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STRONG TiO–RELATED VARIATIONS IN THE DIAMETERS OF MIRA AND R LEONIS*

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ABSTRACT

New speckle interferometer observations in narrow spectral bands show abrupt variations in the diameters of Mira (o Ceti) and R Leonis as a function of wavelength. In strong TiO features, the diameter is two times larger. We have interpreted these variations as due to the large opacity of TiO: in the strong TiO features, the optically thick region of the atmosphere extends from the continuum-formation layer to several AU outward.

Subject headings: stars: diameters — stars: long-period variables

I. INTRODUCTION

Mira (*o* Ceti) and R Leo, the brightest members of the Mira class of long-term variables, have angular diameters well suited to speckle interferometer observations with large existing telescopes. Initial work had produced evidence for color-dependent size in Mira (Bonneau and Labeyrie 1973). With the relatively broad spectral bands used, 200 Å, angular size appeared to decrease rather smoothly from blue to red and infrared wavelengths.

On Betelgeuse, Lynds, Worden and Harvey (1976) have searched for the more abrupt size variations expected at TiO wavelengths in relation to molecular absorption, but did not find large size differences, although a definite small difference appears to be present.

On Mira and R Leo, our recent observations at Mount Palomar showed a similar effect of more striking magnitude: the apparent size increases by as much as a factor of 2 within 100 Å intervals.

In this *Letter*, we present these observations and show that they seem consistent with atmospheric models developed by Tsuji (1973).

II. OBSERVATIONAL RESULTS

The usual digital speckle interferometer and observing procedure were utilized, as previously described, at the 200 inch (5.08 m) telescope. As we had become accustomed to making a quick spectrum scan while observing the speckles, before recording data at selected wavelengths, a rather unusual behavior was immediately noticed on Mira and R Leo. Speckle size increased suddenly as we moved the spectral window across the dark TiO bands at the red end of the spectrum. This prompted a more detailed study, using wavelengths variously located with respect to TiO features and bandwidths adjusted to equalize flux levels in the presence of spectral variations.

The measured angular diameters listed in Table 1 indicate the general tendency: diameters decrease slowly from blue to red wavelengths. At red TiO absorption wavelengths, diameter becomes comparable to the green or yellow value, i.e., appreciably larger than the local average.

We also tried to detect any departure from revolution symmetry. Both Mira and R Leo appear to be revolution symmetrical within the current sensitivity limits of the instrument.

Since these objects are several times larger than the Airy disk, polarized features are likely to be detectable in their atmosphere. Among the most probable of such features is linear polarization at the limb, with radial symmetry. A search for such features was conducted on *o* Ceti at near-maximum brightness. It gave negative results.

The digital correlator described in a previous article (Blazit *et al.* 1977) served to process the data, both online during the observation and when back in the laboratory.

In the recent literature, some confusion has arisen regarding practical ways of determining stellar diameters from speckle data. The basic solution, i.e., observation of a reference star and deconvolution of autocorrelations, is objectionable in practice, since it uses valuable telescope time and assumes a steady type of "seeing." Theoretical knowledge of the reference autocorrelation, as influenced by "seeing" parameters, would eliminate the need for frequent observations of reference stars, especially if the "seeing" parameters can be determined from the data themselves.

An attractive theoretical approach is the analysis of Korff (1973), confirmed by Roddier and Roddier (1975). From their expressions describing the short-exposure optical transfer function, autocorrelation profiles may be computed for different values of the size

* This *Letter* is based upon observations made by A. Blazit, D. Bonneau, L. Koechlin, and A. Labeyrie as Guest Investigators at the Hale Observatories.

TABLE 1

Observed Values of Equipment Uniform-Disk Angular Diameters

Star	Date	Phase	$\lambda/\Delta\lambda$ (Å)	θ_{UD} (arcsec)
Mira*.......	1972.479	0.09	4500/200	0.070±0.010
			5150/200	0.057±0.005
			7500/200	0.051±0.005
			10400/200	0.05
	1972.742	0.38	6700/200	0.062±0.005
			7000/200	0.058±0.005
			7500/200	0.055±0.005
	1977.025	0.11	6080/80	0.071±0.015
			6200/80	0.103±0.020
			6470/30	0.072±0.014
			6720/175	0.075±0.015
			6960/80	0.031±0.006
			7120/80	0.061±0.007
			7400/50	0.034±0.007
R Leo†......	1976.397	0.01	5400/200	0.032±0.005
			7160/200	0.049±0.009
			7300/200	0.054±0.009
			7400/200	0.030±0.007

* Phases calculated from *IAU Inf. Bull. Var. Stars*, No. 513 (1971).

