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NUCLEAR BURNING IN ACCRETING NEUTRON STARS AND X-RAY BURSTS*

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ABSTRACT

We consider the general properties of nuclear burning in accreting neutron stars. We discuss the behavior expected in the pycnonuclear and thermonuclear regimes, the conditions required for thermonuclear runaway, and the energy available from H, He, C, and O burning during an X-ray burst, assuming the burst arises from a thermonuclear flash. By using these results and making conservative assumptions, we derive some constraints placed on such models of X-ray bursts by observation. We find that hydrogen burning appears to be ruled out as the source of the X-ray bursts. Indeed, no nuclear fuel is consistent with observed background luminosities if the fuel burned in a burst must be replenished by the time of the next burst. However, "storage battery" models, in which the fuel is progressively used up, involving the burning of He, C, or O may be consistent with the observed properties of some burst sources, although not with those of the rapid burster MXB 1730−335. The eventual exhaustion of fuel in "battery" models limits both the number of bursts that can occur in a single active state, and the long-term ratio of burst-active to burst-inactive periods. Observational determinations of these quantities are important tests of nuclear burning models.

Subject headings: nucleosynthesis — stars: accretion — stars: neutron — X-rays: bursts

I. INTRODUCTION

In late 1975 the ANS satellite discovered X-ray bursts originating from the previously known X-ray source 3U 1820−30 in the globular cluster NGC 6624 (Grindlay *et al.* 1976). Analysis of earlier SAS-3 observations quickly confirmed this result and led to the remarkable additional discovery that the bursts recurred quasi-periodically on a time scale of $4^{h}.4$ with a phase jitter of a few percent (Clark *et al.* 1976). Many additional burst sources have been reported since then and more than 20 burst sources are now known (Lewin *et al.* 1977). For recent reviews of observations of burst sources, see Grindlay (1977), Clark (1977), Lewin (1977*a*, *b*), and Gursky (1977). Typically, the bursts have rise times $\delta t_b \lesssim 1^s$, durations $\Delta t_b \approx 3^s$–10^s, and repetition time scales $t_b \sim 1^h$–10^h (see Fig. 1). Peak burst luminosities are $L_b \sim 10^{39}$ ergs s^{-1}, and burst energies are $E_b \approx 10^{39}$–10^{40} ergs. Many, and perhaps all, burst sources have "steady" background luminosities $L_0 \lesssim 10^{37}$ ergs s^{-1}, and some have "active" and "inactive" states lasting weeks to months. Table 1 gives repetition time scales t_b, burst energies E_b, and the ratio Λ of the background luminosity L_0 to the time-averaged burst luminosity $\langle L_b \rangle = E_b/t_b$ when the source is in an "active" state for those sources for which this information is available at the time of writing.

Similarities between the burst sources and the "steady" galactic X-ray sources have led to models in which X-ray bursts are produced by modulation of the accretion rate onto a neutron star (Henriksen 1976; Baan 1977; Lamb *et al.* 1977). Here we explore alternate models in which X-ray bursts are produced by thermonuclear flashes in the freshly deposited outermost layers of an accreting neutron star. Models of this type have been discussed by Maraschi and Cavaliere (1977), who considered hydrogen shell flashes, and by Woosley and Taam (1976), who considered helium and carbon shell flashes. In a related model, Woosley and Taam (1976) have also proposed that detonation of carbon in the envelopes of accreting neutron stars might account for the soft γ-ray bursts.

Such models are motivated by the fact that nuclear burning in accreting neutron stars is likely to be unstable. In an important first study of steady-state burning in the surface layers of accreting neutron stars, Hansen and Van Horn (1975) showed that hydrogen is converted to helium, and helium to carbon and oxygen, in thin burning shells. Their results indicate that the burning shells likely are thermally unstable for a wide range of neutron star masses and accretion rates on time scales ranging from $\sim 10^5$ s down to $\sim 10^{-3}$ s. These time scales are similar to the time scales encountered in the rise times, durations, and repetition time scales of the X-ray bursts. Nuclear burning of hydrogen-rich material releases only ~ 6 MeV per nucleon compared to the ~ 140 MeV per nucleon typically liberated by accretion onto neutron stars, and hence can contribute on average only a few percent of the total luminosity (Rosenbluth *et al.* 1973); however, if fuel is stored up and then consumed suddenly, the luminosity from nuclear burning can temporarily dominate that from accretion.

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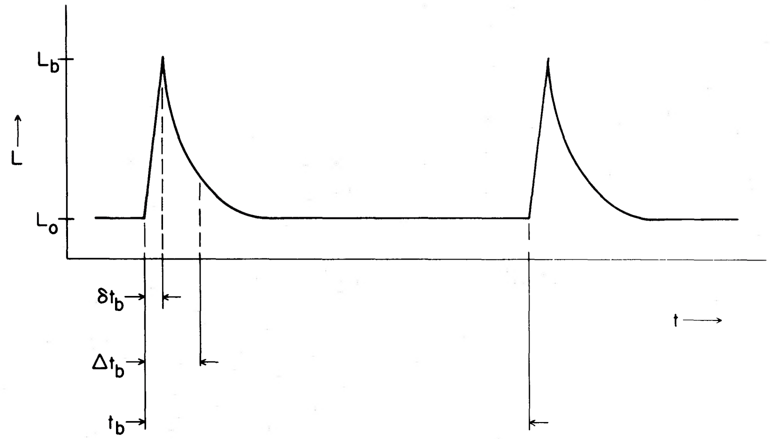

FIG. 1.—Characteristic luminosities and time scales associated with bursting behavior. L_b is the peak luminosity of the burst, and L_0 is the steady background luminosity (if any) of the source. Shown are the burst rise time, δt_b, duration, Δt_b, and repetition time scale, t_b.

In this paper we consider some of the physics relevant to nuclear burning in accreting neutron stars. By making conservative assumptions, we derive some constraints placed on such models of X-ray bursts by observation. We find that no nuclear fuel is consistent with observed background luminosities if the fuel burned in a burst must be replenished by the time of the next burst. However, "storage battery" models, in which the fuel is progressively used up, involving He, C, or O burning may be consistent with the observed properties of some burst sources, although not with those of the rapid burster. On energetic grounds C and O appear to be the most promising fuels, while hydrogen burning appears to be ruled out as the source of the X-ray bursts. The eventual exhaustion of fuel in "battery" models limits both the number of bursts that can occur in a single active state, and the long-term ratio of burst-active to burst-inactive periods. Observational determinations of these quantities are important tests of nuclear burning models. If the X-ray bursts are due to C or O burning, the long-term average accretion rate required for thermonuclear burning of these fuels implies a readily observable background accretion luminosity. Finally, we find that the high internal temperatures required for C and O thermonuclear runaway combined with the large thermal inertia of the neutron star imply a significant X-ray luminosity from the neutron star even when the source is inactive and the star is not accreting.

In a contemporary study, Joss (1978) has also considered nonequilibrium nuclear burning during thermonuclear flashes and the constraint on the burst repetition time scale imposed by the accretion luminosity, as well as the roles of convection and radiative diffusion in determining the time scale for the conversion of nuclear energy into X-radiation.

