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THE LOCAL GROUP: THE SOLAR MOTION RELATIVE TO ITS CENTROID

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ABSTRACT

A new solution for the motion of the local standard of rest (LSR) relative to the centroid of the Local Group (LG) of galaxies, based on 21 cm redshifts for a number of candidates, gives v(LSR) = 300 km s^{-1} toward $l = 107°$, $b = -8°$. Three other solutions are given using different precepts for membership within the LG. This motion of the LSR corresponds to a best-fit *solar* motion relative to the LG centroid of $v(_\odot) = 308$ km s^{-1} toward $l = 105°$, $b = -7°$.

Consideration of the velocity residuals from the ridge-line solution of each candidate galaxy shows that the sometimes-mentioned galaxies IC 342, NGC 6946, NGC 404, and Maffei 1 and 2 are certainly not members. Likely members, on the basis of the kinematics alone, are IC 10, Pegasus dwarf, WLM, DDO 210, Leo A, and IC 5152. Possible, but unlikely, members, again based on kinematics alone, are DDO 187, GR 8, Sextans A and B, and NGC 3109. All five of these latter galaxies have positive residuals of about 125 km s^{-1} relative to the solution, and may be the nearest galaxies that show the cosmological expansion.

A discussion of the error matrix is given, with special emphasis on breaking up the velocity $\boldsymbol{v}_0$ of the LSR relative to the centroid of the LG into the sum of a rotation velocity $\boldsymbol{v}_C$ and the motion of the center of the Galaxy $\boldsymbol{v}_G$. In principle further restrictions on $\boldsymbol{v}_G$ such as requiring $|v_G|$ to be of the same magnitude as the velocity dispersion of other members of the LG, or requiring $\boldsymbol{v}_G$ to be collinear with M31 permit limits to be put on $\boldsymbol{v}_C$. In practice these limits are so wide at the 90% confidence level that from these considerations alone $\boldsymbol{v}_C$ is only restricted to the already known range of $200 \lesssim |v_C| \lesssim 300$ km s^{-1}.

Subject headings: galaxies: clusters of — radio sources: 21 cm radiation — stars: stellar dynamics

I. INTRODUCTION

To understand the velocity field of the expansion for *very nearby* galaxies it is necessary to correct the measured radial velocities for the solar motion with respect to a coordinate frame that has relevance to the Universe itself. For example, the "local standard of rest" (LSR) that is defined by the motion of nearby stars in our Galaxy is rotating and is translating with the peculiar motion of the Galactic center, and is not as fundamental as the frame defined by the centroid of the Local Group (LG) of galaxies. This centroid of the LG may, in fact, approach in first approximation the imaginary Robertson-Walker manifold of an ideal isotropic and homogeneous Universe. It has been argued on observational grounds that (1) the LG itself does not have a large motion relative to an ideally expanding Friedmann frame (i.e., the Hubble flow appears to be isotropic when determined from nearby galaxies outside the LG; e.g., Sandage and Tammann 1975; Sandage 1975), and (2) the mean random motions of nearby field galaxies, relative to the centroid, is small (de Vaucouleurs 1958; Sandage and Tammann 1975; Fisher and Tully 1975). If these two properties are true, then the motion of the LG relative to the underlying inertial manifold would be small, and the solar motion relative to the centroid would be fundamental.

There are, of course, challenges to this point of view. The most serious are due to de Vaucouleurs (1958, 1964, 1966, 1976), Peebles (1976), and Rubin *et al.* (1976), where local anisotropies in the expansion are claimed. If true, then knowledge of the solar motion relative to the frame of the LG is even more important, because it is only by subtracting such motion from the observed velocities of nearby field and group galaxies and then testing for anisotropies that such deviations can be found.

The problem of the rest frame of the local velocity field has a long history. Soon after the first several radial velocities of galaxies were available from Slipher, solutions by Truman (1916), Wirtz (1918), and Slipher (1917) showed a solar motion that was much larger relative to galaxies than to stars. Further solutions from a larger material by Wirtz (1921) with the introduction of a K term to account for an expansion independent of distance, and by Wirtz (1924), Lundmark (1920, 1925), and Stromberg (1925) with the introduction of a Kr term, preceded Hubble's (1929) clear demonstration of the linear expansion. Much later, Hubble (1939) showed that LG members do not partake of the expansion, and that their apparent motions are determined largely by the rotation of the Galaxy.

The nonexpansion of the Group is generally taken to mean that it is bound, and hence that it forms a dynamical unit with negative total energy. With this

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point of view, a new definition of membership can be made on *kinematic* grounds (including only those galaxies that do not partake of the general expansion), rather than on the somewhat arbitrary criterion of spatial proximity to the Galaxy. The question has an immediate application because there exist many very nearby, low-luminosity resolved dwarfs (WLM, Sextans A, Sextans B, Leo A, Pegasus dwarf, NGC 3109, GR 8, and IC 10) whose status as Local Group members is in doubt. If they are bound to the Group, and if the Group has reached dynamical equilibrium, then they should be within the random scatter of the known members once the solar motion is removed, whereas if they partake of the expansion they will show a larger and positive deviation.

In fact, this kinematic criterion may be conservative since some outlying galaxies could still be expanding, but nevertheless with negative total energy for the LG (cf. Gunn and Gott 1972), and hence be members. But because of the difficulty of determining the energy, we will, as a working hypothesis, treat outliers as unbound.

But even so, the gravitational acceleration due to the Galaxy and to M31 could be large enough to slow the expansion in the immediate neighborhood of the LG and a study of the deceleration of the outlying dwarfs could yield information on the mass within the LG.

With these problems in mind we have made another solution for the motion of the Sun. The large available sample of accurate radial velocities, determined primarily from new 21 cm observations, and the previously unavailable data for some of the fainter galaxies justify a new discussion of an old problem.

II. THE DATA

Table 1 lists 26 galaxies (in order of right ascension) that have, at one time or another, been considered as certain, probable, or possible members of the Local Group, mainly on the basis of estimated distance. There are other members such as the dE galaxies of Sculptor, Leo I and II, Ursa Major and Draco systems (cf. Hodge 1971) that are satellites of the Galaxy, and the two dE companions of M31 (van den Bergh 1974). These are not listed because their velocities are unknown. Additional possible dE members have been mentioned by de Vaucouleurs (1975).

Only 10 galaxies in Table 1 (col. [12]) are certain members. (The most distant of these is M33 with $[m - M]_0 \approx 24.56$, or $r = 820$ kpc [Sandage and Tammann 1974], which, in the absence of kinematic data for other candidates, had led in the past to the unwarranted conclusion that all member galaxies had to be within ~ 1 Mpc.)

The columns of Table 1 are generally self-explanatory. The Galactic longitude and latitude are listed in columns (3) and (4). For the LMC we have adopted the radio center of rotation at $\alpha = 5^h20^m$, $\delta = -68.^\circ8$ (1950) from Kerr and de Vaucouleurs (1955). This differs from the center of the bar by $\sim 1°$, but nearly coincides with the centroid of extreme Population I objects (cf. Westerlund 1974), and with the symmetry point of the optical rotation curve (Feast, Thackeray, and Wesselink 1961; Cheriguène 1975).

Columns (5)–(8) list the optical and 21 cm heliocentric radial velocities and their sources. (The systematic velocity of the Large Magellanic Cloud [LMC] is sensitive to the adopted position of the center because the projected rotation component of the LMC itself in its inner part is ~ 24 km s^{-1} deg^{-1} [Cheriguène 1975]. The RCBG [de Vaucouleurs and de Vaucouleurs 1964] gives $v = 265 \pm 1.6$ km s^{-1} for LMC, while the SCBG [de Vaucouleurs, de Vaucouleurs, and Corwin 1976] gives $v = 260 \pm 5$ km s^{-1}; but these velocities refer to the center of the bar.) In principle, the same positional velocity dependence exists for SMC, but here the rotational velocity component is certainly much smaller. The RCBG gives, for SMC, $v = 163 \pm 2$; and the SCBG, 150 ± 5.

Columns (9) and (10) give the weighted mean heliocentric velocity and its mean error from all listed optical and radio observations. The optical and radio velocities agree satisfactorily within the quoted errors, giving confidence that the adopted mean errors are realistic. For LMC and SMC we have adopted somewhat larger errors (± 5 km s^{-1}) than the formal values to account for some of the difficulties described above.

