

STELLAR MODEL CHROMOSPHERES. VI. EMPIRICAL ESTIMATES OF THE CHROMOSPHERIC RADIATIVE LOSSES OF LATE-TYPE STARS

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ABSTRACT

We develop a method for estimating the nonradiative heating of stellar chromospheres by measuring the net radiative losses in strong Fraunhofer line cores. This method is then applied to observations of the Mg II resonance lines in a sample of 32 stars including the Sun. We find at most a small dependence of chromospheric nonradiative heating on stellar surface gravity, contrary to the large effect predicted by recent calculations based on acoustic heating theories.

Subject headings: Ca II emission — stars: chromospheres — stars: emission line — stars: late-type

I. INTRODUCTION

One of the fundamental unresolved questions concerning late-type stars is the energy balance in their outer atmospheres. Some important problems are the amount of convective energy converted to various wave modes, the way in which nonradiative energy is transported (acoustic waves, gravity waves, and various MHD waves), thermalization processes, and where the energy is dissipated. The answers to these questions may depend on the gross properties of individual stars; effective temperature, surface gravity, and chemical composition. Even so, high-resolution observations of the Sun suggest considerable spatial variations in the energy balance across a given stellar surface. Studies of wave modes in the solar atmosphere have been reviewed by Stein and Leibacher (1974), while recent work on the nonradiatively heated outer atmosphere of the Sun and other late-type stars and relevant spectroscopic diagnostics has been reviewed by Linsky (1977).

There have been several attempts to theoretically model the nonradiative heating of stellar chromospheres and coronae, including the work of Kuperus (1965), Ulmschneider (1967), de Loore (1970), Nariai (1969), Renzini *et al.* (1977), and Ulmschneider *et al.* (1977). This work restricts itself to the mechanism suggested by Biermann (1946) and by Schwarzschild (1948) that a turbulent convective zone may generate a flux of acoustic waves. Two fundamental problems with this theoretical approach are that we as yet lack an adequate theory of convection, and that the loca-

tion and amount of heating depend critically on the Mach number of the convective flow which controls the initial spectrum of the acoustic waves. As a result, it is useful to invert the problem by estimating *empirically* the amount of chromospheric heating in a large sample of stars covering a range of stellar properties so as to test our understanding of the generation, propagation, and dissipation of mechanical energy fluxes. For example, it is entirely possible that stars with similar total nonradiative heating may have quite dissimilar outer atmospheres as different proportions of this energy are dissipated in the photosphere, chromosphere, transition region, corona, and wind owing to differing spectral distributions of the initial wave energy, pressure scale heights, magnetic field distributions, rotation, or other properties.

We address in this paper the question of how to estimate the nonradiative heat input into a stellar chromosphere from the measured fluxes in the chromospheric emission cores of strong Fraunhofer lines. Empirical estimates for a sample of 32 stars are then compared directly with the theoretical predictions of Ulmschneider *et al.* (1977).

In § II we describe the measurement of excess chromospheric radiative losses in the particular well-studied case of the Sun, and summarize the likely relative importance of different spectral features in the radiative losses of typical late-type stars. Existing measurements of chromospheric emission lines, primarily the *h* and *k* resonance lines of Mg II, are converted into radiative loss estimates in § III. Finally, in § IV we compare the empirical cooling in the Mg II lines with theoretical predictions of chromospheric heating, and we speculate on this comparison in § V.

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II. THE MEASUREMENT OF CHROMOSPHERIC RADIATIVE LOSSES IN THE CORES OF STRONG FRAUNHOFER LINES

a) Overview

Off-limb spectra obtained during eclipses indicate those emission lines which are the major sources of radiative cooling in the solar chromosphere (Athay 1976). However, we must decide how to estimate the net radiative cooling of these lines from disk-average profiles, so that we may measure such losses in other stars. We must also ask whether a particular cooling feature will have the same or different relative importance in other stellar chromospheres.

A stellar chromospheric plasma converts thermal energy into photons by a variety of processes, but collisional excitation of resonance radiation is probably the most important process for cooling the plasma. However, every photon created by collisional excitation at the expense of thermal energy does not represent net cooling, since some of these photons are eventually converted back into thermal energy by pure absorption. Only those photons able to escape entirely from the chromosphere represent net cooling. In practice, only those photons created within a thermalization length of the outer boundary of the stellar chromosphere have a reasonable chance to escape (see Athay 1976), and only about half ultimately will, because half diffuse inward and are absorbed. On the other hand, the back-warming radiation from the core of one resonance line, after conversion into thermal energy, might be removed from the chromosphere by radiation in a different, less opaque resonance line, or perhaps by optically thin emission in H^- . We can therefore characterize the radiative cooling by a particular spectral line in terms of two complementary processes: *direct* cooling provided by the escape of line-core radiation from the chromosphere, and *indirect* cooling provided by the conversion of thermalized back-warming radiation from one transition into direct cooling in a different transition.

However, we are interested not in the absolute flux from a particular line core but rather in the loss exceeding that which would occur if there were no significant nonradiative heating of the star's outer layers. A good estimate of the "intrinsic" flux profile of a strong Fraunhofer line can be determined using a radiative-equilibrium (RE) model appropriate to the underlying star. In this approach, the direct excess radiative loss is obtained by subtracting the RE profile from the empirical profile. If we also construct a theoretical chromospheric model to match the observed line emission core, we can compare the depth dependence of the radiative losses, $d\mathcal{F}/d\tau$, in the empirical model with those in the RE model, and thereby identify which regions of the chromosphere and upper photosphere are cooled by direct radiation from different parts of the line profile.

