THE ASTROPHYSICAL JOURNAL, 219:504–514, 1978 January 15

IONIZATION CONDITIONS IN THE EXPANDING ENVELOPES OF O AND B STARS

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Received 1977 February 7; accepted 1977 July 20

ABSTRACT

The presence of O VI, N V, and Si IV ions in the expanding envelopes of early-type stars is correlated with their location in the Hertzsprung-Russell diagram. The O VI and N V lines are observed in the region to the left of the dividing line from ($\log T_{\rm eff}$, $M_{\rm bol}$) = (4.50, −6.0) to (4.20, −9.0). The Si IV lines are observed in nearly all stars which show mass-loss effects, i.e., $M_{\rm bol} \leqslant -6.0$. The estimated ionization fractions of these ions in the envelopes were compared with calculations, assuming the temperature of the envelope is $T_{\rm env} = 0.80 T_{\rm eff}$ and that the radiative ionizations are due to the diluted photospheric flux. The observed degree of ionization is much higher than can be accounted for by radiative ionizations, thus indicating that the envelopes are hot. The stars which have O VI and N V lines must have a $T_{\rm env} \approx 4 \pm 2 \times 10^5$ K and the stars which have Si IV lines but no O VI or N V lines must have $T_{\rm env} \approx 7 \pm 3 \times 10^4$ K. Some alternative possibilities to explain the high degree of ionization are discussed and rejected. The consequences of the hot envelopes for the four stellar wind models are discussed.

Subject headings: stars: circumstellar shells — stars: early-type — stars: mass loss — stars: winds

I. INTRODUCTION

Since the first rocket observations of ultraviolet stellar spectra by Morton (1967) indicated conclusively that early-type supergiants are ejecting mass at a rate of about 10^{-5} to $10^{-6}\ \mathfrak{M}_\odot\ {\rm yr}^{-1}$ with velocities up to about 2000 to 3000 km s^{-1} (see Hack 1969 for earlier work), various attempts have been made to explain the mechanism of mass loss. The first model was proposed by Lucy and Solomon (1970), who suggested that the stellar wind is produced by radiation pressure due to the UV resonance lines of abundant ions. The rate of mass loss is determined by the required smooth transition from sub- to supersonic velocities at a critical point where the effective gravity vanishes. The rate at which mass can be ejected is of the order of

$$\dot{\mathfrak{M}} \approx NLc^{-2}\,,$$

where $\dot{\mathfrak{M}}$ is the mass-loss rate (in g s^{-1}), L is the stellar luminosity (in ergs s^{-1}), c is the speed of light (in cm s^{-1}), and N is the effective number of strong resonance lines available. The observed rate of mass loss of the Orion supergiants corresponds to a value of $N \approx 10^2$. As Lucy and Solomon's model assumes $N \approx 1$ or 2, their theory predicts far too low a rate of mass loss.

To overcome this problem Castor, Abbott, and Klein (1975) included the effect of many hypothetical C III lines from excited levels, and found a consequently much larger mass-loss rate. Later, Castor, Abbott, and Klein (1976) rejected the presence of these lines and introduced the effect of resonance lines in the far-UV below the Lyman limit (see also Lucy 1975). This was done independently by Lamers and Morton (1976) on the basis of their analysis of ζ Pup. Lucy and Solomon and Castor *et al.* make the basic assumption that the temperature of the outflowing envelope is determined by radiative equilibrium and thus is approximately equal to the effective temperature of the star. We will refer to these models as the *cool radiation pressure model.*

A completely different model for mass loss was proposed by Hearn (1975*b*). He assumed that the principal cause of the stellar wind is the same as that of the solar wind, namely, the presence of a hot corona which cannot be retained by the stellar gravity. The mass-loss rate is determined in analogy to the Sun by the requirement that at a certain height the flow velocity be equal to both the escape velocity and the sound velocity. In order to explain the observed mass-loss rates, Hearn had to assume a coronal temperature of at least 3.5×10^6 K for the Orion supergiants. The final acceleration in this model is also due to radiation pressure in UV resonance lines in the outer corona. Because of the large radiative losses from the high-density corona, cooling is very rapid in its outer regions. We will refer to this model as the *coronal model.*

In the *coronal model* and in the *cool radiation pressure model*, the mass-loss rate is determined by imposing the boundary condition that the flow should be "perfect," i.e., the velocity should increase smoothly from subsonic to supersonic through the "critical point" of the momentum equation. However, Thomas (1973) and Cannon and Thomas (1977) have argued that in reality the flow will not be "perfect," and that the resulting shocks will create a chromosphere in the region where the flow velocity becomes sonic. In this model the heating occurs very low in the envelope. We will call this the *imperfect flow model.*

Rogerson and Lamers (1975) proposed a model, based on the observations of the UV spectra of τ Sco (B0 V), in which heating and radiation pressure both play an essential role. Owing to the dissipation of mechanical energy, the temperature in the outer atmosphere increases outward. The high temperature gives rise to the presence of highly ionized atoms like O VI, S VI, and N V which have resonance lines in the UV Balmer continuum but no photospheric counterparts. Therefore these ions absorb a strong UV continuum flux and produce a large radiation pressure. The resulting acceleration to supersonic velocities prevents further heating, so the outflowing envelope will be warm, with $T \approx 10^5$ K. We will refer to this as the *warm radiation pressure model.*

In this paper we will study the observational evidence for the presence and absence of warm or hot expanding envelopes and their location in the temperature-luminosity diagram. We will use the ionization balance in the envelope as an indicator of temperature.

II. OBSERVATIONAL DATA

Many of the stars included in the present study are described in the survey of mass-loss effects by Snow and Morton (1976), where tables are presented showing basic data on the stars as well as observed characteristics of the P Cygni profiles.

Additional stars, especially Be and shell stars and others near the lower luminosity limit for observed mass loss to occur (about $M_{\rm bol} = -6.0$; Snow and Morton), have been observed and are also included in the present study. Table 1 lists these new objects, and gives other information such as observation date, spectral class, absolute visual magnitude M_v, the effective temperature ($\log T_{\rm eff}$), the absolute bolometric magnitude $M_{\rm bol}$, the projected rotational velocity $v \sin i$, and brief remarks concerning the presence or absence of mass-loss effects. For most of these stars only the Si IV doublet (1393.755, 1402.770 Å) was observed, since Snow and Marlborough (1976) found this feature to be the most sensitive stellar wind indicator available to *Copernicus* for early to mid-B stars.

