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# ON THE SURFACE COMPOSITION OF THERMALLY PULSING STARS OF HIGH LUMINOSITY AND ON THE CONTRIBUTION OF SUCH STARS TO THE ELEMENT ENRICHMENT OF THE INTERSTELLAR MEDIUM*

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## ABSTRACT

The enhancements of $^3$He, $^4$He, $^{12}$C, $^{13}$C, $^{14}$N, $^{22}$Ne, and *s*-process elements at the surfaces of stars that ultimately experience the thermal-pulse phenomenon is studied in some detail. It is shown that, as a consequence of the dredge-up phenomenon that occurs during the power-down phase of a thermal pulse, the development of a surface ratio of carbon to oxygen greater than 1 may possibly not be achieved for luminosities below $M_{\rm BOL} \sim -5.5$. The enhancement of *s*-process elements that follows as a consequence of the emission of neutrons via the reaction $^{22}$Ne($\alpha$, $n$)$^{25}$Mg is not achieved for luminosities below $M_{\rm BOL} \sim -6$. The thermal pulse mechanism thus appears capable of accounting easily for only the most luminous carbon stars.

The thermal-pulse mechanism predicts comparable enhancements of $^{12}$C and *s*-process elements. Hence, unless $^{12}$C is converted into $^{14}$N at the base of the convective envelope during the inter-pulse phase, it fails to account for two major characteristics of both barium stars and S stars (enhancements of *s*-process elements up to factors of 20 but C/O $\sim$ 1). It is inferred that the infusion of protons into the cores of a fraction of all low-mass stars undergoing the helium flash is responsible for the characteristics of the majority of barium stars and that S stars may be direct (thermally pulsing) descendants of barium stars.

By combining the results of stellar evolution calculations with a standard birthrate function, sources of several elements in the interstellar medium are identified. Stars of low mass ($M \leqslant 2\ M_\odot$) are responsible for most of the $^3$He, stars of high mass ($M \geqslant 8\ M_\odot$) are responsible for $^{16}$O, and stars of intermediate mass are responsible for *s*-process elements with $70 < A < 204$. The elements $^{14}$N and $^{13}$C are contributed fairly evenly by all stars. The elements $^{12}$C and $^{22}$Ne and the progeny of $^{22}$Ne (results of the $^{22}$Ne($\alpha$, $n$)$^{25}$Mg reaction and of subsequent neutron-capture reactions) are contributed in roughly equal amounts by stars of intermediate mass and by stars of high mass.

*Subject headings:* convection — interstellar: abundances — nucleosynthesis — stars: abundances — stars: late-type

## I. INTRODUCTION

It is now well established that the convective shell that forms between the carbon-oxygen core and the hydrogen-rich envelope of thermally pulsing stars is an important site for the production of carbon and of *s*-process elements. By a process of convective dredging, elements freshly synthesized in the shell are first carried to the surface, where they may be observed, and subsequently, as a consequence of mass loss from the stellar surface, contribute to the heavy-element enrichment of the interstellar medium.

There is sufficient quantitative understanding of thermal pulse properties, of cross sections of many important nuclear reactions that occur during pulses, and of surface mass loss rates to justify a zero order description both of (1) the variation with time in surface composition as a consequence of the thermal pulse and convective dredge-up mechanism in stars of different initial mass and of (2) the contribution of thermally pulsing stars of different initial mass to the enrichment of the interstellar medium.

Our principal findings include: (1) Significant enhancement of *s*-process elements does not occur for luminosities less than about $M_{\rm BOL} \sim -6$. (2) For stars of a given initial composition, the surface abundance of $^{12}$C exceeds that of $^{16}$O at roughly the same surface temperature, independent of mass. (3) Stars initially less massive than about 1.5 $M_\odot$ do not, *as a consequence of thermal pulses*, produce $^{12}$C/$^{16}$O > 1 at the stellar surface (become carbon stars). (4) Stars

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initially less massive than about 2.5 $M_\odot$ do not achieve convective shell temperatures sufficiently high for the production of *s*-process elements before losing all of their envelope mass either by quiet mass loss or by the ejection of a planetary nebula. (5) In stars initially more massive than about 3 $M_\odot$, surface enhancements of *s*-process elements ultimately reach values between 4 and 20, with the maximum enhancement occurring in stars of initial mass near 5 $M_\odot$. (6) Stars of initial mass between 3 $M_\odot$ and 8 $M_\odot$ are the major source of *s*-process elements in the Galaxy and are a significant source of $^{12}$C, rivaling in this respect stars initially more massive than 8 $M_\odot$. (7) The luminosity, color, and surface composition characteristics of many carbon stars (N and S types) and of all barium stars are probably not achieved as a consequence of nuclear processing during thermal pulses, and we are obliged to attribute many of the interesting abundances in these stars to some additional process that occurs during a phase of evolution that precedes the asymptotic-branch phase—possibly during the helium-flashing phase in a few stars initially less massive than about 2.25 $M_\odot$.

## II. THE THERMAL PULSE MODEL

We first collect and/or prepare simplified approximations to the pertinent characteristics of stellar models in the asymptotic-branch phase of evolution as presented by Weigert (1966), Paczyński (1970, 1971*a*, *b*, 1974, 1975), Uus (1970), Gingold (1975), Sugimoto and Nomoto (1975), Fujimoto, Nomoto, and Sugimoto (1976), and Iben (1975*a*, 1976, 1977*a*).

Denoting $M_c(0)$ as the mass of the carbon-oxygen core of a star of initial mass $M_*(0)$ when it first re-ignites hydrogen on the asymptotic branch, we have

$$M_c(0) \approx 0.95 + 0.075[M_*(0) - 7]\,, \qquad (1)$$

where masses are in solar units.

As relationships between core mass $M_c$, total stellar mass $M_*$, surface luminosity $L_s$, and hydrogen-burning luminosity $L_H$ during the interpulse phase we adopt

$$L_s = 6.34 \times 10^4(M_c - 0.44)(M_*/7)^{0.19} \qquad (2)$$

and

$$L_H = L_s - 2 \times 10^3(M_*/7)^{0.19} \exp[3.45(M_c - 0.96)]\,, \qquad (3)$$

where luminosities are in solar units. The power to which $M_*$ is raised in these expressions is approximately one-half of that found by Iben (1977*a*) and is a compromise with the results of Paczyński (1970) and Uus (1970) who argue the mass independence of the $M_*$-$M_c$ relationships.

Surface temperature $T_e$, $L_s$, $M_*$, and the abundance by number $Y_{CNO}$ of CNO elements at the surface are assumed to be related by

$$\log T_e = 3.484 - 0.087(\log L_s - 4.5) + 0.08 \log(M_*/7) - 0.075 \log[Y_{CNO}/Y_{CNO}(0)]\,, \qquad (4)$$

where $Y_{CNO}(0)$ is the surface abundance by number of CNO elements at the start of the asymptotic-branch phase of evolution. When $Y_{CNO} = Y_{CNO}(0)$ and $M_* = 1$, this last expression becomes

$$\log T_e = 3.81 - 0.087 \log L_s\,, \qquad (4')$$

which is to be compared with the relationship

$$\log T_e = 3.82 - 0.094 \log L_s \qquad (4'')$$

estimated by Wood and Cahn (1977) on the basis of observational data on asymptotic giant branch stars of near solar mass (Eggen 1975; Johnson 1966).

The mass $\Delta M_{CSH}$ contained in a convective shell at maximum size during a pulse, the duration $\Delta t_{CSH}$ of the convective shell phase, the maximum temperature $T_m$ achieved at the base of the convective shell, and the fractional overlap $r$ of successive convective shells are taken to be (Iben 1977*a*):

$$\log \Delta t_{CSH} = 9.66 + 0.9M_c - 2.06M_c^2\,, \qquad (5)$$

$$\log \Delta M_{CSH} = -1.835 + 1.73M_c - 2.67M_c^2\,, \qquad (6)$$

$$T_m = 310 + 285(M_c - 0.96)\,, \qquad (7)$$

and

$$r = 0.43 - 0.795(M_c - 0.96) + 0.346(M_c - 0.96)^2\,, \qquad (8)$$

where $\Delta t$ is in seconds, $\Delta M_{CSH}$ is in solar units, and $T$ is in $10^6$ kelvins.

Following the disappearance of the convective shell, the abundances by mass of carbon and oxygen in the region below the hydrogen-helium discontinuity are chosen as (Iben 1977*a*)

$$X_{12} \approx 0.25(M_c - 0.16)\,,$$
$$X_{16} \approx 2.5 \times 10^{-4} \exp[3.45(M_c - 0.96)]\,, \qquad (9)$$

and the ratio $f_{22}$ of $^{22}$Ne remaining relative to initial $^{22}$Ne in the region of size $\Delta M_{CSH}$ just below the hydrogen-helium interface following a pulse is approximated by (Iben 1977*a*)

$$f_{22} = Y_{22}/Y_{22}(0) = \exp(-\Delta t_s/46\tau_m)\,, \qquad (10)$$

where $\tau_m$ is the lifetime of $^{22}$Ne at the base of the convective shell at temperature maximum and $\Delta t_s$ is the duration of the convective shell following temperature maximum. In first approximation (Iben 1977*a*)

$$\Delta t_s/\Delta t_{CSH} \approx 0.66 + 1.025(M_c - 0.96) - 1.125(M_c - 0.96)^2\,. \qquad (11)$$

If we adopt the rate for the $^{22}$Ne$(\alpha, n)^{25}$Mg reaction given by Fowler, Caughlan, and Zimmerman (1975), then

$$\tau^{-1} = 1.04 \times 10^{21}(\rho X_4/T_6^{2/3}) \exp(-470.0/T_6^{1/3})\,, \qquad (12)$$

where $\rho$ = density in g cm$^{-3}$, $T_6$ = temperature in $10^6$ kelvins, and $X_4$ = abundance by mass of $^4$He.