The velocity in column (9) is reduced to the local standard of rest (LSR) in column (11) by using a solar peculiar motion with respect to the LSR of 16.5 km s^{-1} toward $l = 53°$, $b = +25°$, corresponding to $U = +9$ km s^{-1} (toward the galactic center), $V = +12$ km s^{-1} (toward the direction of rotation), and $Z = +7$ km s^{-1} (Delhaye 1965). Column (12) identifies the *certain* members of the Local Group.

III. MOTION OF THE LOCAL STANDARD OF REST

The motion $\boldsymbol{v}_0$ of the LSR relative to the *centroid of the LG* can be decomposed into two vectors: (1) the circular rotation $\boldsymbol{v}_C$ about the galactic center, and (2) the velocity $\boldsymbol{v}_G = \boldsymbol{v}_0 - \boldsymbol{v}_C$ of the galactic center *relative to the centroid.* We can solve for $\boldsymbol{v}_0$ by finding the apparent anisotropy over the celestial sphere of the velocities (listed in col. [11] of Table 1) for those galaxies that are moving randomly in the gravitational field of the Local Group. Satellite galaxies cannot be used because they are likely to be bound to the primaries, and will reflect orbital motions about them rather than peculiar velocities relative to the centroid. Hence, we exclude LMC, SMC, M32, NGC 185, and NGC 205 from the solution for $\boldsymbol{v}_0$ (but keep Fornax because its orbital status is not so clear).

Although we are principally interested in $\boldsymbol{v}_0$, it is useful to consider limits on the rotation velocity $\boldsymbol{v}_C$, and on the motion of the center ($\boldsymbol{v}_G$), and to compare this latter motion with the velocity dispersion of other LG members.

The data in Table 1 are used to discuss three problems, two in this section, and the third in § VI. (*a*) Galaxies that are *certain* members of the LG are used to determine $\boldsymbol{v}_0$, and to estimate the velocity dispersion σ within the LG itself. (*b*) The possibility

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TABLE 1

LOCAL-GROUP MEMBERS AND SOMETIME CANDIDATES WITH KNOWN RADIAL VELOCITIES

Galaxy (1)	Other Name (2)	ℓ (3)	b (4)	v (opt) km s^{-1} (5)	Source (6)	v (21 cm) km s^{-1} (7)	Source (8)	v (adopt) km s^{-1} (9)	ε km s^{-1} (10)	v (LSR) km s^{-1} (11)	Member (12)
IC 10		118°.96	- 3°.32	- 343 ± 12	6	- 343 ± 2	11,12,14,15, 18,19,20	- 343	2	- 337	
NGC 185		120.77	-14.48	- 252 ± 34	6,7,8			- 252	34		certain
NGC 205		120.71	-21.13	- 240 ± 12	6,7,41			- 240	12		certain
NGC 221	M32	121.16	-21.97	- 193 ± 4	6,7,8,41,45			- 216	10		certain
NGC 224	M31	121.17	-21.57	- 298 ± 3	see 1,2,3,45	- 302 ± 2	see 1	- 301	2	- 298	certain
SMC	A0051	302.80	-44.30	+ 166 ± 3	25	+ 161 ± 2	29,31	+ 163	5	+ 154	certain
IC 1613	DDO 8	129.87	-60.55	- 238 ± 10	6	- 235 ± 3	9,11	- 235	3	- 239	certain
NGC 404		127.02	-27.01	- 23 ± 15	6,8,13			- 23	15	- 22	
NGC 598	M33	133.63	-31.33	- 178 ± 4	see 4	- 180 ± 1	see 4,5	- 180	1	- 182	certain
Maffei 1		136.09	- 0.17	- 10 ± 50	35			- 10	50	- 8	
Fornax	A0237	237.31	-65.72	+ 49 ± 10	6, 34			+ 49	10	+ 36	certain
Maffei 2		136.87	- 0.06	+ 1 ± 20	40	- 18 ± 5	(36),37,38, 39	- 17	5	- 15	
IC 342		138.16	+10.58	+ 10 ± 20	6,7,21	+ 31 ± 3	11,16,17,22	+ 31	3	+ 34	
LMC	A0524	279.33	-33.45	+ 270 ± 2	25,26,27,28	+ 270 ± 2	29,30	+ 270	5	+ 257	certain
Leo A	LeoIII, DDO 69 A0956	196.91	+52.41			+ 26 ± 10	32	+ 26	10	+ 24	
Sextans B	DDO 70, A0957	233.19	+43.76			+ 295 ± 10	32	+ 295	10	+ 289	
NGC 3109	DDO 236	262.10	+23.07	+ 441 ± 20	7	+ 403 ± 1	11,32,42, 43,44	+ 403	1	+ 394	
Sextans A	DDO 75, A1009	246.23	+39.92	+ 370 ± 30	6	+ 323 ± 5	11,12,32	+ 324	5	+ 317	
GR8	DDO 155	310.70	+77.00	+ 257 ± 30	33	+ 216 ± 5	32	+ 217	5	+ 223	
DDO 187		25.60	+70.50			+ 153 ± 5	32	+ 153	5	+ 164	
NGC 6822	DDO 209	25.38	-18.38	- 32 ± 20	6,7,8	- 59 ± 3	9,10,11,12	- 58	3	- 48	certain
NGC 6946		95.71	+11.66	- 16 ± 35	6,7	+ 42 ± 3	(11),12,(14), (15),16,17	+ 42	3	+ 54	
DDO 210		34.10	-31.30			- 131 ± 10	32	- 131	10	- 123	
IC 5152		343.91	-50.22	+ 78 ± 30	23, 24			+ 78	30	+ 76	
Pegasus	DDO 216, A2326	94.79	-43.54			- 181 ± 10	14,32	- 181	10	- 178	
WLM	DDO 221, A2359	75.71	-73.60	- 78 ± 20	6	- 124 ± 9	12,32	- 116	8	- 119	

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TABLE 2

SOLUTIONS FOR MOTION OF THE LOCAL STANDARD OF REST USING FOUR DIFFERENT DATA SAMPLES

Solution	Sample	v_0 (LSR) (km s^{-1})	l	b	σ (km s^{-1})
1............	M31, M33, I1613, N6822, Fornax	287 ± 33*	106° ± 8°	−13° ± 8°	66 (+28, −16)
2............	1 + I10, Peg, WLM, DDO 210, Leo A, I5152	300 ± 22	107 ± 5	−8 ± 4	45 (+12, −9)
3............	2 + GR8, Sex A, Sex B, N3109	343 ± 30	101 ± 6	−11 ± 5	68 (+15, −11)
4............	3 + DDO 187	334 ± 30	105 ± 6	−16 ± 5	72 (+15, −11)
5............	2 + SMC, LMC, N185, N205, N221	287 ± 15	105 ± 5	−7 ± 4	43 (+9, 7)
6............	1 + WLM	264 ± 30 $K = -50 \pm 45$ km s^{-1} Mpc^{-1}	113 ± 7	−4 ± 8	48 (+19, −12)

* One sigma errors of individual parameters. For the complete variance matrix see § VI.

that other galaxies in Table 1 may be members is analyzed on the basis of kinematic arguments by performing additional solutions for $\boldsymbol{v}_0$ with various trial memberships. (*c*) The errors in our estimate of $\boldsymbol{v}_0$ are discussed in detail, with special emphasis on its breakup into $\boldsymbol{v}_C$ and $\boldsymbol{v}_G$ under different dynamical hypotheses.

The notation in what follows is that v_i is the observed radial velocity of galaxy *i* relative to the LSR (col. [11] of Table 1) and $\boldsymbol{n}_i$ is the unit vector in the direction of galaxy *i*. (Components of vectors are characterized by upper indices, e.g., $v_0{}^x$, the *x*-axis is toward the Galactic center, the *y*-axis in the direction of rotation, and the *z*-axis toward the north galactic pole.

a) Motion of the Local Standard of Rest Determined from Known Members of the Local Group

The velocity of the LSR relative to the velocity centroid of the Local Group, $\boldsymbol{v}_0$ is determined by a least squares fit, minimizing the statistic

$$\Lambda = \sum (v_i + \boldsymbol{v}_0 \cdot \boldsymbol{n}_i)^2 \,, \tag{1}$$

and the velocity dispersion σ is estimated from the rms residuals

$$\sigma^2 = \Lambda_{\min}/(N - 3) \,, \tag{2}$$

where N is the number of data points, and $N - 3$ is the number of degrees of freedom. In principle the observational errors of v_i should also be taken into account in the fit, but they turn out to be negligible compared with the velocity dispersion σ, and are ignored. A detailed discussion of the confidence limits is given in § VI. In this section we are primarily concerned with the residuals of individual galaxies from the ridge-line solution *as indicators of membership*. Because these residuals are not significantly altered when $\boldsymbol{v}_0$ is varied within reasonable confidence limits, the best-fit values may be used for this purpose, before the discussion of the confidence limits is made.