† Phase calculated from elements given by Lockwood and Wing 1971.

of a turbulent element. If these theoretical profiles prove consistent with observation, it should become possible to derive the desired reference profiles from measurements of the size of a turbulent element, which is related to the seeing angle.

Pending such experimental verifications of Korff's analysis, we have adopted a more primitive reduction scheme, the validity of which is believed to hold within the stated accuracy: in the autocorrelation profile, the pedestal component, which may be approximately identified with the long-exposure autocorrelation, was delineated visually and subtracted. The remaining peak was fitted to a set of curves representing the convolution of Airy disk and uniform disk autocorrelations. The uniform-disk diameter giving the best fit was adopted as representing the object's diameter. In contrast to the refined reduction scheme suggested above, this implies assumptions, of peak additivity and of a peak shaped according to an autocorrelated Airy disk, which are in slight conflict with Korff's results. Similar assumptions had already been made by Lynds, Worden, and Harvey in their attempt to produce Betelgeuse images (1976), and by Blazit *et al.* (1977).

III. PHYSICAL INTERPRETATION

The following analysis, worked out by one of us (R.F.), attempts to interpret the observed wavelength-dependent size of Mira and R Leo in terms of the very strong opacity in TiO bands. Our purpose is not to determine accurate atmospheric parameters, but to show that the order of magnitude of the proposed effect is right.

TiO opacities have been computed from 4100 to 10500 Å at 50 Å intervals, utilizing (1) radiative opacity tables $\kappa(T, \lambda)$ calculated per TiO molecule by Collins and Faÿ (1974); (2) electronic-band oscillator strengths, $f(\lambda)$, measured by Price, Sulzmann, and Penner (1974); (3) three atmospheric models typical of Mira, kindly computed by Tsuji (1973) for one of us (R.F.). Owing to the lack of laboratory data, the tables of Collins and Faÿ include only the α and γ bands of TiO. These band systems dominate respectively the blue and near-infrared parts of the spectrum, while the γ' system which is not accounted for in our calculations is strongest near the 0–0 band head at 6200 Å, as apparent in the spectra of Mira.

The oscillator strengths measured by Price *et al.* at three wavelengths cannot be considered significantly different. We have therefore taken the weighted mean, resulting in $f_\alpha = 0.18$ and $f_\gamma = 0.50$, respectively, for the α and γ band systems.

Atmospheric models are characterized by their effective temperature $E_{eff} = 2600$ K, their gravity $g = 0.1$, and a solar chemical composition. They differ in turbulent velocity, the total Doppler velocity v_D being respectively 2.0, 5.0, and 10 km s^{-1}. The input physics of these models is described in Tsuji (1966, 1971). Let us briefly recall here that the models are multilayer, assuming a plane-parallel stratification, constant flux in radiative equilibrium, and LTE. They take into account the molecular blanketing due to H_2O, TiO, SiH, CaH, and CO.

From these models, we have the depth distribution of partial pressure for TiO and temperature. Then we obtain the optical depth in the TiO features as a function of wavelength, $\tau_{TiO}(\lambda)$, using the $\kappa(T, \lambda)$ tables. $\tau_{TiO}(\lambda)$ is plotted in Figure 1 against the geometrical depth z (increasing outward from the layer