In § II we discuss the general properties of nuclear burning in accreting neutron stars, including both pycnonuclear and thermonuclear regimes, the conditions required for thermonuclear runaway, and the energy available from H, He, C, and O burning during an X-ray burst, assuming the burst arises from a thermonuclear flash. In § III we compare the energetics of observed X-ray bursts with these results, including the maximum burst energy possible for each nuclear fuel, the constraint on the burst repetition time scale imposed by the accretion luminosity, and

TABLE 1*

BURST REPETITION TIME SCALES, ENERGIES, AND LUMINOSITIES

Source	t_b	E_b (10^{39} ergs)†	$\Lambda \equiv L_0/\langle L_b \rangle$
MXB 1728−34......	$3^{h}0$–$7^{h}8$	~6.0	50–100
MXB 1730−335.....	6^{s}–450^{s}	0.08–12	< 2
MXB 1735−44......	50^{m}–$7^{h}5$	1–3	≲100
MXB 1742−29......	~13^{h}	0.5–3	≲100
MXB 1743−28......	~10^{m}	~2	...
MXB 1743−29......	~35^{h}	~5	≲100
3U 1820−30.........	$2^{h}2$–$4^{h}4$	1.3–1.8	~35
MXB 1837+05......	~$6^{h}3$ (Irr. ≳ $1^{h}4$)	~1	~150
MXB 1906+00......	~$8^{h}9$	~1.4	~80

* Data for all sources are taken from Lewin 1977*a*, except for MXB 1735−44 (Lewin *et al.* 1977).

† All sources are assumed to be at a distance d = 10 kpc, except for 3U 1820−30 = NGC 6624 (6 kpc) and MXB 1730–335 (11 kpc).

the background luminosity due to energy outflow from the interior of the neutron star.

II. NUCLEAR BURNING IN ACCRETING NEUTRON STARS

a) *Overview*

Let us consider first the overall picture of nuclear fusion in accreting neutron stars. Infalling matter reaching the surface of the neutron star liberates an amount of energy per nucleon equal to the gravitational potential energy at the surface,

$$\phi_{\rm grav} \equiv GMm_p/R = 140(M/M_\odot)R_6^{-1}\ \text{MeV per nucleon}\,, \quad (1)$$

where M is the stellar mass and R_6 is the stellar radius in units of 10^6 cm. This heats the surface to temperatures $T \sim 10^7$–10^8 K, and the liberated energy is efficiently and rapidly radiated away as X-rays. If the internal temperature of the neutron star is low, there will be a heat flux from the hot surface into the interior, but this is expected to be small. If the internal temperature is higher, the hot X-ray emission region is expected to have very little effect on the deeper layers of the envelope because of the well-known insensitivity of radiative envelopes to the precise form of the outer boundary condition (cf. Schwarzschild 1958). In either case the effect of the high-temperature surface region on the internal thermal structure of the star can, to a first approximation, be neglected. We shall also neglect the effect of compressional heating (Giannone and Weigert 1967).

After matter has passed through the high-temperature X-ray emission region, it quickly cools and becomes buried by newly accreted material. As the matter continues to be compressed, its density increases. When the density becomes sufficiently large, pycnonuclear and electron capture reactions transmute the hydrogen first to helium, then to carbon and oxygen, and finally to the composition of iron-peak elements characteristic of the deep crust of a neutron star, even at zero temperature (Rosenbluth *et al.* 1973; Van Horn and Hansen 1974). If the internal temperature is sufficiently high, these same reactions will occur at somewhat lower densities via thermonuclear reactions (Hansen and Van Horn 1975). In either case, the nuclear burning is expected to occur in a series of thin shells due to the large value of the gravitational acceleration, g, and the resulting very small density scale height of neutron star envelopes. Figure 2 illustrates the thin thermonuclear burning shells found by Hansen and Van Horn (1975).

One can estimate the maximum possible total nuclear energy that can be released via these reactions by taking the difference between the binding energy per nucleon of hydrogen-rich matter ($X = 0.7$, $Y = 0.3$) and that of ^{56}Fe, neglecting neutrino energy losses. This gives a total energy $(\phi_{\rm nuc})_{\rm max}$ of 6.0 MeV per nucleon, or a "specific Q-value" of

$$q_{\rm nuc} = 5.8 \times 10^{18}\ \text{ergs g}^{-1}\,. \quad (2)$$

The total rate of nuclear energy production, assuming a steady state, can then be written as

$$L_{\rm nuc}/L_\odot = 0.95(\dot{M}/10^{-11}\ M_\odot\ \text{yr}^{-1})\,, \quad (3)$$

where $\dot{M}$ is the mass accretion rate.

Again assuming a steady state and that nuclear fusion is the only source of energy in the neutron star interior, one can derive approximate expressions relating both the mass accretion rate $\dot{M}$ and the observed accretion luminosity L to the central temperature T_c. Using power law fits to the curves of central temperature T_c versus surface temperature given by Tsuruta (1974) for the extreme cases of low and high internal magnetic field strength B and $M = 1.07\ M_\odot$, we find

$$L_{\rm cool}/L_\odot = 2.2(T_c/10^8\ \text{K})^{2.8}R_6^2 \quad (\text{low } B)$$
$$= 6.8(T_c/10^8\ \text{K})^{2.5}R_6^2 \quad (\text{high } B)\,, \quad (4)$$

where $L_{\rm cool}$ is the luminosity produced by cooling alone. The expression for use in the case of high internal magnetic fields applies only in the temperature range $4 \leqslant \log T_c \leqslant 9$; outside this range the relevant expression is that for low B. Since the two expressions converge at the high temperatures ($T \geqslant 10^8$ K) of interest here, the uncertainty in the central temperature corresponding to a given cooling luminosity is relatively small. Now equating $L_{\rm cool}$ and $L_{\rm nuc}$, we obtain

$$T_c = 7.5 \times 10^7(\dot{M}/10^{-11}\ M_\odot\ \text{yr}^{-1})^{5/14}R_6^{-5/7}\ \text{K} \quad (\text{low } B)$$
$$= 4.6 \times 10^7(\dot{M}/10^{-11}\ M_\odot\ \text{yr}^{-1})^{2/5}R_6^{-4/5}\ \text{K} \quad (\text{high } B)\,, \quad (5)$$

or

$$T_c = 4.1 \times 10^8 L_{37}^{5/14}(M/M_\odot)^{-5/14}R_6^{-5/14}\ \text{K} \quad (\text{low } B)$$
$$= 3.1 \times 10^8 L_{37}^{2/5}(M/M_\odot)^{-2/5}R_6^{-2/5}\ \text{K} \quad (\text{high } B)\,, \quad (6)$$

where L_{37} is the accretion luminosity in units of 10^{37} ergs s^{-1}. Thus the steady-state central temperature increases with increasing mass accretion rate.

