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CARBON, NITROGEN, AND OXYGEN ABUNDANCES IN 11 G AND K GIANTS

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ABSTRACT

Carbon, nitrogen, and oxygen abundances have been determined for 11 G and K giants. High-resolution photoelectric scans of C_2, CH, CN, [O I], and [C I] lines have been combined with recent model atmospheres. Relative to the solar atmosphere, the C abundances are depressed, the N abundances enhanced, and the O abundances unchanged. The observed CNO abundances and the previously obtained $^{12}C/^{13}C$ ratios are in good agreement with the predictions for a giant after the convective envelope has mixed material to the surface from the zone which was partially processed during the star's main-sequence lifetime. These are the first results to demonstrate this agreement between theory and observation.

Subject headings: stars: abundances — stars: late-type

I. INTRODUCTION

The surface abundances of C, N, and O for a red giant should differ from those of the main-sequence progenitor because, on the first ascent of the giant branch, the convective envelope mixes material to the surface from a zone which underwent partial processing through the CNO cycle during the main-sequence lifetime. Iben (1964, 1967) discovered this convective episode and pointed out the importance of observational checks through the C/N and $^{12}C/^{13}C$ ratios.

Recent observations of the $^{12}C/^{13}C$ ratio in giants (Lambert 1976; Tomkin, Luck, and Lambert 1976) show that the ^{13}C abundance is enhanced in qualitative agreement with the convective mixing predictions. About one-half of the nearly 100 giants and supergiants for which $^{12}C/^{13}C$ has been measured show a $^{12}C/^{13}C$ ratio in quantitative agreement with the predictions (Iben 1964; Dearborn, Eggleton, and Schramm 1976); i.e., $^{12}C/^{13}C \approx 20$–30, which corresponds to an initial ratio $^{12}C/^{13}C \approx 40$–90.

However, the other stars in the sample show a $^{12}C/^{13}C$ ratio unexplained by the standard calculations; e.g., the K0 II–III star 37 Com has $^{12}C/^{13}C = 3.4$. Clearly the calculations must be modified; Dearborn, Eggleton, and Schramm (1976) and Tomkin, Luck, and Lambert (1976) sketch possibilities involving mass loss, meridional mixing, an H shell instability, and the effects of a close companion. Refinement and elimination of theoretical possibilities require more complete information on the surface composition of red giants.

Predicted abundance changes for a convectively mixed giant are modest; e.g., Dearborn (1976) reports that the C/N ratio for a 2 $M_\odot$ giant is reduced to 1.4 from an assumed initial value of 3 (C decreases by 25% and N increases by 75%). Although larger changes may have occurred in those giants with an anomalously low $^{12}C/^{13}C$ ratio, such modest abundance changes offer a severe observational challenge. Results of a CNO abundance study for 11 giants are presented in this paper.

II. CNO ANALYSIS: DESIGN AND EXECUTION

a) *Line Selection*

A search was conducted for atomic and molecular signatures exhibiting the least sensitivity to the primary sources of uncertainty in an abundance analysis—i.e., the errors in effective temperature (T_{eff}), surface gravity (g), and microturbulence (ξ_{mic}) as well as possible departures from local thermodynamic equilibrium (LTE). Additional constraints included the requirement that the line be observable with the Tull (1972) coudé scanning spectrometer; recent installation (Tull 1976) of a Reticon detector extends the wavelength range to ~11,000 Å.

Permitted C I and O I lines were rejected on account of a severe temperature dependence and the likely intrusion of departures from LTE. However, if LTE were applicable, these lines could give a C/O ratio with minimal temperature sensitivity. N I lines are undetectable. Other rejected lines include the CN violet system; the CH A–X, $\Delta v = 0$, sequence near 4300 Å; and other CH, NH, and OH electronic transitions in the ultraviolet. A common factor applicable to these transitions is the uncertainty of the equivalent widths for weak lines resulting from continuum location difficulties in the crowded spectrum below 4500 Å.