In all cases studied by Iben (1977*a*), $\rho X_4$ is approximately equal to $2.2 \times 10^3$ g cm$^{-3}$ when $T = T_m$. Hence

$$\tau_m^{-1} \approx 2.3 \times 10^{24} T_m^{-2/3} \exp(-470/T_m^{1/3}) . \quad (12')$$

For the amount of freshly processed material dredged into the envelope from the region occupied by the convective shell at maximum size we are forced to adopt an exceedingly crude estimate. Any estimate is, of course, uncertain if for no other reason than that convective overshoot affects the dredge-up process in a manner that is difficult to assess (Iben 1976; see also Paczyński 1977). Paczyński (1977) has argued that the energy made available during the phase of pulse power-down is insufficient to carry the freshly made carbon into the convective envelope. The study by Iben (1976) suggests that Paczyński has overstated the case. The amount of energy absorbed locally in the region in which the mixing between carbon and hydrogen takes place depends on the composition profiles in this region. Iben shows that if the profile extends over 2 pressure scale-heights, the rate of absorption due to mixing is at maximum approximately 30% of the rate at which energy is transferred to this region by the transfer of entropy from the helium-burning region below it. Furthermore, the integrated absorption rate due to mixing is independent of the profile and is a small fraction of the total rate of energy supply by entropy transfer. It is therefore simply not true that the dredge-up phenomenon is prevented by lack of energy for mixing. If anything, as a consequence of convective overshoot (Iben 1976), the extent of dredge-up is expected to be *larger* than is calculated in the simplest approximation.

After 18 pulses in the case $M_c = 0.96$, the mass $\Delta M_{\rm dredge}$ of the dredged-up material relative to the mass $\Delta M_c$ through which the hydrogen-burning shell passes during the interpulse phase is estimated by Iben (1976) to be $\lambda = \Delta M_{\rm dredge}/\Delta M_c = 0.33$. Gingold (1975) finds that no dredging up occurs for at least the first nine pulses when $M_c \sim 0.6$, and Fujimoto, Nomoto, and Sugimoto (1976) estimate that $\lambda$ reaches $\sim 0.3$ after five pulses when $M_c = 1.26$. Since $\lambda$ continues to increase with each pulse, these last two values of $\lambda$ are really only lower limits. We adopt, conservatively,

$$\lambda = 0 , \quad M_c < 0.6 ;$$

$$\lambda = 0.33\{1 + \sin [\pi(M_c - 0.96)/0.72]\} , \quad 0.6 < M_c < 0.96 ;$$

$$\lambda = 0.33 + 0.17 \sin [\pi(M_c - 0.96)/0.6] , \quad 0.96 < M_c < 1.26 ;$$

and

$$\lambda = 0.50 , \quad M_c > 1.26 . \quad (13)$$

The increase $\Delta M_c$ in core mass during the interpulse phase is related to the mass $\Delta M_{\rm CSH}$ by

$$\Delta M_c = (1 - r)\Delta M_{\rm CSH} , \quad (14)$$

and the time interval $\Delta t_{ip}$ during quiescent hydrogen burning between pulses is

$$\Delta t_{ip} = (3.14 \times 10^{18}\ {\rm s})\Delta M_c (L_{\rm H} X_{\rm H})^{-1} , \quad (15)$$

where $X_{\rm H}$ is the abundance by mass of hydrogen at the surface.

Following each pulse, core mass is decreased by

$$\delta M_{\rm core} = \lambda \delta M_c . \quad (16)$$

The final ingredient, except for further nucleosynthesis algorithms, is an approximate mass loss rate. In common with others who have studied the effects of mass loss along the asymptotic branch (e.g., Fusi-Pecci and Renzini 1975*a*, *b*, 1976; Scalo 1976*a*, *c*; Wood and Cahn 1977), we adopt the Reimers (1975, 1977) rate

$$\dot{M}_* = -1.26 \times 10^{-20} \alpha L_s R_s / M_* , \quad (17)$$

where $R_s$ is stellar radius in solar units, $\dot{M}_*$ is in $M_\odot$ s$^{-1}$, and $\alpha$ is a parameter whose value appears to be (Fusi-Pecci and Renzini 1976; Wood and Cahn 1977; Reimers 1977) somewhere between 0.5 and 0.25. We are aware of the qualitative nature of the Reimers rate (e.g., Bernat 1977), but have no alternative rate to offer. In order to make headway, we make a concrete (though uncertain) choice: $\alpha = \frac{1}{3}$.

### III. THREE STAGES OF ELEMENT ENHANCEMENT AT THE SURFACE

In order to achieve a simple, broad-brush understanding of element enrichment of both the stellar envelope and the interstellar medium, we shall confine our treatment of nucleosynthesis to several of the more significant and well studied elements. Before embarking on a discussion of surface changes during the thermal pulse phase, we define our initial abundances and discuss changes in surface abundances that arise during two previous post–main-sequence phases.

As relative initial main-sequence abundances by number of $^1$H, $^3$He, and $^4$He we choose $Y_{\rm H} = 0.7$, $Y_3 = 8 \times 10^{-6}$ (Cameron 1973), and $Y_4 = 0.07$. Main-sequence abundances of $^{12}$C, $^{14}$N, and $^{16}$O are chosen in solar system ratios (Cameron 1973), as $Y_{12} = 2.6 \times 10^{-4}$, $Y_{14} = 8.0 \times 10^{-5}$, and $Y_{16} = 4.7 \times 10^{-4}$. During the early main-sequence phase $^{12}$C is converted into $^{14}$N and most of the initial $^3$He is destroyed over the inner portions of the star. Simultaneously, fresh $^3$He is created between mass fractions 0.3 and 0.6 and $^{13}$C is created in a narrow region centered on the $^{12}$C–$^{14}$N transition edge. As each star rises along the giant branch for the first time, convection carries freshly made $^3$He, $^{13}$C, and $^{14}$N to the surface and depletes the abundance of $^{12}$C there.

From Iben's (1964, 1965; 1966*a*, *b*, *c*, 1967*a*, *b*) results we find for the final surface abundance of $^3$He

$$Y_3 \sim 1.8 \times 10^{-4}(M_\odot/M_*)^2 + 0.7 Y_3{}^0 , \quad (18)$$

where the second term is the initial $^3$He abundance adjusted to take into account the destruction of original $^3$He over approximately the inner 30% of the region covered by convection at its maximum inward extent on the red-giant branch. The first term is smaller by $\sqrt{5}$ than that given by Iben's results simply because the estimate for the cross section for the $^3$He($^3$He, 2$p$)$^4$He reaction, which is the primary reaction responsible for limiting the production of new $^3$He, has increased by a factor of 5 in the intervening time.

Again from Iben's results it follows that the decrease in the surface abundance of $^{12}$C is, to within 10%, given by

$$\Delta Y_{12}/Y_{12} \sim -0.19 - 0.26(M_* - 1) , \quad M_* < 1.5 ,$$
$$\Delta Y_{12}/Y_{12} \sim -0.32 , \quad M_* > 1.5 . \quad (19)$$

Nitrogen-14 is, of course, enhanced by $\Delta Y_{14} = |\Delta Y_{12}|$. We shall neglect the small contribution to $^{14}$N that is made by the burning of $^{16}$O and its products in stars near the upper end of the initial mass range in which we are most interested ($1 < M_* < 8$).

As a star ascends the giant branch for the first time, the ratio of $^{12}$C to $^{13}$C at its surface becomes

$$C_{\rm RG} \approx C_{\rm MS}/(1 + C_{\rm MS}/X) , \quad (20)$$

where $X$ is the initial main-sequence ratio and $C_{\rm MS}$ varies from about 36 when $M_*(0) = 1$ to about 25 when $M_*(0) = 15$ (Iben 1966*a*, 1969, 1977*b*). Since $X$ is in the range 90 (Cameron 1973) to, say, 45, as inferred from several isotope studies of the interstellar medium, $C_{\rm RG}$ lies between 16 and 26.

To complete the surface abundance set as it is in models on the first red-giant branch, we choose for $^{56}$Fe, the light elements between $^{22}$Ne and $^{56}$Fe, $^{22}$Ne, and heavy ($A > 56$) *s*-process elements (Cameron 1973), respectively: $Y_{56} \sim Y_{\rm CNO}/42.5$, $Y_l \sim Y_{56}$, $Y_{22} \sim Y_{56}/2$, and $Y_s \sim Y_{56}/200$, where $Y_{\rm CNO} = Y_{12} + Y_{14} + Y_{16}$.

Following exhaustion of central helium and extinction of shell hydrogen burning, stars again assume a giant structure. Prior to the reignition of hydrogen in a shell, convection in stars more massive than about 3 $M_\odot$ once again dredges up products of hydrogen burning, this time those created in the hydrogen-burning shell during the core helium-burning phase. Since almost all initial $^{12}$C and $^{16}$O are converted into $^{14}$N in this shell, convective rearrangement on the initial ascent of the asymptotic branch leads to a further enhancement of surface $^{14}$N at the expense of both $^{12}$C and $^{16}$O. In addition, the surface abundance of $^4$He is also slightly increased at the expense of $^1$H.

In rough approximation the mass of $^4$He and $^{14}$N-rich matter that is incorporated into the envelope of intermediate-mass stars during this second dredge-up phase is (in solar units)

$$\Delta M_{\rm He} \approx 0.135 M_* - 0.415 , \quad 3.08 < M_* \leqslant 8 ,$$
$$\Delta M_{\rm He} = 0 , \quad M_* < 3.08 , \quad (21)$$

and the change $\Delta Y_i{}^e$ in the envelope abundance of the $i$th element is given by

$$\Delta Y_i{}^e = (Y_i{}^{\rm He} - Y_i{}^e)\Delta M_{\rm He}/(M_* - M_c) , \quad (22)$$

where $Y_i{}^{\rm He}$ is the abundance of the $i$th element in the helium zone and $Y_i{}^e$ is the abundance of the $i$th element in the hydrogen-rich envelope prior to the second dredge-up phase. We have $Y_{\rm H}{}^{\rm He} \approx Y_3{}^{\rm He} \approx Y_{12}{}^{\rm He} \approx Y_{13}{}^{\rm He} \approx Y_{16}{}^{\rm He} \approx 0$, $Y_{14}{}^{\rm He} \approx Y_{\rm CNO}{}^e$, and $Y_4{}^{\rm He} \sim 0.245$.

Having completed a description of envelope abundances at the start of the thermal pulse phase, we next describe abundance changes in the convective shell that appears during the course of a thermal pulse.

In § II we have given estimates of the fraction $f_{22}$ of $^{22}$Ne remaining in the shell and of the final abundances of $^{12}$C and $^{16}$O in the shell after shell convection ceases. At the beginning of each burning epoch, the initial abundance of $^{22}$Ne is composed of (1) $^{22}$Ne left over from the previous epoch in the region of overlap and (2) new $^{22}$Ne produced in the shell as a consequence of the conversion of fresh $^{14}$N incorporated into the shell at the beginning of the epoch. If we label by superscripts $B$ and $A$ all abundances that appear in the convective shell before and after the nucleosynthesis episode, then

$$Y_{22}{}^B = Y_{\rm CNO}{}^e(1 - r)(1 - f_{22}r)^{-1} ,$$
$$Y_{22}{}^A = f_{22} Y_{22}{}^B , \quad (23)$$

where $Y_{\rm CNO}{}^e$ is the abundance of CNO elements in the envelope.