The spectral types, absolute visual magnitudes, and absolute bolometric magnitudes in Table 1 are taken primarily from the compilations of Lesh (1968, 1972), using intrinsic colors from Johnson (1966). For one star not included in these compilations (β Tau), the spectral type from Slettebak *et al.* (1975) was adopted, and the other stellar parameters were taken from stars of identical spectral type in Lesh's work. For η Ori (B0.5 Vnn), the absolute visual magnitude adopted by Lesh seems unreasonably bright, possibly owing to misassignment of this star to a background association, and values of M_v and hence $M_{\rm bol}$ for other B0.5 V stars were adopted instead.

All of the effective temperatures in Table 1 were taken from the empirical scale of Code *et al.* (1976), again using the intrinsic colors adopted by Johnson (1966). For two stars, γ Ori and γ Crv, empirical effective temperatures and bolometric corrections were taken from Code *et al.* (1976); these quantities are underlined in the table.

The values of $v \sin i$ which are listed in Table 1 are taken from the catalog of Uesugi and Fukuda (1970). It was decided not to use the new calibration of Slettebak *et al.* (1975) because several of the stars from Table 1 are not included in that compilation, whereas all of them are in Uesugi and Fukuda. For the present discussion and that of Snow, Lamers, and Marlborough (1978), it is important to properly rank the stars in order of projected rotational velocity, even if the absolute calibration of the $v \sin i$ scale is inaccurate. Comparisons of the data in Uesugi and Fukuda and those in Slettebak *et al.* show that this goal is being achieved.

The remarks in the final column of Table 1 are used primarily to indicate whether Si IV is present in the spectrum, and if it is, whether it is photospheric or is formed in a stellar wind. The chief means of distinction between photospheric and circumstellar Si IV is the presence or absence of extended short-wavelength wings, or, in more extreme cases, a shortward shift of the line centers. In the spectra of some B dwarfs of class B3 or later, shifted Si IV is present, while a photospheric component is clearly absent. According to Kamp (1975), the total equivalent width of the 1393.755 and 1402.770 Å lines is less than 2 Å for dwarfs with $T_{\rm eff} \leq 20{,}000$ K, corresponding to a B2.5 V classification. This seems consistent with the *Copernicus* data, which show that photospheric Si IV is either very weak or absent in class IV and V stars of this type or cooler. Shifted Si IV absorption is seen in stars as late as B5 V, however.

The remarks also indicate whether photomultiplier U1 (resolution 0.05 Å) or U2 (0.2 Å) was used in making the scans. The spectrometer is described by Rogerson, Spitzer *et al.* (1973).

The stellar winds and other characteristics of Be star ultraviolet spectra will be discussed more fully in later papers by Marlborough (1977) and Snow, Peters, and Mathieu (1978). In this paper the primary concern is for the distribution of warm stellar winds in the H-R diagram, although some discussion of Be stars is included (§ V).

Figure 1 shows in the temperature-luminosity diagram where stars exhibiting shifted lines from O VI (Fig. 1*a*), N V (1*b*), and Si IV (1*c*) are located. In these diagrams only stars which show mass loss are included; reference to Table 1 and to Snow and Morton (1976) shows the distribution of stars examined which are not seen to be ejecting material. In Figure 1 open and filled circles represent the absence or presence of each ion, respectively, and horizontal bars through

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TABLE 1
Properties of Stars not Tabulated by Snow and Morton (1976)

Star	HD	Year	Days	Spectrum	M_V	$\log T_{eff}$ (°K)	M_{bol}	vsini (km s^{-1})	Remarks
υ Ori	36512	1975	328-329	B0 V	-3.92	4.51	-6.98	17	OVI present and asymmetric (U1)
γ Cas	5394	1976	34-35	B0.5 IVe	-4.10	4.45	-6.85	300	SiIV asymmetric (U1)
η Ori	35411	1976	38-39	B0.5 Vnn	-3.7:	4.45	-6.5:	47	SiIV unshifted, symmetric (U1)
25 Ori	35439	1976	52-52	B1 Vn	-3.13	4.41	-5.71	316	SiIV shifted and asymmetric (U2)
π Aqr	212571	1976	188-189	B1 Ve	-3.50	4.41	-6.08	278	SiIV shifted and asymmetric (U2)
α Pyx	74575	1976	129-130	B1.5 III	-3.66	4.34	-5.86	1	SiIV unshifted, symmetric (U2)
	28497	1973	280-285	B1.5 Ve	-2.80	4.38	-5.21	340	SiIV shifted (U2)
59 Cyg	200120	1972	289-294	B1.5 Ve	-2.80	4.38	-5.21	450	N V and SiIV strongly asymmetric (U2)
γ Ori	35468	1976	57-58	B2 III	-3.21	4.33	-5.30	64	SiIV asymmetric (U2)
δ Cen	105435	1976	142-143	B2 IVne	-3.00	4.36	-5.32	215	SiIV shifted and asymmetric (U1)
δ Cru	106490	1976	78-79	B2 IV	-3.00	4.36	-5.32	218	SiIV asymmetric (U2)
γ Lup	138690	1976	161-162	B2 IV	-3.00	4.36	-5.32	283	SiIV unshifted, symmetric (U1)
ω CMa	56139	1976	61-62	B2 IV-Ve	-2.44	4.32	-4.76	137	SiIV unshifted, symmetric (U2)
μ Cen	120324	1975	142-143	B2IV-Ve	-2.40	4.36	-4.72:	191	SiIV unshifted, symmetric (U2)
$υ^1$ Cen	121790	1976	77-78	B2 IV-V	-2.40	4.36	-4.72	153	SiIV unshifted, symmetric (U2)
φ Per	10516	1973	242-245	B2 Vep	-2.20	4.36	-4.52	450	SiIV strongly asymmetric (U2)
υ Cyg	202904	1976	183-184	B2 Ve	-2.20	4.36	-4.52	261	SiIV possibly asymmetric (U2)
ζ Cen	121263	1976	162-163	B2.5 IV	-2.10	4.32	-4.22	191	SiIV possibly asymmetric (U1)
ο Vel	74195	1976	118-119	B3 IV	-1.30	4.26	-3.10	0	SiIV present; unshifted, symmetric(U2)
α Eri	10144	1972	280-285	B3 Vp	-1.30	4.26	-3.10	411	SiIV present and asymmetric (U1)
	102776	1976	83-84	B3 V	-1.30	4.26	-3.10	270	SiIV absent (U2)
ζ Tau	37202	1976	66-66	B4 IIIp	-4.70	4.16	-5.9:	310	SiIV shifted, asymmetric (U1)
48 Lib	142983	1976	210-210	B5 IIIp	-2.51	4.12	-3.21	400	SiIV possibly present, shifted (U2)
ψ Per	22192	1976	40-41	B5 Ve	-2.26	4.18	-3.61	398	SiIV present, shifted (U2)
ρ Lup	128345	1976	81-82	B5 V	-0.90	4.18	-2.25	240	SiIV absent (U2)
β Tau	35497	1976	59-59	B7 III	-2.7:	4.07	-3.3:	68	SiIV possibly present, unshifted, symmetric (U1)
α Col	37795	1973	8-16	B7 IV	-0.60	4.11	-1.50	155	SiIV probably absent (U2)
γ Crv	106625	1976	173-173	B8 III	-1.7:	4.10	-2.42	41	SiIV absent (U2)