Since in equations (4)–(6) we have used the results reported by Tsuruta (1974) for the cooling of a $1.07\ M_\odot$ neutron star, these equations become inaccurate for neutron stars with masses $M \leqslant 0.15\ M_\odot$ which have large radii and consist mostly of solid crust. However, it is unlikely that such extremely light neutron stars can be formed (Ruderman 1972).

b) *Pycnonuclear Regime*

If the mass accretion rate is sufficiently low, the steady-state central temperature (eq. [5]) will be so low that all nuclear burning will occur via pycnonuclear or electron capture reactions. In the pycnonuclear regime, nuclear reaction rates scale as

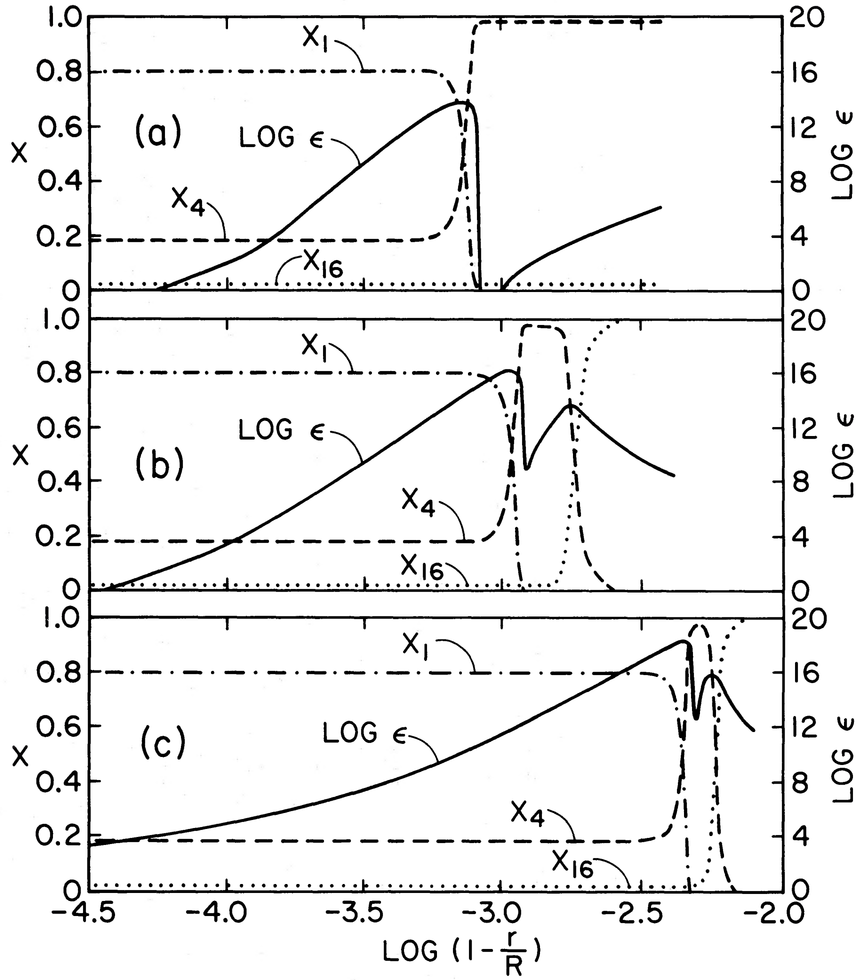

FIG. 2.—The structure of the shell-burning regions in the envelope of a 0.476 $M_{\odot}$ neutron star for three mass accretion rates: (*a*) 10^{-11} $M_{\odot}$ yr^{-1}, (*b*) 10^{-9} $M_{\odot}$ yr^{-1}, and (*c*) 10^{-7} $M_{\odot}$ yr^{-1}. Solid line shows the logarithm of the nuclear energy generation rate; dotted line, the composition of ^{16}O; dashed line, that of ^{4}He; dot-dashed line, that of ^{1}H. Note the presence of two distinct shells and the tendency for the shells to lie deeper for higher mass accretion rates. (From Hansen and Van Horn 1975.)

$\exp(-c/\rho^{1/6})$, where c is a constant, so that the rate of nuclear burning is independent of temperature but increases extremely rapidly beyond a certain density (Cameron 1959). As a result, there exists a characteristic maximum density ρ_{max} beyond which a given element has been consumed entirely, and this characteristic density is essentially independent of the mass accretion rate, or equivalently, the nuclear luminosity.

Consider now the behavior of each of the various nuclear fuels.

i) *Hydrogen*

As accreted matter is compressed to higher density, the first reaction to proceed will be electron capture by protons to form deuterium, followed rapidly by fusion of the deuterium with protons to form He (Van Horn and Hansen 1974). Electron capture by protons increases rapidly above the characteristic density $\rho_{max}(H) \approx 1 \times 10^{7}$ g cm^{-3} (Bahcall 1964; Van Horn and Hansen 1974).

ii) *Helium*

The pycnonuclear rate for the triple-α reaction $3\,^{4}He \rightarrow {}^{12}C$ increases rapidly above $\rho_{max}(He) \approx 2 \times 10^{9}$ g cm^{-3} (Cameron 1959).

iii) *Carbon and Oxygen*

As the accreted matter is compressed to still higher densities, electron capture by C, O, and heavier elements occurs before the time scales for pycnonuclear reactions involving these elements become short enough (i.e., $\lesssim 10^{3}$ s) to be of interest (cf. Salpeter 1961). We take $\rho_{max}(C) \approx 3 \times 10^{10}$ g cm^{-3} and $\rho_{max}(O) \approx 2 \times 10^{10}$ g cm^{-3}, the thresholds for electron capture by ^{12}C and ^{16}O, as representative of the reactions involving heavier elements.

These characteristic maximum densities are listed in Table 2. The envelope masses M_{env} lying outside $\rho_{max}(H)$ and $\rho_{max}(He)$ are shown in Figure 3 for Baym, Pethick, and Sutherland (1971) and for Pandharipande and Smith (Pandharipande, Pines,

TABLE 2

PROPERTIES OF NUCLEAR FUELS

Quantity	H*	He	C	O
$\log \rho_{max}$	7.0†	9.3	10.5†	10.2†
$\log \rho_s$	5.0	7.0	9.3	9.3
$\log T_{min}$	7.3	8.0	8.6	8.9
$\dot{M}_{min}$ (10^{-11} $M_\odot$ yr^{-1})‡	0.024	2.2	110	730
$(L_{acc})_{min}$ (10^{37} ergs s^{-1})‡	2.1×10^{-4}	0.019	0.91	6.3
$(L_{cool})_{min}$ (10^{36} ergs s^{-1})‡	9.1×10^{-5}	8.4×10^{-3}	0.42	2.8
ϕ_{nuc} (MeV per nucleon)	4.7×10^{-4} (0.49)	0.61	1.08	0.79
q (10^{17} ergs g^{-1})	4.5×10^{-3} (4.7)	5.9	10.4	7.6

* Numbers in parentheses are for a composition consisting of equal numbers of ^{1}H and ^{14}N nuclei.

† This density is that at which the electron Fermi energy equals minus the Q-value of the appropriate electron capture reaction.

‡ The tabulated values of these quantities are for a neutron star of mass 1 $M_\odot$ and radius 10^6 cm.

and Smith 1976) neutron star models. These models, hereafter denoted as BPS and PS, span the range of neutron star models currently under study, and the corresponding envelope masses M_{env} therefore provide an estimate of the amount of H, He, C, and O present at zero temperature. Since at finite temperatures lesser amounts will be present, these envelope masses represent absolute upper limits to the amounts of H, He, C, and O available as nuclear fuel for X-ray bursts.

c) *Thermonuclear Regime*

At sufficiently high temperature, nuclear burning will occur via thermonuclear, rather than pycnonuclear, reactions. An approximate criterion for thermonuclear burning is that $\Theta/kT \lesssim 1$ in the burning region, where $\Theta = \hbar\Omega_p$ is the Debye temperature for the nucleus of interest (Salpeter and Van Horn 1969). This criterion can be rewritten as

$$T \gtrsim T_{min} \equiv 1.23 \times 10^8(\mu_e/2)^{-2} \times (\rho/10^9\ \mathrm{g\ cm^{-3}})^{1/2}\ \mathrm{K}\,, \qquad (7)$$

where $\mu_e = A/Z$. If, beginning in the pycnonuclear regime, the temperature in the neutron star is gradually increased, equation (7) will be satisfied first at the characteristic ρ_{max} associated with the given nuclear species. This point will be reached first for H, and then for He, C, and O, and defines for each nuclear species the approximate minimum temperature T_{min} required for thermonuclear burning. These T_{min} are listed in Table 2.