Examination of model atmosphere predictions led to the following list of potential abundance indicators: the [C I] line at 8727 Å; the C_2 Swan system; the CH $A\,^2\Delta$–$X\,^2\Pi$, $\Delta v = -1$, sequence near 4850 Å; the CN red system; and the [O I] lines at 6300 and 6363 Å. Unfortunately, the [C I] line is blended (see Appendix) and was dropped from the active line list.

The abundance analysis for the giants is basically a differential one with respect to the Sun. A thorough rediscussion of the solar CNO abundances including an examination of the role of the present line list is given by Lambert (1977). In the following paragraphs the selected lines are discussed.

The C_2 Swan band lines are the primary source of the C abundance. Tull scanner observations refer to two C_2 features at 5086 Å [the $R(35)$ triplet] and 5135 Å [the $P(44)$ triplet] from the (0, 0) band. Both contain weaker blends of either $^{12}C_2$ or $^{12}C^{13}C$ lines which have been taken into account in the analysis. At spectral types of K2 and earlier, the two C_2 features give a C abundance differing by less than 0.05 dex. An unidentified blend affects the 5135 Å feature at spectral types later than K2. In K4 and K5 giants, the C_2 lines are too blended to serve as a reliable abundance indicator. The observed line profiles of these triplets are approximately consistent with the predicted profiles. Small discrepancies equivalent to only 0.05 dex in the C abundance are probably attributable to weak unidentified blends and the inevitable uncertainties in the continuum location. Other C_2 lines in the Arcturus spectrum and the (0, 1) band head in several other stars were examined. The 5086 and 5135 Å features were quite consistent with this more extensive line list. The analysis is based upon the dissociation energy $D_0{}^0 = 6.11$ eV (Messerle and Krauss 1967) and upon a band oscillator strength $f(0, 0) = 0.0239$ derived by Lambert (1977) from accurate radiative lifetime measurements (Curtis, Engman, and Erman 1976) and from the relative band f-values provided by Danylewych and Nicholls (1974). These parameters, the solar equivalent widths (Grevesse and Sauval 1973), and the Holweger-Müller (1974) model solar atmosphere give a C abundance $\log \epsilon(C) = 8.73$ on the usual scale $\log \epsilon(H) = 12.00$. As Lambert (1977) shows, this abundance is within 0.06 dex of the final recommended C abundance based upon the [C I] lines, upon the CH $A-X$ and C_2 Swan lines, and upon the best of two from a set of five recent model atmospheres; i.e., the use of the above parameters in a stellar abundance analysis is equivalent to a differential analysis with respect to the Sun.

Although the CH lines from the $\Delta v = 0$ sequence of the $A-X$ transition are second-class abundance indicators, lines from the weak $\Delta v = -1$ sequence around 4800 Å may be exploited. The selected feature is a CH blend at 4835 Å comprising two lines from the (0, 1) band and one from the (1, 2) band (see Moore and Broida 1959). Two other lines near 4879 Å are used in a confirming role. A careful search of the new Liège Atlas (Delbouille, Neven, and Roland 1973) provided seven (0, 1) CH solar lines which yielded $f(0, 1) = (1.49 \pm 0.06) \times 10^{-4}$ with the Holweger-Müller model atmosphere, a carbon abundance $\log \epsilon(C) = 8.67$, and a dissociation energy $D_0{}^0 = 3.465$ eV (Brzozowski *et al.* 1976), with account taken of the vibration-rotation correction to the Hönl-London factor (Bell, Branch, and Upson 1976). A similar result was obtained by Gustafsson, Kjaergaard, and Andersen (1974).

This solar f-value may be compared with predictions derived using the accurate f-value for the (0, 0) band: $f(0, 0) = 4.93 \times 10^{-3}$ (see Lambert 1977). Standard RKR calculations show that the Franck-Condon factor for the (0, 1) band is very small. Therefore, the f-value prediction is sensitive to the variation of the transition moment with internuclear separation. An *ab initio* calculation (Hinze, Lie, and Liu 1975) predicts $f(0, 1)/f(0, 0) = 0.0274$. The solar lines correspond to $f(0, 1)/f(0, 0) = 0.0302$. Agreement is satisfactory considering the small equivalent widths of the few solar lines. The standard recipe using the RKR Franck-Condon factors and the assumption of a constant transition moment provides the ratio $f(0, 1)/f(0, 0) = 0.0073$, which is in poor agreement with the solar prediction. The *ab initio* calculations predict $f(1, 2)/f(0, 1) = 1.9$, which was adopted in the analysis. From solar lines, Gustafsson, Kjaergaard, and Andersen (1974) obtained $f(1, 2)/f(0, 1) = 1.25$, a value which is in reasonable agreement with the theoretical value. The CH $A-X$, $\Delta v = 0$, sequence with the Holweger-Müller atmosphere gives a solar C abundance [$\log \epsilon(C) = 8.67$] in agreement with the recommended value.