In a similar fashion we can write

$$Y_{56}{}^B = Y_{56}{}^e(1-r)(1 - f_{56}r)^{-1} ,$$
$$Y_{56}{}^A = f_{56} Y_{56}{}^B , \quad (24)$$

where $Y_{56}$ is the abundance of $^{56}$Fe. In first approximation $f_{56}$ is (Iben 1975*b*),

$$f_{56} = \exp(-\sigma_{56}\phi) , \quad (25)$$

where $\sigma_{56}$ is the cross section for neutron capture on $^{56}$Fe at 30 keV, and the exposure $\phi$ is defined by (Truran and Iben 1977)

$$\phi = -(\sigma_l - 2\sigma_{22})^{-1} \ln [1 - (1 - r)/\beta] \quad (26)$$

and

$$\beta = 1 + (\sigma_l - 2\sigma_{22})^{-1}[(n_s\sigma_s + n_{ln}\sigma_{ln})n_0{}^{-1} + 2\sigma_{22}]$$
$$\times [1 + f(1 - f)^{-1}(1 - r)] . \quad (27)$$

We choose $\sigma_{56} = 13.5$ mb and, for other neutron capture cross sections required for determining $\phi$, we choose $\sigma_{22} = 0.5$ mb (for $^{22}$Ne), $\sigma_s = 13.5$ mb (for $^{56}$Fe *and* its neutron capture progeny), $\sigma_l = 3.8$ mb (for light elements that appear in the convective shell as direct progeny of $^{22}$Ne), and $\sigma_{ln} = 13$ mb (for all light elements between $^{22}$Ne and $^{56}$Fe that are not direct progeny of $^{22}$Ne). The abundance $Y_l$ of light elements in the shell is defined so that, at all times in the shell, $Y_l + Y_{22} = Y_{\text{CNO}}^e$. Similarly, for the abundance of heavy *s*-process elements in the shell, $Y_s + Y_{56} = Y_{56}^e(0)$.

To complete the specification of needed shell abundances, we have (see the previous section) $Y_{14}^A = Y_{\text{H}}^A = Y_3^A = 0$ and $Y_{12}^A$ and $Y_{16}^A$ follow from equations (9). The abundance of $^4$He follows from nucleon conservation.

After the active nucleosynthesis phase of the pulse, the dredging-up process increases the envelope mass by $\Delta M_e = \lambda \Delta M_c$ and the abundance of the *i*th element is altered by

$$Y_i^e = (Y_i^A - Y_i^e)\Delta M_e/(M_e + \Delta M_e) . \quad (28)$$

The abundance of *s*-process elements and the abundance of light elements in the surface are enhanced, respectively, by

$$\Delta Y_s^e = (Y_{56}^B - Y_{56}^A)\Delta M_e/(M_e + \Delta M_e) ,$$

and

$$\Delta Y_{ln} = (Y_{22}^B - Y_{22}^A)\Delta M_e/(M_e + \Delta M_e) . \quad (29)$$

The total envelope abundance of CNO elements is, of course, enhanced by the addition of fresh carbon. Whether or not this $^{12}$C is partially converted into $^{13}$C and $^{14}$N and whether or not $^3$He is destroyed at the base of the convective envelope during the interpulse phase is a function of uncertain parameters in the treatment of convection (e.g., Iben 1975*a*, 1976). We shall ignore for the moment the possibility of burning in the envelope.

During the dredge-up phase and the subsequent interpulse phase, we assume a mass loss of

$$\Delta M_* = -\dot{M}_* \Delta t_{ip} \quad (30)$$

and adjust abundances in the "wind" (defined as all matter which has left the model star since birth) according to

$$\Delta Y_i^w = (Y_i^e - Y_i^w)\Delta M_*/(M_w + \Delta M_*) , \quad (31)$$

where $M_w$ is the mass of the wind.

Finally, core mass is increased by $\Delta M_c$, the mass of the envelope is reduced by $(\Delta M_c + \Delta M_*)$, and the mass of the wind is increased by $\Delta M_*$. We are then ready to recalculate envelope abundances after the next pulse, and so forth.

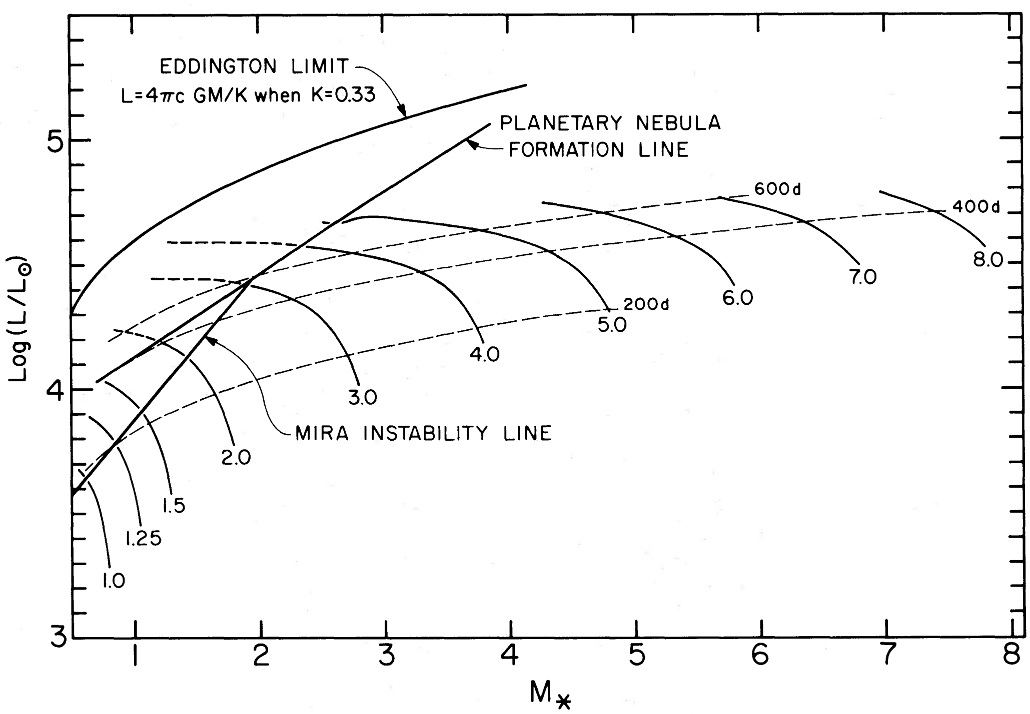


FIG. 1.—Evolutionary tracks in the luminosity-mass plane. The solid curves are tracks of model stars that suffer mass loss at the Reimers rate while on the asymptotic branch. Initial masses are indicated. It has been assumed that every star loses a mass $=0.2M_0$ prior to the asymptotic-branch phase. Dashed lines connect points of the same first-harmonic pulsation period. It is assumed that whenever a track crosses the Mira instability line (which terminates where it crosses the planetary formation line), the star will pulsate in the first harmonic mode and that, whenever a track crosses the planetary nebula formation line, all matter outside of the carbon-oxygen core is ejected.

In following the progression of our models we also compute the period for pulsation in the first harmonic radial mode using

$$P(\text{days}) \approx 0.038 R_s^{3/2}/M_* \quad (32)$$

as given by Wood (1976) and determine the characteristics of the star at the onset of Mira variability and at the occurrence of planetary nebula ejection, following the prescriptions of Wood and Cahn (1977).

### IV. EVOLUTION IN THE *M-L* PLANE AND IN THE H-R DIAGRAM

Evolutionary tracks in the mass-luminosity (*M-L*) plane are shown in Figure 1. The number beside each track is the initial mass in solar units. It has been assumed that 0.2 $M_\odot$ is lost by each model star during its first ascent of the giant branch (Fusi-Pecci and Renzini 1975*a*, *b*). Dashed curves in Figure 1 are loci of constant period for pulsation in the first harmonic mode. We have assumed that when a track reaches the straight line of larger positive slope, Mira variability is initiated, and that when a track reaches the straight line of smaller positive slope, a planetary nebula is ejected. The ejected nebula is assumed to contain all of the matter between the location of the hydrogen-burning shell and the surface at the time of ejection.

We emphasize that the location of the Mira instability line and the location of the planetary nebula