The parameters $\boldsymbol{v}_0$ and σ, determined only from the certain members of the LG (M31, M33, IC 1613, NGC 6822, and Fornax, but excluding the companions to M31 [NGC 185, NGC 205, M32] and the companions to the Galaxy [LMC and SMC]) are $\boldsymbol{v}_0 = 287$ km s^{-1} toward $b = -13°$, $l = 106°$, with a dispersion of $\sigma = 66$ km s^{-1}. These values, along with later solutions, are summarized in Table 2 to be discussed in the next section. The solution is illustrated in Figure 1 where the v_i (LSR) velocities from Table 1 (col. [11]) are plotted against the angle λ_i of the given galaxy from the quoted apex. The two parallel lines bordering the central solution are drawn at ± 66 km s^{-1} which is the 1 σ dispersion. The details of the deviations (D) from the ridge-line solution for this and for the other solutions are listed in Table 3 in terms of the normalized residuals (D/S), which are the number of standard deviations (S) that any given galaxy falls from the ridge-line.

Noteworthy features of Figure 1 are: (1) In addition to the five certain members (*filled circles*) used for the solution, six additional galaxies (IC 10, Pegasus, WLM, DDO 210, IC 5152, and Leo A) fall within the 1 σ scatter band ($\Delta v = \pm 66$ km s^{-1}), and hence are candidates for membership on kinematic grounds. (2) The five galaxies near cos $\lambda = +0.85$ (IC 342, Maffei 1,

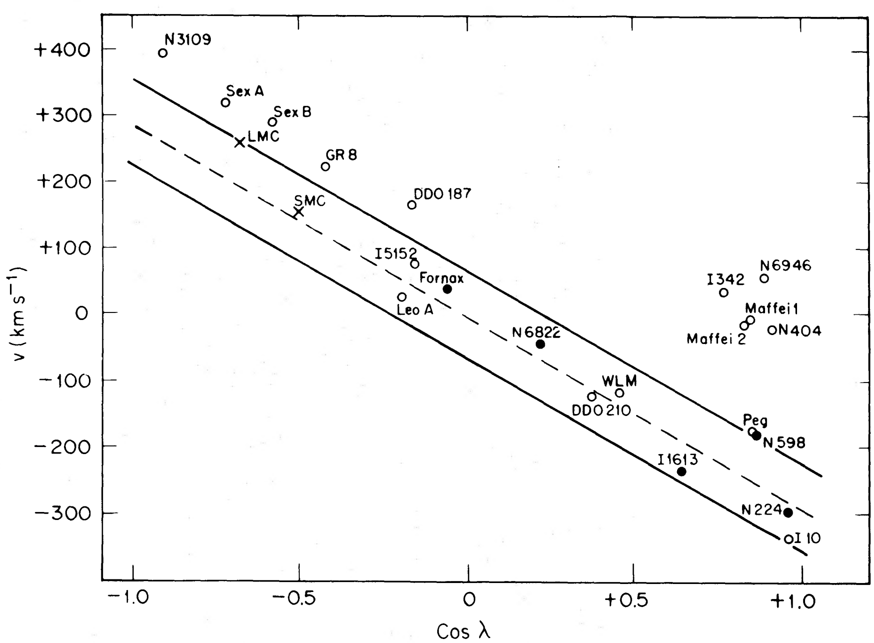

FIG. 1.—Solution 1 (Table 2) based on the five certain members of the Local Group (LMC and SMC excluded). The two solid lines are the ± 1 σ deviations from the ridge-line solution. Closed circles are the five certain members. LMC and SMC are shown as crosses; all other galaxies, as open circles. The abscissa is the cosine of the angle from the galaxy in question to the apex (Table 2).

TABLE 3

ANGLE FROM THE APEX AND NORMALIZED RESIDUALS FOR THE FOUR SOLUTIONS

	SOLUTION 1		SOLUTION 2		SOLUTION 3		SOLUTION 4	
NAME	cos λ	D/S	cos λ	D/S	cos λ	D/S	cos λ	D/S
M33	+0.857	+0.99	+0.832	+1.50	+0.810	+1.42	+0.863	+1.48
Fornax	−0.053	+0.32	−0.131	−0.06	−0.110	−0.02	−0.015	+0.43
NGC 6822	+0.222	+0.25	+0.178	+0.13	+0.288	+0.75	+0.253	+0.51
M31	+0.959	−0.35	+0.946	−0.32	+0.930	+0.30	+0.959	+0.30
IC 1613	+0.639	−0.85	+0.574	−1.47	+0.595	−0.52	+0.669	−0.23
Pegasus	+0.851	+1.01	+0.800	+1.34	+0.842	+1.62	+0.876	+1.58
Leo A	−0.192	−0.47	−0.111	−0.19	−0.214	−0.72	−0.239	−0.77
DDO 210	+0.377	−0.21	+0.320	−0.57	+0.427	+0.35	+0.414	+0.21
WLM	+0.459	+0.20	+0.376	−0.13	+0.438	+0.46	+0.502	+0.67
IC 10	+0.960	−0.94	+0.976	−0.98	+0.944	−0.20	+0.947	−0.30
IC 5152	−0.154	+0.44	−0.237	+0.09	−0.138	+0.39	−0.104	+0.53
GR 8	−0.424	+1.54	−0.344	+2.62	−0.384	+1.35	−0.464	+0.95
Sextans A	−0.721	+1.68	−0.665	+2.58	−0.742	+0.93	−0.753	+0.92
Sextans B	−0.584	+1.83	−0.519	+2.87	−0.609	+1.18	−0.621	+1.12
NGC 404	+0.915	+3.58	+0.895	+5.15	+0.877	+4.01	+0.919	+3.86
NGC 6946	+0.890	+4.73	+0.921	+7.27	+0.916	+5.43	+0.873	+4.80
IC 342	+0.768	+3.88	+0.808	+6.07	+0.735	+4.22	+0.739	+3.89
Maffei 1	+0.843	+2.84	+0.867	+3.74	+0.806	+3.19	+0.823	+3.04
Maffei 2	+0.836	+3.42	+0.860	+5.32	+0.798	+3.80	+0.816	+3.56
NGC 3109	−0.908	+2.03	−0.881	+2.85	−0.929	+1.11	−0.923	+1.19
DDO 187	−0.164	+1.78	−0.088	+3.02	−0.105	+1.89	−0.201	+1.34

Maffei 2, NGC 6946, and NGC 404) stand between 3 σ and 4 σ from the solution, and are clearly not members. They evidently have Hubble velocities of ~ 250 km s^{-1} relative to the LG centroid. (3) Five galaxies (DDO 187, GR 8, Sextans A and B, and NGC 3109) fall within $\sim 2\ \sigma$ of the mean relation, *but all have positive residuals.* Because no galaxy in our sample deviates toward negative velocities relative to the mean line by more than 1 σ (66 km s^{-1}), it seems likely that most of the five are not bound to the LG, but rather show an expansion component of the order of ~ 100 km s^{-1}.

b) *Other Solutions for v_0 from an Increased Sample*

These points suggest that other solutions, using different galaxy subsamples, might add to the evidence for membership. Figure 2 shows a solution from 11 galaxies (five certain members used in Fig. 1, and the

