportional to $r^{-1/2}$) decrease by a factor of 3. These figures suggest that the collision frequency is too low to maintain equilibrium; i.e., T_d is not approximately equal to T_e.

In summary, an excitation temperature $T_{\mathrm{exc}} \approx 100$ K appears to be a plausible estimate for SiO throughout a major portion of the shell. A substantially lower T_{exc} demands that the electron (and H) densities be higher than those predicted by Bernat's model and that the electrons maintain a low kinetic temperature. This combination can probably be excluded, because higher densities enhance the probability of equilibration with the dust grains which are heated through absorption of infrared radiation to $T_d \approx 100$ K across the major part of the shell (Dyck and Simon 1975).

c) Possible Explanations for the SiO *Problem*

Predicted and observed antenna temperatures for the SiO 86 GHz line can be reconciled by a factor of 300 reduction in the predicted SiO column density. Two possibilities are discussed: (i) Si is depleted by silicate grain formation, and (ii) formation of SiO molecules is suppressed or inhibited.

Dust grains offer a simple and direct explanation for the SiO deficiency. Dyck and Simon (1975) estimate $m_d \approx 10^{-4}\ g\ cm^{-2}$ for the mass column density of dust in the circumstellar shell. The grains are almost certainly a type of silicate. The mass fraction of Si in the grains is assumed to be $f \approx 0.15$; enstatite ($MgSiO_3$) provides $f \approx 0.3$, and fayalite [$(FeMg)_2SiO_4$] provides $f \approx 0.1$. Then the dust grains represent a Si column density N(Si) $\approx 3 \times 10^{17}\ cm^{-2}$; the uncertainty in N(Si) is about a factor of 2. N(Si) is close to the value of $5 \times 10^{17}\ cm^{-2}$ for the predicted SiO column density in the gas. Since the dust grains are driven by radiation pressure through the gas, the Si content of the grains is a true measure of the depletion in the gas. Clearly, the silicate grains can in principle account for the absence of the 86 GHz SiO line. A comparison of the predicted $\tau_0 \approx 30$ and the observed limit $\tau_0 \lesssim 0.06$ suggests that the grains remove more than 99% of the Si in the shell. This may be a surprising and demanding requirement. If grains are the sole agent responsible for the absence of SiO, this requirement represents a constraint on their composition. Fayalite grains, $(FeMg)_2SiO_4$, would leave about 50% of the Si in the gas phase, because the total abundances of Fe, Mg, and Si are approximately equal. Without another reason for the absence of SiO in the gas, the grains cannot be fayalite. Cosmic abundance tabulations (e.g., Cameron 1973) show that the Mg abundance exceeds the Si abundance by 5%. If these relative abundances are applicable to Betelgeuse, enstatite grains, $MgSiO_3$, should satisfy the requirement. Silica grains, SiO_2, would achieve the desired efficiency, because the oxygen-to-silicon abundance ratio probably exceeds 2.

Although depletion by dust grain formation is obviously important, the complete absence of SiO from the circumstellar gas may also be attributable to the inhibition of SiO formation in the shell. SiO is present in the photosphere in about the predicted abundance (Beer, Lambert, and Sneden 1974), but the gas must pass through a warm chromosphere ($T \leqslant 5000$ K, according to Bernat 1977) which, if equilibrium were achieved, would dissociate the SiO. However, the chromosphere may not cover the entire stellar surface. Above the chromosphere, the temperature must drop in order to permit dust grain formation. One conjectures that molecule formation may also occur. For the chromospheric temperature adopted by Bernat, the radiation field within the shell in the photodissociation (or photoionization) continuum of SiO (the dissociation energy $D_0{}^0 \approx 8.3$ eV) is dominated by the interstellar radiation field. The gas expanding at 10 km s^{-1} takes about 1000 years to move through a distance equivalent to 80% of the predicted SiO column density above r_i. Although lifetime calculations for SiO in the interstellar radiation field are unavailable, an appreciable amount of SiO should survive in spite of photodissociation and photoionization. Perhaps other processes, for example, ion-molecule reactions, remove SiO. Clearly, it would be of interest to examine the chemistry of Si in the circumstellar shell.