Observations of CN red system (2, 0) and (4, 0) band lines are already available. These lines, when combined with the C and O abundances, are the only reliable source of a N abundance in G and K giants. Since the CN dissociation energy and the red system f-value are shrouded in uncertainty, the solar spectrum is employed as a calibrating source. A dissociation energy $D_0{}^0 = 7.66$ eV (Engleman and Rouse 1975) is adopted. Grevesse and Sauval's (1973) equivalent widths for the (0, 0) band; the Holweger-Müller atmosphere; and the abundances $\log \epsilon(C) = 8.67$ and $\log \epsilon(N) = 7.99$ give $f(0, 0) = 2.6 \times 10^{-3}$.[1] This result, with the relative f-values measured by Arnold and Nicholls (1972), provides the f-values $f(2, 0) = 9.16 \times 10^{-4}$ and $f(4, 0) = 1.01 \times 10^{-4}$. Since the (2, 0) band lines are very weak, this slightly indirect derivation of the empirical f-value is preferred over a direct deduction from the (2, 0) solar lines. This use of the solar spectrum should be discarded when the CN dissociation energy and the CN red system f-value are finally determined with appropriate precision. A reference to the Sun is a potential source of a somewhat significant uncertainty; i.e., substitution of the Harvard-Smithsonian Reference Atmosphere (Gingerich *et al.* 1971) would provide $f(0, 0) = 1.8 \times 10^{-3}$ and a consequent increase of the stellar N abundances by 0.15 dex.

The CN dissociation energy has been put as high as $D_0{}^0 = 7.9$ eV (see Arnold and Nicholls 1973; Lambert 1977); the effect of this higher value on the stellar N abundances is very slight after the solar f-value is recalculated.

The [O I] magnetic dipole transitions at 6300 and 6363 Å monitor the O abundance. Theoretical f-values (Garstang 1976) are adopted of $\log gf = -9.75$ and $\log gf = -10.25$ for the 6300 and 6363 Å lines,

[1] This f-value is to be used with Hönl-London factors normalized such that $S_{J'J''} \approx J$ for Q_1 or Q_2; and $S_{J'J''} \approx J/2$ for P_1, R_1, P_2, and R_2. The exact values are to be calculated from the standard expressions (see Kovács 1969).