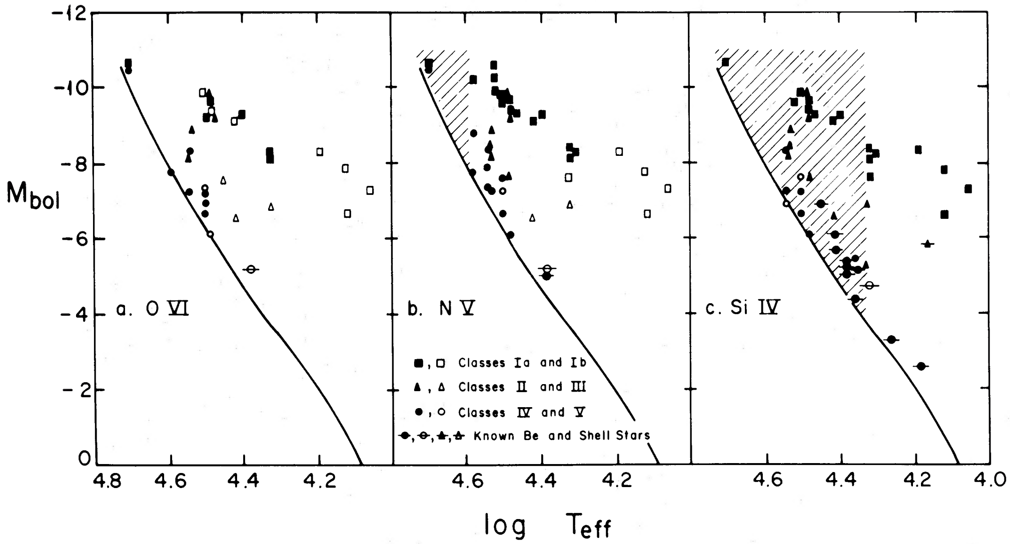

FIG. 1.—The location in the temperature-luminosity diagram of the stars which have O VI, N V, or Si IV ions in their envelopes. *Filled symbols*, those stars where the lines of these ions are present or probably present. *Open symbols*, those stars where the lines are absent or probably absent. The hatched area shows the location of the stars for which one would expect to observe the lines if the envelopes were cool ($T \approx 0.8T_{\rm eff}$).

the symbols indicate known Be or shell stars. The zero-age main sequence adopted by Snow and Morton is shown in each diagram.

Snow and Morton found that observational evidence for mass loss generally occurs for stars brighter than $M_{\rm bol} = -6.0$, and is usually absent below that luminosity. Figure 1 demonstrates that some stars fainter than $M_{\rm bol} = -6.0$ do show P Cygni profiles. Most of these objects are Be or shell stars which were excluded by Snow and Morton. The addition of these stars along with the use of U1 in several cases has revealed shifted or asymmetric Si IV λ1400 features in stars as faint as $M_{\rm bol} = -3.61$ (ψ Per).

One star brighter than $M_{\rm bol} = -6.0$, δ Sco, was found by Snow and Morton not to show mass loss in the U2 data available to them. New U1 scans of the Si IV doublet reveal extended short-wavelength wings of these features, establishing that mass ejection is occurring in δ Sco, although the maximum observed outward velocity, about 420 km s^{-1}, is less than the escape velocity at the surface of the star.

III. THE PREDICTED IONIZATION BALANCE

The presence of the observed ions in the envelopes can be related to the physical conditions in the envelope. Therefore we calculated the ionization balance in the envelopes for two different models: (*a*) the envelope is cool ($T_e \approx 0.80T_{\rm eff}$) and optically thin; (*b*) the envelope is hot ($T_e > T_{\rm eff}$) and optically thin. We will show in § V that the optically thin approximation is justified.

a) *Ionization in a Cool Envelope*

In a cool envelope, where $T_e \approx T_{\rm eff}$, the ionization balance is mainly determined by radiative processes. We calculated the ionization fractions in the envelope taking into account radiative ionization, collisional ionization, direct recombination, and dielectronic recombination. As the collisional ionizations and the dielectronic recombination coefficients are temperature-dependent, we adopted an electron temperature of $T_e = 0.8T_{\rm eff}$. The atomic data are the same as used by Lamers and Morton (1976). The recombination coefficients are from Aldrovandi and Péquignot (1973) and their corrections (1976). At electron densities of the order of 10^9 to 10^{11} cm^{-3}, the dielectronic recombination is reduced to about 20% or 40% of the values given by Aldrovandi and Péquignot (Jordan 1969). Therefore we calculated the ionization equilibrium with and without dielectronic recombination.