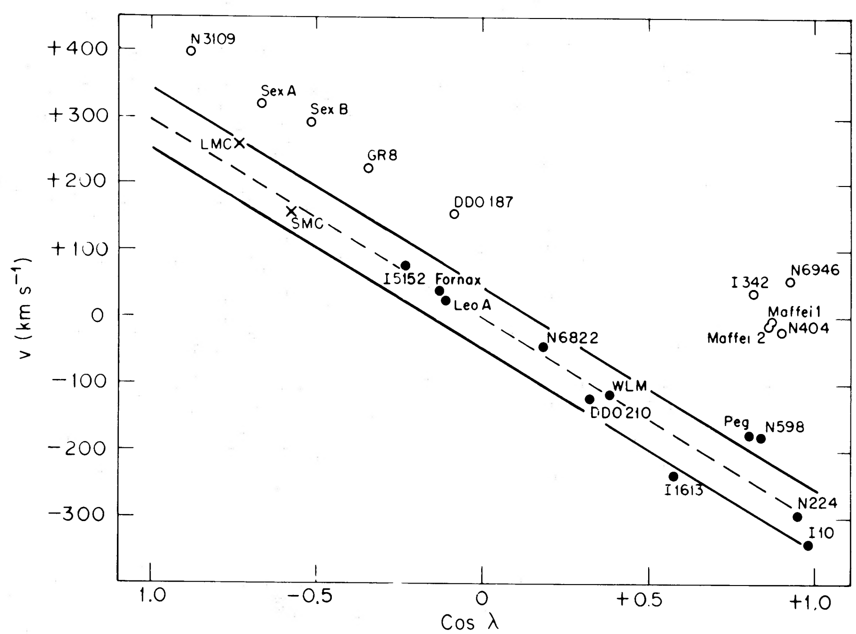

FIG. 2.—Same as Fig. 1 but with six additional galaxies as postulated members of the Local Group. The ridge-line is solution 2 of Table 2.

six additional candidates discussed in point 1 above). The parameters, listed in Table 2, solution 2, show an increase in $|v|_0$(LSR) to 300 km s^{-1}, a 5° change in the latitude of the apex, and a reduced radial velocity dispersion of $\sigma = 45$ km s^{-1}.

Figure 2 and Table 3 show that the solution is well defined. The lower value of σ for this solution, while in agreement with the previous value within the error, seems more reasonable. In the first solution which gives $\sigma = 66$ km s^{-1}, all 11 residuals would be within 1.0 σ, and the probability of such an eventuality is only 0.015 for Gaussian residuals. On the other hand, in the second solution where $\sigma = 45$ km s^{-1}, three out of the 11 residuals are in the 1–2 σ range, which is the expected ratio for a Gaussian distribution. The smaller σ increases the significance of the deviations from the ridge line, and reinforces the second and third conclusions (above) concerning the IC 342 group and the nonmembership of DDO 187, GR 8, Sextans A and B, and NGC 3109, all of which have positive residuals above 2.5 σ.

However, this last point is still not settled by Figure 2 alone, and we have made two other solutions: first including four of the five last mentioned galaxies (solution 3), and then all five (solution 4). The results, listed in Table 2 and illustrated in Figure 3 for the third solution, show an increase of v_0(LSR) to $\sim$340 km s^{-1}, only a moderate change in the apex, and, as expected, an increase in σ.

For completeness we have considered two additional solutions: (1) The five companion galaxies (SMC, LMC, NGC 185, NGC 205, and NGC 221), which were excluded in solutions 1–4 on the grounds that their radial velocities reflect essentially their orbital motion about the Galaxy and M31 and the solar motion, were treated in solution 5 as independent members of the LG. The result, set out in Table 2, is in close agreement with solution 2. This shows that $\boldsymbol{v}_0$ is quite independent of whatever assumptions are made about the status of the companion galaxies. (2) To test Hubble's supposition that the velocity-distance relation does not apply *within* the LG, we have made a sixth solution including a Kr term using the five certain members (M31, M33, IC 1613, NGC 6822, and Fornax) and the possible member WLM. This sample is restricted to the galaxies for which reliable distances are known (Sandage and Tammann 1971, 1974; Hodge 1971; Sandage and Katem 1976, 1977). In agreement with Hubble's result, we find no evidence for a significant Kr term. Our formal value of K is -50 ± 45 km s^{-1} Mpc^{-1}, which clearly is a null result.

IV. THE SOLAR MOTION RELATIVE TO THE CENTROID OF THE LOCAL GROUP

Our adopted motion of the LSR of 300 km s^{-1} toward $l = 107°$, $b = -8°$ (solution 2), can be changed to a motion of the *Sun* relative to the LG centroid. The result is a solar motion of 308 km s^{-1} toward $l = 105°$, $b = -7°$, found by using the motion of the Sun relative to the LSR adopted in § II.

This solution, *which we adopt*, can be compared with earlier determinations listed in Table 4. The values are all surprisingly concordant in view of discordant precepts (e.g., inclusion or exclusion of LMC and SMC).

The observed heliocentric redshifts of non–Local Group galaxies have normally been corrected for the solar motion of $\Delta v = 300 \sin l \cos b$ km s^{-1}. Our adopted present solution corresponds to a correction

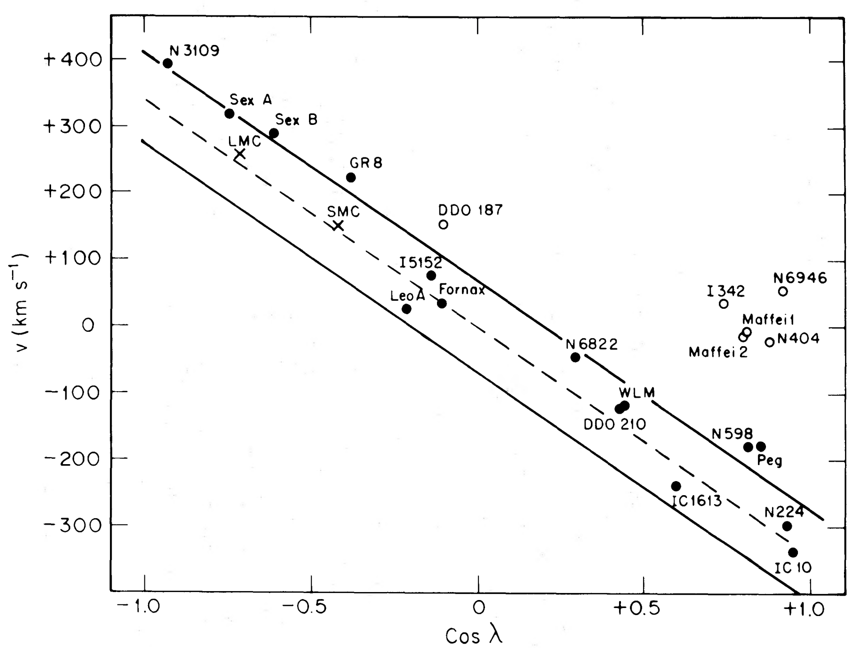

FIG. 3.—Same as Figs. 1 and 2 but for solution 3 of Table 2

TABLE 4

OUR ADOPTED SOLUTION COMPARED WITH PREVIOUS SOLUTIONS FOR THE SOLAR MOTION

Source	v_0 (km s^{-1})	l	b
Mayall 1946.......	300 ± 25	93° ± 6°	−14° ± 4°
Humason and Wahlquist 1955..	292 ± 32	106 ± 6	−7 ± 4
Byrnes 1966.......	280 ± 23	107 ± 5	−7 ± 4
de Vaucouleurs and Peters 1968.....	315 ± 15	95 ± 6	−8 ± 3
This paper........	308* ± 23	105 ± 5	−7 ± 4

* For the complete variance matrix see § VI.

of $\Delta v = -79 \cos l \cos b + 296 \sin l \cos b - 36 \sin b$ which, although close to the conventional value, differs by significant amounts in certain directions. The maximum deviation from the old solution is ± 87 km s^{-1} in the directions $l = 3°$, $b = 24°$, and $l = 183°$, $b = -24°$. This difference may be important for nearby galaxies with small redshifts. The change of the solar motion relative to the LG centroid has, of course, also some influence on the possible anisotropy of the local expansion field. For instance, the Rubin-Ford effect requires a solar motion of 600 ± 125 km s^{-1} toward $l = 135°$, $b = -8°$ (Rubin *et al.* 1976), which, with the new motion of the Sun within the LG, leaves a component of anisotropy of only 370 ± 125 km s^{-1}.

V. DISCUSSION ON THE DIFFERENT SOLUTIONS

The fact that there are no significant negative residuals in any of the solutions (Figs. 1–3) strengthens a current belief that observed radial velocities of nearby galaxies outside the LG can have only small random components (≤ 50 km s^{-1} according to Sandage and Tammann 1975, and to Fisher and Tully 1975). The point is that if the peculiar motions were large, then in some of the nearer galaxies they would exceed the expansion component, and the net velocity (relative to the LG centroid) could be negative. Because this does not happen in Figures 1–3, or in the list of dwarf galaxies by Fisher and Tully (1975), a limit can be placed on the component of the random velocity σ_r. While it is difficult to calculate exactly this limit, it is clear that the local velocity field is more regular than $\sigma_r \approx 100$ km s^{-1}; otherwise Figures 1–3 would be very much more chaotic.