Hansen and Van Horn (1975) have investigated steady-state thermonuclear burning in accreting neutron stars for a wide range of neutron star masses (0.09–1.41 $M_\odot$) and mass accretion rates (10^{-11}–10^{-7} $M_\odot$ yr^{-1}). They found densities in the H-burning shell ranging from 7×10^2 to 2×10^5 g cm^{-3} and densities in the He-burning shell ranging from 6×10^4 to 1×10^6 g cm^{-3}. Thus the characteristic maximum densities encountered in the burning shells in their models are $\rho_s(\mathrm{H}) \approx 10^5$ g cm^{-3} and $\rho_s(\mathrm{He}) \lesssim 10^7$ g cm^{-3}. If these results are taken as characteristic of the conditions in a neutron star just prior to an X-ray burst, the envelope masses M_{env} lying outside the densities ρ_s(H) and ρ_s(He) provide an estimate of the maximum amounts of H and He available, under conditions of thermonuclear burning, as nuclear fuel for X-ray bursts. These envelope masses are shown in Figure 3 for BPS and PS neutron star models.

Analogous estimates can be made of the maximum amounts of C and O available as nuclear fuel. The maximum density for C ignition (Graboske 1973), which corresponds approximately to the density beyond which plasma neutrino energy losses are suppressed, can be taken as an estimate of the highest densities of C and O that can be encountered under thermonuclear burning conditions. This density is

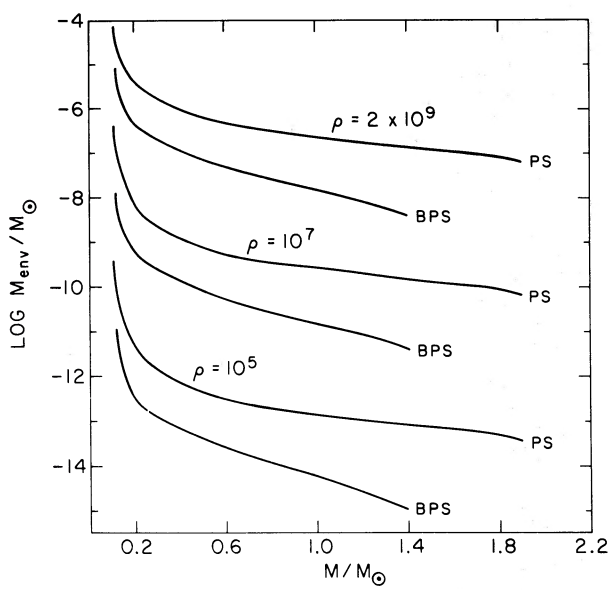

FIG. 3.—The envelope mass exterior to the characteristic densities 10^5, 10^7, and 2×10^9 g cm^{-3} as a function of neutron star mass, for PS and BPS model stars.

$\rho_s(\mathrm{C, O}) \approx 2 \times 10^9$ g cm^{-3}, and the envelope mass M_{env} lying outside this density for BPS and PS neutron star models is also shown in Figure 3.

Having used the results of the study by Hansen and Van Horn to estimate the amounts of H and He available as nuclear fuel, let us note some modifications of their results which can be expected in more detailed future studies of thermonuclear burning in accreting neutron stars.

i) *Temperature Structure*

If nuclear burning is the only energy source within the neutron star, we expect that the temperature will be highest in the burning shells. Interior to the burning shells, the temperature will be nearly constant because of the high conductivity of degenerate matter but may decrease slightly toward the center of the star due to neutrino energy losses. Thus heat will flow from the nuclear-burning region toward both the surface and the center of the neutron star. In a few cases at high mass accretion rates, the models of Hansen and Van Horn have temperatures at the base of the envelope that are as much as 2–3 times larger than the temperatures in the He-burning shell. This produces an extra outward flux of heat, and we therefore expect that in improved models the H- and He-burning shells will occur at somewhat lower temperatures and higher densities.

ii) *Presence of Carbon*

Since the triple-α reaction rate scales as ρ^2 whereas the rate for α-capture on C and O scales as ρ, the triple-α reaction is favored at high densities. The most recent published nuclear reaction rates (Fowler, Caughlan, and Zimmerman 1975) show that at the high densities encountered in the thermonuclear burning regions of neutron stars the triple-α reaction will proceed much faster than α capture on C, except at the very highest temperatures. Carbon will therefore likely be a major product of He burning in neutron stars, in contrast to the situation suggested by Hansen and Van Horn.

iii) *Hydrogen Burning*

At the high temperatures ($T \gtrsim 10^8$ K) encountered in thermonuclear burning in neutron stars, hydrogen will burn primarily via the CNO cycle rather than via the *p-p* chain. However, because the CNO cycle "hangs up" while waiting for the positron decays of ^{13}N, ^{15}O, and ^{17}F, the CNO cycle operating in equilibrium has a maximum energy generation rate of $\epsilon_{\max}(\mathrm{CNO}) \approx 1.8 \times 10^{14}$ ergs g^{-1} s^{-1} (Fowler 1966). Half of the models discussed by Hansen and Van Horn have peak energy generation rates in the H-burning shell well in excess of $\epsilon_{\max}(\mathrm{CNO})$. Therefore we expect that the properties of the H-burning region will be altered in models incorporating a more detailed treatment of hydrogen burning.

In particular, if the peak nuclear energy generation rate is much smaller, the H-burning region might not be thermally unstable (Hansen and Van Horn 1975). Also proton capture on ^{13}N and ^{17}F, α capture on ^{14}O and ^{15}O, and other nuclear reactions, may be important (Caughlan and Fowler 1972; Audouze, Truran, and Zimmerman 1973). Finally, the H-burning region itself may be broadened and may extend to densities as high as $\sim 10^6$ g cm^{-3}. For example, if one takes a solar abundance of CNO nuclei and a positron decay time scale $\tau_\beta \gtrsim 100$ s, and neglects changes in the nuclear reaction processes, a characteristic time $t_{\mathrm{H}} = (N_{\mathrm{H}}/N_{\mathrm{CNO}})\tau_\beta \sim 10^3 \cdot 10^2 \sim 10^5$ s is required to consume the hydrogen, where N_{H} and $\mathrm{N}_{\mathrm{CNO}}$ are the fractional abundances by number of hydrogen and of CNO nuclei. From the continuity equation we have, assuming steady-state accretion,

$$v(\rho) = \dot{M}/4\pi R^2\rho = 5.1(\rho/10^3 \text{ g cm}^{-3})^{-1} R_6^{-2} \times (\dot{M}/10^{-9}\, M_\odot \text{ yr}^{-1}) \text{ cm s}^{-1} . \tag{8}$$