The mean intensity of the radiation was expressed in terms of the stellar photospheric flux $F_\nu{}^*$ and a geometrical dilution factor $W(r)$

$$J_\nu(r) = W(r)F_\nu{}^* \, . \qquad (1)$$

The photospheric flux distribution was taken from the line-blanketed models of Kurucz, Peytremann, and Avrett (1974). Since the ionization rate is proportional to W and the recombination rate is proportional to the electron density n_e, the ratio of two successive ionization stages n_{i+1}/n_i will be proportional to W/n_e. In Figures 2*a*, *b*, *c* we show the fraction n_i/n_t of the observed ions (i) relative to the total number of particles of that element (t), for a series of model atmospheres at three values of $n_e/W = 10^9, 10^{10}, 10^{11}$ cm^{-3}. (For characteristic values in the envelopes of $n_e \approx 10^9$ and $W \approx 0.1$, i.e., $r/R_* \approx 3$, the value of $n_e/W \approx 10^{10}$ should be more or less characteristic; see § V.) Since any ratio n_{i+1}/n_i is proportional to W/n_e, the ratio n_{i+m}/n_i is proportional to $(W/n_e)^m$; therefore, if n_i is

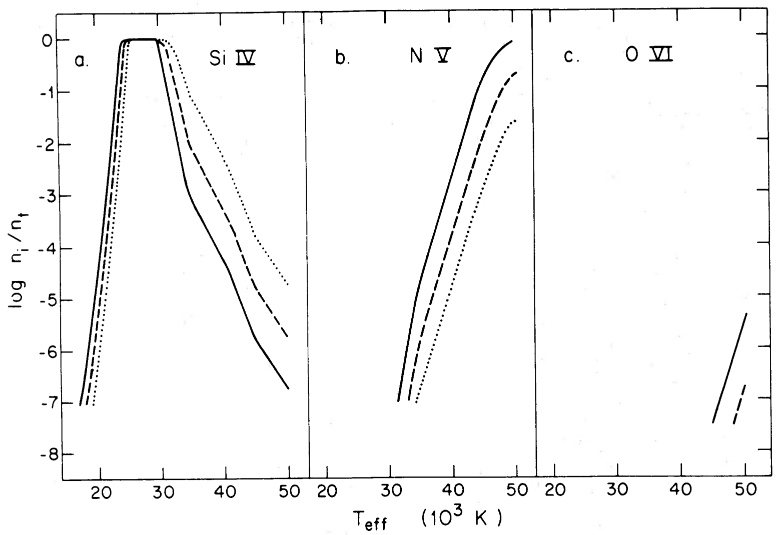

FIG. 2.—The predicted ionization fractions of the Si IV, N V, and O VI ions in cool envelopes around early-type stars. The ionization is in radiative equilibrium with the diluted photospheric flux. The ordinate gives the effective temperature of the adopted line-blanketed model atmosphere with $\log g = 4.5$. *Solid line*, $n_e/W = 10^9$ cm^{-3}; *dashed line*, $n_e/W = 10^{10}$ cm^{-3}; *dotted line*, $n_e/W = 10^{11}$ cm^{-3}.

the dominant stage of ionization, n_i/n_t is nearly insensitive to W/n_e, but n_{i+m}/n_t is proportional to $(W/n_e)^m$. The curves in Figure 2 were calculated without dielectronic recombination and with $F_\nu{}^*$ from models of $\log g = 4.5$. We also calculated the ionization for lower-gravity models, but the difference with data in Figure 2 was always less than $\Delta \log n_i/n_t < 0.2$. Therefore we can adopt Figure 2 for all luminosity classes. Notice that we expect $n(\text{O VI})/n(\text{O}) \leq 10^{-5}$ for models with $T_{\text{eff}} = 50{,}000$ K and $< 10^{-10}$ for models with $T_{\text{eff}} \geq 45{,}000$ K. If dielectronic recombination had been taken into account, the ascending branches in Figure 2 would have been lower by about 0.5 to 1 dex.

b) *Collisional Ionization in an Optically Thin Hot Envelope*

If the temperature in the envelope is higher than T_{eff}, the ionization balance will shift to higher stages of ionization when the collisional ionization rate dominates the radiative ionization rate. Summers (1974*a*, *b*) calculated ionization factors for various values of n_e and a large range of electron temperatures. We adopt his values for $n_e \approx 10^9$ cm^{-3} which is about a typical density in the envelopes (§ V, Table 3). In any case, if the collisional ionizations are more important than the radiative ionizations, the fractions n_i/n_t are nearly independent of n_e, since the ionization rates and the recombination rates are both proportional to n_e. In Table 2 are listed the electron temperature $T_{\max}$ where the ionization fraction of the ions reaches its maximum value $(n_i/n_t)_{\max}$. We also indicate the lower and upper temperatures T_l and T_u for values of $n_i/n_t = 10^{-2}$ and 10^{-4}. We also include the data for He II from House (1964).

IV. COMPARISON BETWEEN PREDICTED AND OBSERVED IONIZATION BALANCE

The result of the calculated ionization balance is shown in Figure 1, where the hatched areas indicate the regions in the temperature-luminosity diagram where one might expect to find the three ions *if the envelopes were cool.* This region was derived from Figure 2 and defined by $n_i/n_t \geq 10^{-4}$ if $n_e/W \geq 10^{10}$ cm^{-3}.

It is clear that the region where the ions are actually observed extends to much lower temperatures than predicted. For instance, the cool boundary of the region where O VI and N V lines were observed in the envelopes occurs for stars of types as late as B1 Ib ($T_{\text{eff}} = 20{,}000$ K) where the predictions for a cool envelope result in $n(\text{O VI})/n(\text{O}) < 10^{-10}$ and $n(\text{N V})/n(\text{N}) < 10^{-10}$. Similarly, the cool boundary

TABLE 2

THE PREDICTED IONIZATION BALANCE IN A HOT ENVELOPE

		$n_i/n_t = 10^{-4}$		$n_i/n_t = 10^{-2}$	
ION	$\log T_{\max}$	$\log T_l$	$\log T_u$	$\log T_l$	$\log T_u$
He II...	4.65	4.20	5.70	4.30	5.15
C III....	4.90	4.25	5.45	4.40	5.20
C IV....	5.05	4.65	6.20	4.80	5.40
N III....	4.90	4.30	5.55	4.45	5.35
N IV....	5.20	4.65	5.85	4.90	5.50
N V....	5.30	4.95	6.45	5.05	5.70
O IV....	5.25	4.70	5.85	4.85	5.60
O V....	5.45	5.00	6.20	5.15	5.70
O VI....	5.50	5.15	6.70	5.30	6.00
Si III....	4.70	4.05	5.45	4.20	5.05
Si IV....	4.90	4.45	6.05	4.65	4.60

of the region where the Si IV lines are observed in the envelopes occurs in stars of types as late as B8 Ia ($T_{\rm eff} \approx 12{,}000$ K) where the predicted ratio of n(Si IV)/n(Si) $< 10^{-10}$. As noted in § II, Si IV lines are seen in the envelopes of some main-sequence stars which are too cool to have any Si IV even in their photospheres.