The most straightforward conclusion from Figures 1–3 is again that the five points clustered about IC 342 are clearly field galaxies. They stand far from the ridge line of the solution. Furthermore, independent evidence from the stellar content in IC 342 and NGC 6946, both ScI galaxies, show that at least these two systems are far beyond the Local Group. The angular size of the H II regions, together with the calibration given elsewhere (Sandage and Tammann 1974), show beyond doubt that these two galaxies are more distant than any that lie within the 1 σ scatter lines in Figures 1–3. Although the cases of Maffei 1 and 2 are more difficult because of the high extinction in this low-latitude field, inspection of the available photographs clearly shows them to be more distant also. The $S0_3$ galaxy NGC 404 is unresolved, and there is no reason to doubt that its distance is larger than $r \gtrsim 3$ Mpc.

We consider next the six possible members that lie within the scatter (IC 10, Pegasus, WLM, DDO 210, Leo A, and IC 5152). All of these, for which adequate large-scale plate material exists, are *highly* resolved (we have no long focal-length reflector plates of DDO 210). Furthermore, photometry of the brightest stars in WLM (Sandage and Katem 1977) gives $(m - M_0) = 26.0 \pm 0.2$, or $r = 1.6 \pm 0.2$ Mpc, which clearly puts this galaxy within the spatial confines of the LG. From the available plate material, resolution into brightest stars occurs at nearly equally bright magnitudes for IC 10, WLM, and IC 5152, and somewhat fainter for Pegasus and Leo A. But even for these galaxies, the resolution begins at brighter magnitudes than for members of the NGC 2403 group (Ho I, Ho II, IC 2574, NGC 2366, etc.) which is at a distance of $r = 3.25$ Mpc, or $(m - M)_0 = 27.56$ (Tammann and Sandage 1968). Hence, the direct evidence from large-scale Hale reflector 5 m plates is that at least four of the five galaxies are closer than members of the M81–NGC 2403 group, and hence could be members of the LG, based on proximity alone; and, clearly, the kinematic evidence, summarized in Table 3, supports this identification.

Finally, consider an argument concerning the velocities themselves. First, let us assume that these five members are *not* members of the LG. Then they must show some Hubble motion. Hence, if indeed the random velocities of field galaxies are $\sigma_r = \pm 50$ km s^{-1} and even under the extreme assumption that they are *all* approaching us with this random velocity, then their Hubble distances of $(v_0 + 50)/H_0$ are still very small—in five out of the six cases less than 1.6 Mpc. (The calculations are set out in Table 5, with $H_0 = 50$ km s^{-1} Mpc^{-1}.) Therefore, if $\sigma_r \leq 50$ km s^{-1}, then most of these galaxies must be members, because they are so close, and the initial hypothesis is invalid. However, this argument is weakened if the LG considerably decelerates the local expansion flow, or if σ_r is larger.

The final consideration from Figures 1–3 is the difficult decision on the status of DDO 187, GR 8, Sextans A and B, and NGC 3109. Figures 1 and 2 (solutions 1 and 2) show that the five galaxies stand well above the 1 σ scatter line. The D/S values are in the interval between 1.5 and 2 for the first solution, and between 2.5 and 3 (all positive) for the second (cf. Table 3). The third and fourth solutions (cf. Fig. 3 for the third solution), made on the basis that most or all of the five galaxies are members, still show that all five candidates lie on the *positive* side of the ridge line, again all with deviations $\gtrsim 1\ \sigma$. Hence, it seems likely that at least some of these candidates indeed are not members of the LG and are expanding with the Hubble flow, although perhaps decelerated.

To illustrate the problem further, we list in Table 6 various parameters for the five galaxies in question, based on the second solution (Fig. 2), which is the

TABLE 5

KINEMATIC PARAMETERS FOR THE SIX NEW PROBABLE MEMBERS OF THE LOCAL GROUP

Galaxy (1)	v(LSR) (2)	287 cos λ (3)	v_0 (4)	$(v_0 + 50)/H_0$ (Mpc) (5)	Other Distance (Mpc) (6)	Source (7)
IC 10	−337	276	−62	< 0	(2.8), (1.3)	1, 4
Leo A	+24	−55	+31	1.6	...	
DDO 210	−123	108	−14	1.1	...	
IC 5152	+76	−44	+32	1.6	(4.6)	2
Pegasus	−178	244	+67	2.3	...	
WLM	−119	132	+13	1.3	1.6	3

SOURCE.—(1) Sandage and Tammann 1975. (2) Sérsic 1968. (3) Sandage and Katem 1977. (4) de Vaucouleurs and Ables 1965.

least controversial. Assuming $H_0 = 50$ and $\sigma^r = 50$ km s^{-1} the distances of all five galaxies are $\sim 2.5 \pm 1.0$ Mpc, which is entirely consistent with the ease with which individual stars resolve across the face of those four systems for which 5 m telescope plate material exists; i.e., inspection of Sextans A and B and NGC 3109 shows that their stars resolve more readily than for any member of the M81–NGC 2403 group, but not quite as easily as for NGC 6822, M33, M31, and WLM. However, until accurate measurements of the stellar content of all the relevant galaxies are made, it is premature to discuss the direct evidence for distance *differences* between Tables 5 and 6 to the necessary accuracy that will decide the question of membership on that basis alone.

Hence, although the question is still open, we conclude on *kinematic* grounds that most of the galaxies in Table 5 are probably members, while most in Table 6 are not, but we also note that information on distances to these systems (from current studies of the brightest stars) can be expected to clarify the problem.[1]

Finally, it is of interest to inquire into the spatial extent of the LG, based on the membership from the present discussion. We adopt solution 2 (Fig. 2) as defining the certain members (including of course LMC and SMC) and plot the spatial distributions in Figure 4. The (X, Y)- and (X, Z)-projections are in the Galactic plane (X toward $l = 0°$), and perpendicular to it. The distances are in megaparsecs. The certain members from solution 2 are shown as closed circles, and the five questionable members, just discussed, as open circles. Where the distances are not known by fundamental indicators (Cepheids or brightest stars), they have been assumed to be 1.5 Mpc as for WLM (the assumption is generally supported by the high resolution into individual stars). Those galaxies whose distances are thus assumed are DDO 210, Pegasus, IC 10, DDO 187, GR 8, IC 5152, Leo A, Sextans A, Sextans B, and NGC 3109.

Because the assumed distances are minimum values (WLM resolves most easily of the list), the spatial extent of the LG is at least that shown by the projected circles in Figures 4*a* and 4*b*. These circles have radii of 1.5 Mpc, giving a diameter of the Local Group of not less than 3 Mpc, which compares reasonably with the diameter of ~ 1.6 Mpc for the M101 group (the latter, containing only one giant galaxy, is, expectedly, relatively small).

[1] If, as we suspect, some of the galaxies in Table 6 are not bound to the LG, then they represent the very faint end of the distribution of field galaxies (Table 6, cols. [6]–[8]), and a serious study of their distances is expected to improve our knowledge of the general galaxian luminosity function.

VI. ERROR ANALYSIS AND THE DECOMPOSITION OF v_0 INTO v_C AND v_G

The result of decomposing $\boldsymbol{v}_0$ into $\boldsymbol{v}_C + \boldsymbol{v}_G$ is quite uncertain because both the size of the variance of $\boldsymbol{v}_0$ in our solution and the large range of $|v_C|$ ($200 \leqslant v_C \leqslant 300$ km s^{-1} from other considerations) permit no useful subtraction for $\boldsymbol{v}_G$ directly. However, we can use

TABLE 6

PARAMETERS FOR THE FIVE IMPROBABLE MEMBERS OF THE LOCAL GROUP

Name (1)	v(LSR) (km s^{-1}) (2)	300 cos λ (km s^{-1}) (3)	v_0 (km s^{-1}) (4)	v_0/H_0* (Mpc) (5)	m_{pg} (6)	Source (7)	$M_{pg}{}^0$ (8)
Sextans B	+289	−156	+133	2.7	11.82	1	−15.3
NGC 3109	+394	−264	+130	2.6	10.6	2	−16.5
Sextans A	+317	−200	+118	2.4	11.55	1	−15.4
GR 8	+223	−103	+120	2.4†	14.51	3	−12.4
DDO 187	+164	−26	+138	2.8	13.9	4	−13.3

* H_0 adopted as 50 km s^{-1} Mpc^{-1} (Sandage and Tammann 1976).