Blanco *et al.* (1974) have used a variant of this "intrinsic profile" subtraction procedure to estimate chromospheric radiative losses in observed stellar

Ca II H and K profiles. They subtracted an "interpolated" intrinsic profile from the empirical profiles, obtained by extending the photospheric inner wings into line center. As will be shown below, the intrinsic RE profile of the solar Ca II K line is substantially different from that obtained from using the Blanco *et al.* (1974) interpolation technique.

b) Estimates of Excess Radiative Losses in the Solar Ca II K and Mg II k Lines

We have plotted in Figure 1 several thermal models of the Sun's outer atmosphere. The solid curve is Kurucz's (1976) LTE line-blanketed RE model, which includes mixing-length convection in the deeper layers and which has a boundary temperature of ~ 3400 K at small continuum optical depths. The dot-dashed extension of the RE model with a boundary temperature of 4300 K is intended to schematically represent Athay's (1970) non-LTE line-blanketed RE model. The latter is essentially identical to Kurucz's model in the upper photosphere ($\tau_{5000} > 10^{-4}$; see, e.g., Avrett 1977), but may lie substantially above the LTE model at small optical depths owing to non-LTE effects. The remaining two models of Figure 1—

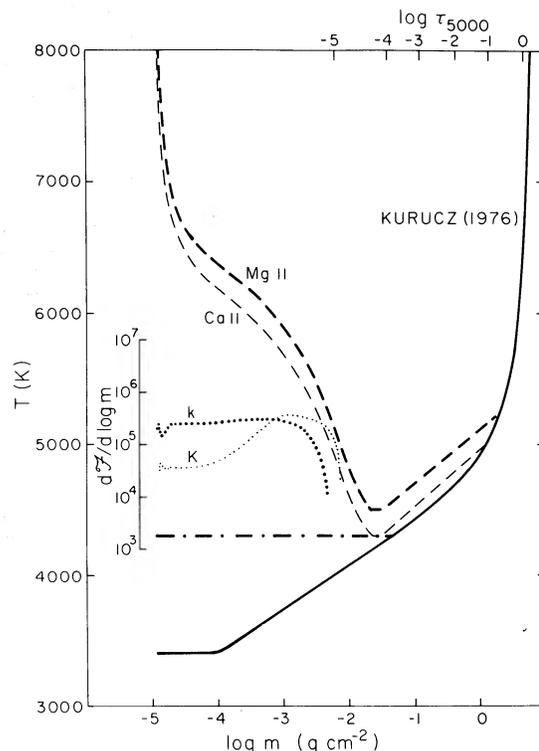


FIG. 1.—Schematic solar outer atmosphere models. *Solid curve*, Kurucz's (1976) LTE line-blanketed RE model; *dash-dot curve*, his model modified for a 4300 K temperature minimum. Dashed curves represent chromospheric thermal structures constructed to fit flux-average profiles of the solar Ca II K and Mg II k resonance lines. Also shown are the net radiative losses in K and k (*dotted curves*). The units of $d\mathcal{F}/d\log m$ are $\text{ergs s}^{-1} \text{g}^{-1}$.

the dashed curves labeled “Ca II” and “Mg II”—are schematic models of the solar chromosphere constructed to fit the integrated core emission in disk average profiles of K and k . For simplicity we use two-level-plus-continuum representations of the radiating ions, but include partial redistribution effects (Milkey and Mihalas 1974) and the abundances, atomic parameters, and photoionization rates used by Ayres and Linsky (1976). The “Mg II” thermal structure for $T > 4500$ K is similar to that of the Vernazza, Avrett, and Loeser (1976) model M chromosphere.

Figure 2 compares Ca II and Mg II profiles synthesized by using the empirical and RE solar models. The observed flux profile of Ca II K is taken from the Beckers, Bridges, and Gilliam (1976) integrated sunlight atlas (Labs and Neckel absolute calibration), while the Mg II k profile is based on a flux average of Kohl and Parkinson's (1976) specific intensity measurements of the quiet Sun.

The profiles synthesized using the empirical “Ca II” and “Mg II” models include a microturbulence distribution which is similar to that advocated by Canfield and Beckers (1976). The synthetic profiles have not been degraded to the resolution of the observations, nor smeared with any macroturbulence. The

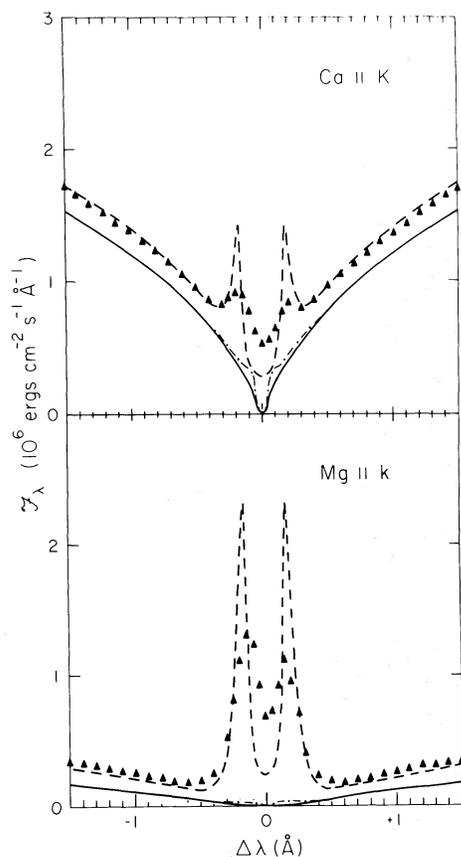


FIG. 2.—Flux profiles of Ca II K and Mg II k synthesized using the chromospheric and RE models of Fig. 1 (with same coding). Filled triangles, observed profiles.

profiles synthesized by using the RE solar models include van der Waals and natural damping but only the purely thermal component of the Doppler broadening.