An estimate of the minimum width $\Delta r_{\min}$ of the H-burning region is then

$$\Delta r_{\min} \sim v(\rho_b) t_{\mathrm{H}} \sim 5 \times 10^3 (\rho_b/10^5 \text{ g cm}^{-3})^{-1} \times R_6^{-2} (\dot{M}/10^{-9}\, M_\odot \text{ yr}^{-1}) \text{ cm} , \tag{9}$$

where ρ_b is the characteristic density of the burning region. An estimate of the minimum density $\rho_{\min}$ to which the H-burning region extends is given implicitly by the density at which $\Delta\rho/\rho = v(\rho)t_{\mathrm{H}}/h_\rho \sim 1$, where $\Delta\rho$ is the change in density through the burning zone and $h_\rho = -(d \ln \rho/dr)^{-1}$ is the density scale height there. We then have

$$\rho_{\min} \sim 5 \times 10^5 (t_{\mathrm{H}}/10^5 \text{ s})(h_\rho/10^3 \text{ cm})^{-1} \times R_6^{-2} (\dot{M}/10^{-9}\, M_\odot \text{ yr}^{-1}) \text{ g cm}^{-3} . \tag{10}$$

These values of $\Delta r_{\min}$ and $\rho_{\min}$ are, for example, substantially larger than the width $\Delta r \sim 10^2$ cm (see Fig. 2) and characteristic density $\rho \sim 2 \times 10^4$ g cm^{-3} found by Hansen and Van Horn (1975) for the H-burning shell in a $M = 0.476\, M_\odot$ neutron star accreting at a rate $\dot{M} = 10^{-9}\, M_\odot$ yr^{-1}. If the H-burning region does, in fact, extend to densities as large as $\sim 10^6$ g cm^{-3}, many unusual nuclear reactions may occur, particularly if hydrogen is burned in the same region as helium.

d) *Conditions Required for Nuclear Runaway*

In order to clarify the conditions required for nuclear runaway, it is useful to consider a cold star and an initial mass accretion rate sufficiently small that when a steady state is reached, all nuclear burning occurs via pycnonuclear reactions. Following the onset of accretion, the rate of nuclear reactions in the accreted matter slowly increases until eventually each species of nucleus reaches the depth and the density

ρ_{max}, discussed earlier, at which it is burned as quickly as it arrives. The nuclear energy generation rate then approaches that given by equation (3). Although the temperature may be somewhat higher in the regions of nuclear burning than at the center, as noted earlier, the interior of the neutron star will at all times remain nearly isothermal because of the high conductivity of degenerate matter. Thus the whole interior will slowly heat up until the central temperature approaches that in the burning zone. As long as $\dot{M}$ is sufficiently small that the burning remains pycnonuclear, steady nuclear burning will occur, since the nuclear reaction rates are independent of temperature and nuclear runaway is not possible.

If now the mass accretion rate $\dot{M}$ is gradually increased, the temperature of the neutron star interior will increase until eventually a point is reached at which the criterion for thermonuclear burning (eq. [7]) is reached for a given nuclear species. This point is reached first for H, and then for He, C, and O. Once the thermonuclear regime is reached, the nuclear burning shells are thermally unstable, according to the criterion of Giannone and Weigert (1967), if $t_{nuc} \lesssim t_c$, where t_{nuc} is the time scale for the thermal energy content of a shell to increase as a result of nuclear burning and t_c is the time scale for removal of energy from the shell by photon or electron conduction.

The temperature profile in the envelope exterior to the first nuclear burning shell will be the same as in the simple cooling model with the same luminosity, $L_{cool} = L_{nuc}$. The profile interior to the first shell will differ somewhat from that in the cooling model with the same luminosity; but if the luminosity of the innermost burning shell is dominant, the temperature profile exterior to the innermost shell will still be very similar to that in the corresponding cooling model. One expects the temperature interior to the innermost burning shell to be roughly constant or even to decrease slightly toward the center, if neutrino losses are appreciable, whereas in the corresponding cooling model the temperature will tend to increase inward. Therefore the temperature T_s at the innermost shell is expected to be less than the central temperature T_c of the corresponding cooling model. If T_s is maintained at or above the temperature T_{min} required for thermonuclear burning by the current nuclear energy release, this implies, using equation (5), that the average mass accretion rate must equal or exceed a certain minimum value, $\dot{M}_{min}$. This minimum mass accretion rate in turn implies a minimum accretion luminosity, $(L_{acc})_{min}$. Even if T_s is *not* being maintained by the current nuclear energy release, the requirement that thermonuclear runaway be possible implies a surface luminosity due to cooling alone which must exceed $(L_{cool})_{min} = L_{cool}(T_{min})$. Approximate values of $\dot{M}_{min}$, $(L_{acc})_{min}$, and $(L_{cool})_{min}$ are listed in Table 2 for H, He, C, and O burning for the conservative case of a low magnetic field. The values of $\dot{M}_{min}$ range from $\sim 10^{-13}\ M_\odot\ yr^{-1}$ for H burning to $\sim 10^{-8}\ M_\odot\ yr^{-1}$ for O burning. These values assume spherically symmetric accretion; if accreting matter settles over only a fraction of the stellar surface, these estimates can be scaled to give the appropriately modified values (Hansen and Van Horn 1975).

If X-ray bursts originate from nuclear burning in accreting neutron stars, the internal temperature must reach a high value because thermonuclear reactions are required for unsteady burning. It will then remain high between bursts because of the large thermal inertia of the neutron star. To see this, consider first the case if the neutrons were normal. They would then contain most of the thermal energy in the heated star. The heat capacity per neutron of a nonrelativistic, partially degenerate free neutron gas is (Landau and Lifshitz 1969)

$$c_v^{neu} = (\pi/3)^{2/3}(m_p/\hbar^2)n^{-2/3}k^2T\,, \qquad (11)$$

where n is the neutron number density, and therefore the thermal energy content of the neutron star is approximately

$$E_{th} \approx (M/m_p)c_v^{neu}T = 5.0 \times 10^{45}\bar{\rho}_{15}^{-2/3}(T_c/10^8\ K)^2(M/M_\odot)\ ergs\,, \qquad (12)$$

where $\bar{\rho}_{15}$ is the mean density of the neutron star in units of 10^{15} g cm^{-3} (Hansen and Van Horn 1975). If the neutrons are instead superfluid as expected, their heat capacity is reduced exponentially (Tsuruta *et al.* 1972). Then, independent of whether the protons are normal or not, the dominant contribution to the heat capacity of the neutron star comes from the relativistic, partially degenerate electrons. Their heat capacity per electron is (Landau and Lifshitz 1969)

$$c_v^{elec} = (3\pi^2)^{2/3}(3\hbar c)^{-1}n_e^{-1/3}k^2T\,, \qquad (13)$$

where n_e is the electron number density. In this case the thermal energy content of the neutron star is approximately

$$E_{th} \approx Y_e(M/m_p)c_v^{elec}T = 3.2 \times 10^{44}(Y_e/0.04)^{2/3}\bar{\rho}_{15}^{-1/3}(T_c/10^8\ K)^2 \times (M/M_\odot)\ ergs\,, \qquad (14)$$

where Y_e is the number of electrons per nucleon. In either case, the heating time scale t_{th} for the neutron star, if neutrino energy losses are neglected, is approximately

$$t_{th} = 320(E_{th}/10^{45}\ ergs) \times (L_{nuc}/10^{35}\ ergs\ s^{-1})^{-1}\ yr\,, \qquad (15)$$

which is extremely long compared with typical X-ray burst repetition time scales of 1^h–10^h.