† Hodge (1974) has determined a distance of 1 ± 0.5 Mpc for GR 8.

NOTE.—Sources for the apparent magnitudes: (1) Holmberg 1958; (2) RCBG; (3) Hodge 1967; (4) Fisher and Tully 1975.

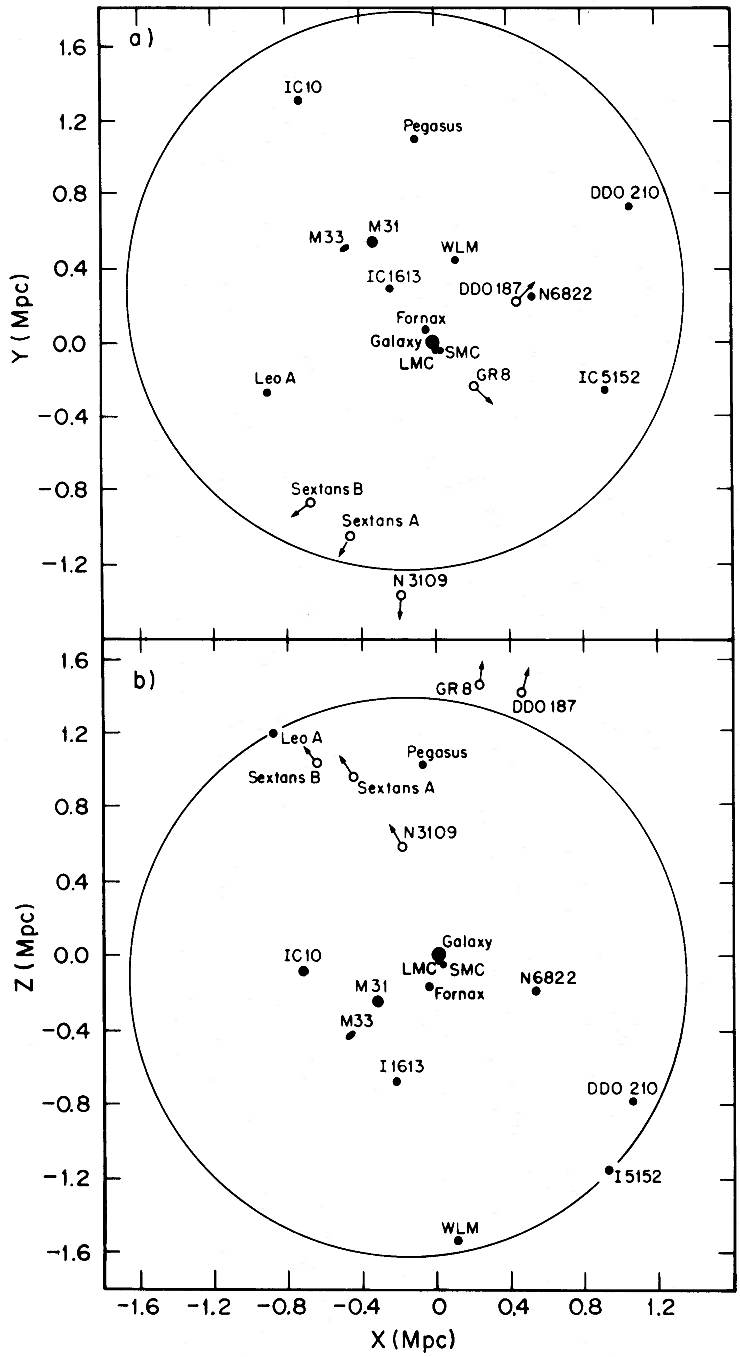

FIG. 4*a*.—Projection of certain probable and possible members of the Local Group onto the (X, Y)-plane. The direction X is toward the Galactic center, Y toward $l = 90°$. *Closed circles*, certain and probable members. *Open circles*, possible members. Galaxies with unknown distances are shown with arrows that begin at a distance of $r = 1.5$ Mpc taken from the presently estimated distance of WLM. The circle has a radius of 1.5 Mpc. The companions of M31 and some dwarf ellipticals with unknown velocities are not shown.

FIG. 4*b*.—Same as 4*a*, but in the (X, Z)-projection, where Z is the direction of the North Galactic Pole.

the variance matrix of $\boldsymbol{v}_0$ to put limits on the ranges of the $\boldsymbol{v}_C$ and $\boldsymbol{v}_G$ components in various directions by considering the decomposition of the variance matrix along the particular axes.

These limits are, in fact, interesting when certain constraints are put on the kinematics and the dynamics of the M31–Galaxy pair. In the next sections we consider several possibilities for the decomposition of $\boldsymbol{v}_0$, and finally discuss (§ VI*f*) a firm lower limit for the sum of the masses of M31 and the Galaxy, determined from the observed radial velocity of M31 and the assumption that the total energy of the pair is negative.

a) *The Variance Matrix*

The variance matrix is calculated in the usual way from its inverse, given by

$$[V^{-1}]^{\lambda\mu} = \sigma^{-2} \sum n_i^{\lambda} n_i^{\mu} . \tag{3}$$

In equation (3), components of vectors and tensors are denoted by upper indices, and σ is the velocity dispersion given by equation (2). The variance matrix for solution 2, as derived from equation (3), is listed in Table 7. Specific confidence limits, such as those discussed below, can be determined by considering the variation of the sum of the squares of the residuals around the best fit

$$\Lambda = \sigma^2(N - 3 + \Delta v_0^{\lambda}[V^{-1}]^{\lambda\mu}\Delta v_0^{\mu}) , \tag{4}$$

where the tensor summation convention is implied. It will be noticed from equation (3) that, apart from a scale factor depending on σ, the variance matrix is determined entirely by the direction cosines of the galaxies. If the sky coverage is good, the off-diagonal elements are small compared with the diagonal ones. This sample is reasonable in this respect.

A rough measure of the allowed variation in $\boldsymbol{v}_0$ is offered by the trace of $\boldsymbol{V}$ which is $(47 \text{ km s}^{-1})^2$. For a scalar matrix $\boldsymbol{V}$ this corresponds to an increase of Λ by $3\,\sigma^2$. However, as recently emphasized by Avni (1976), it is often desirable to split parameter space into the sum of two subspaces, one containing the "interesting" ones and the other containing the "uninteresting" ones. For example, in the next subsection we are interested in the components of $\boldsymbol{v}_0$ perpendicular to $\boldsymbol{v}_C$, i.e., v_0^x and v_0^z, and consider v_0^y to be "uninteresting." In that case the proper method for obtaining a confidence limit $\alpha (0 \leq \alpha \leq 1)$ is to require

$$\Delta v_0^{\lambda}[V^{-1}]^{\lambda\mu}\Delta v_0^{\mu} \leq \Delta(q, \alpha) , \tag{5}$$

where $\Delta(q, \alpha)$ is the range of the χ^2 statistic, such that

$$\text{Probability } \{\chi^2(q \text{ degrees of freedom}) \leq \Delta\} = \alpha , \tag{6}$$

and q is the number of *interesting* parameters. This method is exact for linear fits such as ours, provided that the residuals are Gaussian distributed. It is reputed to apply to a variety of nonlinear fits as well (cf. Avni 1976).