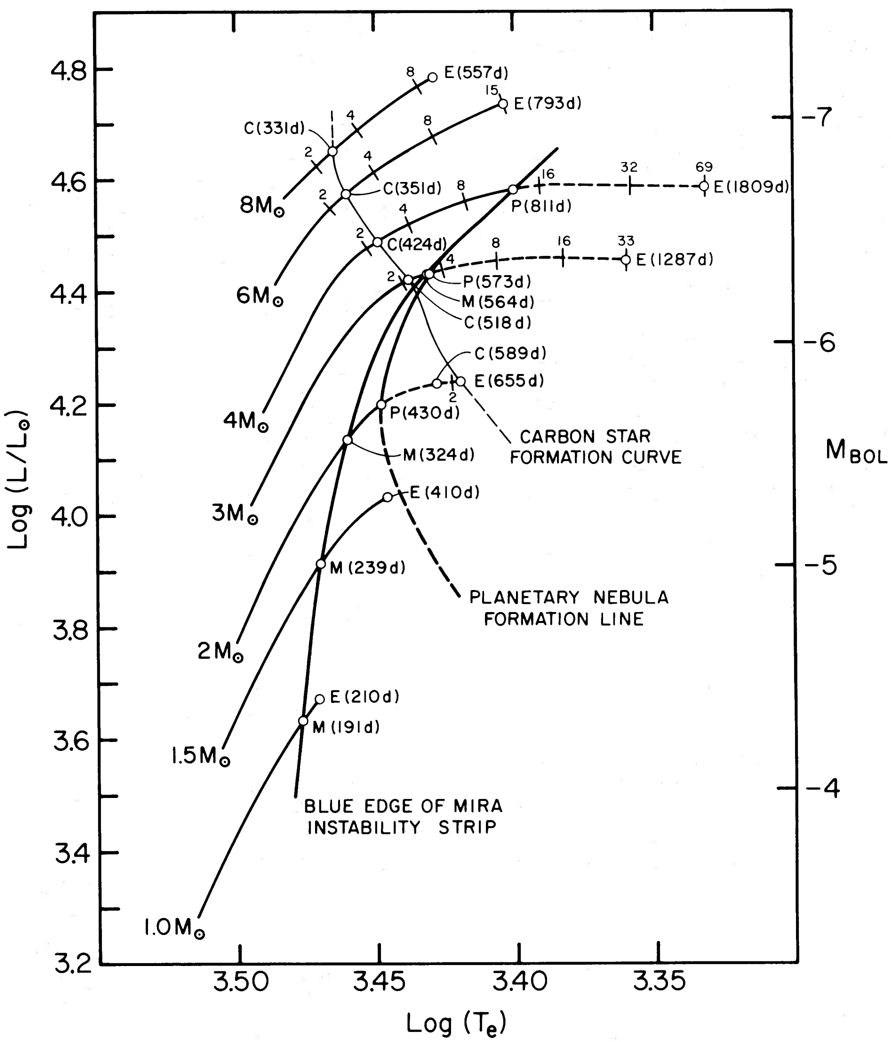


FIG. 2.—Evolutionary tracks of asymptotic branch stars in the H-R diagram. The initial mass of a model star is given in solar units at the start of its evolutionary track. At point M a star is assumed to start pulsating in the first harmonic mode, and at point P all matter outside the carbon-oxygen core is assumed to be ejected as a planetary nebula. A dashed curve joining points P and E is the path a model would follow if planetary ejection did not occur and evolution were instead terminated when the entire hydrogen-rich envelope had been lost from the star, thanks to mass loss by a steady wind. Along the carbon star formation line, the abundance of carbon first exceeds the abundance of oxygen at the surface. At each tick mark along the track of a more massive model, the abundance of *s*-process elements at the surface is enhanced, relative to the solar system abundance, by the indicated amount. Numbers in parentheses give the period for pulsation in the first-harmonic mode, whether or not the star pulsates in that mode.

formation line are extremely speculative, although they do conform to the algorithm devised by Wood and Cahn (1977) to match the observational data on Mira variables: The two lines intersect on the 600 day period curve, Mira variability is confined to periods between about 200 days and 600 days, and the bulk of all Miras have periods between 250 and 450 days.

Evolutionary tracks are shown in the H-R diagram of Figure 2, together with the Mira instability blue edge and the planetary nebula ejection curve. Again, initial model mass labels the beginning of each track. The point along each track where the surface carbon abundance exceeds the surface oxygen abundance is marked by a C, and the formal period for pulsation in the first harmonic mode is contained within the following parentheses. The point where Mira variability begins and the point of planetary nebula formation are respectively labeled M and P, and the final position that the model would reach if planetary formation did not take place is labeled E. Finally, the numbers along the tracks for the more massive models give the factor whereby *s*-process elements have been enhanced at the model surface.

Further details for individual models are given in Tables 1–8 and characteristics of final models (at point E, or at point P if P precedes E) are summarized in Table 9. In these tables, $N$ = number of pulses since the beginning of the thermal pulse phase, $\Delta t$ = interval between adjacent phases (in years), $\bar{Y}_i^e$ = abundance of $i$th element in the envelope relative to solar abundance, $\bar{Y}_i^W$ = relative abundance of $i$th element in the wind ejectum, and $\Lambda_e$ and $\Lambda_w$ = characteristics of the *s*-process distribution in the envelope and in the wind, respectively. Here $\Lambda$ is given by (Truran and Iben 1977)

$$\Lambda = 2\beta(1 + r)^{-1}(\sigma_l - 2\sigma_{22})/(1 + x/2 + x^2/3 + \cdots)\,, \quad (33)$$

where

$$x = (1 - r)/\beta\,. \quad (34)$$

In Table 9, $M_R$ = mass of the final remnant, $M_w$ = mass of matter lost (via a quiet "wind") prior to the ejection of a planetary nebula or formation of a C-O core with $M_c = 1.4$, $M_P$ = mass of ejected planetary or [for $M_*(0) \geqslant 5$] supernova shell, $M_{\rm PR}$ = mass processed through a convective shell during thermal pulses, and $M_{\rm DG}$ = total mass of processed matter dredged up into envelopes.

From Figure 2 it is clear, that, if they obey the conservative dredge-up law that we have assumed, stars of Population I composition do not become carbon stars via the thermal pulse mechanism until they are more luminous than about $2 \times 10^4\, L_\odot$ ($M_{\rm BOL} \leqslant -6$). Most models that do develop carbon star characteristics appear to be outside of the Mira instability strip (between points M and P along any track); but if they do pulsate in the first harmonic mode (say, as irregular variables), their periods should initially lie between 350 days and 500 days and the maximum period should not exceed 800 days. If pulsation occurs in the fundamental mode, then periods may be as much as twice as large as these.

If we adopt a less conservative dredge-up law that permits more extensive dredge-up for small cores, the "carbon star curve" in Figure 2 rotates clockwise. In fact, if we choose

$$\lambda = 0\,, \quad M_c < 0.64\,,$$

and

$$\lambda = 0.33 + 0.73(M_c - 0.96) - 0.531(M_c - 0.96)^2\,, \quad M_c > 0.64\,, \quad (13')$$

the carbon star curve becomes nearly vertical (log $T_e \sim 3.45$) and extends downward into the Mira instability strip. The luminosity lower limit for achieving carbon star characteristics becomes $M_{\rm BOL} \sim -5$, and on reaching the carbon star line at log $T_e \sim 3.45$ a Mira variable brighter than $M_{\rm BOL} = -5$ will become a carbon star.

As pointed out earlier, the occurrence of convective overshoot at the base of the convective envelope during the dredge-up phase will increase the degree of dredge-up over extant estimates. It is conceivable that a correct treatment of the dredge-up phenomenon may predict the formation of carbon star characteristics for magnitudes as dim as $\sim -4$, which is the magnitude at which a star of initial mass $\sim 1.5\, M_\odot$ first reaches the asymptotic branch.

The enhancement of *s*-process elements due to the thermal-pulse, $^{22}$Ne neutron-source mechanism cannot in any case occur until the maximum temperature at the base of the convective shell that appears during a pulse exceeds about 300 million kelvins. It is for this

TABLE 1

CHARACTERISTICS OF MODELS OF INITIAL MASS $M_*(0)$ = 1, 1.5, AND 2.0

| $M_*(0)$ | Phase | $N$ | $\Delta t$ | $M_*$ | $M_c$ | $\bar{Y}_{12}^e$ | $\bar{Y}_{12}^w$ | $\bar{Y}_s^e$ | $\bar{Y}_s^w$ |
|---|---|---|---|---|---|---|---|---|---|
| 1......... | Mira | 21 | $1.75 \times 10^6$ | 0.61 | 0.55 | 0.81 | 0.81 | ... | ... |
| | End | 23 | $1.24 \times 10^5$ | 0.56 | 0.56 | 0.81 | 0.81 | ... | ... |
| 1.5....... | Mira | 27 | $1.37 \times 10^6$ | 1.05 | 0.63 | 0.68 | 0.68 | ... | ... |
| | End | 45 | $5.71 \times 10^5$ | 0.70 | 0.70 | 0.83 | 0.70 | ... | ... |
| 2.0....... | Mira | 43 | $1.39 \times 10^6$ | 1.41 | 0.73 | 0.75 | 0.69 | ... | ... |
| | Planetary | 61 | $2.88 \times 10^5$ | 1.18 | 0.79 | 0.97 | 0.73 | 1.05 | 1.00 |
| | "Carbon" | 78 | $2.25 \times 10^5$ | 0.92 | 0.84 | 1.86 | 0.86 | 1.56 | 1.05 |
| | "End" | 82 | $4.19 \times 10^4$ | 0.85 | 0.85 | 2.61 | 0.94 | 2.14 | 1.10 |

TABLE 2

CHARACTERISTICS OF A MODEL OF INITIAL MASS $M_*(0) = 2.5$

| Phase | $N$ | $\Delta t$ (yr) | $M_*$ | $M_c$ | $\bar{Y}_{12}^e$ | $\bar{Y}_{22}^e$ | $\bar{Y}_l^e$ | $\bar{Y}_s^e$ | $\Lambda_e$ | $\bar{Y}_{12}^w$ | $\bar{Y}_{22}^w$ | $\bar{Y}_l^w$ | $Y_s^w$ | $\Lambda_w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mira....... | 76 | $1.45 \times 10^6$ | 1.71 | 0.85 | 1.07 | 1.67 | 1.00 | 1.17 | 12.5 | 0.76 | 1.14 | 1.00 | 1.02 | 7.30 |
| Planetary... | 92 | $1.25 \times 10^5$ | 1.58 | 0.89 | 1.33 | 2.09 | 1.01 | 1.42 | 13.7 | 0.82 | 1.24 | 1.00 | 1.06 | 8.30 |
| Carbon star | 113 | $1.29 \times 10^5$ | 1.42 | 0.92 | 1.81 | 2.84 | 1.05 | 2.01 | 13.4 | 0.93 | 1.42 | 1.01 | 1.15 | 9.50 |
| $\bar{Y}_s^e \approx 4$..... | 146 | $1.53 \times 10^5$ | 1.16 | 0.96 | 3.29 | 5.22 | 1.27 | 4.18 | 10.9 | 1.22 | 1.88 | 1.03 | 1.48 | 10.5 |
| $\bar{Y}_s^e \approx 8$..... | 158 | $4.52 \times 10^4$ | 1.05 | 0.97 | 5.73 | 10.1 | 1.96 | 8.03 | 9.15 | 1.43 | 2.24 | 1.06 | 1.77 | 10.4 |
| "End"..... | 166 | $2.70 \times 10^4$ | 0.98 | 0.98 | 10.4 | 23.8 | 4.20 | 15.6 | 8.07 | 1.71 | 2.82 | 1.14 | 2.20 | 9.98 |

NOTE.—$t$ (asymptotic branch) = $1.58 \times 10^6$ yr.