Silicate grains probably do not suffice to remove sufficient oxygen to cut the CO column density by the desired factor. The explanation may be that other types of grains (carbonates?) remove CO. Alternatively, CO formation is inhibited.

Additional observational and theoretical evidence will be needed before the relative contributions of the two proposed explanations can be assessed. In particular, attempts should be made to detect other molecules. High-resolution observations of the CO and SiO vibration-rotation bands may reveal circumstellar absorption/emission cores; the abundance and oscillator strengths favor CO over SiO. Electronic transitions constitute an attractive probe, because their oscillator strengths are usually much larger than those of the vibration-rotation transitions.

Unfortunately, the electronic structure of SiO and CO places electronic transitions from their ground $^1\Sigma^+$ states in the currently inaccessible ultraviolet. TiO does have electronic transitions accessible from the ground. A calculation based on the assumption that all Ti is associated into TiO predicts line-center optical depths $\tau_0 \approx 20$ for lines in the γ-system (0, 0) band near 7100 Å. The electronic oscillator strength is taken from Krupp, Collins, and Johnson (1977).

A high-resolution spectrum of Betelgeuse obtained at the McDonald Observatory fails to show circumstellar blueshifted TiO lines. A conservative estimate $\tau_0 \leqslant 0.2$ corresponds to an upper limit for the TiO column density about 100 times smaller than the prediction.

Titanium is present in the shell; both Ti I and Ti II absorption lines are observed. Bernat (1977) finds that three Ti I lines give a column density which is within a factor of 2 of the predicted value based upon a shell model which is matched to the absorption lines of other elements. Titanium is predominantly ionized. Ti II lines provide a column density a factor of 16 below the predicted value, but Bernat (1976) cautions that better spectra are needed before this underabundance is given great weight.

The nondetection of TiO gives $N(\text{TiO}) \leq 10^{13}$ cm^{-2}, a value corresponding to a TiO/Ti ratio from less than $\sim$0.1 to less than $\sim$0.01, where the latter limit is preferred because it is based on the more reliable Ti I lines. These figures indicate that only a small fraction of Ti atoms in the shell are associated into TiO. A plausible inference is that the SiO–Si relationship may be similar. Quantitative comparison is difficult. SiO formation may be favored by the higher dissociation energy ($D_0{}^0 = 8.3$ and 6.8 eV for SiO and TiO, respectively) and the higher ionization potential of the neutral atoms ($I = 8.2$ and 6.8 eV for Si and Ti, respectively). If the Ti abundance given by the Ti I lines is correct, the grains cannot contain a large fraction of the available Ti.

Analyses of the shell absorption/emission lines seen in the cores of resonance and low-excitation lines often assume that the relative abundances of the elements in the circumstellar gas are normal (i.e., solar). If Si is depleted through grain formation, depletions of other elements should occur either as atoms are incorporated into grains or as they adhere to the surfaces of grains. Inspection of Bernat's (1977) abundances for eight elements in the Betelgeuse shell shows that Ca is significantly underabundant (a factor of 30). A large Ca depletion is a common characteristic of interstellar clouds. However, the small depletions of the remaining elements may show little correlation with the depletion factors derived by Morton (1974) for an H I region near ξ Oph; in particular, Al has a normal relative abundance, but Al and Ca are underabundant by similar factors in interstellar clouds. The anomalous Ca abundance in the Betelgeuse shell and other shells analyzed by Bernat (1977) could indicate an underestimate of the photoionization rate; Ca is the only element examined by Bernat for which the dominant species, the single charged ion, has an ionization potential less than that of hydrogen. The shell is most probably highly opaque to radiation below the Lyman limit, so that the other ions are not photoionized. In order to provide a good test of the dust grain explanation of the SiO depletion, ultraviolet high-resolution spectra should be obtained of the appropriate resonance lines of Si, Mg, Fe, and other possible major constituents of the grains.