Once the range of the χ^2 statistic Δ (available in tables) has been established, the confidence limits on the "interesting" parameters are determined from equation (5) by optimizing the "uninteresting" parameters (in the sense of minimizing Λ in eq. [1]) for each choice of the "interesting" ones. In general, this is a complicated procedure, but for linear fits it is fortunately simple. The recipe is to project the variance matrix on the "interesting" subspace, to invert the projected matrix in that subspace, and then to apply equation (5) again in that subspace. Returning to the example of the next subsection, where the interesting subspace is the (X, Z)-plane, we project V from Table 7 to the (X, Z)-plane by crossing out the second row and second column, invert the 2×2 submatrix, and obtain the error ellipse in the (X, Z)-plane from equation (5). If the "interesting" subspace is not spanned by a subset of the axes by which the components of V are defined, a rotation to a new set of axes is required. However, if the "interesting" subspace is one-dimensional, i.e., if we are interested only in the component of $\Delta\boldsymbol{v}_0$ in the direction of a unit vector $\boldsymbol{n}$, then the confidence limit is simply

$$(\Delta\boldsymbol{v}_0 \cdot \boldsymbol{n})^2 \leq \Delta(1, \alpha) n^{\lambda} V^{\lambda\mu} n^{\mu} . \tag{7}$$

TABLE 7

VARIANCE MATRIX FOR $\boldsymbol{v}_0$ (in km² s⁻²)

	x*	y	z
x	773	247	185
y	247	907	361
z	185	361	558

* The x-axis is toward the Galactic Center, the y-axis toward Galactic rotation, and the z-axis toward the North Galactic Pole.

b) *Velocity Components of $\boldsymbol{v}_G$ Perpendicular to Galactic Rotation*

A firm lower limit on $|v_G|$ can be put by considering the components of $\boldsymbol{v}_0$ perpendicular to $\boldsymbol{v}_C$, since at least these must be attributed to $\boldsymbol{v}_G$ alone. Using the condition of equation (5) at the 90% confidence level ($\alpha = 0.9$) with $q = 2$ gives the area in Figure 5 within which the vector sum of v_G^x and v_G^z must lie. The length of this sum varies between 35 km s^{-1} and 160 km s^{-1} if it is to cover all parts of the area. The line marks the relation between v_G^x and v_G^z imposed by requiring $\boldsymbol{v}_G$ to be in the direction of M31 (as in § VI*e*).

Remembering that any v_G^y component must be added to the above limits, we conclude that $|v_G|$ must be larger than 35 km s^{-1} at the 90% confidence level.

c) *Limits on the Galactic Rotation $\boldsymbol{v}_C$*

If we can identify the velocity centroid frame of the LG with the center-of-mass frame of the M31–Galaxy double, then a relation exists between (1) the circular velocity of the LSR ($\boldsymbol{v}_C$), (2) the observed radial velocity ρ_A of M31, and (3) $\boldsymbol{v}_0$. The correspondence

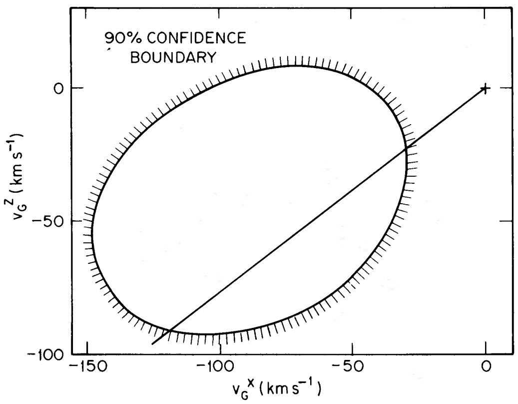

FIG. 5.—Components $v_G{}^x$ and $v_G{}^z$ of the motion of the galactic center perpendicular to the rotation vector $\boldsymbol{v}_C$. The value of $|v_G|$ must be larger than this projection. The variance of $\boldsymbol{v}_0$ projected into the (X, Z)-plane defines the 90% confidence boundary as the hatched curve. The straight line is the relation between $v_G{}^x$ and $v_G{}^z$ that is imposed if $\boldsymbol{v}_G$ is constrained to lie in the direction of M31 (e.g., § VI*e*).

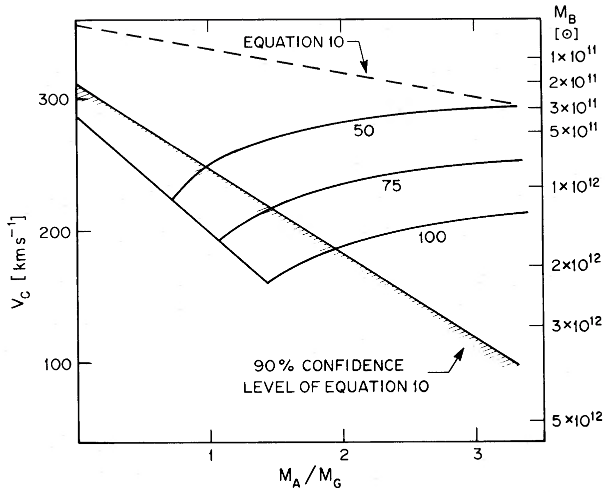

FIG. 6.—The heavy dashed line is eq. (10) using solution 2 for $\boldsymbol{v}_0$. The lower bound of the 90% confidence limit for eq. (10) is shown by the hatched line. (There is a corresponding upper bound line off the top of the graph.) Solid lines are lower limits obtained by requiring $|v_G|$ to be less than 50, 75, and 100 km s^{-1}, and $\boldsymbol{v}_0$ to be within the 90% confidence limit (see text).

of the two frames is not unreasonable. M31 and the Galaxy probably contain most of the mass in the LG (but cf. Herbst 1975), and the other galaxies, being distributed reasonably isotropically around them, are not expected to have a systematic translational velocity relative to them. (If this is not so, then the following argument breaks down, since $\boldsymbol{v}_0$ is determined by all the galaxies in the LG, and not by M31 alone.)

The absence of total linear momentum in the center-of-mass frame imposes the relation

$$m_G \boldsymbol{v}_G + m_A \boldsymbol{v}_A = 0 \tag{8}$$

between the peculiar velocities of the two galaxies. In practice, of course, we only have knowledge of the radial velocity component, which is obtained from equation (8) by taking the dot product with the unit vector $\boldsymbol{n}_A$ toward M31. Noting that the observed radial velocity of M31 is

$$\rho_A = \boldsymbol{v}_A \cdot \boldsymbol{n}_A - \boldsymbol{v}_0 \cdot \boldsymbol{n}_A \,, \tag{9}$$

and using $\boldsymbol{v}_0 = \boldsymbol{v}_G + \boldsymbol{v}_C$ to eliminate $\boldsymbol{v}_G$ from equation (7), gives the useful result

$$\boldsymbol{v}_C \cdot \boldsymbol{n}_A = \boldsymbol{v}_0 \cdot \boldsymbol{n}_A + \frac{m_A}{m_G} (\rho_A + \boldsymbol{v}_0 \cdot \boldsymbol{n}_A) \,. \tag{10}$$

Note that the right side of equation (10) contains only known quantities (aside from the mass ratio), either from our solution for $\boldsymbol{v}_0$ or from data of observation. (The radial velocity of M31, its direction, and solution 2 for $\boldsymbol{v}_0$ are used.)

Although in principle equation (10) gives a unique solution for $\boldsymbol{v}_C$ (once m_A/m_G is specified), in practice the variance-limits on $\boldsymbol{v}_0$ are wide enough to give a substantial range of $|v_C|$ at the 90% confidence level. Equation (10) is plotted in Figure 6 as a dashed line near the top of the diagram, using $\boldsymbol{v}_0$ from our second solution (300 km s^{-1} toward $l = 107$, $b = -8°$), and $\rho_A = -298$ km s^{-1}. The best-fit value of $|v_C|$ is higher than usual estimates ($v_C > 320$ km s^{-1} for $m_A/m_G < 2$). But the 90% confidence limit on $\boldsymbol{v}_0 \cdot \boldsymbol{n}_A$, obtained from equation (5) with $q = 1$, is plotted as the hatched line, below which is forbidden. (There is a similar upper limit which is off scale on the plot.) It is seen that $|v_C|$ could be as low as 246 km s^{-1} and 182 km s^{-1} for mass ratios of 1 and 2. A similar conclusion has been reached by Lynden-Bell and Lin (1977). They favor a lower mass ratio, and hence an increased lower limit to $|\boldsymbol{v}_C|$. Note that at $m_A/m_G = 0.5$, our lower limit becomes $|v_C| = 280$ km s^{-1}.

The other lines in Figure 6 are discussed in the next section, and the mass scale along the right ordinate is from § VI*c*.

d) The Peculiar Velocity $\boldsymbol{v}_G$ of the Galaxy

Another limit of interest can be obtained by requiring $|v_G|$ to be comparable to the velocity dispersion (σ) in the Local Group. The assumption is ad hoc, since dynamically the mass is dominated by M31 and the Galaxy, yet σ measures the dispersion of the dwarf galaxies. Furthermore, in view of the long crossing time of the LG ($\sim 10^{10}$ years), the motion of the galaxies probably still reflects initial conditions at the time of galaxy formation rather than an equilibrium condition after relaxation. Nevertheless, it is of interest to put limits on $|v_C|$ and $|v_G|$ by requiring $\sigma \sim |V_G|$, since it is unlikely that $|V_G| > 100$ km s^{-1} in view of the low mean random motion of field galaxies.