As noted above, the differences between the “empirical” and “RE” profiles represent the direct, excess chromospheric radiation loss in the particular line core. Qualitatively, we see that the empirical k_1 index for Mg II k —the total flux emitted in the line core between the k_1 minimum features—is an accurate measure of the net chromospheric cooling in this line:

$$\mathcal{F}^{\text{net}}(k) \sim \mathcal{F}(k_1) \sim 6.0 \times 10^5 \text{ ergs cm}^{-2} \text{ s}^{-1}.$$

On the other hand, the K_1 index for Ca II K is apparently an overestimate of the direct, excess radiative cooling, since the true net loss in the K core is about 60% of the K_1 index:

$$\mathcal{F}^{\text{net}}(K) \sim 0.6\mathcal{F}(K_1) \sim 2.4 \times 10^5 \text{ ergs cm}^{-2} \text{ s}^{-1}.$$

The effect on both $\mathcal{F}^{\text{net}}(k)$ and $\mathcal{F}^{\text{net}}(K)$ of changing the RE boundary temperature by nearly 10^3 K is relatively small.

By comparison, Blanco *et al.* (1974) obtained $\mathcal{F}^{\text{net}}(K) \sim 1.2 \times 10^4 \text{ ergs cm}^{-2} \text{ s}^{-1}$ using their photospheric profile subtraction procedure. This value is a factor of 20 smaller than the cooling we estimate, and perhaps explains why they concluded that the Sun has an anomalously low K-index (see their Fig. 2), as compared with their sample of stars chosen for large Ca II emission contrasts.

We also point out that the measured wings of both Mg II k and Ca II K are systematically brighter than the RE profiles (Ayres 1975a) and may indicate non-radiative heating of the upper photosphere, but this question is beyond the scope of our present discussion.

c) Location of the Radiative Cooling

i) Analytic Arguments

It is easy to show (e.g., Athay 1976, eq. [VI-68]) that the total radiation loss is essentially equal to the number of photons created by collisional excitations within one thermalization length of the surface,

$$\mathcal{F}(\text{total}) \sim h\nu_{12} \int_{\tau_{10}=0}^{\tau_{10}=\Lambda} n_1 C_{12} dz. \quad (1)$$

Here, z is the vertical height scale, n_1 is the number density of absorbers, and $C_{12} = n_e \Omega_{12}(T)$ is the collisional excitation rate. If the transition arises from the ground state of a majority species, it follows that

$$\mathcal{F}(\text{total}) \sim \text{const. } h\nu_{12} n_e \langle \Omega_{12} \rangle A_{e1} m_\Delta. \quad (2)$$

In the above A_{e1} is the abundance of the absorbing species relative to hydrogen, m_Δ is the mass column density (g cm^{-2}) at $\tau_{10} = \Lambda$, $\langle \Omega_{12} \rangle$ is a thermal average of $\Omega_{12}(T)$, and we have used the result that n_e is relatively constant with depth in the solar chromosphere (e.g., Vernazza, Avrett, and Loeser 1976).

We can evaluate m_Λ by noting that $\Lambda \sim 1/\epsilon$ for a Doppler profile (Avrett and Hummer 1965) and $\epsilon = C_{21}/(A_{21} + C_{21})$, where $C_{21} = n_e \Omega_{21}(T)$ is the collisional deexcitation rate and A_{21} is the spontaneous decay rate:

$$\begin{aligned} \tau_{lc}(m_\Lambda) &\sim \text{const. } A_{e1} m_\Lambda \Delta \nu_D^{-1} \\ &\sim \Lambda \sim A_{21}/C_{21} \quad (\text{for } \epsilon \ll 1). \end{aligned} \quad (3)$$

Therefore

$$m_\Lambda \sim \text{const. } \frac{A_{21} \Delta \nu_D}{A_{e1} n_e \langle \Omega_{21} \rangle}. \quad (4)$$

Combining equations (2) and (4), we find that

$$\mathcal{F}_{12} \sim \text{const. } h \nu_{12} A_{21} \Delta \nu_D \exp(-h \nu_{12}/kT^*). \quad (5)$$

The exponential thermal emission term arises from the ratio $\langle \Omega_{12} \rangle / \langle \Omega_{21} \rangle$; hence T^* is a characteristic temperature of the effectively thin region of line formation.

From equation (5) we note a very important property of effectively thick chromospheric cooling: *the total radiation loss is relatively independent of density* provided that the radiating atom or ion is the dominant ionization stage and the transition becomes effectively thick somewhere within the chromosphere. Notice also that the total loss is independent of the chemical abundance, although the depth of the effectively thin region of the chromosphere within which the losses are important is, of course, sensitive to A_{e1} and n_e . For example, the solar Ca II lines are nearly as important as the Mg II lines in cooling the middle chromosphere [$\mathcal{F}(\text{Ca II}) \sim \frac{1}{2} \mathcal{F}(\text{Mg II})$; see § IIb, above], despite the factor of ~ 15 difference in abundance (Withbroe 1976). But cooling by Mg II should be more important at higher levels of the chromosphere than Ca II, because the more opaque h and k lines become effectively thick at smaller mass column densities than H and K.

The cooling function of equation (5) contrasts sharply with that of optically thin H^- emission. In particular, $\mathcal{F}(\text{H}^-)$ is proportional to the *square* of the density (through the product $n_e n_{\text{H}^-}$) and decreases rapidly with height—compared to the effectively thick line core case—above a critical temperature owing to the low ionization potential of H^- . In fact, the very existence of the stellar chromospheric temperature rise is likely related to the vastly different density and temperature dependences of effectively thick line core cooling and optically thin H^- cooling.

ii) *Quantitative Estimates of $d\mathcal{F}/d \log m$ for Ca II K and Mg II k*

In order to check the analytical results of the previous section concerning the depth dependence of the radiative losses for effectively thick line cores, we have calculated the net losses for Mg II k and Ca II K using the empirical and RE models of § IIa above:

$$\left. \frac{d\mathcal{F}}{d \log m} \right|_{\text{net}} = \left. \frac{d\mathcal{F}}{d \log m} \right|_{\text{emp}} - \left. \frac{d\mathcal{F}}{d \log m} \right|_{\text{RE}}. \quad (6)$$

These results are plotted in Figure 1 for a bandpass corresponding to the emission cores and inner wings ($\Delta\lambda = \pm 0.6 \text{ \AA}$) of both resonance lines. Note that the core and inner wing losses in Ca II K arise primarily from deeper layers of the chromosphere than the losses in Mg II k . Small regions of net *heating* by Ca II and Mg II in the vicinity of T_{min} where $J > B$ have been omitted from Figure 1, for clarity. Our results confirm that the bulk of the apparent radiative loss in the emission cores and inner wings of k and K is indeed chromospheric in origin.

d) *Relative Importance of Mg II and Ca II in the Total Radiative Cooling of Solar and Stellar Chromospheres*

In order to estimate the total radiative cooling of a chromosphere empirically, we must sum the direct radiative losses in as many strong line cores as feasible, recognizing that the back-warming of one strong transition may appear as direct cooling in the core of a less opaque transition or as optically thin continuum emission. Therefore, a sum of direct losses for a large sample of strong line cores is a lower limit to the true total cooling and indeed could be a reasonably close approximation to it.