Thus, if X-ray bursts are due to thermonuclear runaway in accreting neutron stars, the large thermal inertia of the star implies a minimum background luminosity $(L_{cool})_{min}$ even when the source is in an inactive state or the star is not accreting. The values of $(L_{\rm cool})_{\rm min}$ corresponding to different possible nuclear fuels are given in Table 2.

e) Energy Available during a Burst

Let us now estimate the nuclear energy available from H, He, C, and O burning during an X-ray burst, assuming the burst arises from a thermonuclear flash. Because the nuclear energy must be released within the $\lesssim 1^s$ rise time of the X-ray burst, the direct role of weak interactions in the CNO cycle affects hydrogen burning even more profoundly here than in the case of steady-state burning, and significantly reduces the amount of nuclear energy available. A reduction has also been found by Joss (1978). Consider each of the possible nuclear fuels in turn.

i) *Hydrogen*

The reaction rates for the *p-p* chains are too slow at any temperature to release significant amounts of energy within the short time required for X-ray bursts and we shall henceforth ignore them. As noted above, the CNO cycle cannot operate fully because the positron decay lifetimes involved [$\tau(^{13}\rm N) = 598^s$, $\tau(^{14}\rm O) = 71^s$, $\tau(^{15}\rm O) = 122^s$, and $\tau(^{17}\rm F) = 66^s$; Starrfield *et al.* (1972)] are much longer than the observed X-ray burst rise times, which are $\lesssim 1^s$. The principal source of energy will therefore be prompt proton capture on CNO nuclei. If the temperature reaches $\sim 10^9$ K during the flash, additional proton captures as well as α-particle captures can occur (Audouze, Truran, and Zimmerman 1973) and in subsequent shell flashes daughter nuclei may play an important role. We also expect extensive convective mixing of the envelope following the onset of a flash. Therefore the nuclear energy liberated during a shell flash will depend not only on the temperature T_s and density ρ_s in the burning shell at the time of the flash, but also on the nuclear burning history of the neutron star.

Nevertheless, we can make fairly secure estimates of the minimum and maximum energy available from hydrogen burning in an X-ray burst. If the CNO cycle is operating near equilibrium prior to the flash, one expects that nearly all CNO nuclei will have been converted to ^{14}N (Clayton 1968). And prompt proton capture on ^{14}N is also the most exothermic reaction among reactions involving ^{12}C, ^{14}N, and ^{16}O in the CNO cycle, yielding an energy $Q = 7.293$ MeV per reaction. Additional proton captures, if they occur, yield less energy per reaction. Therefore we consider two possible compositions: (*a*) a fraction of ^{14}N nuclei by number which is solar, i.e., 9×10^{-5} (composition H) (this composition corresponds to the *minimum* energy likely available in a flash); (*b*) equal numbers of protons and ^{14}N nuclei (composition H′) (this composition corresponds to the *maximum* energy that can be made available in a flash). By assuming all CNO nuclei are ^{14}N and scaling from composition H′, one can also estimate the maximum energy available in a flash for any fractional abundance of CNO nuclei.

The available nuclear energy $\phi_{\rm nuc}$ in MeV per nucleon and the corresponding "specific Q-value" q in ergs g^{-1} for compositions H and H′ are listed in Table 2. Note that even for composition H′, consisting of equal numbers of protons and ^{14}N nuclei, $q_{\rm H} = 4.7 \times 10^{17}$ ergs g^{-1} compared with $q_{\rm nuc} = 5.8 \times 10^{18}$ ergs g^{-1} (eq. [2]) for the CNO cycle operating in equilibrium.

ii) *Helium*

Under conditions of thermonuclear burning in neutron stars the triple-α reaction is expected to proceed more rapidly than α-capture on C or O, as discussed earlier. We can therefore estimate the energy available from helium burning by taking the difference in binding energy per nucleon between He and ^{12}C. The resulting values of $\phi_{\rm nuc}$ and q are listed in Table 2.

iii) *Carbon and Oxygen*

In a thermonuclear flash, C and O are expected to burn to iron-peak nuclei. We can estimate an upper bound on the energy available from C and O by taking the difference between the binding energy per nucleon of C and O, and that of ^{56}Fe, since ^{56}Fe has the highest binding energy per nucleon of any nucleus. The resulting values of $\phi_{\rm nuc}$ and q are listed in Table 2.

The above estimates of q represent the maximum energy per gram available from nuclear burning, since in all cases we have neglected any energy losses due to neutrinos. Using these values of q and our earlier calculated values of $M_{\rm env}$, we can estimate the largest amount of nuclear energy $E_{\rm max}$ that can be stored on a neutron star in the form of a given nuclear fuel. This energy is given by

$$E_{\rm max} = qM_{\rm env}\,, \tag{16}$$

where both q and $M_{\rm env}$ depend on the particular nuclear fuel, and $M_{\rm env}$ also depends on the neutron star mass and the particular stellar model used. Figures 4 and 5 show the maximum nuclear energy that can be stored on PS and BPS stars of a given mass. The curves in Figure 4 are for fuel accreted onto cold stars in which the accumulation of fuel is limited by the onset of pycnonuclear burning. $M_{\rm env}$ in this case is the mass exterior to the radius where $\rho = \rho_{\rm max}$. However, in order for nuclear runaway to occur, the star must be hot enough to burn the fuel in the thermonuclear regime, as discussed above. In this case the accumulation of fuel is limited by the onset of thermonuclear burning, and the values of $E_{\rm max}$ for a given stellar mass and nuclear fuel are very much reduced, as shown in Figure 5. Here $M_{\rm env}$ is the mass exterior to the radius where $\rho = \rho_s$. The estimates of $E_{\rm max}$ shown in Figure 5 for the fuels H and He are generous, since the values of ρ_s used are larger than the corresponding values found by Hansen and Van Horn (1975) in any of their models, and neutrino losses have been neglected.

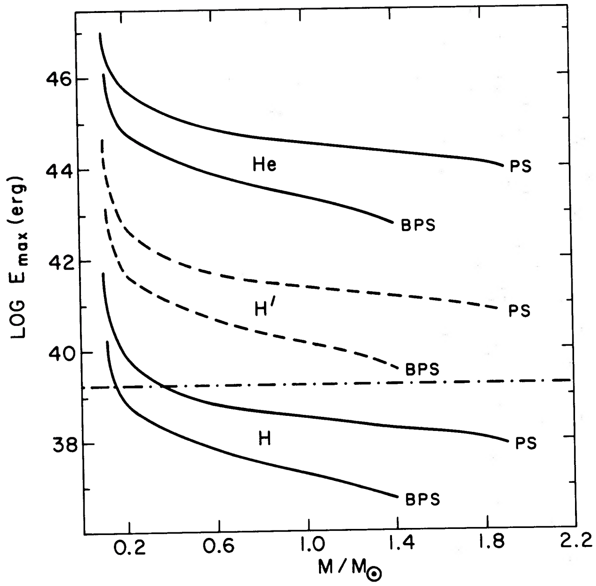

FIG. 4.—The maximum nuclear energy that can be stored on the surface of cold PS and BPS model neutron stars as a function of stellar mass for three fuels: composition H, composition H′, and pure helium (see text).