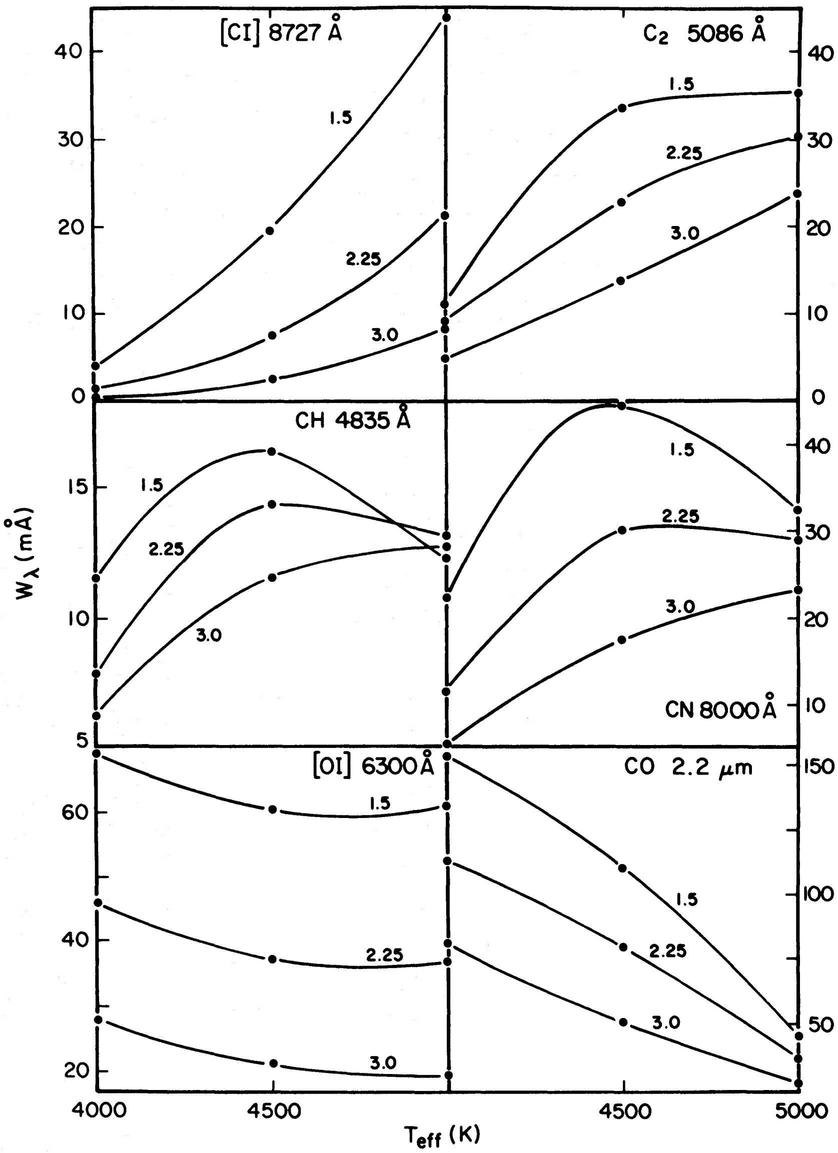

FIG. 1.—Predicted equivalent widths for program lines as a function of effective temperature, T_{eff}, and surface gravity, g. Curves are labeled according to log g.

respectively. These lines are the primary source of the solar O abundance, log ϵ(O) = 8.92, recommended by Lambert (1977); so adoption of the same f-values ensures that the present analysis is a differential one. An Ni I line at 6300.336 Å (Kurucz and Peytremann 1975) is blended with the [O I] line. The Liège Atlas (Delbouille, Neven, and Roland 1973) shows that the solar line is within 5 mÅ of the predicted [O I] line position as derived from the accurate laboratory wavelength (Eriksson 1965) and from the gravitational redshift. The line is also very symmetrical. These constraints convert to the limit $W_\lambda \leq 0.5$ mÅ for the solar Ni I line, which corresponds to a negligible contribution of 1 mÅ or less in the typical red giant. The 6363 Å line is corrected for a small contribution from the ^{12}CN $Q_2(28)$ (10, 5) red system line. Also, the line is in the red wing of a broad Ca I autoionization line (Griffin 1964). Test calculations show that the O abundance can be derived from an equivalent width measurement with respect to the local continuum provided by the autoionization line.

A third [O I] line, the electric quadrupole transition at 5577 Å, was observed in a few stars. Unfortunately, the line is weak and blended with the C_2 doublet $P_1(27) + P_2(26)$ from the (1, 1) Swan band. In a typical K giant, the 5577 Å line has an equivalent width $W_\lambda \approx 6$ mÅ to which the C_2 lines contribute ~3 mÅ. The resultant [O I] contribution is only 3 mÅ with an uncertainty of ± 1 to ± 2 mÅ. Therefore, the 5577 Å [O I] line was not used in this analysis.