Therefore the conclusion that the ionization balance is not dominated by radiative processes seems unavoidable. The high degree of ionization points to the presence of a high-temperature envelope rather than a cool one.

To estimate the temperature in the envelopes of the stars, we use the data from Summers's calculations as summarized in Table 2. We can distinguish three regions in the H-R diagram, where we can approximately indicate the temperature.

In the region $\log T_{\rm eff} > 4.5$ the stars which show O VI also have N V. In only one of them (ζ Pup, O4f) was the Si IV region scanned and the lines are strongly present. If we adopt as a criterion for the presence of lines that $n_i/n_t \geq 10^{-2}$, the stars in this region must have envelopes with $5.30 \leq \log T \leq 5.70$. This agrees with the temperature derived for ζ Pup from a detailed study of the envelope by Lamers and Morton (1976), who found $\log T = 5.28$. We assign an average temperature of $4 \pm 2 \times 10^5$ K to this region.

The stars with $\log T_{\rm eff} < 4.3$ have Si IV in their envelopes but no O VI or N V, and consequently the temperature is probably on the order of $7 \pm 3 \times 10^4$ K. Toward high temperatures this region extends to $M_{\rm bol} = -6$ and $\log T_{\rm eff} = 4.5$.

In the region in between, from $\log T_{\rm eff} < 4.5$ to the boundary, $M_{\rm bol} = -6$, and $\log T_{\rm eff} > 4.5$ to $M_{\rm bol} = -8$ and $\log T_{\rm eff} = 4.3$, all stars except δ Sco have N V lines. Most of the stars in this region, 11 out of 16, have O VI lines. All of them except μ Col and ζ Oph have Si IV lines showing mass-loss effects. The temperature of the envelopes in this region is probably of the order of $2 \pm 1 \times 10^5$ K.

The three regions and their approximate temperatures are plotted in Figure 3.

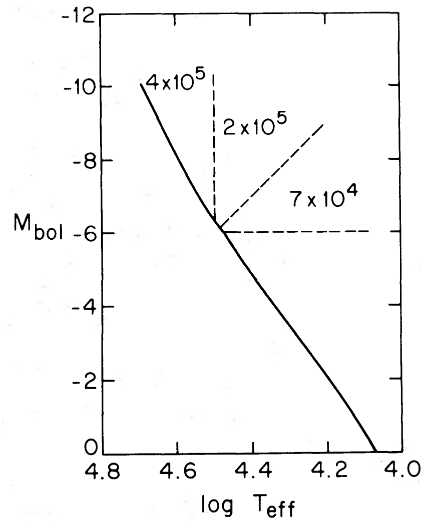

FIG. 3.—The three regions in the temperature-luminosity diagram which were studied. The derived approximate temperatures of the envelopes are indicated.

V. STUDIES OF INDIVIDUAL STARS

In the previous section we showed that the presence of O VI and N V ions in the envelopes of early B and O supergiants and the presence of Si IV ions in the envelopes of middle B and late B supergiants indicates a high temperature ($T > T_{\rm eff}$) of the envelope. In this section we will study the ionization balance of a few stars in more detail. In particular, the optical depth of the envelope in the He II continuum will be estimated to demonstrate that the optically thin approximations used in § IV are justified.

a) *Mass-Loss Rates and Electron Densities*

In Table 3 we listed the stars which were observed by the *Copernicus* satellite and for which the mass-loss rates have been determined by Barlow and Cohen (1977) from the infrared excess. The first six columns give the basic data, taken from Barlow and Cohen except for ζ Pup, where the data are taken from Lamers and Morton (1976). For the terminal velocity v_∞ of the Orion supergiants, we did not adopt the high value derived from the N V lines, since this may be due to some blending line (Snow and Morton 1976). Instead we adopted the edge velocity derived from the other lines, given by Barlow and Cohen.

The density in a spherically symmetric expanding envelope is given by the equation of mass continuity

$$\rho(r) = 1.04 \times 10^3\, \dot{\mathfrak{M}}(r/R_*)^{-2} \times (R_*/R_\odot)^{-2} v(r)^{-1}\ \mathrm{g\ cm^{-3}}\,, \qquad (2)$$

where $\dot{\mathfrak{M}}$ (in $\mathfrak{M}_\odot$ yr^{-1}) is the mass-loss rate, R_* and $R_\odot$ (in cm) are the stellar and solar radius, and $v(r)$ (in cm s^{-1}) is the wind velocity. An approximate velocity law for the outer layers of the envelope, where $v \geq 0.5 v_\infty$, can be derived. The radiative pressure force, which produces the acceleration to large velocities, decreases outward with the flux as r^{-2}, so

$$v dv/dr (:) r^{-2}\,. \qquad (3)$$

The solution of this equation, with the boundary conditions $v = v_\infty$ at $r = \infty$ and $v = 0.5 v_\infty$ at $r = r_{0.5}$ implied, gives

$$v(r) = v_\infty (1 - 0.75 r_{0.5}/r)^{1/2} \quad \text{for } 0.5 \leq v/v_\infty \leq 1\,. \qquad (4)$$

The studies of the Orion supergiants by Hutchings (1970, 1976) indicate that $v = 0.5 v_\infty$ is reached at $r_{0.5} \approx 3 R_*$. Barlow and Cohen (1977) derived a velocity law for P Cygni and found $r_{0.5} \approx 5 R_*$, while Lamers and Morton (1976) derived a value of $r_{0.5} \approx 2 R_*$ for ζ Pup. We adopt a mean value of $r_{0.5} \approx 3 R_*$. The resulting electron density in the envelopes at a