TABLE 3

CHARACTERISTICS OF A MODEL OF INITIAL MASS $M_*(0) = 3$

| Phase | $N$ | $\Delta t$ (yr) | $M_*$ | $M_c$ | $\bar{Y}_{12}^e$ | $\bar{Y}_{22}^e$ | $\bar{Y}_l^e$ | $\Lambda_e$ | $\bar{Y}_{12}^w$ | $\bar{Y}_{22}^w$ | $\bar{Y}_l^w$ | $\bar{Y}_s^w$ | $\Lambda_w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{Y}_s^e = 2$................. | 136 | $1.46 \times 10^6$ | 2.05 | 0.96 | 1.64 | 2.39 | 1.06 | 11.0 | 0.92 | 1.37 | 1.01 | 1.18 | 8.72 |
| Carbon star ($\bar{Y}_s^e = 2.35$).. | 143 | $2.51 \times 10^4$ | 2.02 | 0.97 | 1.82 | 2.61 | 1.09 | 10.6 | 0.94 | 1.40 | 1.01 | 1.21 | 8.84 |
| Mira ($\bar{Y}_s^e = 3.1$)......... | 164 | $6.57 \times 10^4$ | 1.93 | 0.98 | 2.37 | 3.29 | 1.20 | 9.69 | 1.04 | 1.53 | 1.02 | 1.33 | 9.06 |
| Planetary ($\bar{Y}_s^e = 3.3$)..... | 168 | $1.20 \times 10^4$ | 1.91 | 0.99 | 2.48 | 3.43 | 1.22 | 9.54 | 1.06 | 1.56 | 1.02 | 1.36 | 9.08 |
| $\bar{Y}_s^e = 8$................ | 257 | $2.09 \times 10^5$ | 1.55 | 1.04 | 5.44 | 7.21 | 2.44 | 7.40 | 1.77 | 2.48 | 1.19 | 2.43 | 8.56 |
| $\bar{Y}_s^e = 16$................ | 337 | $1.28 \times 10^4$ | 1.23 | 1.07 | 10.0 | 13.6 | 6.55 | 6.46 | 2.75 | 3.76 | 1.67 | 4.02 | 7.71 |
| $\bar{Y}_s^e = 32.5$.............. | 371 | $4.22 \times 10^4$ | 1.08 | 1.08 | 19.7 | 32.5 | 24.5 | 6.21 | 3.49 | 4.84 | 2.35 | 5.26 | 7.30 |

NOTE.—$t$ (asymptotic branch) = $1.57 \times 10^6$ yr.

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TABLE 4

CHARACTERISTICS OF A MODEL OF INITIAL MASS $M_*(0) = 4$

| Phase | $N$ | $\Delta t$ (yr) | $M_*$ | $M_c$ | $\bar{Y}_{12}^e$ | $\bar{Y}_{22}^e$ | $\bar{Y}_l^e$ | $\Lambda_e$ | $\bar{Y}_{12}^w$ | $\bar{Y}_{22}^w$ | $\bar{Y}_l^w$ | $\bar{Y}_s^w$ | $\Lambda_w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{Y}_s^e = 2$ | 147 | $9.93 \times 10^5$ | 3.32 | 0.98 | 1.49 | 2.03 | 1.08 | 9.02 | 0.84 | 1.25 | 1.01 | 1.16 | 7.63 |
| Carbon star ($\bar{Y}_s^e = 2.44$) | 172 | $6.22 \times 10^4$ | 3.27 | 1.00 | 1.76 | 2.30 | 1.13 | 8.63 | 0.90 | 1.32 | 1.02 | 1.24 | 7.79 |
| $\bar{Y}_s^e = 4$ | 265 | $1.72 \times 10^5$ | 3.09 | 1.05 | 2.70 | 3.10 | 1.43 | 7.57 | 1.16 | 1.59 | 1.06 | 1.62 | 7.88 |
| $\bar{Y}_s^e = 8$ | 536 | $2.74 \times 10^5$ | 2.71 | 1.13 | 5.02 | 4.38 | 2.85 | 6.35 | 1.94 | 2.24 | 1.35 | 2.89 | 7.23 |
| $\bar{Y}_s^e = 12$ | 851 | $1.70 \times 10^5$ | 2.39 | 1.17 | 7.32 | 5.01 | 5.19 | 5.89 | 2.78 | 2.73 | 1.86 | 4.30 | 6.70 |
| Planetary ($\bar{Y}_s^e = 12.3$) | 877 | $1.08 \times 10^4$ | 2.37 | 1.18 | 7.50 | 5.04 | 5.42 | 5.87 | 2.84 | 3.76 | 1.91 | 4.41 | 6.67 |
| $\bar{Y}_s^e = 16$ | 1187 | $1.06 \times 10^5$ | 2.11 | 1.21 | 9.63 | 5.32 | 8.44 | 5.69 | 3.61 | 3.09 | 2.57 | 5.72 | 6.37 |
| $\bar{Y}_s^e = 24$ | 1794 | $1.36 \times 10^5$ | 1.69 | 1.25 | 14.3 | 5.55 | 17.7 | 5.61 | 5.11 | 3.52 | 4.37 | 8.28 | 6.05 |
| $\bar{Y}_s^e = 68.5$ | 2451 | $9.27 \times 10^4$ | 1.28 | 1.28 | 41.7 | 6.28 | 167. | 6.92 | 7.21 | 3.84 | 8.72 | 11.8 | 5.93 |

NOTE.—$t$ (asymptotic branch) = $1.68 \times 10^6$ yr.

TABLE 5

CHARACTERISTICS OF A MODEL OF INITIAL MASS $M_*(0) = 5$

| Phase | $N$ | $\Delta t$ (yr) | $M_*$ | $M_c$ | $\bar{Y}_{12}^e$ | $\bar{Y}_{22}^e$ | $\bar{Y}_l^e$ | $\Lambda_e$ | $\bar{Y}_{12}^w$ | $\bar{Y}_{22}^w$ | $\bar{Y}_l^w$ | $\bar{Y}_s^w$ | $\Lambda_w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{Y}_s^e = 2$ | 155 | $6.89 \times 10^5$ | 4.47 | 1.00 | 1.39 | 1.82 | 1.09 | 7.87 | 0.81 | 1.19 | 1.01 | 1.17 | 6.88 |
| Carbon star | 203 | $8.82 \times 10^4$ | 4.41 | 1.03 | 1.71 | 2.07 | 1.17 | 7.52 | 0.89 | 1.28 | 1.02 | 1.29 | 7.04 |
| $\bar{Y}_s^e = 4$ | 355 | $1.88 \times 10^5$ | 4.24 | 1.09 | 2.57 | 2.59 | 1.50 | 6.76 | 1.16 | 1.51 | 1.09 | 1.72 | 7.06 |
| $\bar{Y}_s^e = 8$ | 889 | $3.30 \times 10^5$ | 3.85 | 1.19 | 4.87 | 3.15 | 3.10 | 5.85 | 2.03 | 1.99 | 1.47 | 3.16 | 6.49 |
| $\bar{Y}_s^e = 16$ | 3425 | $3.86 \times 10^5$ | 3.14 | 1.32 | 9.53 | 3.20 | 8.90 | 5.44 | 3.99 | 2.46 | 3.08 | 6.53 | 5.83 |
| Planetary ($\bar{Y}_s^e = 21.2$) | 6800 | $1.58 \times 10^5$ | 2.71 | 1.37 | 12.7 | 3.11 | 14.5 | 5.45 | 5.32 | 2.59 | 4.67 | 8.79 | 5.67 |
| $\bar{Y}_s^e = 24.3$ | 9171 | $6.96 \times 10^4$ | 2.52 | 1.40 | 14.5 | 3.05 | 18.6 | 5.49 | 6.09 | 2.63 | 5.76 | 10.1 | 5.63 |

NOTE.—$t$ (asymptotic branch) = $1.84 \times 10^6$ yr.

TABLE 6

CHARACTERISTICS OF A MODEL OF INITIAL MASS $M_*(0) = 6$

| Phase | $N$ | $\Delta t$ (yr) | $M_*$ | $M_c$ | $\bar{Y}_{12}^e$ | $\bar{Y}_{22}^e$ | $\bar{Y}_l^e$ | $\Lambda_e$ | $\bar{Y}_{12}^w$ | $\bar{Y}_{22}^w$ | $\bar{Y}_l^w$ | $\bar{Y}_s^w$ | $\Lambda_w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{Y}_s^e = 2$ | 158 | $4.75 \times 10^5$ | 5.58 | 1.02 | 1.31 | 1.64 | 1.10 | 6.90 | 0.79 | 1.14 | 1.01 | 1.18 | 6.21 |
| Carbon star ($\bar{Y}_s^e = 2.6$) | 238 | $1.12 \times 10^5$ | 5.50 | 1.06 | 1.68 | 1.87 | 1.21 | 6.64 | 0.89 | 1.23 | 1.03 | 1.35 | 6.35 |
| $\bar{Y}_s^e = 4$ | 460 | $1.93 \times 10^5$ | 5.35 | 1.12 | 2.48 | 2.20 | 1.55 | 6.14 | 1.17 | 1.42 | 1.11 | 1.80 | 6.36 |
| $\bar{Y}_s^e = 8$ | 1641 | $2.06 \times 10^5$ | 4.96 | 1.24 | 4.78 | 2.44 | 3.23 | 5.50 | 2.09 | 1.78 | 1.56 | 3.38 | 5.93 |
| $\bar{Y}_s^e = 12$ | 4486 | $1.64 \times 10^5$ | 4.56 | 1.34 | 7.12 | 2.41 | 5.72 | 5.33 | 3.14 | 1.95 | 2.34 | 5.19 | 5.64 |
| $\bar{Y}_s^e = 15.2$ | 9142 | $2.60 \times 10^5$ | 4.27 | 1.40 | 9.01 | 2.37 | 8.27 | 5.30 | 4.03 | 2.03 | 3.17 | 6.71 | 5.51 |

NOTE.—$t$ (asymptotic branch) = $1.57 \times 10^6$ yr.