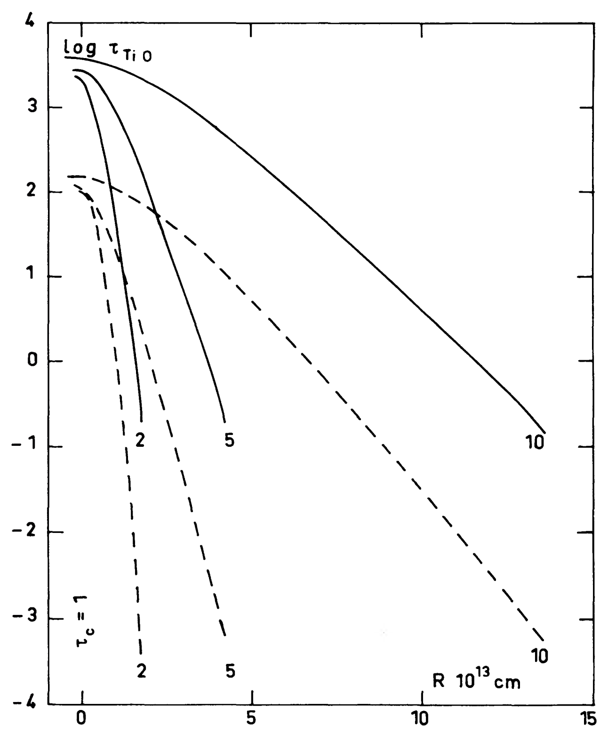

Fig. 1.—Optical depth τ_{TiO} as a function of the geometrical depth z in a strong TiO feature (λ = 7120 Å, *solid curve*) and outside (λ = 7400 Å, *dotted curve*). 2, 5, and 10 refer to the Doppler velocity adopted in the atmosphere model.

$\tau_{\text{continuum}} = 1$) for two wavelengths in the three models. It clearly appears that the geometrical depth where $\tau_{\text{TiO}}(\lambda) = 1$, i.e., where the atmosphere becomes optically thick, varies strongly with λ: the optically thick atmospheric region is very much larger in strong TiO features (e.g., 7120 Å) than outside (e.g., 7400 Å). This implies theoretical diameters larger in these strong features than in the weak ones, as observed. Limb-darkening effects have not been taken into account. They also depend on wavelength and opacity, but this cannot modify the dominant effect produced by TiO opacity.

In order to avoid any hypothesis on the stellar radius defined by $\tau_c = 1$, we have chosen to compare theoretical values of the radius $R(\lambda)$ to observed values by plotting the differences $R(\lambda) - R(7500)$ (Fig. 2). This requires distance values for Mira and R Leonis, which we have taken as 77 pc and 250 pc, respectively, from the absolute visual magnitude versus period relation by Foy, Heck, and Mennessier (1975). Different distance values change the vertical scale of measured radii in Figures 2 and 3.

From comparison with observation, it emerges that the best choices in v_D are the interpolated values $v_D = 4$ km s^{-1} for Mira (Fig. 2) and $v_D = 6.5$ km s for R Leonis (Fig. 3). These values seem reasonable for stars of such late spectral types. Similar theoretical curves $R(\lambda) - R(7500)$ would be obtained by adjusting the gravity at given v_D.

The theoretical functions $R(\lambda) - \mathrm{R}(7500)$ for Mira (Fig. 2) depart from the observed values at wavelengths 6080, 6200, and 6460 Å, near the (0–0) band head for the γ' system (6200 Å). This departure was expected, since the γ' system is not included in our calculations.

Also plotted in Figure 2 are the earlier observations by Bonneau and Labeyrie (1973).

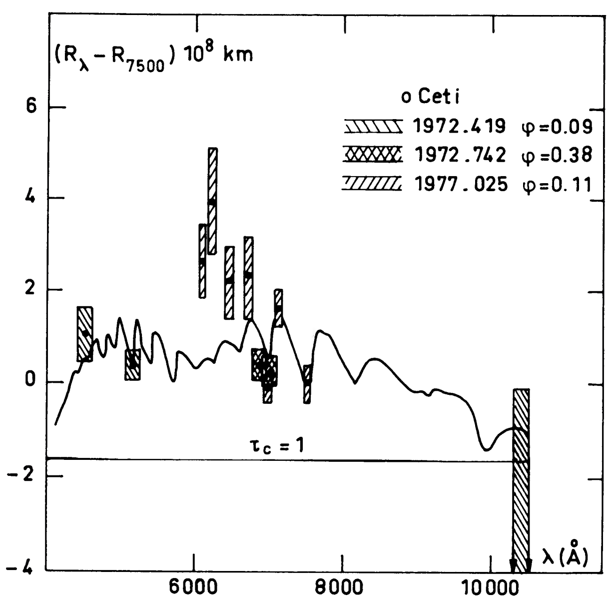

FIG. 2.—Radius at wavelength λ minus radius at 7500 Å, versus λ, for Mira. The solid curve is computed by assuming a Doppler velocity $v_D = 4$ km s^{-1}. Rectangles indicate the observed values; $z = 0$ at $\tau_c = 1$.