TABLE 1

ADOPTED BASIC PARAMETERS

Star	Spectral Type	T_{eff} (K)	log g	ξ (km s^{-1})	$^{12}C/^{13}C$	Ref.
Sun............	G2 V	5780	4.44	0.9	89	
ϵ Vir...........	G9 II–III	5000	3.0	1.3	20	*a, b, c, d, e, f*
ϵ Cyg..........	K0 III	4800	2.85	1.2	11.5	*f, g, k*
β Gem..........	K0 III	4700	2.7	1.2	16	*c, f, h, i, j*
γ Tau..........	K0 III	4960	2.6	1.7	19	*c, d, e, f, k, l, m*
δ Tau..........	K0 III	4940	2.7	1.6	23	*c, d, e, f, k, m*
ϵ Tau..........	K0 III	4910	2.6	1.7	22	*c, d, e, f, k, l, m*
θ^1 Tau..........	K0 III	5000	2.8	1.4	20	*c, d, f, k, l, m*
γ Cep..........	K1 IV	4770	3.2	1.0	24	*c, n, o*
ν^2 CMa.........	K1 IV	4700	2.9	1.4	51	*c, o*
μ Leo..........	K2 III	4540	2.35	1.3	18	*c, e, f, j, l, p, q*
α Boo..........	K2 IIIp	4250	1.7	2.4	7.2	See text

REFERENCES.—(*a*) Cayrel and Cayrel 1963. (*b*) Glebocki 1972. (*c*) Gustafsson, Kjaergaard, and Andersen 1974. (*d*) van Paradijs 1973. (*e*) Strom, Strom, and Carbon 1971. (*f*) Williams 1971. (*g*) VPK. (*h*) Greene 1969. (*i*) Griffin 1976. (*j*) Tomkin and Lambert 1974. (*k*) Tomkin, Luck, and Lambert 1976. (*l*) Oinas 1974. (*m*) Helfer and Wallerstein 1964. (*n*) Herbig and Wolff 1966. (*o*) Dearborn, Lambert, and Tomkin 1975. (*p*) Blanc-Vaziaga, Cayrel, and Cayrel 1973. (*q*) Day 1976. (*r*) Peterson 1976.

The variation of the predicted equivalent widths with T_{eff} and log g is shown in Figure 1 for the selected lines. The figure also includes a representative CO line, because these infrared lines may soon be observable in a wide sample of giants. The model atmospheres used in the calculations are discussed in a later section.

b) *Defining Parameters for a Model Atmosphere*

The parameter set—T_{eff}, log g, ξ_{mic}, and the chemical composition—defines the model stellar atmosphere. Both T_{eff} and log g (see Table 1) were selected after a careful scrutiny of the literature was made. In averaging published values, we assigned double weight to high-quality spectroscopic analyses. If the formal errors were less than 100 K in T_{eff} or 0.25 in log g, these minimum values were adopted. The actual errors may be larger.

A controversial set of basic parameters for α Boo, $T_{eff} = 4260$ K and log $g = 0.90 \pm 0.35$, was obtained by Mäckle *et al.* (1975) from a fine analysis and an empirical model atmosphere. The low surface gravity and associated low stellar mass challenged the traditional higher surface gravity and the standard picture of stellar evolution without excessive mass loss.

Several new analyses appear to quench the controversy. Van Paradijs and Meurs (1974) using the Carbon-Gingerich (1969) model atmosphere grid found that neutral and ionized lines of a species gave identical abundances for the parameters $T_{eff} = 4350$ K and log $g = 1.95 \pm 0.2$, or $T_{eff} = 4260$ K and log $g = 1.6 \pm 0.2$. Gustafsson, Kjaergaard, and Andersen (1974) derived log $g = 1.9$. Most recently, Ayres and Johnson (1977) obtained log $g = 1.6 \pm 0.2$ from "opacity sampling" model atmospheres and from the Ca I and Ca II resonance line wings. In this CNO analysis, $T_{eff} = 4250$ K and log $g = 1.7$ are adopted.

The microturbulent velocity ξ_{mic} was derived from the CN lines used in the $^{12}C/^{13}C$ analyses. These values are in good agreement with values obtained from metal lines by Gustafsson, Kjaergaard, and Andersen (1974). Fortunately, the CNO line list provides abundances with only a slight sensitivity to errors in ξ_{mic}.