TABLE 7

CHARACTERISTICS OF A MODEL OF INITIAL MASS $M_*(0) = 7$

| Phase | $N$ | $\Delta t$ (yr) | $M_*$ | $M_c$ | $\bar{Y}_{12}^e$ | $\bar{Y}_{22}^e$ | $\bar{Y}_l^e$ | $\Lambda_e$ | $\bar{Y}_{12}^w$ | $\bar{Y}_{22}^w$ | $\bar{Y}_l^w$ | $\bar{Y}_s^w$ | $\Lambda_w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{Y}_s^e = 2$ | 162 | $3.24 \times 10^5$ | 6.65 | 1.05 | 1.26 | 1.49 | 1.12 | 6.22 | 0.78 | 1.10 | 1.02 | 1.19 | 5.83 |
| Carbon star ($\bar{Y}_s^e = 2.66$) | 283 | $1.27 \times 10^5$ | 6.57 | 1.09 | 1.65 | 1.68 | 1.25 | 6.04 | 0.90 | 1.19 | 1.05 | 1.39 | 5.92 |
| $\bar{Y}_s^e = 4$ | 599 | $1.92 \times 10^5$ | 6.43 | 1.15 | 2.42 | 1.87 | 1.61 | 5.70 | 1.18 | 1.34 | 1.14 | 1.87 | 5.89 |
| $\bar{Y}_s^e = 6$ | 1381 | $2.21 \times 10^5$ | 6.23 | 1.23 | 3.57 | 1.95 | 2.36 | 5.42 | 1.64 | 1.49 | 1.35 | 2.68 | 5.72 |
| $\bar{Y}_s^e = 8$ | 2811 | $1.79 \times 10^5$ | 6.03 | 1.30 | 4.72 | 1.96 | 3.32 | 5.27 | 2.16 | 1.58 | 1.66 | 3.57 | 5.56 |
| $\bar{Y}_s^e = 10$ | 5450 | $1.54 \times 10^5$ | 5.82 | 1.36 | 5.90 | 1.94 | 4.46 | 5.21 | 2.71 | 1.65 | 2.04 | 4.51 | 5.44 |
| $\bar{Y}_s^e = 11.6$ | 9097 | $1.10 \times 10^5$ | 5.67 | 1.40 | 6.86 | 1.92 | 5.53 | 5.19 | 3.17 | 1.70 | 2.41 | 5.31 | 5.37 |

NOTE.—$t$ (asymptotic branch) = $1.31 \times 10^6$ yr.

TABLE 8

CHARACTERISTICS OF A MODEL OF INITIAL MASS $M_*(0) = 8$

| Phase | $N$ | $\Delta t$ (yr) | $M_*$ | $M_c$ | $\bar{Y}_{12}^e$ | $\bar{Y}_{22}^e$ | $Y_l{}^{-e}$ | $\Lambda_e$ | $\bar{Y}_{12}^w$ | $\bar{Y}_{22}^w$ | $\bar{Y}_l^w$ | $\bar{Y}_s^w$ | $\lambda_w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{Y}_s^e = 2$ | 206 | $2.39 \times 10^5$ | 7.68 | 1.09 | 1.22 | 1.28 | 1.16 | 5.68 | 0.77 | 1.06 | 1.03 | 1.18 | 5.58 |
| Carbon star ($\bar{Y}_s^e = 2.73$) | 406 | $1.27 \times 10^5$ | 7.60 | 1.14 | 1.64 | 1.39 | 1.33 | 5.52 | 0.89 | 1.11 | 1.07 | 1.41 | 5.58 |
| $\bar{Y}_s^e = 4.0$ | 905 | $1.37 \times 10^5$ | 7.47 | 1.20 | 2.37 | 1.47 | 1.70 | 5.38 | 1.16 | 1.19 | 1.17 | 1.89 | 5.51 |
| $\bar{Y}_s^e = 6.0$ | 2369 | $2.26 \times 10^5$ | 7.27 | 1.28 | 3.52 | 1.49 | 2.47 | 5.16 | 1.65 | 1.27 | 1.42 | 2.74 | 5.36 |
| $\bar{Y}_s^e = 8.0$ | 5550 | $1.89 \times 10^5$ | 7.07 | 1.36 | 4.96 | 1.48 | 3.42 | 5.10 | 2.20 | 1.32 | 1.76 | 3.69 | 5.26 |
| $\bar{Y}_s^e = 9.19$ | 9007 | $9.97 \times 10^4$ | 6.94 | 1.40 | 5.39 | 1.47 | 4.07 | 5.08 | 2.54 | 1.34 | 1.99 | 4.28 | 5.21 |

NOTE.—$t$ (asymptotic branch) = $1.06 \times 10^6$ yr.

TABLE 9

CHARACTERISTICS OF FINAL MODELS

| $M_*(0)$ | $M_R$ | $M_W$ | $M_P$ | $\bar{Y}_{12}^e$ | $\bar{Y}_s^e$ | $\bar{Y}_{12}^w$ | $\bar{Y}_s^w$ | $M_{PR}$ | $M_{DG}$ | $\Lambda_e$ | $\Lambda_w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0....... | 0.56 | 0.44 | 0 | 0.81 | 1.0 | 0.81 | 1.0 | 0.08 | 0.0 | 5.5 | 5.5 |
| 1.25...... | 0.63 | 0.62 | 0 | 0.75 | 1.0 | 0.75 | 1.00 | 0.13 | 0.0 | 5.5 | 5.5 |
| 1.50...... | 0.70 | 0.80 | 0 | 0.83 | 1.0 | 0.70 | 1.00 | 0.18 | 5.32/–4 | 9.1 | 5.9 |
| 2.0....... | 0.79 | 0.82 | 0.39 | 0.97 | 1.05 | 0.73 | 1.01 | 0.24 | 3.47/–3 | 12.0 | 6.65 |
| 2.5....... | 0.89 | 0.91 | 0.70 | 1.33 | 1.38 | 0.82 | 1.05 | 0.30 | 1.20/–2 | 13.7 | 8.30 |
| 3.0....... | 0.99 | 1.09 | 0.92 | 2.48 | 3.31 | 1.06 | 1.36 | 0.39 | 3.73/–2 | 9.5 | 9.1 |
| 3.5....... | 1.08 | 1.35 | 1.07 | 4.87 | 7.57 | 1.79 | 2.55 | 0.51 | 0.10 | 6.8 | 7.8 |
| 4.0....... | 1.18 | 1.64 | 1.18 | 7.50 | 12.3 | 2.84 | 4.41 | 0.65 | 0.18 | 5.9 | 6.7 |
| 5.0....... | 1.38 | 2.28 | 1.34 | 12.65 | 21.2 | 5.32 | 8.80 | 0.97 | 0.38 | 5.4 | 5.7 |
| 6.0....... | 1.40 | 1.73 | (2.87) | 9.01 | 15.2 | 4.03 | 6.71 | 0.94 | 0.40 | 5.3 | 5.5 |
| 7.0....... | 1.40 | 1.33 | (4.27) | 6.86 | 11.6 | 3.17 | 5.31 | 0.86 | 0.39 | 5.2 | 5.4 |
| 8.0....... | 1.40 | 1.06 | (5.54) | 5.39 | 9.9 | 2.54 | 4.28 | 0.75 | 0.36 | 5.1 | 5.2 |

reason that *s*-process elements are not significantly enhanced for models less massive than about 3 $M_\odot$ and at luminosities less than $M_{bol} \approx -6$, whatever the dredge-up law adopted. From Figure 2 and Tables 3–8 it appears that, in luminous carbon stars of initial mass larger than about 3 $M_\odot$, typical *s*-process enhancement factors should lie between about 4 and 8. Typical $^{12}$C enhancements are from 2.5 to 5.

From Table 9 we see that the enhancement of *s*-process elements at the stellar surface and the enhancement of the $^{12}$C abundance there are well correlated and simultaneously reach maxima toward the end of the evolution of a star of initial mass $M_*(0) \sim 5$. The occurrence of a maximum follows from the facts that (1) neither dredge-up nor $^{22}$Ne burning occurs for C-O core masses below a critical core mass, (2) with increasing initial stellar mass, the amount of matter between the initial C-O core mass (at the start of the thermal pulsing phase) and the Chandrasekhar mass ($M_c = 1.4$) decreases, and (3) beyond another critical mass, the amount of "diluting" matter between the thermally pulsing region and the stellar surface increases with increasing initial stellar mass.

Of interest is the value of the parameter $\Lambda$ achieved in the stellar envelope and in the wind ejectum. For initial masses greater than about 5 $M_\odot$, $\Lambda$ in both the envelope and the ejectum reach final values within 10% of 5.5, the value we have assumed for *s*-process elements in the solar system distribution.

However, $\Lambda$ is significantly larger than this in the lowest mass stars capable of developing carbon star characteristics via the dredge-up mechanism and simultaneously large enhancements of *s*-process elements. For example, for $3 < M_*(0) < 3.5$, where final surface abundances of $^{12}$C and *s*-process elements respectively reach $2.5 < \bar{Y}_{12}^e < 5.0$ and $3 < \bar{Y}_s^e < 8$, we find $6.8 < \Lambda_e < 11.4$. Thus, the *s*-process elements in the lowest-luminosity carbon stars for which the dredge-up phenomenon is responsible should be characterized by a $\Lambda$ that is observationally distinguishable from the $\Lambda$ characterizing *s*-process elements in the highest-luminosity carbon stars.

The surface ratio of $^{12}$C to $^{13}$C is also of some interest. We have seen that this ratio should initially be in the neighborhood of $20 \pm 5$ and, for low-mass stars, remain at this value for the entire asymptotic-branch phase. However, in more massive stars, $Y_{12}^e/Y_{13}^e$ should increase with time, unless $^{12}$C is converted into $^{13}$C and $^{14}$N in substantial amounts at the base of the convective envelope. For example, in the model of initial mass $M_*(0) = 5$, the ratio $^{12}$C/$^{13}$C increases from 22 to 57 during the asymptotic-branch phase if $^{12}$C does not burn in the envelope.

Whether or not $^{12}$C will burn at the base of the convective envelope is something which can be determined only from the observations, if at all. Burning in stellar models can be controlled simply by adjusting the ratio of mixing length to scale height (Iben 1975*a*, 1976).

Several interpretations of the observational evidence (Fujita, Tsuji, and Maehara 1966; Scalo 1977) suggest typical values of $^{12}$C/$^{13}$C near 20, consistent with theoretical predictions for the early asymptotic-branch phase. However, they also suggest that ratios significantly smaller than this can occur. If much smaller ratios do occur, they may either be a consequence of meridional mixing during the main-sequence phase (e.g., Iben 1969; Paczyński 1973; Dearborn and Eggleton 1977) or be a consequence of burning at the base of the convective envelope during the asymptotic-branch phase. In this latter instance, the abundance of $^{14}$N at the surface must increase dramatically, a circumstance which should be subject to observational test. In fact, the abundances of carbon and nitrogen in planetary nebulae perhaps already supply such a test. In one such nebula, carbon and nitrogen are enhanced, respectively, by factors of 9 and 4 (Torres-Peimbert and Peimbert 1977). Whereas the enhancement of carbon is not unexpected, the enhancement of nitrogen is a factor of 2 larger than is expected to occur in the absence of a substantial conversion of $^{12}$C into $^{14}$N during the asymptotic-branch phase, and one infers that roughly 10% of the fresh $^{12}$C injected into the convective envelope has been converted into $^{14}$N prior to the formation of the planetary nebula.