The estimated excess radiative losses in Mg II k [$\mathcal{F}(k) \sim 6 \times 10^5 \text{ ergs cm}^{-2} \text{ s}^{-1}$] and Ca II [$\mathcal{F}(K) \sim 2.4 \times 10^5$] can be compared with the middle chromosphere losses in various spectral features given by Athay (1976; his Table IX-1) based on eclipse measurements. The result for Ca II K is consistent with the total loss cited for H and K together, $\mathcal{F}(H + K) \sim 3.8 \times 10^5$. The $\text{H}\alpha$ loss is roughly twice the K line cooling and is comparable to Mg II k . Combining h and k gives a total Mg II loss of $\sim 1 \times 10^6$, while adding the Ca II infrared triplet to H and K gives a total Ca II loss of $\sim 0.8 \times 10^6$. The combined radiative loss for the other important middle chromospheric lines (including Mg I, Na I, Ca I, and Fe I and II) is about $3\text{--}4 \times 10^5 \text{ ergs cm}^{-2} \text{ s}^{-1}$. The Mg II lines thus provide roughly 30% of the total, while the Ca II resonance lines provide about 10%. By comparison, the quiet Sun $\text{L}\alpha$ flux, which is the dominant source of cooling in the upper chromosphere ($T > 10^4 \text{ K}$), is only about 10% of the total middle chromosphere radiative loss.

The next question is whether the large relative contributions of Ca II and Mg II to the total solar line core radiative losses will be similar in other late-type stellar chromospheres, particularly those with much different surface gravities. If such chromospheres are effectively thick in the Ca II and Mg II cores, then we feel that the arguments of § IIc above suggest that at least the ratio of Ca II/Mg II losses should be maintained and perhaps also the $\mathcal{F}(\text{Ca II} + \text{Mg II})/\mathcal{F}(\text{total})$ ratio, since the effectively thick cooling is roughly density-independent. We cite some evidence in § IIIe that chromospheric emission line flux ratios are indeed density-independent, but this presumption should be tested for a wide range of stellar parameters.

We believe that the core emission indices of Ca II and Mg II should be good indicators of nonradiative

heating in stellar chromospheres as they represent about half of the radiative loss independent of chromospheric density so long as the chromospheres are effectively thick in these lines. We must avoid, however, systematic errors arising from corrections to the K and k indices to account for the intrinsic RE core profile. Typically, cool stars with large K_2/K_1 contrasts (e.g., α Boo, α Tau) should require only small corrections to the measured K indices, whereas hotter stars with lower contrast emission may require large corrections to the measured K index. Since observed k_2/k_1 contrasts are generally larger than K_2/K_1 contrasts for all cool stars, we base our discussion of empirical radiative losses mainly on calibrated measurements of the Mg II h and k lines, in order to avoid the problem of estimating RE core corrections in stars for which reliable RE models are not yet available.

III. EMPIRICAL ESTIMATES OF CHROMOSPHERIC RADIATIVE LOSSES

a) The Mg II h and k Lines

Fluxes of Mg II have been measured in 31 stars and the Sun. The stellar data are the result of observations by OAO-2 (Doherty 1972), OAO-3 (McClintock *et al.* 1975*a, b*; Evans, Jordan, and Wilson 1975; Bernat and Lambert 1976; Baliunas, Dupree, and Lester 1976), and the BUSS experiment (Kondo *et al.* 1972, 1976; Kondo, Morgan, and Modisette 1975, 1976*a, b*). Listed in Table 1 are the integrated fluxes of the Mg II h and k lines at Earth in units of $\text{ergs cm}^{-2} \text{s}^{-1}$ as given in each paper. Also given are the Bright Star Catalog numbers (Hoffleit 1964), Wilson-Bappu intensities, and He I $\lambda 10830$ equivalent widths (Zirin 1976).

The criteria for measuring integrated line fluxes are not uniform among these sources. As shown in § II, an accurate measure of chromospheric radiative losses in Mg II is the total flux between k_{1R} and k_{1V} and between h_{1R} and h_{1V} . Unfortunately, the integrated Mg II fluxes found in the literature are often obtained by subtracting an assumed "photospheric line profile" from the observed profile as described in § II for Ca II, and this procedure can underestimate the chromospheric radiative loss in the Mg II lines by up to a factor of 2 in F and G stars where the h_1 and k_1 intensities are large. Fluxes designated by R in Table 1 are revised values obtained by planimetry of the published profiles and using the published absolute intensity calibration factors. Fluxes which could not be revised and which we felt are seriously underestimated by the photospheric line profile subtraction procedure are designated by a colon. The OAO-2 fluxes we have used are the MAX values cited by Doherty (1972) on the basis that they probably come closer to estimating the chromospheric radiative losses in h and k . There are no other observations of F and G-type OAO-2 stars whereby we may check this assumption, but for α Tau and α Ori the OAO-2 fluxes are similar to the values obtained by planimetry of the BUSS data. When more than one set of

measurements is available, we average the data, except when one set is felt to be a serious underestimate.