The chemical composition adopted in the construction of the model atmosphere should be consistent with the deduced abundances. In principle, an iteration on the composition should be pursued. Two groups of elements are identifiable: the major electron donors (Si, Mg, Fe, Ca, Na) and the CNO group which provides the major molecules (CO, CN) affecting the line blanketing. A brief remark appears below on the iteration on the CNO abundances and the use of a model atmosphere for which the adopted and derived CNO abundances are consistent. Metal abundances were taken to have the solar values, except for the Hyades stars (metals up by 0.13 dex) and α Boo (metals down by 0.5 dex).

c) *Model Stellar Atmospheres*

A model atmosphere grid (Bell *et al.* 1976) constructed by Gustafsson *et al.* (1975) was the basis for the final abundance analyses. Their calculations use opacity distribution functions to represent the atomic (approximately 50,000 lines) and molecular (^{12}CO, ^{13}CO, ^{12}CN, ^{13}CN, CH, NH, OH, MgH) line blanketing.

Additional models were constructed with the ATLAS (Kurucz 1970) code as modified by Dragon (1974). Molecular (^{12}CO, ^{12}CN, TiO, and H_2O) line blanketing is included in the form of straight mean opacities. A recipe described by Mutschlechner and Keller (1972) is used to represent atomic line blanketing. The deficiencies (Carbon 1973) of the straight mean representation are here minimized because the ATLAS models are primarily an interpolation device.

The two families of model atmospheres were compared in analyses of ϵ Vir and μ Leo. Models for

$T_{\rm eff}$ = 5000 K, log g = 3.0, and solar metal abundance were compared for ϵ Vir. Logarithmic abundance differences in the sense ATLAS *minus* Gustafsson *et al.* (1975) were 0.03, 0.04, and 0.03 dex for C, N, and O, respectively. With models for $T_{\rm eff}$ = 4500 K, log g = 2.25, and solar metal abundance to mimic μ Leo, larger differences were found: -0.02, 0.12, and 0.08 dex for C, N, and O, respectively. Here, the larger C/O ratio increases the difference between the models. For the other stars, except α Boo, small interpolated corrections were applied to results obtained from ATLAS models.

Recently, Peterson (1976) has proposed that the temperature profile in super-metal-rich (SMR) stars like μ Leo is steepened and the boundary lowered by increased CN line blanketing. To assure self-consistency, initial CNO abundances for μ Leo were adopted as the input values for the ATLAS code, a new model atmosphere was calculated, and the abundance analysis repeated. One iteration gave small ($\sim$0.05 dex) corrections. An inspection of the models shows that CN exerts a minor influence on the boundary temperature, for the simple reason that CO formation inhibits CN formation in the upper photosphere.

Observations of α Boo were reduced with models for $T_{\rm eff}$ = 4250 K, log g = 1.7, and a reduced ($Z = \frac{1}{3}Z_\odot$) metal abundance. An ATLAS model was compared with one calculated by Professor H. Johnson using the opacity sampling (Sneden, Johnson, and Krupp 1976) code. Differences (ATLAS *minus* Johnson) were small: 0.05, -0.03, and 0.07 dex for C, N, and O. Ayres and Linsky (1975) have given an empirical atmosphere for the low chromosphere and upper photosphere of α Boo. Their model extends down to $\tau_{5000} \approx 0.3$. A composite model constructed by grafting their temperature profile onto the Johnson model was compared with the Johnson model; logarithmic differences in the sense Ayres-Linsky *minus* Johnson were 0.03, 0.00, and 0.06 for C, N, and O.

These comparisons suggest that the model atmospheres are not the source of a significant abundance error; in particular, the dramatic reduction in the C/N ratio (see below) in giants is unlikely to be attributable to a model atmosphere error. Of course, the theoretical atmospheres share a common set of assumptions (radiative and convective equilibrium in a plane-parallel homogeneous layered atmosphere in LTE), and the above comparisons do not test for departures from these assumptions. The Ayres-Linsky empirical atmosphere provides a partial test of the common assumptions.