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TABLE 3

ELECTRON DENSITY AND OPTICAL DEPTHS OF THE ENVELOPES

HD	Star	Type	$R_*/R_\odot$	v_∞	$\dot{M}$	$n_e(r_{0.5})$	$\tau(\nu)/\kappa(\nu)$	Observed ions in envelope						τ(N IV)	τ(O V)
				km s^{-1}	$M_\odot$ yr^{-1}	cm^{-3}	at $r_{0.5}$	Si III	C III	Si IV	C IV	N V	O VI	at $r_{0.5}$	at $r_{0.5}$
66811	ζ Pup	O4f	15.6	2660	7.2×10^{-6}	1.4×10^{10}	5.6×10^{-2}	-	+	+	+	+	+	21.8	9.5
36861	λ Ori	O8 III((f))	11.5	3050	3.8×10^{-7}	1.1×10^{9}	3.5×10^{-3}	-	w 1)	-		+	+	1.4	0.6
36486	δ Ori	O9.5 I	22.8	2700	9.2×10^{-7}	7.9×10^{8}	4.8×10^{-3}	+	+	+	+	+	+	1.9	0.8
37742	ζ Ori	O9.5 I	23.9	2700	1.0×10^{-6}	7.9×10^{8}	5.0×10^{-3}	+	+	+	+	+	+	2.0	0.9
37128	ε Ori	B0 Ia	34.4	2300	1.7×10^{-6}	7.6×10^{8}	6.9×10^{-3}	+	+	+	+	+	+	2.7	1.2
38771	κ Ori	B0.5 Ia	30.7	1900	7.0×10^{-7}	4.7×10^{8}	3.9×10^{-3}	+	+	+		+	+	1.5	0.7
91316	ρ Leo	B1 Iab	31.0	1580	8.6×10^{-7}	6.9×10^{8}	5.7×10^{-3}	+	+	+		+	+	2.2	1.0
53138	o^2CMa	B3 Ia	64.6	580	1.7×10^{-6}	8.5×10^{8}	1.5×10^{-2}	+		+	+	-	-		
58350	η CMa	B5 Ia	59.1	590	4.2×10^{-7}	2.5×10^{8}	3.9×10^{-3}	+			-	-	-		
164353	67 Oph	B5 Ib	37.3	500	1.2×10^{-7}	2.1×10^{8}	2.1×10^{-3}	+		+		-			
34085	β Ori	B8 Ia	135	530	$1\ \times10^{-6}$	$1\ \times10^{8}$	$5\ \times10^{-3}$	+		+		-	-		

\+ line present - with $v_{edge} \simeq v_\infty$

\- line not present

w line weakly present

1) lines of this ion extend to $v < v_\infty$

distance $r_{0.5}$ where $v = 0.5v_\infty$ is given in column (7). A ratio of $n_e/\rho = 5.22 \times 10^{23}$ g^{-1} was adopted.

Columns (9) through (14) in Table 3 indicate stars in which the resonance lines of Si III, C III, Si IV, C IV, N V, and O VI (in order of increasing ionization potential of the preceding stage) are observed in the envelope.

b) *The Optical Depth of the Envelopes in the Ultraviolet Continuum*

The continuum optical depth of the envelope at a distance r from the stellar center is

$$\tau(\nu) = \int_r^\infty \kappa(\nu) \times \rho(r) dr = \sum_i \int_r^\infty a_i(\nu) \times n_i dr \quad (5)$$

where $\kappa(\nu)$ is the absorption coefficient cm^{-3}, $a_i(\nu)$ (in cm^2) is the absorption cross section, and n_i is the number density of the ions i. Combining equations (2) and (5) yields

$$\tau(\nu) = 7.24 \times 10^{13}\, \dot{\mathfrak{M}} (R_*/R_\odot)^{-1} \kappa(\nu) \times \int_r^\infty v(r)^{-1} (r/R_*)^{-2} d(r/R_*)\,, \quad (6)$$

if we assume that the ionization equilibrium and $\kappa(\nu)$ do not change with distance from the star.

We want to find $\tau(\nu)$ in the layer of the envelope where the O VI, N V, and Si IV lines are formed. The absorption components of O VI, N V, and Si IV extend from $v = 0$ to $v \approx v_\infty$ in the spectra of those stars where they are observed (Snow and Morton 1976). Therefore we adopt $v = 0.5v_\infty$ as the characteristic velocity, and the corresponding $r_{0.5} = r(v = 0.5v_\infty)$ as the characteristic distance of the layers where the lines are formed. So the integration in equation (6) should go from $r = r_{0.5}$ to $r = \infty$. With the velocity law of equation (4) and assuming $r_{0.5} = 3R_*$, the value of the integral in equation (6) is $(2.25v_\infty)^{-1}$. The values of $\tau(\nu)/\kappa(\nu)$ at $r_{0.5}$ are given in Table 3, column (8).

The absorption coefficient $\kappa(\nu)$ contains contributions from many ions

$$\kappa(\nu) = 4.42 \times 10^{23} \sum_i a_i(\nu) \times (n_i/n_t) \times A_i\,, \quad (7)$$

where n_i/n_t is the ionization fraction and A_i is the abundance relative to hydrogen. The constant gives the number of hydrogen atoms per gram of stellar material. The abundances were taken from Withbroe (1971).

Since we want to study the formation of the N V, O VI, and Si IV ions, we are especially interested in the optical depths at the ionization edges of N IV, O V, and Si III. The cross sections are taken from the references listed by Aldrovandi and Péquignot (1973). The following ions were taken into account: He I, II; C II–IV; N II–V; O III–V; Ne II; Si III. The following expressions were derived for the absorption coefficients κ_0 at the ionization edges of N IV, O V, and Si III.

$$\begin{aligned}
\kappa_0(\text{N IV}) = {} & 1.70 \times 10^4(\text{He II/He}) + 8.8 \times 10^1(\text{C IV/C}) + 4.0 \times 10^1(\text{N IV/N}) \\
& + 5.1 \times 10^2(\text{O III/O}) + 3.9 \times 10^2(\text{O IV/O})\,, \\
\kappa_0(\text{O V}) = {} & 5.57 \times 10^3(\text{He II/He}) + 4.0 \times 10^1(\text{C IV/C}) + 1.8 \times 10^1(\text{N IV/N}) \\
& + 1.9 \times 10^2(\text{O III/O}) + 1.1 \times 10^2(\text{O IV/O}) + 2.4 \times 10^2(\text{O V/O})\,,
\end{aligned}$$

and

$$\kappa_0(\text{Si III}) = 1.36 \times 10^5(\text{He I/He}) + 4.2 \times 10^2(\text{C II/C}) + 2.1 \times 10^2(\text{N II/N}) + 7.5 \times 10^2(\text{Si III/Si})\,. \quad (8)$$

Only the main contributors are included.