Surface fluxes of Mg II for the average Sun were estimated from the quiet Sun rocket spectra of Kohl and Parkinson (1976) using a flux-weighting of their $\mu = 1$ and $\mu = 0.23$ intensity profiles. The assumption that the average and quiet Sun Mg II fluxes are essentially the same is based on the better than 10% agreement between the Ca II H and K average and quiet Sun fluxes.

We wish to express the observed Mg II fluxes in a manner which is independent of uncertain stellar angular diameters, so as to minimize systematic errors in our comparison of measured radiative losses with theoretical acoustic heating rates. Our approach is to normalize the Mg II fluxes to the total radiative output of the star, both measured at the Earth. This ratio $f(\text{Mg II})/f_*$ is independent of distance for nearby stars which have insignificant interstellar extinction, and is therefore equivalent to $\mathcal{F}(\text{Mg II})/\sigma T_{\text{eff}}^4$, where $\mathcal{F}(\text{Mg II})$ is the surface flux of Mg II radiation at the star and σT_{eff}^4 is the total surface luminosity.

The ratio $f(\text{Mg II})/f_*$ follows from the measured Mg II flux, the apparent stellar visual magnitude V_* , and the bolometric correction,

$$\begin{aligned} \frac{\mathcal{F}(\text{Mg II})}{\sigma T_{\text{eff}}^4} &= \frac{f(\text{Mg II})}{f_{\odot}} \left(\frac{f_{\odot}}{f_*} \right) \\ &= f(\text{Mg II}) \left[\frac{\text{dex}(-V_{\odot}/2.5)}{f_{\odot}} \right] \\ &\quad \times \text{dex} \left[\frac{V_* + \delta(\text{B.C.})_*}{2.5} \right]. \quad (7) \end{aligned}$$

Here $V_{\odot} \approx -26.76$ (Flannery and Ayres 1978) and $f_{\odot} = 1.36 \times 10^6 \text{ ergs cm}^{-2} \text{ s}^{-1}$ (Labs 1975). The quantity $\delta(\text{B.C.})_*$ is a bolometric correction for the stellar energy distribution relative to the Sun. We obtain $\delta(\text{B.C.})$ from the $V - I$ colors of Johnson *et al.* (1966), using the $\delta(\text{B.C.}) - (V - I)$ transformation given by Johnson (1966).

We estimate effective temperatures for the program stars using the mean of the Johnson (1966) $T_{\text{eff}} - (V - I)$ transformations, since they are essentially independent of luminosity. We chose $V - I$ to establish the T_{eff} and $\delta(\text{B.C.})$ scales, because $V - I$ is the widest color separation available for all of the stars in the Johnson *et al.* tables that does not include bands heavily affected by line blanketing. The $\mathcal{F}(\text{Mg II})/\sigma T_{\text{eff}}^4$ ratios obtained using equation (7) are presented in Table 1 and also in Figure 3. Several conclusions can be drawn from an examination of these data:

1. Stars with known very active chromospheres as evidenced by large Wilson-Bappu Ca II intensities for their spectral type clearly lie above the mean run of stars. The two obvious examples here are β Dra and λ And.

2. Stars cooler than the Sun which do not have active chromospheres show a systematic trend of decreasing $\mathcal{F}(\text{Mg II})/\sigma T_{\text{eff}}^4$ with decreasing $\log T_{\text{eff}}$. The

TABLE 1
Mg II *h* AND *k* FLUXES AT EARTH (ergs cm⁻² s⁻¹)

STAR	Sp	BS	Ca II	$\lambda 10830$	T_{eff} (K)	OBSERVED FLUXES			$\frac{\mathcal{F}(\text{Mg II})}{\sigma T_{\text{eff}}^4}$
						OA0-2	BUSS	OA0-3	
α Car.....	F0 Ib-II	2326	7420	9.1 (-10)R	1.9 (-5)
β Cas.....	F2 IV	21	7060	2.8 (-5)
α CMi.....	F5 IV-V	2943	0	0.16A	6450	3.6 (-5)
α Per.....	F5 Ib	1017	5980	1.9 (-5)
α UMi.....	F8 Ib	424	5930	4.7 (-6)
Average Sun.....	G2 V	5770	2.0 (-5)
α Aur.....	G5 III+	1708	...	0.70A	5280	4.5 (-5)
γ Dra.....	G8 III	6132	1	0	5220	2.4 (-5)
δ Her.....	G8 III	6148	1	0	5120	2.8 (-5)
β Dra.....	G2 II	6536	3	1.00A	5010	1.3 (-4)
ζ Cyg.....	G8 II	8115	1	0	4970	5.2 (-5)
ϵ Eri.....	K2 V	1084	4	0.30A	4950	4.4 (-11):	4.1 (-5)
δ Dra.....	G9 III	7310	4910	2.7 (-5)
β Gem.....	K0 III	2990	1	0.14A	4830	1.1 (-5)
ϵ Cyg.....	K0 III	7949	2	0	4790	1.3 (-10)R	4.0 (-5)
θ Cen.....	K0 III-IV	5288	1	0	4750	1.4 (-5)
λ And.....	G8 III-IV	8961	5	0.90A	4640	~1.7 (-10)	1.8 (-4)
α Cas.....	K0 II-III	168	2	0	4580	2.6 (-5)
α UMa.....	K0 III	4301	1	0	4570	1.7 (-5)
α Ari.....	K2 III	617	2	0	4450	1.1 (-10)R	1.7 (-5)
ϵ Gem.....	K2 III	2473	4	0.2E	4300	3.9 (-5)
α Boo.....	K2 IIIp	5340	3	0	4240	1.1 (-5)
α Hya.....	K3 II-III	3748	3	0	4030	5.4 (-10)	8.7 (-6)
ϵ Peg.....	K2 Ib	8308	4	0.35A	4030	1.2 (-10)	2.3 (-5)
γ Aql.....	K3 II	7525	3	0.1E	4020	2.0 (-5)
ζ Cep.....	K1 Ib	8465	3	0	3970	> 2.7 (-5)
β UMi.....	K4 III	5563	3	0	3690	5.3 (-6)
α Tau.....	K5 III	1457	3	0	3640	6.7 (-6)
β And.....	M0 III	337	4	0	3280	2.3 (-5)
α Sco.....	M1 Ib	6134	4	0	3200	5.0 (-10)	6.9 (-6)
β Peg.....	M2 II-III	8775	4	0	3240	7.8 (-6)
α Ori.....	M2 Iab	2061	2	0.1A	6.1 (-10)	3.5 (-6)