IV. THE LONG-PERIOD VARIABLES

Infrared observations of the circumstellar dust emission and optical observations of the gas in the expanding shell indicate that a LPV is immersed in a more substantial shell than that surrounding the M supergiant Betelgeuse. Detailed analyses of the shell absorption lines have not been reported. However, physical conditions should be similar so that a major fraction of the Si should be locked up in silicate grains.

The observed antenna temperatures for the SiO 86 GHz emission from R Cas, R Leo, and χ Cyg are similar. These T_A^* depend upon the circumstellar shell (SiO density, excitation temperature, etc.) and, if the shell is unresolved by the antenna, upon the distance. Distance estimates (Barnes 1977) place R Cas, R Leo, and χ Cyg at 300, 270, and 400 pc, respectively. The observed T_A^* show that the circumstellar shell SiO emission from three LPVs is quite similar; the range is less than a factor of 2. Mira (o Cet) at a distance of 90 pc is an obvious exception; $T_A^* \approx 1$ K is predicted from the R Cas and R Leo observations and from the inverse-square law, but SiO emission is not detected or $T_A^* < 0.1$ K. One explanation is that the hot companion to Mira provides sufficient ultraviolet radiation to dissociate most of the SiO molecules. This results in undetectable thermal emission but still allows for strong maser lines, because the strength of the maser lines depends primarily on the pumping mechanism rather than the column density.

Approximate limits to the optical depth of the shell in the 86 GHz ^{28}SiO line can be obtained from the nondetection of the isotopic ^{30}SiO $J = 2 \rightarrow 1$ line at 84.7 GHz. Reid and Dickinson (1976) obtained a satisfactory fit to the 86 GHz line profile on the assumption that the profile was parabolic, the shape expected for an optically thick shell. However, if R/B is small, an optically thin shell gives a flat-topped profile (Morris 1975). The observed profiles are sufficiently noisy that they cannot yet be used to discriminate between the optically thick and thin limiting cases. If the isotopic abundance ratio ^{28}Si/^{30}Si = 30 (the terrestrial value), then $\tau(86\ \mathrm{GHz}) \approx 30\ \tau(84\ \mathrm{GHz})$. Inspection of slightly smoothed profiles obtained with both the 500 kHz and 256 kHz filters gives T_A^* (84 GHz) $\leqslant T_A^*$ (86 GHz)/5. This translates to $\tau_0 \leqslant 10$ for the ^{28}SiO line.

The detection of CO emission from R Cas is a new result. The ratio T_A^* (SiO)/T_A^* (CO) ≈ 2, if typical of LPVs, shows that searches for molecular line emission from circumstellar shells should select the SiO line. If all or similar fractions of C and Si in a shell are associated into CO or SiO and if the excitation temperatures are similar, the optical depths in the CO $J = 1 \rightarrow 0$ and SiO $= 2 \rightarrow 1$ lines should satisfy the relation

$$\frac{\tau(\mathrm{CO})}{\tau(\mathrm{SiO})} \approx \frac{1}{600} \frac{N(\mathrm{C})}{N(\mathrm{Si})} \, .$$

If the relative abundance is approximately solar, i.e., $N(\mathrm{C})/N(\mathrm{Si}) \approx 10$, then $\tau(\mathrm{SiO}) \approx 60\tau(\mathrm{CO})$. Since the ^{30}SiO line for which $\tau(^{28}\mathrm{SiO}) \approx 30\tau(^{30}\mathrm{SiO})$ was not found, the detection of a CO line requires either some depletion of SiO relative to CO or a lower excitation temperature for SiO. The effect is probably larger than indicated here, because the atmospheres of LPVs have been contaminated with CNO–cycled material (i.e., ^{12}C/^{13}C ≈ 7, according to Hinkle 1978) which will be carbon deficient. Morris and Alcock (1977) argue that the SiO molecules are excited by 8 μm radiation. They fit the observed (Buhl *et al.* 1975) SiO 86 GHz profiles to their predicted profiles for an expanding shell. Their mass-loss rates are about 1000 less than the rates estimated from previous observations (dust + gas). They point out that the silicate grains are probably the source of the "missing" silicon.