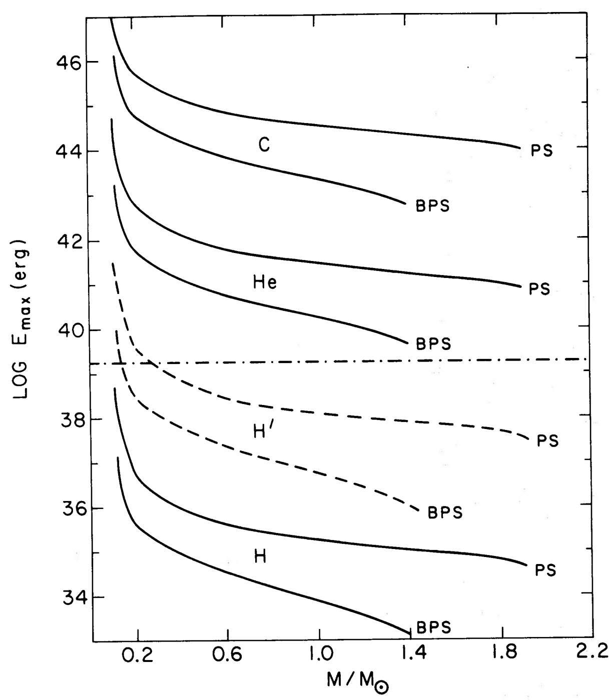

FIG. 5.—The maximum nuclear energy that can be stored on the surfaces of PS and BPS model neutron stars that are hot enough for thermonuclear runaway to occur. The four fuels are those in Fig. 4, plus pure carbon. Note the great reduction in the amount of energy that can be stored under these conditions compared to the energy that can be stored on cold stars.

III. COMPARISON WITH OBSERVATION

Let us now compare the energetics of observed X-ray bursts with our results:

a) *Burst Energy*

Since a sudden release of nuclear energy is possible only in the thermonuclear regime, the maximum energy that can be extracted from a given fuel in a single burst is that shown in Figure 5. The dash-dotted line corresponds to an energy 2×10^{39} ergs, typical of observed values of E_b for sources associated with the galactic center (Lewin 1977*a*). Figure 5 shows that the amount of hydrogen that can be stored on the surface of a neutron star falls short of that required to produce a single burst of this energy by several orders of magnitude, assuming the accreted matter has composition H. For the most efficacious composition (H′), sufficient hydrogen can be stored only on very light neutron stars ($M \leqslant 0.3\, M_\odot$). Whether such light neutron stars are ever formed is at present unknown (Arnett 1977). Neutron stars less massive than $0.15\, M_\odot$ would be less tightly bound than either degenerate dwarfs of the same mass or dispersed helium (Baym, Pethick, and Sutherland 1971; Ruderman 1972). In order to form such stars in a supernova event, the expanding outer envelope must do significant work on the collapsing core, and the formation of such stars is therefore thought to be unlikely (Ruderman 1972). With the (deliberately optimistic) assumptions made here, sufficient quantities of He, C, or O can be stored up to account for the total energies observed to be liberated in a single burst.

b) *Background Luminosity*

Since the deposition of nuclear fuel on the neutron star surface releases a very large quantity of gravitational energy, the observed background luminosity L_0 of the source between bursts places an interesting upper bound on the rate of deposition. The development of a thermonuclear runaway requires that the interior of the star be maintained at a temperature greater than T_{min}, which in turn implies a long-term average accretion luminosity greater than $(L_{acc})_{min}$ (see Table 2). This luminosity is relatively low for the fuels H and He, but for C it is comparable to observed background luminosities, while for O it is so large ($\sim 6 \times 10^{37}$ ergs s^{-1}) that it may be inconsistent with the observed luminosities of some sources. If the magnetic field in the crust is large, $(L_{acc})_{min}$ will be greater than the values quoted here, whereas if fuel is deposited only over a fraction of the star's surface or if the star is very light (but see above), $(L_{acc})_{min}$ will be less. In addition, energy outflow from the stellar interior implies a background luminosity $L_0 \geqslant (L_{cool})_{min}$ for $T_s \geqslant T_{min}$ (see Table 2).

The rate of fuel deposition must also be sufficient to supply the observed luminosity in bursts. If we assume that the layer of fuel is maintained during a burst-active state, i.e., that the fuel used up in a given burst is replenished before the next burst occurs, the energy released as the fuel is laid down implies a background accretion luminosity

$$L_0 \gtrsim (\phi_{\text{grav}}/\phi_{\text{nuc}})\langle L_b\rangle \,, \tag{17}$$

where $\langle L_b\rangle = E_b/t_b$ is the average luminosity in bursts. The observational parameter $\Lambda = L_0/\langle L_b\rangle$ must therefore equal or exceed $\Lambda_c \equiv \phi_{\text{grav}}/\phi_{\text{nuc}}$ in order to be consistent with nuclear burning with steady replenishment. Rigorous lower bounds on Λ_c can be obtained by assuming nuclear yields of 0.61 MeV per nucleon for He burning and 1.08 MeV per nucleon for C burning (cf. Table 2). These lower bounds are displayed in Figure 6 for BPS and PS neutron star models. The value of Λ_c will be substantially larger than these lower bounds if there are significant neutrino energy losses or, in the case of C burning, if the carbon does not burn all the way to ^{56}Fe. Figure 6 shows that Λ must exceed 200 to be consistent with He burning and even 100 to be consistent with C burning, for a 1.4 $M_\odot$ PS model neutron star. Table 1 shows that there are several sources with Λ's significantly less than this. (Note that Λ is independent of the source distance.) Therefore, if the burst sources are physically similar to one another, burning with steady replenishment does not appear to be consistent with observation.

The implications of the observed values of E_b and Λ may be seen clearly in Figure 7, which shows the allowed values of E_b and t_b for each fuel. The vertical line bounding the region allowed for a given fuel is the value of $E_{\max}$ for that fuel, while the line sloping from lower left to upper right is the function

$$t_b(E_b) = (\phi_{\text{grav}}/\phi_{\text{nuc}})L_0^{-1}E_b \,, \tag{18}$$

which does not depend on the distance to the source. The values of $E_{\max}$ and ϕ_{grav} used are those appropriate to the 1.0 $M_\odot$ PS neutron star model, while the value of ϕ_{nuc} used is 0.7 MeV per nucleon. L_0 has been set equal to 10^{37} ergs s^{-1}. The hatched region in the lower right of the figure represents values of E_b and t_b that are inconsistent with nuclear burning with steady replenishment. The allowed regions shown are optimistic in the sense that (1) neutrino energy losses have been ignored, (2) the values of ρ_s(H) and ρ_s(He) used are larger than the corresponding values found by Hansen and Van Horn in any of their models, (3) ^{12}C has been assumed to burn to ^{56}Fe (if it were to burn to other iron-peak nuclei, the yield would be smaller), and (4) a PS neutron star model, which is based on a stiff equation of state and therefore has a large value of M_{env}, has been used. The observed values of E_b and t_b for the burst sources listed in Table 1 are shown as filled circles, with the exception of the rapid burster which is shown as a heavy line. The observed values of E_b are inconsistent with hydrogen burning as a source of the bursts, but are consistent with He, C, or O burning. The observed (E_b, t_b) points for four sources lie near the regions allowed for He, C, and O burning, but those for the other five sources lie well outside.