All stars in Table 3 which have O VI and N V lines also show C III and C IV lines. The Si III lines are not observed in the two hottest stars. The spectrum of ζ Pup also showed weak lines from excited levels of C III, N IV, O IV, and O V. Lamers and Morton concluded that an envelope temperature of $\log T = 5.30$ K is consistent with the observed ionization balance in ζ Pup. As the same ions are observed in the other stars, we adopt this envelope temperature for all stars which show O VI and N V lines (see also § IV). At 2×10^5 K, the dominant ionization stages are He III, C V, N IV, V, and VI, O III, IV, and V, and Si V. Using Summers's (1974*a*, *b*) ionization fractions and equation (8), we find the opacities at the ionization edges of N IV and O V of $\kappa(\text{N IV}) = 3.9 \times 10^2$ and $\kappa(\text{O V}) = 1.7 \times 10^2$ cm^2 g^{-1}; the largest contributions due to O IV. At $\log T_e = 5.0$ they are about a factor of 2 larger, and at $\log T = 5.50$ they are about a factor of 2 smaller. The resulting values of τ at $r_{0.5}$ are given in columns (15) and (16) of Table 3. Except for ζ Pup, the optical depths are between 0.6 and 2.7, i.e., of the order of unity. The optical depths for ζ Pup are about 10 times higher, due to its larger mass-loss rate.

The supergiants of types B3 and later do not have N V and O VI ions in their envelopes, but they do show lines of Si IV originating in their envelopes. Underhill (1974) found components of C II, N II, and Mg II shifted up to about -230 km s^{-1} (about

$0.5v_\infty$) in the *Copernicus* spectrum of η CMa. Lamers, Stalio, and Kondo (1978) found Mg II components up to -250 km s^{-1} ($\sim 0.5v_\infty$) in the high-resolution balloon-borne ultraviolet stellar spectrograph (BUSS) spectrum of β Ori. However, in both stars the envelope lines seem to be drastically variable, so it is dangerous to derive general conclusions for late B supergiants based on a few observations.

The simultaneous presence of Mg II, Si III, and Si IV ions in an optically thin envelope requires a temperature in the range of $4.45 \leq \log T \leq 5.20$ if $n_i/n_t > 10^{-4}$, but there is no range where the three ions reach $n_i/n_t > 10^{-2}$ simultaneously. A temperature as high as $\log T_e = 4.90$ is unlikely, however, since the N V lines are absent. The C IV lines, which could also give an indication of the temperature, are present in the spectrum of o^2 CMa but probably absent in the spectrum of η CMa. In the range of $4.45 < \log T < 4.90$, silicon is mainly in the Si III stage (Si III/Si ≈ 0.5). The optical depth at the Si III edge depends critically on the He I/He ratio. If He I/He ≈ 0.5 ($\log T = 4.45$) then κ_0(Si III) $\approx 7 \times 10^4$ cm^2 g^{-1} and the optical depth will be large: τ(Si III) $\approx 1 \times 10^3$ for o^2 CMa to 4×10^2 for β Ori. If, on the other hand, He I/He $\lesssim 10^{-3}$ ($\log T = 4.90$), the optical depth will be considerably smaller: τ(Si III) ≈ 8 for o^2 CMa to 3 for β Ori.

c) *Deviations from LTE in the Far-ultraviolet Continuum*

The optical depth of the envelopes at the ionization edges of O V, N IV, and Si III is of the order of unity. At such an optical depth very large deviations from LTE are to be expected for the radiation in the continuum. The continuum approaches LTE (i.e., J_ν approaches B_ν) for an optical depth of the order of

$$\tau \approx \epsilon^{-1/2}\,, \tag{9}$$

where

$$\epsilon = n_e \Omega_{ik}/4\pi \int_{\nu_0}^{\infty} (\alpha_\nu B_\nu / h\nu) d\nu \tag{10}$$

is the ratio of collisional to radiative ionization (e.g., Mihalas 1970, p. 227). An approximate expression for Ω_{ik} has been given by Allen (1973, p. 42). The denominator can be estimated by assuming Kramers's opacity law and the Wien distribution for the radiation. This yields

$$\epsilon = 6.2 \times 10^{-28} n \chi^{-4} a(\nu_0)^{-1} T^{-1/2} n_e\,, \tag{11}$$

where n is the number of optical electrons, χ is the ionization energy in eV, and $a(\nu_0)$ is the absorption coefficient at the ionization edge. For ions of interest O V, N IV, and Si III we find that $\epsilon \approx 10^{-9}$ to 10^{-11} if $n_e \approx 10^9$ cm^{-3} (Table 3) and $T \approx 10^4$ to 10^5 K. For such small values of ϵ and τ of the order of unity, the continuum will deviate very strongly from LTE (eq. [9]). Consequently, the assumption of an optically thin envelope in the calculation of the ionization balance is justified, except possibly for the late B stars, if the He I/He ratio is larger than 10^{-3} in their envelopes.

VI. DISCUSSION AND CONCLUSIONS

a) *Discussion*

We have collected data on the presence of O VI, N V, and Si IV lines in the expanding envelopes of early-type stars from the catalogue of Snow and Morton (1976), extended with recently available *Copernicus* spectra, listed in Table 1. In order to derive criteria for the electron temperature in the envelope, $T_{\rm env}$, we compare these data with predicted ionizations for two different situations: (*a*) radiative and collisional ionizations in a cool optically thin envelope where $T_{\rm env} = 0.80 T_{\rm eff}$, with radiative fluxes predicted for line-blanketed models; (*b*) collisional ionizations in a hot optically thin envelope for a range of $10^4 \leq T_{\rm env} \leq 10^7$ K.

In case (*a*) the ionization balance depends on n_e/W, for which we adopted a range of $10^9 \leq n_e/W \leq 10^{11}$ cm^{-3}, based on detailed studies of the expanding envelopes of a few stars. In case (*b*) the ionization balance depends only very weakly on n_e, since both the ionization and recombination rates are approximately proportional to n_e (Summers 1974*a*, *b*).

In the above calculations we have explicitly assumed that the high degree of ionization in the envelopes is not a result of an equilibrium which is frozen-in. This is a valid assumption if the recombination time scales are short, compared with the expansion time scale. The recombination time scale, $t_{\rm rec}$, can be defined as

$$t_{\rm rec} = [\partial \ln (n_{i+1}/n_i)/\partial t]_{\rm rec}{}^{-1} = (\alpha n_e)^{-1}\,, \tag{12}$$

where α is the recombination coefficient.