A plausible interpretation requires that the observed SiO emission come from a large shell; e.g., if $T_{\mathrm{exc}} = 200$ K, $T_A^* = 0.2$ K, and $\phi_A = 78''$, then $\phi_{\mathrm{sh}} \approx 6''$. This angular size can be resolved by optical telescopes. In the case of R Leo, fluorescent emission in the potassium resonance lines at 7665 and 7669 Å has been detected from the gas in the shell. Spectra were obtained with a small slit centered on the star and on a point 3″ north of the star. The latter spectrum is presumed to be a blend of the circumstellar emission-line spectrum and a spectrum of the star resulting from

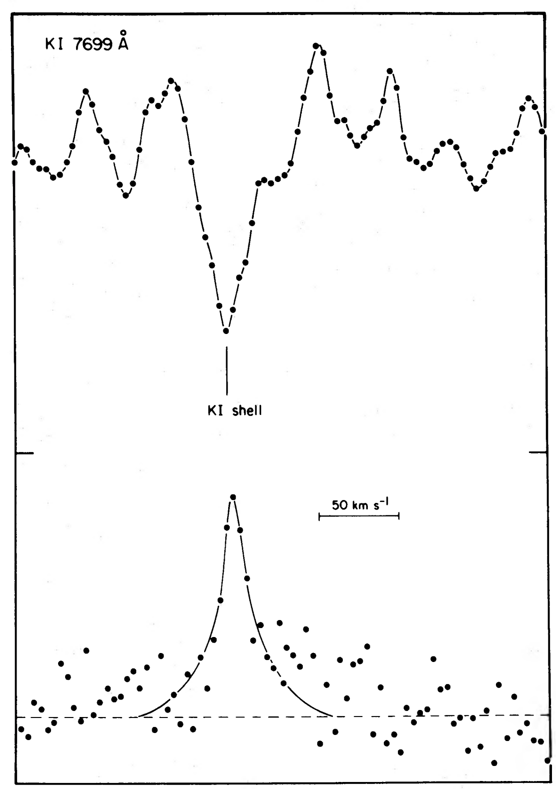

FIG. 2.—The spectrum of R Leo in the region of the K I 7699 Å resonance line. The upper spectrum shows the observed stellar spectrum; the short tick mark indicates the zero intensity level. The lower spectrum is the difference spectrum between the "on star" and "off star" spectra after they are matched to about 50 channels to the red and the blue of the illustrated portion. The K I line is clearly in emission in the difference or circumstellar spectrum.

scattering in the Earth's atmosphere and the telescope. The circumstellar shell may also act like a reflection nebula and contribute a Doppler-shifted and Doppler-broadened stellar spectrum. This possibility is ignored here. Inspection of the "off star" spectrum shows that the only difference between it and the stellar spectrum occurs in the two K I resonance lines. Both are filled in, and, after removal of the scattered light, they appear as emission lines (Fig. 2). No other stellar features in the observed 100 Å interval appear in corrected spectra. K I emission has previously been detected around Betelgeuse (Bernat and Lambert 1975, 1976*b*; Bernat *et al.* 1978). These observations indicate that the circumstellar shell around R Leo is at least several arcsec in extent. Attribution of the SiO emission to a cool, extended shell is confirmed.

V. CONCLUDING REMARKS

A complete understanding of the physical and chemical conditions in the circumstellar shell around a late-type giant, such as Betelgeuse, and the LPVs will come from a synthesis of observations across the electromagnetic spectrum. Millimeter SiO and CO emission lines should be looked for from other nearby LPVs. The present analysis suggests that the SiO molecules represent a small proportion of the Si in the shell; the major proportion appears to be locked into the grains. These grains cannot be fayalite. It will be of interest to investigate the relative intensities of the SiO and CO lines. Accurate line profiles will yield information on the velocities within the shell and on the systemic velocity (Reid and Dickinson 1976).

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