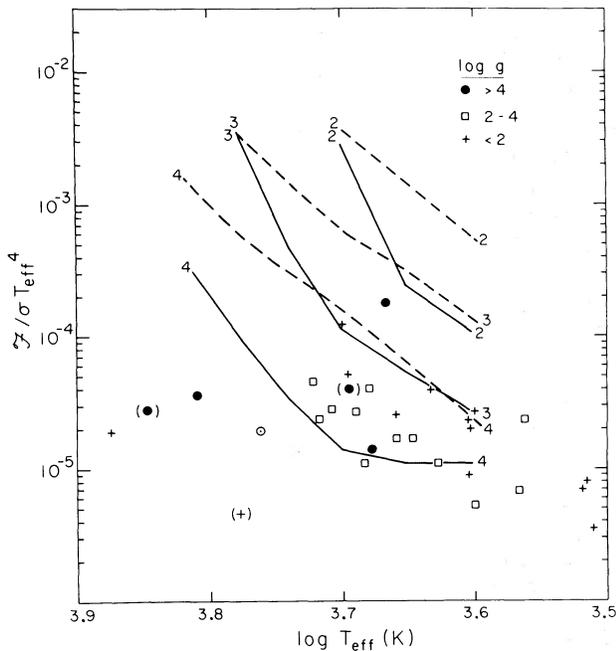


FIG. 3.—Measured radiative losses in Mg II h and k normalized to the surface luminosity σT_{eff}^4 and plotted versus effective temperature. Stellar gravities were crudely estimated according to luminosity class and spectral type (Allen 1973). Solar Mg II fluxes are plotted for the average Sun (circle with dot), and the two stars well above the mean are λ And (filled circle) and β Dra (plus sign). Those data which we feel to be underestimates are in parentheses. Also shown are the total acoustic flux generation curves of Renzini *et al.* (1977; dashed) designated by log g , and the acoustic flux available at the temperature minimum after photospheric radiative damping losses as computed by Ulmschneider *et al.* (1977; solid).

scatter about the mean trend of the data is about a factor of 2.

3. For nonactive chromosphere stars cooler than the Sun there appears to be no large gravity dependence. A plot of the linear regression curve of $\mathcal{F}(\text{Mg II})/\sigma T_{\text{eff}}^4$ against $\log T_{\text{eff}}$ for the supergiant stars in the range $3.5 \leq \log T_{\text{eff}} \leq 3.7$ lies only about a factor of 2 above a similar curve drawn through

the giant star data. The fluxes of the two nonactive chromospheric dwarf stars near $\log T_{\text{eff}} = 3.7$ are consistent with the giant star fluxes.

4. $\mathcal{F}(\text{Mg II})/\sigma T_{\text{eff}}^4$ appears to be comparable or slightly larger in the F stars than in nonactive chromosphere G stars. We stress the importance in future work of properly measuring $\mathcal{F}(\text{Mg II})/\sigma T_{\text{eff}}^4$ in the F and late A stars in order to assess whether the chromospheric heating function turns over in the hotter stars, and, if so, at what effective temperature.

b) The $L\alpha$ Line

$L\alpha$ fluxes have now been published for six stars and the Sun. These data were obtained by McClintock *et al.* (1975b), Evans, Jordan, and Wilson (1975), and Dupree (1975), using *Copernicus*. Even for nearby stars interstellar absorption is important. Each of these authors has corrected the observed $L\alpha$ flux for this absorption using the interstellar hydrogen densities listed in Table 2. The average $L\alpha$ surface flux from the Sun is based on the solar irradiance value of 3.1×10^{11} photons $\text{cm}^{-2} \text{s}^{-1} = 5.1$ ergs $\text{cm}^{-2} \text{s}^{-1}$ given by Timothy (1977).

c) The Ca II H and K Lines

Dravins (1976), Blanco *et al.* (1974), and Blanco, Catalano, and Marilli (1976) have measured Ca II K line fluxes (above the estimated “photospheric” absorption line profile) for a wide range of late-type stars. Dravins (1976) finds that the K line surface fluxes measured this way are approximately constant between spectral classes F0 and K0. Blanco *et al.* (1974, 1976) find that the ratio of K-line surface flux to σT_{eff}^4 goes through a maximum at $T_{\text{eff}} = 4500$ K for giants and at 5000 K for main-sequence stars.

These studies have a systematic bias resulting from the “photospheric” absorption-line subtraction procedure. In the late K and M stars the K_2/K_1 ratio is large and the ignored flux is usually small compared to that in the emission feature above the intrinsic absorption profile, but in G and especially in F stars the ignored contribution can be the dominant term. Thus in the hotter stars the K-line fluxes should be systematically larger, and the trend of decreasing

TABLE 2
STELLAR SURFACE FLUX RATIOS

Star	Sp	$\frac{\mathcal{F}(\text{K})^\dagger}{\mathcal{F}(\text{Mg II})}$	$\frac{\mathcal{F}(L\alpha)}{\mathcal{F}(\text{Mg II})}$	$\frac{\mathcal{F}(\text{O I, C II, Si II})}{\mathcal{F}(\text{Mg II})}$	$\frac{\mathcal{F}(\text{He I, He II})}{\mathcal{F}(\text{Mg II})}$
α CMi	F5 IV–V	0.54	0.05 [0.03]‡
Average Sun	G2 V	0.43	0.22	0.011	0.026
α Aur	G5 III+	0.85	0.20 [0.01]	0.41	...
ϵ Eri	K2 V	...	0.55 [0.10]
β Gem	K0 III	0.57	0.20 [0.10]
α Boo	K2 IIIp	0.34	0.28 [0.10]	0.02	...
α Tau	K5 III	0.43	0.33 [0.10]

† Upper limit (not corrected for RE profile).