The solid lines in Figure 6 are lower limits in the $(|v_C|, m_A/m_G)$-plane obtained by requiring $|v_G|$ to be less than 50, 75, and 100 km s^{-1}, and $\boldsymbol{v}_0$ to be within the 90% confidence limit as determined from equation (5) with $q = 3$. It is seen that these reasonable limits on

$|V_G|$ are consistent with the usual intervals of $200 < |v_C| < 300$ km s^{-1} and $0.5 < m_A/m_G < 2$.

(It will be noticed that part of the plane allowed by this limit is excluded by the previous direct limit on $\boldsymbol{v}_0 \cdot \boldsymbol{n}_A$ discussed in § VI*c*, shown by the hatched line. This is caused by the subtle difference resulting from different requirements. The limit on $\boldsymbol{v}_0 \cdot \boldsymbol{n}_A$ was applied with $q = 1$ in § VI*c*, i.e., $\boldsymbol{v}_0 \times \boldsymbol{n}_A$ could assume any value, and the allowed values of $\boldsymbol{v}_0 \cdot \boldsymbol{n}_A$, within the given confidence limit of 90%, were more restricted. In the present limit, all components of $\boldsymbol{v}_0$ are constrained simultaneously and there is therefore a little more leeway in $\boldsymbol{v}_0 \cdot \boldsymbol{n}_A$ at the expense of $\boldsymbol{v}_0 \times \boldsymbol{n}_A$. This illustrates the point emphasized by Avni (1976), that the confidence limits obtained depend on the questions asked.)

e) Angular Momentum in the Local Group

The analysis so far has not attempted to constrain the angular momentum of the LG, for which the chief contributors are again M31 and the Galaxy. Traditionally, the two galaxies have been considered to be moving collinearly (e.g., Kahn and Woltjer 1959). This is equivalent to taking the angular momentum of the M31–Galaxy pair to be zero (i.e., $\boldsymbol{v}_G \times \boldsymbol{n}_A = 0$), as for two masses initially receding from each other with the Hubble expansion. But again, the assumption is ad hoc because there are many rationalizations that can be made why it need not be.

Some spin-orbit coupling may have occurred at an early epoch (Hoyle 1951; Peebles 1969; Thuan and Gott 1977); but unless much spin angular momentum is contained in postulated massive halos (Ostriker and Peebles 1973; Ostriker, Peebles, and Yahil 1974), the orbital angular momentum gained by the process is still very small because the known spin angular momentum itself is so much less than the orbital, even for very small deviations from collinearity.

Initial angular momentum due to tidal forces caused by neighboring protogroups is also not excluded, and the orbital angular momentum of M31 and the Galaxy could be considerable if they originated at different positions in the protocloud.

Direct observational data that seem to rule out very eccentric orbits in other binaries (Turner 1976), are, however, not consistent with dynamical requirements of binary models (Yahil 1977). Hence, the situation concerning their net orbital angular momenta is not now clear.

In view of these uncertainties, we sought yet another solution with no orbital angular momentum. With the condition $\boldsymbol{v}_G \times \boldsymbol{n}_A = 0$, it is evident that

$$\boldsymbol{v}_0 = \boldsymbol{v}_C + |v_G|\boldsymbol{n}_A\,, \tag{11}$$

which is used to replace $\boldsymbol{v}_0$ in equation (1), and then the resulting Λ is minimized with respect to $|v_C|$ and $|v_G|$. The "best-fit" values from this procedure are unrealistic ($|v_C| = 152$ km s^{-1}, $|v_G| = 151$ km s^{-1}), but the 90% confidence limit for $q = 2$, illustrated in Figure 7, encompasses more acceptable values. The

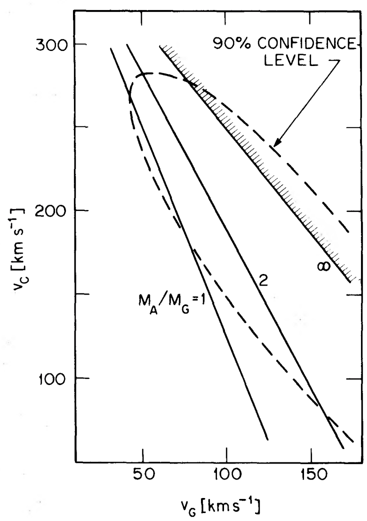

FIG. 7.—Zero orbital angular momentum solutions. The 90% confidence limit is shown as the very elongated ellipse. Straight lines are the relations between $|v_C|$ and $|v_G|$ imposed by fixing the mass ratio to various values. The regime to the right of the hatched line is forbidden.

point is that the small angular separation between M31 and the direction of Galactic rotation results in a strong negative correlation between $|v_C|$ and $|v_G|$, and hence an elongated error ellipse.

In the above analysis the mass ratio m_A/m_G was arbitrary. Fixing it requires the solution to lie along a straight line, illustrated for three mass ratios in Figure 7. Consideration of the mass ratio will also change the error analysis slightly, since, as plotted in Figure 7, it did not take into account the forbidden region of the (v_C, v_G)-plane in which there is no positive mass ratio, but the result that $|v_C| \lesssim 280$ km s^{-1} is not strongly affected by this.

The conclusion from this section is that the ad hoc, strong dynamical requirement here of zero orbital angular momentum has not resulted in more useful limits on $|v_G|$ and $|v_C|$ than we obtained in previous sections. The same point is illustrated by Figure 5 where we have plotted as a line the relation between v_G^x and v_G^z which results by the requirement of this section that $\boldsymbol{v}_G$ be parallel to $\boldsymbol{n}_A$. The fact that this line passes more or less centrally through the error ellipse of solution 2, which has no restriction on the direction of $\boldsymbol{v}_G$, shows independently that such a restriction does not lead to more useful limits.

f) Mass of the Local Group

A lower limit on the mass of the LG can be obtained by requiring that the gravitational potential is sufficient to bind M31 and the Galaxy, i.e., $\Omega + T < 0$. From a straightforward calculation for a two-body system in the center-of-mass frame it follows that

$$M_T > 7.7 \times 10^7\, \mathfrak{M}_\odot |v_G|^2 (1 + m_G/m_A)^2\,, \tag{12}$$

where the distance to M31 is taken to be 667 kpc

[$(m - M)_0 = 24.12$ from Sandage and Tammann 1974], and $|v_G|$ is in km s^{-1}.

For a given rotation velocity $\boldsymbol{v}_C$, the extreme lowest bound M_B occurs when $\boldsymbol{v}_G$ is parallel to $\boldsymbol{n}_A$ because the kinetic energy of the pair is then a minimum. In this case, equation (12) can be simplified using equation (9) with $\rho_A = -298$ km s^{-1}, and making use of equation (8), with the knowledge that $\boldsymbol{v}_A$ and $\boldsymbol{v}_G$ are collinear, and that $\boldsymbol{v}_C \cdot \boldsymbol{n}_A = 0.796|v_C|$, to give

$$M_B = 4.9 \times 10^7\, \mathfrak{M}_\odot (375 - |v_C|)^2 \,. \qquad (13)$$

The mass from equation (13) is plotted along the right-hand ordinate of Figure 6 for various $|v_C|$ values.

This absolute lower bound in equation (13) is interesting because it may be low enough to alleviate the need for unseen mass in the LG. (If $|v_C| = 300$ km s^{-1}, then $M_B = 2.8 \times 10^{11}\, \mathfrak{M}_\odot$; and if $|v_C| = 250$ km s^{-1}, $M_B = 7.7 \times 10^{11}\, \mathfrak{M}_\odot$.) But it should also be stressed that our M_B value is an absolute minimum, which can only be approached by a very eccentric binary system close to perigalacticon. Timing arguments (Kahn and Woltjer 1959) suggest that M31 may be closer to apgalacticon, in which case the mass of the LG is higher than M_B. But independent of these types of arguments, which depend on the largely unknown formation history of the LG, the higher mass limit in equation (12) should be used if $\boldsymbol{v}_G \times \boldsymbol{n}_A$ is nonzero.

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