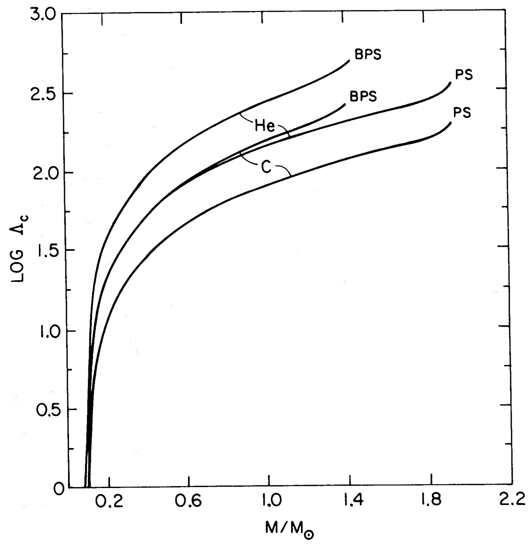

FIG. 6.—Lower bounds on Λ_c for He and C burning for BPS and PS neutron star models. The observational parameter $\Lambda = L_0/\langle L_b\rangle$ must equal or exceed Λ_c in order to be consistent with nuclear burning with steady replenishment.

c) Frequencies and Durations of Burst-active States

Although nuclear burning *with steady replenishment* appears to be inconsistent with the observed values of Λ, nuclear burning sources can exhibit relatively small values of Λ for some time if the fuel is not replenished between bursts but is instead progressively used up (Woosley and Taam 1976). Nevertheless, even in such "storage battery" models the duration of burst-active states and the fraction of the time that the source is in such a state are both subject to upper bounds similar to those which apply in the case of burning with steady replenishment. Thus, if fuel is burned significantly faster than it is accreted, all the fuel must be consumed after a time

$$t_{\text{active}} \sim (E_{\max}/E_b)t_b \,. \tag{19}$$

Hence if $N_{\max}$ is the number of bursts in the *longest* observed burst-active period of a given source, it must satisfy

$$N_{\max} \lesssim E_{\max}/E_b \tag{20}$$

in order to be consistent with nuclear burning models (it can, of course, be much greater than $E_{\max}/E_b$ for accretion models). Figure 5 shows that $N_{\max}$ is *at most* $\sim 10^2$ for He, and $\sim 10^5$ for C and O. So far 18 nearly consecutive bursts have been reported from MXB 1837+05 (Li *et al.* 1977), 22 from 3U 1820−30 = NGC 6624 (Clark *et al.* 1977), 21 from MXB 1728−34 (Hoffman, Lewin, and Doty 1977), and 35 from MXB 1659−29 (Lewin 1977*b*). These observations are inconclusive with regard to He, C, and O, but even a fivefold increase in the number of consecutive bursts would place severe restrictions on He-burning models.

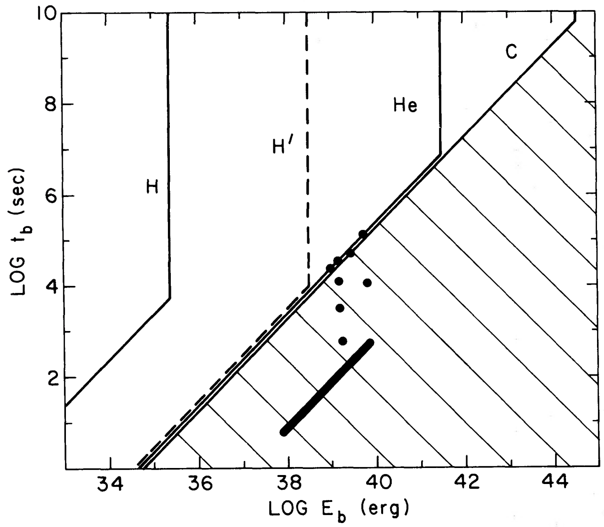

FIG. 7.—The allowed values of E_b and t_b for the fuels of Figs. 4 and 5 deposited on a 1.0 $M_\odot$ PS star. The vertical line bounding the allowed region for a given fuel on the right is the value of E_{max} for that fuel (see Fig. 4), while the line sloping from lower left to upper right is the constraint imposed by the background luminosity L_0, here assumed to be 10^{37} ergs s^{-1}. The hatched region in the lower right of the figure represents values of E_b and t_b that are inconsistent with nuclear burning with steady replenishment of fuel. The observed values of E_b and t_b for the burst sources listed in Table 1 are shown as filled circles with the exception of the rapid burster, which is shown as a heavy line.

The long-term average background luminosity $\langle L_0 \rangle$ places an upper bound on the fraction f of the time that a nuclear-powered burst source can be in an active state. To see this, note that the total nuclear energy that can be accumulated during a long period T is

$$E_{nuc} = \dot{M}\phi_{nuc}T\,, \tag{21}$$

which can produce at most $N_b = E_{nuc}/E_b$ bursts. Thus the total time that the source can be in a burst-active state is at most $T_{active} = N_b t_b$, so that $f = T_{active}/T$ must satisfy

$$f \leq \frac{\langle L_0 \rangle}{\langle L_b \rangle} \frac{\phi_{nuc}}{\phi_{grav}} = \frac{\langle \Lambda \rangle}{\Lambda_c}\,. \tag{22}$$

The burst-active states of NGC 6624 ($\langle L_0 \rangle / \langle L_b \rangle \sim 10^3$ and $f \sim 0.5$; Lewin 1977*a*) appear to be just consistent with inequality (22), but the data on most sources are still too sparse to estimate f.

IV. CONCLUDING REMARKS

Hydrogen-burning on the surfaces of neutron stars appears to be ruled out as the cause of the X-ray bursts. Indeed, no nuclear fuel is consistent with observed background luminosities if the fuel burned in a burst must be replenished by the time of the next burst. However, "storage battery" models of He, C, or O burning may be consistent with the observed properties of some burst sources, although not with those of the rapid burster. On energetic grounds C and O appear to be the most promising fuels. The unavoidable eventual exhaustion of fuel in "battery" models limits the number of bursts that can occur in a single active state, as well as the long-term ratio of burst-active to burst-inactive periods. Thus, observational determinations of the quantities N_{max} and f are important tests of nuclear burning models. The average accretion rate required for C or O thermonuclear runaway implies a readily observable background accretion luminosity. Finally, the high envelope temperatures required for C and O thermonuclear runaway combined with the large thermal inertia of the neutron star imply a significant X-ray luminosity even when the source is inactive and the star is not accreting.

Within the framework of the nuclear burning model of X-ray bursts, there are many questions for future study. Many of these questions are also relevant to the nuclear burning behavior of accreting neutron stars in contexts other than that of X-ray bursts, such as the transient X-ray sources (Van Horn and Hansen 1974) and even the known neutron star pulsating X-ray sources. Some of the most important questions are the following: (1) What is the structure of the H-burning region and what nuclear reactions occur there? Is the H-burning region thermally unstable? (2) What are the processes governing the time scale between flashes? Does thermal instability lead to rapid flashing or to more violent or longer-time scale nuclear burning? (3) What is the time scale on which the energy released by a thermonuclear flash can get out of the neutron star? What roles do convection and radiative diffusion play in this? (4) What are the effects of very strong ($B \sim 10^{12}$ gauss) magnetic fields on fuel deposition, the structure of the burning regions, and the time scale for transport of energy to the stellar surface? (5) Why might nuclear bursts occur in some accreting neutron star X-ray sources but not in all? Detailed numerical calculations now in progress (Joss 1977; Starrfield 1977; Taam 1977) may soon answer some of these questions.

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