‡ Assumed interstellar n_{H} (cm^{-3}).

$\mathcal{F}(K)/\sigma T_{\text{eff}}^4$ for $T_{\text{eff}} > 4500$ K (giants) and $T_{\text{eff}} > 5000$ K (main-sequence stars) noted by Blanco *et al.* (1974, 1976) may not be real. The trend of decreasing $\mathcal{F}(K)/\sigma T_{\text{eff}}^4$ toward cooler temperatures is most likely real, however, and probably is more exaggerated than they state, as the ignored contribution is relatively smaller as T_{eff} decreases.

We derive the Ca II H and K surface flux in integrated sunlight from Beckers, Bridges, and Gilliam (1976; Labs and Neckel calibration). H and K to Mg II surface flux ratios are also given in Table 2 for α Boo (Ayres and Linsky 1975, Paper III), α CMi (Ayres, Linsky, and Shine 1974, Paper II) and α Aur, β Gem, and α Tau (Kelch *et al.* 1978). The cited fluxes include all of the emission between K_{1V} and K_{1R} , with no correction for the intrinsic RE profile, so that we may compare the solar data with stellar fluxes for which reliable RE profiles are not yet available.

d) Other Chromospheric Lines

There exist measurements of a few additional contributors to the chromospheric radiative loss. Emission fluxes of the resonance lines of O I, C II, and Si II are given in Table 2 for α Aur (Vitz *et al.* 1976), α Boo (Weinstein, Moos, and Linsky 1977), and the average Sun (Vitz *et al.* 1976). Also given are values for the He I and He II resonance lines in the average Sun (Timothy 1977).

e) Relative Strengths of Chromospheric Emission Lines

The flux ratio $\mathcal{F}(K)/\mathcal{F}(\text{Mg II})$ for six stars and the average Sun lies in the range 0.34–0.57, with the exception of α Aur. Allowing for measurement errors and the lack of RE corrections, the prediction in § II d that $\mathcal{F}(\text{Ca II})/\mathcal{F}(\text{Mg II})$ should be relatively independent of gravity appears reasonable, given the few data available. $\mathcal{F}(\text{La})/\mathcal{F}(\text{Mg II})$ is more difficult to measure owing to uncertain corrections for interstellar absorption, but this ratio for five of the stars in Table 2 lies in the narrow range 0.20–0.33, while the remaining two stars have ratios of 0.05 (α CMi) and 0.55 (ϵ Eri). We feel that this is additional evidence that the relative strengths of chromospheric emission lines do not vary greatly from star to star. Therefore, we feel reasonably justified in estimating trends of chromospheric cooling using only the Mg II lines.

IV. DISCUSSION

Figure 3 compares the measured direct middle chromosphere cooling in the Mg II resonance lines with theoretical estimates of the total acoustic flux produced in stellar convection zones, and the fraction of that energy which is available for heating in the stellar chromosphere. The dashed curves, labeled with values of the stellar surface gravity $\log g = 2, 3, 4$ cm s^{-2} , refer to the total acoustic flux calculations of Renzini *et al.* (1977) based on the Lighthill theory of turbulent sound generation and the mixing-length

theory of convection. In all cases, observed and theoretical fluxes have been normalized to the total surface luminosity of the star, σT_{eff}^4 . The acoustic fluxes obtained by Renzini *et al.* (1977) agree reasonably well with the earlier calculations of de Loore (1970).

In the Renzini *et al.* calculations the total acoustic flux reaches a maximum value near F5 ($T_{\text{eff}} = 6500$ K), relatively independent of luminosity class. As surface temperatures increase above 6500 K, the acoustic flux generation decreases as convection becomes a progressively less efficient energy transport mechanism compared with radiation. The acoustic flux generation decreases toward cooler spectral types because the maximum convective velocity $\langle V \rangle$ for efficient convection scales as the square root of the mean temperature excess $\langle \Delta T \rangle$ of the convecting fluid elements, whereas $\langle V \rangle \langle \Delta T \rangle$ is proportional to σT_{eff}^4 since the convective flow is carrying virtually all of the stellar luminosity at the depth where $\langle V \rangle$ peaks. In other words, $\langle V \rangle$ is approximately proportional to T_{eff} . In practice, additional factors of T_{eff} and surface gravity g must be included to account for the specific heat and density in the transport equation, and Renzini *et al.* obtain

$$\langle V \rangle / V_s \sim g^{-0.2} T_{\text{eff}}^2, \quad (8)$$

where V_s is the sound velocity. Since the acoustic flux \mathcal{F}_m is proportional to a large positive power of $\langle V \rangle / V_s$, $\mathcal{F}_m / \sigma T_{\text{eff}}^4$ will also be proportional to a large power of T_{eff} . In fact, $\mathcal{F}_m / \sigma T_{\text{eff}}^4 \sim T_{\text{eff}}^\beta$ and $\beta \sim 8$ –14, depending on whether T_{eff} is near 6500 K or is much cooler. For example, with $\beta = 10$ the decrease in \mathcal{F}_m is roughly two orders of magnitude from $T_{\text{eff}} = 6000$ to $T_{\text{eff}} = 4000$, in agreement with the behavior of the acoustic flux curve for the high-gravity stars ($\log g \approx 4$ in Fig. 3), for which convection is expected to be most efficient.

We are, however, interested in the fraction of total acoustic energy which propagates through the stellar photosphere and is available for shock heating of the outer atmosphere. The solid curves in Figure 3 are the Ulmschneider *et al.* (1977) calculations of the energy remaining after the initial acoustic flux spectra of Renzini *et al.* are attenuated by radiative damping in RE stellar photospheres. (Both sets of acoustic flux calculations plotted in Fig. 3 are based on a ratio of mixing length to scale height $l/H = 1$.)