## V. ON THE NECESSITY FOR AN ALTERNATE SOURCE OF $^{12}$C AND OF *S*-PROCESS ELEMENTS

Because of quite large observational and theoretical uncertainties, it is difficult to place carbon stars reliably in the theoretical H-R diagram for comparison with models. Scalo (1976*b*) has collated much of the available information and has attempted to locate systematically many known N-type carbon stars (C > O) and S stars (C ~ O) in this diagram. It would appear that the distribution of S stars, although overlapping the distribution of N-type carbon stars, is on the whole somewhat cooler and less luminous than the N-type star distribution. One infers that S stars are on the whole somewhat less massive and/or somewhat more metal rich than N stars.

One clear fact emerges. If the luminosity estimates can be trusted, and if the dredge-up law we have adopted is approximately correct, *then the bulk of all N-stars and S-stars do not owe their high surface abundances of* $^{12}$C *and of s-process elements to the thermal-pulse*, $^{22}$Ne-*source mechanism*, which produces large enhancements of $^{12}$C and of *s*-process elements only when $M_{\rm BOL} \leqslant -6$. On the other hand, it is definitely established that at least some carbon stars in the Magellanic Clouds are sufficiently bright to be explicable by this mechanism (Crabtree, Richer, and Westerlund 1976).

The apparent fact that S stars can exhibit enhancements of *s*-process elements as large as 20 (Boesgaard 1970) and yet have surface ratios of C to O close to one (Greene and Wing 1975; Wyckoff and Wehinger 1976) is further very strong evidence that these two S-star characteristics cannot be produced primarily as a consequence of the thermal pulse, $^{22}$Ne-source mechanism which predicts a tight correlation between the surface $^{12}$C abundance and the abundances of *s*-process elements. However, the existence of $^{99}$Tc in many S stars (Merrill 1952*a*, *b*) is a clear indication that such stars are experiencing thermal pulses at intervals of less than $10^5$ yr.

Perhaps the simplest explanation of S stars and of many N-type carbon stars is that their surface composition characteristics originate during a helium flash of an infrequently occurring nature (e.g., Truran and Iben 1977). We suppose that, in a few low-mass stars [$M_*(0) \leqslant 2.25$], angular momentum and magnetic field configurations are such that the convective core that is engendered by the flash engulfs a small part of the hydrogen-rich envelope. The protons injected into the core will react with the $^{12}$C already there or produced there as a consequence of the triple-$\alpha$ reactions to form, after a $\beta$-decay, $^{13}$C. The $^{13}$C in turn will capture $\alpha$-particles to act as a neutron source for the production of *s*-process elements. The *s*-process elements will then be convected into the hydrogen-rich envelope in exchange for the core-injected hydrogen.

The abundance by number of fresh $^{12}$C produced in the core during the flash is on the order of $Y_{12} \sim 10^{-3}$ (e.g., Iben and Rood 1970), which is comparable to the initial abundance of CNO nuclei in stars of solar-type initial composition. Thus, during the helium flash in a disk-population star, the maximum number of neutrons that may be emitted per seed nucleus [$Y_{56}(0) \sim Y_{\rm CNO}(0)/40$] is on the order of 40, and perhaps half of this number is captured by those $^{14}$N nuclei (and their progeny) that are not converted into $^{22}$Ne during the flash. Further, since *s*-process elements are produced at the expense of $^{12}$C, the ultimate surface ratio of C to O should not exceed the initial ratio by much (say a factor of 2).

In a star of lower metal abundance, however, although the maximum number of neutrons that can potentially be captured by seed nuclei is roughly the same as in a Population I star, the abundance of seed nuclei is much smaller, and the extent to which the *s*-process may be driven is consequently much greater. Further, the ultimate surface ratio of C to O could greatly exceed unity, since the amount of fresh $^{12}$C made in the core far exceeds the initial abundance of CNO elements there.

We suggest, then, that S stars and many N stars are low-mass stars in the early asymptotic-branch phase of evolution (following a helium flash of an anomalous type) and are characterized by a surface *s*-process distribution in which the abundance of heavier neutron-rich elements (say in the Ba peak) is larger relative to the abundance of lighter neutron-rich elements (say in the Sr peak) for stars with a lower surface abundance of $^{56}$Fe. Further, the abundance of $^{12}$C relative to the abundance of $^{16}$O should also be inversely correlated with the abundances of metals (actually the initial abundance of CNO nuclei). On this picture, then, S stars should, as a class, be more metal-rich than N stars. Two additional pieces of evidence support this interpretation. First, for a given luminosity, S stars tend to be cooler than N stars; theoretical models of a given luminosity are cooler, the larger the assumed abundance of heavy elements. Second, the velocity dispersion of early (spectral) type carbon stars is significantly larger than that of late-type carbon stars (Dean 1976); it is known that kinematical properties and population types are related in the sense that the smaller the mean metal abundance of a group, the greater is its velocity dispersion.

If many carbon stars and S stars owe their peculiar abundance properties to a mechanism that operates during the helium flash, then progenitors of such stars should exhibit similar peculiar abundances during the core helium phase which follows the helium flash. We suggest that barium stars (of a luminosity characteristic of core helium-burning stars) may be the progenitors of S stars and that S stars are simply barium stars that are in the double-shell-source stage of evolution with a steadily increasing core mass. Mass loss and/or planetary nebula formation terminates their evolution before large quantities of fresh carbon produced during thermal pulses can be dredged up into the envelope.

One strong argument in favor of the suggested temporal relationship between S stars and barium stars is the fact that the abundance of C to O does not differ much from unity in both types of star and the enhancement of *s*-process elements embraces the same range in both cases, with the upper end of the range exceeding the typical enhancements expected on the thermal pulse model. The studies of Williams (1975) and Pilachowski (1977) reveal that the enhancement of Ba in barium stars ranges continuously up to a factor of about 20, and Boesgaard (1970) finds that the ratio of Zr to Ti ranges continuously between 3 and 20.

An argument against a temporal relationship is based on the apparent frequencies of the two stellar types. MacConnell, Frye, and Upgren (1973) argue that Ba stars comprise roughly 1% of G5–K5 giants, whereas Keenan (1954) estimates that S stars are roughly 1/20 as frequent as normal M stars and carbon stars. If one assumes that all low-mass horizontal-branch stars become supergiants, then, since supergiants are formed also from stars of intermediate and high mass, less than 1% of all supergiants can have descended from Ba stars. Thus only a fraction of all S stars could have evolved from Ba stars. The only way to avoid this conclusion is if one or the other (or both) of the frequency estimates is in error by an order of magnitude.

## VI. ELEMENT ENHANCEMENT IN THE INTERSTELLAR MEDIUM

In order to make a crude estimate of the contribution of stars to the element enrichment of the interstellar medium, we have adopted a birthrate function consistent with the Salpeter (1955) mass function (but see Becker, Iben, and Tuggle 1977) and have assumed that the probability for stellar formation in the interval $d \log M_*(0)$ about $M_*(0)$ is

$$dP = [M_*(0)]^{-1.35} d \log [M_*(0)] . \qquad (35)$$

We neglect the contribution of stars less massive than 1 $M_\odot$ (lifetime greater than the age of the Galaxy) and assume that no stars are born with initial mass larger than 64 $M_\odot$. We assume further that stars with $M_*(0) > 8$ do not develop an electron-degenerate carbon-oxygen core and therefore do not pass through a thermal pulse phase but continue to burn nuclear fuel under nondegenerate conditions until a $^{56}$Ni–$^{56}$Fe core of mass 1.4 $M_\odot$ is formed. If we assume that all mass in excess of 1.4 $M_\odot$ is returned to the interstellar medium via a supernova explosion, the contribution in mass to the interstellar medium by stars with $M_*(0) \geq 8$ may be measured by

$$M_{\text{out}}{}^B = \int_{M_*(0)=8}^{M_*(0)=64} [M_*(0) - 1.4] dP \approx 0.285 . \qquad (36)$$

For stars initially less massive than 8 $M_\odot$ we must perform a numerical integration to obtain

$$M_{\text{out}}{}^A = \int_{M_*(0)=1}^{M_*(0)=8} [M_*(0) - M_R] dP , \qquad (37)$$

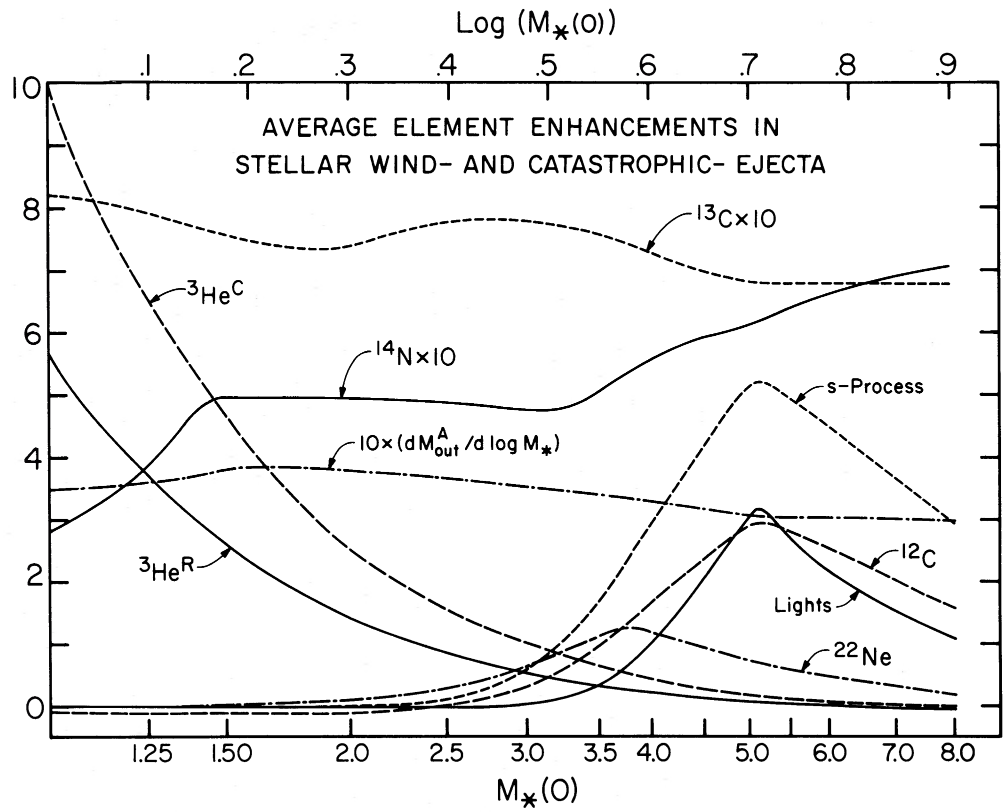


FIG. 3.—Relative contributions of stars of low and intermediate mass to the enhancement of several elements. Actually shown are differential contributions $\bar{Y}_i^{\text{net}}$ assuming that the average stellar birthrate is proportional to $dP$ in eqs. (35).

where $M_R$ is a remnant mass that is given in Table 1 for various choices of $M_*(0)$.