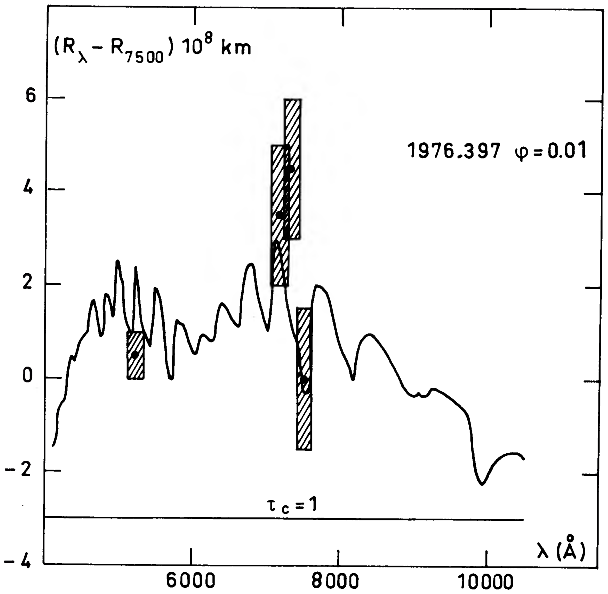

FIG. 3.—Same as Fig. 2, but for R Leo, with $v_D = 6.5$ km s^{-1}

Comparison of the theoretical curve and observational points in Figure 2 shows reasonable agreement, except in the γ' band system. Thus, TiO absorption is probably responsible for the slow increase of diameter toward the blue found in 1973. No broad-band scattering in the atmosphere of Mira is needed to explain this blueward increase of diameter.

The stellar radius R_c, defined by $\tau_c = 1$, may be derived from $R(\lambda)$ and the observed diameters. The average value resulting from the observed wavelengths, except those in the γ' system, is $R_c = 5 \pm 8 \times 10^{12}$ cm, i.e., $\lesssim 180\, R_\odot$. This value is significantly lower than the radius commonly assumed for Miras, $R \approx 320\, R_\odot$ (Allen 1973); but agrees with that proposed by Koester (1974), $R \approx 150\, R_\odot$ at maximum of the light curve.

IV. CONCLUSION

To conclude, we wish to point out some consequences of our results.

1. Since the radius of Mira variables is smaller, these stars are hotter than currently assumed. This supports the results obtained by Tsuji (1976), indicating that the temperature scale of cool stars has to be shifted toward hotter temperatures.

2. The assumption of a plane-parallel atmosphere is not realistic for such stars. Indeed, the stellar radius defined by $\tau_c = 1$ is smaller by one order of magnitude than the geometrical depth of the atmospheric region where $\tau_{\text{TiO}} \geq 1$. Therefore the atmospheric density decreases more rapidly outward than predicted by the plane-parallel atmosphere model.

3. There are very large differences in the geometrical depths of the line-, band-, or continuum-formation regions. Therefore the gravity should not be taken as constant in the whole atmosphere, as generally assumed. The approximation of a constant gravity is also conducive to an underestimate of the density gradient in the atmosphere.

4. No continuum can be seen in the visible or near-infrared spectral region as shown by the line $\tau_c = 1$ in Figures 2 and 3. Therefore the observed lines are formed in the highest atmospheric layers, above the TiO band-formation region. This could cause the veiling effect observed for Miras.

5. The expected stellar radius variations with phases are probably attenuated by the large extension of the optically thick region. It is unfortunate that sufficient observational data are not yet available on diameter variations with phase. Continued observations should fill this gap.

6. Large diameters can also be expected at the infrared wavelengths where H_2O and CO cause strong absorption.

Finally, we wish to emphasize the power of multi-wavelength interferometer observations as a tool to investigate the different atmospheric layers of cool stars.

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