The important point here is the large radiative damping of the short period acoustic waves as they propagate from the deepest layers of the photosphere out into the chromosphere. According to Ulmschneider *et al.* most of this damping occurs between $\tau_{5000} = 10$ and 0.1, whereas the middle photosphere layers above $\tau_{5000} = 0.1$ are relatively transparent to the short-period waves. This behavior is presumably due to the large decrease in H^- radiative efficiency with decreasing density.

Ulmschneider *et al.* estimate a flux of $\mathcal{F}_m \sim 3$ – 8×10^6 $\text{ergs cm}^{-2} \text{s}^{-1}$ ($l/H = 1.0$ – 1.5) available for heating at and above the solar temperature minimum. This is 3–8 times larger than the measured direct radia-

tion losses in the cores of the solar Mg *h* and *k* lines, and comparable to or twice as large as the total line losses of $3\text{--}4 \times 10^6$ estimated in § II. We feel that the measured radiative cooling and calculated acoustic heating of the solar chromosphere are in reasonable agreement since we have not included H^- losses or cooling by the wings of resonance lines (e.g., Fig. 2) in the empirical estimate.

We find that the Mg II flux measurements in the cool ($T_{\text{eff}} < 5000$ K) giants and supergiants of Figure 3 show decreasing radiative losses with decreasing effective temperature, in qualitative agreement with the trends of the Ulmschneider *et al.* (1977) calculations. However, Ulmschneider *et al.* predict an order of magnitude increase in heating between $\log g = 4$ and $\log g = 2$, and by extrapolation an increase of at least two orders of magnitude between dwarfs ($\log g = 4.5$) and supergiants ($\log g = 0$). This large gravity dependence is not seen in our data, although there is evidence for a factor of 2 increase in $\mathcal{F}(\text{Mg II})/\sigma T_{\text{eff}}^4$ between the $\log g = 2\text{--}4$ and $\log g < 2$ data sets.

Cram and Ulmschneider (1977) have pointed out that the Ulmschneider *et al.* (1977) models predict Ca II K line widths inconsistent with observations, except for the Sun, and in the case of giants as much as an order of magnitude larger than observed. Cram and Ulmschneider suggest that much of this discrepancy arises from computed mass column densities at the temperature minima of giant stars which are a factor of 100 too large. This discrepancy and our result that $\mathcal{F}(\text{Mg II})/\sigma T_{\text{eff}}^4$ increases only slowly at most with decreasing stellar gravity, contrary to the results of Ulmschneider *et al.* (1977), both call attention to uncertainties in the existing theory of acoustic flux generation in stellar atmospheres.

Finally, we note that two of the stars of our sample (λ And, β Dra) lie significantly above the mean trends. These are examples of the so-called active-chromosphere stars (Linsky 1977), in analogy with solar active regions which show much larger apparent radiative losses than the quiet Sun (Athay 1976). One possible explanation is that the magnetic field configuration in the photosphere serves to channel acoustic flux through the radiative damping zone or in some other way alters the transmission coefficient of those layers, permitting a larger fraction of the initial acoustic flux to reach and heat the chromosphere. Since only about 20% of the acoustic flux generated in the solar convection zone actually reaches the temperature minimum region (Ulmschneider *et al.* 1977), a relatively small change in the transmission coefficient could greatly alter the energy balance of the outer atmosphere. Alternatively, magnetic fields may influence the convective dynamics directly and thereby produce enhanced acoustic fluxes, or heat the chromosphere by field annihilation (microflares).

On the other hand, many active chromosphere stars like λ And are members of close binary systems. Interactions between the components of such systems could produce excess heating by completely non-magnetic processes such as mass transfer and accretion, tidal interactions, and resonances. To test these

and other possibilities, we encourage monitoring of the chromospheric emission of such systems and the search for surface magnetic fields.

V. CONCLUSIONS

This paper attempts to begin a dialogue between theoretical estimates and empirical measurements of nonradiative heating in stellar chromospheres. The dialogue has barely begun, and our empirical estimates of chromospheric heating are based on few data. Nevertheless, we feel some confidence in drawing from the present discussion the following conclusions:

1. The Mg II *h* and *k* lines represent roughly 30% of the identifiable emission-line cooling of the solar chromosphere, and the Ca II H and K lines represent about 10%. Furthermore, the ratio of $\mathcal{F}(\text{Mg II} + \text{Ca II})/\mathcal{F}(\text{total})$ in stellar chromospheres should be relatively independent of density, and therefore stellar gravity, so long as the chromospheres are effectively thick in these lines. For the few stars for which we have Mg II, Ca II, and $L\alpha$ fluxes, the line ratios are indeed approximately constant. Thus the Mg II lines should be good indicators of nonradiative heating in stellar chromospheres.

2. The proper procedure for measuring the chromospheric radiative loss in emission lines like Mg II *k* is to integrate the emission between k_{1R} and k_{1V} , subtracting the line core computed for a purely radiative equilibrium model. The Mg II lines are convenient to use because the computed RE line profiles are generally negligible, hence large corrections to the measured k_1 or h_1 fluxes are not required.

3. Measured surface fluxes in the Mg II lines normalized to σT_{eff}^4 show no large dependence on stellar gravity but do decrease slightly from early G to M stars.

4. Theoretical estimates of the acoustic flux available at and above stellar temperature minima, computed on the basis of the Lighthill theory of turbulent sound generation, the mixing-length theory of convection, and allowing for photospheric radiative damping, predict a large increase in chromospheric heating with decreasing stellar gravity. Since no large effect is observed, we should question the significance of this mechanism and the physical basis upon which these calculations are made. Cram and Ulmschneider (1977) have also called attention to inadequacies in present shock heating methods by noting discrepancies between computed and measured K line widths. We suggest that alternatives to acoustic wave heating should be more actively studied—in particular, Alfvén waves or annihilation of magnetic fields in small regions (microflares) because of the close correlation of field strength and chromospheric line intensities seen in the Sun (e.g., Skumanich, Smythe, and Frazier 1975).

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