As shown in Figure 3, the derivative $dM_{\rm out}{}^A/d \log M_*(0)$ remains relatively constant over the entire range. Integrating under the curve gives

$$M_{\rm out}{}^A \sim 0.436\,. \quad (38)$$

The total mass ejected by all stars is therefore measured by

$$M_{\rm out} = M_{\rm out}{}^A + M_{\rm out}{}^B \approx 0.72\,. \quad (39)$$

A measure of the contribution of stars to the abundance of a given element in the interstellar medium is given by the difference $\bar{Y}_i^{\rm net}$ between

$$\bar{Y}_i^{\rm out} = \int_{M_*(0)=1}^{M_*(0)=64} [Y_i^e M_e + Y_i^w M_w] dP/(Y_i^s M_{\rm out}) \quad (40)$$

and

$$\bar{Y}_i^{\rm in} = Y_i^0/Y_i^s = \bar{Y}_i\,, \quad (41)$$

where $Y_i^s$ is the abundance by number of the $i$th element in the solar system distribution, $Y_i^0$ is the abundance by number of the $i$th element characterizing stars of a given generation at birth, and $Y_i^e$ and $Y_i^w$ are, respectively, the abundances by number of the $i$th element in the envelope (mass $M_e$) of a star just prior to its ejection of a planetary or of a supernova shell (of mass $M_e$) and in the matter (of mass $M_w$) ejected via a stellar wind prior to catastrophic envelope ejection.

Adopting the values of $Y_i^s$ and $Y_i^0$ given in § II, we show in Figure 3 the products $M_{\rm out}\, d\bar{Y}_i^{\rm net}/d \log [M_*(0)]$ that characterize several elements.

As an illustration of the uncertainties involved we shall first discuss the abundance of $^3$He. If the abundance of $^3$He in the solar system is $Y_3{}^s \sim 8 \times 10^{-6}$, as given by Cameron (1973), then the total contribution to $\bar{Y}_3{}^{\rm net}$ from stars less massive than 8 $M_\odot$ is equal to the area under the dashed curve labeled $^3$He$^C$ in Figure 3, namely, $\Delta \bar{Y}_3{}^{\rm net} \sim 2.10/M_{\rm out}$. If the abundance of $^3$He in the envelope of massive stars is approximated by equation (18), then stars initially more massive than 8 $M_\odot$ contribute $\Delta \bar{Y}_3{}^{\rm net} \approx -0.04/M_{\rm out}$. Thus, the net contribution of all stars to the enhancement of $^3$He is

$$\bar{Y}_3{}^{\rm net} \approx 2.86\,. \quad (42)$$

There is considerable uncertainty as to the appropriate choice for the solar system $^3$He abundance (Truran and Cameron 1975; Reeves 1974; Rood, Steigman, and Tinsley 1976). If we choose, with Reeves, $Y_3{}^s \approx 1.4 \times 10^{-5}$, a repeat of the exercise we have just completed gives

$$\bar{Y}_3{}^{\rm net} \sim 1.59\,. \quad (42')$$

In a similar fashion we can estimate the enrichment in $^{14}$N and $^{12}$C. Stars less massive than 8 $M_\odot$ contribute $\bar{Y}_{14}{}^{\rm net} \sim 0.47\,\bar{Y}_{12}{}^0/M_{\rm out}$ to the net enrichment of $^{14}$N. In more massive stars (Iben 1966*a*, *b*; Lamb, Iben, and Howard 1976), the original CNO elements are destroyed over the inner one-third of the star's mass prior to an explosive event. In the outer two-thirds of the star, roughly one-third of the original $^{12}$C and roughly one-sixth of the original $^{16}$O have been converted into $^{14}$N. Thus the net contribution to $^{14}$N of stars between 8 $M_\odot$ and 64 $M_\odot$ can be estimated as $\Delta \bar{Y}_{14}{}^{\rm net} \sim (-0.08\,\bar{Y}_{14}{}^0 + 0.22\,\bar{Y}_{12}{}^0 + 0.20\,\bar{Y}_{16}{}^0)/M_{\rm out}$, giving a total net enhancement of

$$\bar{Y}_{14}{}^{\rm net} \sim -0.11\,\bar{Y}_{14}{}^0 + 0.96\,\bar{Y}_{12}{}^0 + 0.28\,\bar{Y}_{16}{}^0\,; \quad (43)$$

or, if we adopt solar system ratios for the initial CNO elements,

$$\bar{Y}_{14}{}^{\rm net} \sim 1.1\,\bar{Y}_{\rm CNO}{}^0\,. \quad (43')$$

The contribution of stars less massive than 8 $M_\odot$ to the enhancement of $^{12}$C is

$$\Delta \bar{Y}_{12}{}^{\rm net} \approx 1.27 - 0.20\,\bar{Y}_{12}{}^0\,, \quad (44)$$

whereas stars more massive than 8 $M_\odot$ contribute

$$\Delta \bar{Y}_{12}{}^{\rm net} \approx (0.285/0.72)\,\bar{Y}_{12}{}^m - 0.10\,\bar{Y}_{12}{}^0$$
$$\approx 0.4\,\bar{Y}_{12}{}^m - 0.10\,\bar{Y}_{12}{}^0\,, \quad (45)$$

where $\bar{Y}_{12}{}^m$ is the mean abundance of $^{12}$C that is freshly created in massive stars during helium burning and subsequently incorporated into the ultimate ejecta of such stars. The total net enhancement is thus

$$\bar{Y}_{12}{}^{\rm net} \approx 1.27 + 0.4\,\bar{Y}_{12}{}^m - 0.3\,\bar{Y}_{12}{}^0\,. \quad (46)$$

The total enhancement of $^{13}$C is found to be

$$\bar{Y}_{13}{}^{\rm net} \approx 1.5\,\bar{Y}_{12}{}^0 \quad (47)$$

if it is assumed that $Y_{13}{}^s/Y_{12}{}^2 = 90$ and

$$\bar{Y}_{13}{}^{\rm net} \approx 0.7\,\bar{Y}_{12}{}^0 \quad (47')$$

if it is assumed that $Y_{13}{}^s/Y_{12}{}^s = 45$.

If we neglect the creation of *s*-process elements in the range $90 < A < 204$ in stars initially more massive than 8 $M_\odot$, then the enhancement of these elements in the interstellar medium due to less massive stars follows from our model as

$$\bar{Y}_s{}^{\rm net} \approx 2.0 \quad (48)$$

if $Y_s{}^s \sim Y_{56}{}^s/200$, and as half of this is $Y_s{}^s \sim Y_{56}{}^s/100$. Similarly, the contribution of models less massive than 8 $M_\odot$ to the enhancement of $^{22}$Ne and of the light-element progeny of $^{22}$Ne is

$$\Delta \bar{Y}_{22}{}^{\rm net} \approx 0.5\,, \quad (49)$$

and

$$\Delta \bar{Y}_l{}^{\rm net} \approx 0.9\,. \quad (50)$$

For $^4$He we find that stars of mass less than 8 $M_\odot$ contribute an enhancement of $\Delta \bar{Y}_4{}^{\rm net} = 0.05$. In more massive stars (e.g., Lamb, Iben, and Howard 1976) approximately 2 $M_\odot$ of nearly pure helium remains outside nuclear-burning regions prior to an explosive event. Hence such stars contribute $\Delta Y_4^{\rm net} \sim 0.05$ also, so that, from all stars,

$$\bar{Y}_4^{\rm net} \sim 0.1 \,. \tag{51}$$

A comparison of equations (42)–(51) suggests that, in order to achieve solar system abundances of $^3$He, $^{13}$C, $^{14}$N, and *s*-process elements over the lifetime of the Galaxy, it is necessary that the average enhancement in stars be on the order of $\bar{Y}_i^{\rm net} \sim 1$–2. From equations (49) and (50) we conclude that stars less massive than 8 $M_\odot$ contribute from one-quarter to one-half of the solar system abundances of $^{22}$Ne and its light-element progeny, and from equations (44) and (45) we infer that such stars also produce at least one-half of the solar system abundance of $^{12}$C. Very little new $^4$He is produced (see also Gingold 1978). Since some $^{12}$C may be converted into $^{14}$N at the base of the convective envelope during the asymptotic-branch phase, our estimate of $^{14}$N enrichment is a lower limit.

The enrichments we have obtained are all due to models of Population I composition. This is appropriate for the "second generation" elements $^{13}$C, $^{14}$N, and the heavy *s*-process elements which require the presence of progenitors made in an earlier generation. It is probably all right also for $^3$He which is made during the main-sequence phase and therefore is not affected by the mass-loss rate on the asymptotic branch. However, the enrichments in $^{12}$C, $^{22}$Ne, and the light-element progeny of $^{22}$Ne and $^{25}$Mg could be a strong function of composition. For a given luminosity, the radius of a star on the asymptotic branch roughly halves for every factor of 100 decrease in the metal abundance (Iben 1977*a*). If the mass-loss rate is the same function of mass, luminosity, and radius for a Population II star as it is for a Population I star, then a Population II star will lose mass on the asymptotic branch less rapidly than will a Population I star. The net result will be the production of more $^{12}$C, $^{22}$Ne, and "lights." On the other hand, the temperature at the base of the convective envelope increases with decreasing radius, all other things being equal. Hence the conversion of $^{12}$C into $^{13}$C and $^{14}$N might proceed more rapidly and further in Population II asymptotic-branch stars. An adequate answer to questions such as these will require a more complete understanding of the composition dependence of the characteristics of asymptotic-branch models.

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