We find that for stars of types B1 and earlier in Table 3 $t_{\rm rec}$ (O VI/O V) ≤ 520 s and $t_{\rm rec}$ (N V/N IV) ≤ 300 s. In such a short time the flow travels less than $0.1 R_*$. Similarly, we found that for stars of types B3 and later $t_{\rm rec}$ (Si IV/Si III) ≤ 3000 s, which also corresponds to a flow distance less than $0.1 R_*$. So the recombination times are short compared with the expansion time.

From a comparison of the observations and predictions we can eliminate the first model, (*a*). We found that N, O, and Si are ionized to higher stages than expected on basis of the photospheric flux, if the envelopes are cool and optically thin for UV radiation. (*b*) The simultaneous presence of high (N V and O VI) and low (Si IV and C III) ionization stages in the envelopes of types B1 and earlier can be explained by assuming an optically thin envelope with a temperature of about 2–4 $\times 10^5$ K. The optically thin approximation is justified, since the optical depth of the envelopes in the layers where the lines are formed is of the order of unity at the ionization edges of O V and N IV (§ V*b*), whereas an optical depth of the order of 10^5 would be required for LTE (§ V*c*). (*c*) The envelopes around late B supergiants are considerably cooler, since the N V lines and probably also the C IV lines are absent. The ionization balance can be explained by assuming an optically thin envelope with a temperature $4.45 < \log T < 4.90$. However, the optically thin approximation is only justified if the He I/He ratio is smaller than about 10^{-3}. Moreover, these stars have variable envelope lines, which makes the result uncertain. If the Si III continuum were in LTE, a temperature of $1 \cdot 10^4 < T < 2 \cdot 10^4$ K would be required to explain the presence of Si IV lines.

The boundary between the stars with envelopes of about 2–4 × 10^5 K and those with lower temperatures occurs for supergiants between B1 and B5. The star o^2 CMa (B3 Ia) may be an intermediate star, as it shows envelope lines of C IV but no N V lines (Table 3). The data in Figure 1*b* suggest that this boundary shifts to earlier spectral types with decreasing luminosities, reaching the main sequence at $\log T_{\mathrm{eff}} \approx 4.5$ (type B0).

Figure 3 shows the location of the regions in the luminosity-temperature diagram where the envelopes are warm and cool, and the possible intermediate region. In this respect it is interesting to note that Rosendhal (1973) found an abrupt change in the velocity gradient luminosity relationship between the supergiants of types B3 and B8. He suggested that the driving mechanism for mass loss may change in this range.

As the envelopes of O and early B supergiants and possibly also of late B supergiants are hotter than the photospheres, the problem of the energy balance arises. The most plausible solution is to assume that the envelope is heated by dissipation of mechanical energy which could be generated either in the photosphere or in the envelope as a result of instabilities in the flow. From the theoretical point of view, Hearn (1972, 1973) has predicted a mechanical flux in the photospheres by radiation-driven sound waves. Spiegel (1976) has reviewed a larger class of instabilities in the presence of a strong radiation field. From the observational point of view, Lamers and De Loore (1976) have pointed out that the observed turbulence in supergiants, if interpreted in terms of sound waves, would represent a substantial mechanical flux.

Four models for the stellar winds were mentioned in § 1. The high temperatures in the envelopes of O and early B stars are in disagreement with the *cool radiation pressure model*. The temperatures of the envelopes are too low compared with the predictions of the *coronal* model, unless the temperature of the corona decreases very rapidly outward and the observed ions are located in the cooler region above the thin, hot corona. This last possibility, however, would put severe restrictions to the extent of the hot region, as does the observed upper limit of the soft X-ray flux of ζ Pup (Mewe *et al.* 1975).

The observations are in agreement with the *warm radiation pressure model*, which predicts temperatures of the order of 10^5 K to produce N V and O VI ions in the envelope. It also agrees with the *imperfect flow model*, which predicts that any star with an appreciable mass-loss rate has a chromosphere. The difference between these last two models is the *origin* of the wind: in the *warm radiation pressure model* the wind is a result of the radiation pressure force, which is increased by the heating; in the *imperfect flow model* the wind is produced by an imposed outward velocity in the subatmospheric regions, and the heating is due to shocks in the trans-sonic region. To distinguish these models observationally will require quantitative theoretical models that can be compared with the observational models.

Additional evidence for these last two models can be found from the study of mass loss from rapidly rotating stars. Figure 1 shows that below $M_{\mathrm{bol}} = -6.0$, nearly every class IV and V star which has a warm wind containing Si IV is a Be or shell star, i.e., a rapidly rotating star. Hence it seems that the presence of an extended atmosphere giving rise to emission or shell absorption lines is strongly linked to the presence of mechanical heating above the photosphere, which in turn plays a role in driving a stellar wind. The enhanced tendency for Be stars to have winds has already been pointed out by Snow and Marlborough (1976), and the possible relationship of stellar rotation to heating, mass loss, and Be star characteristics will be further discussed by Snow, Lamers, and Marlborough (1978).

b) *Conclusions*

1. The spectrum of supergiants of type B1 and earlier and the main-sequence stars of types B0 and earlier show absorption lines of C III, C IV, N V, O VI, and Si IV which are formed in the expanding envelopes. The spectrum of the supergiants of types B3 and later do not have N V and O VI lines, but they do show Si III and Si IV lines originating in the envelope. Be and shell stars of types later than B0 have shifted components of Si IV lines (Fig. 1).
2. The ionization equilibrium in the envelopes of stars with N V and O VI lines can be explained if the envelope has a temperature of about 2–4 × 10^5 K. Such an envelope is optically thin in the He II continuum ($\lambda < 228$ Å).
3. The ionization equilibrium in the envelopes of stars with Si III and Si IV lines, but without N V lines, can be explained if the envelope has a temperature of about 2.8–8.0 × 10^4 K and is optically thin in the He I continuum, or a temperature of 1–2 × 10^4 K and is optically thick in the He I continuum.
4. The presence of an envelope which is hotter than the photosphere around stars of types B1 and earlier is not in agreement with the cool radiation pressure model, but it is indicative of the warm radiation pressure model and the imperfect model.

We are indebted to Dr. N. Panagia for a lively and stimulating discussion concerning the optical depth of the envelopes, and to R. N. Thomas for many critical comments. An unknown referee is thanked for valuable suggestions.

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