d) *The* CNO *Observations and Analysis*

Sample observations are shown in Figure 2. Equivalent widths appear in Table 2. Observations were obtained with the McDonald Observatory 2.7 m reflector and Tull coudé spectrometer in a single-channel mode (Tull 1972) with a RCA gallium arsenide photomultiplier. A resolution of $\sim$50 mÅ was employed, and integration was continued until the signal-to-noise ratio in the continuum was 100 or better. Scan lengths were kept short. For the C_2 and CH scans, test scans of longer length were examined to ensure that the highest points on the short program scans were at the continuum level. Recent observations have been obtained with a Reticon 1024 element silicon diode array (Tull 1976) which, in a single exposure, provides a spectrum of $\sim$30 Å length at a resolution

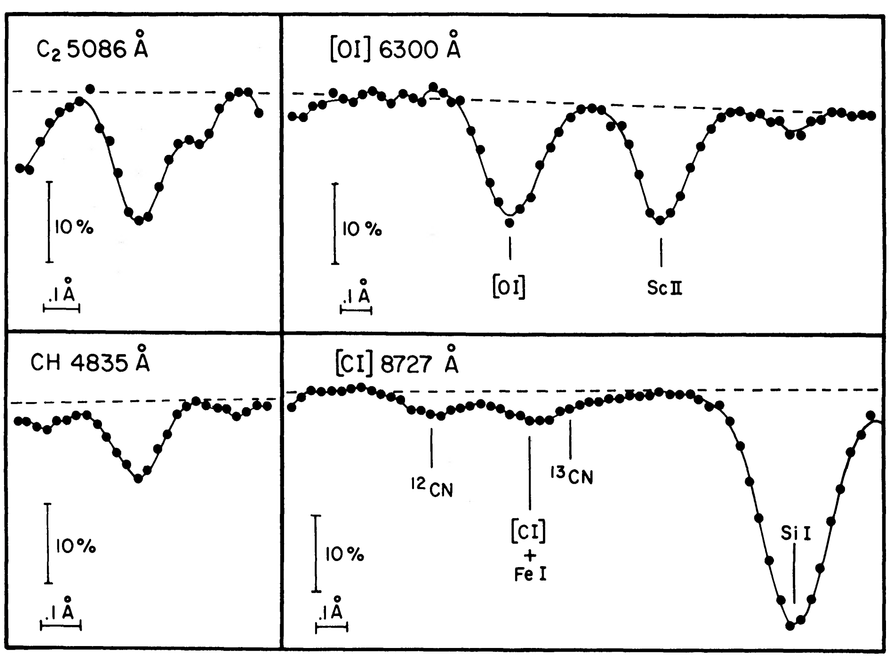

FIG. 2.—Observations of program lines in β Gem

TABLE 2

EQUIVALENT WIDTHS FOR THE CNO LINES

	W_λ (mÅ)				
STAR	C_2* (5086 Å)	C_2* (5135 Å)	CH (4835 Å)	[O I] (6300 Å)	[O I]† (6363 Å)
ε Vir.............	24.1	19.2	6.9	24.6	10.0
ε Cyg............	26.0	22.8	11.4	32.3	12.8
β Gem...........	27.8	23.3	11.9	23.1	8.7
ν^2 CMa..........	55.6	54.7	...	22.3	...
γ Cep............	45.4	41.0	13.8	20.1	5.9
μ Leo............	64.5	62.5	...	41.4	13.4
α Boo............	21.8	...	12.8	61.2	31.4
γ Tau............	34.1	30.2	...	26.1	10.2
δ Tau............	31.7	25.6	...	26.6	9.7
ε Tau............	33.6	29.7	...	28.7	10.1
θ^1 Tau...........	32.0	28.9	...	24.3	9.5

* The observed W_λ including small contributions from C_2 lines other than the dominant (0, 0) triplet.

† The observed W_λ less a small correction (2–6 mÅ) for the blending CN line.

of ~0.06 Å and a concomitant easing of the continuum definition problem.

Versatile computer programs (Sneden 1973; Luck 1977) extract abundances from the observations and model atmosphere. Molecule formation was considered with four molecules contributing to the equilibrium calculations. Test calculations show that the four molecule equilibrium calculation reproduces results obtained with a much larger number of molecules. The dominant troika are CO, N_2, and H_2. Abundances of C_2, CH, and CN were calculated from the partial pressures of C, N, and H derived from the molecular equilibrium calculations.

CO controls the C and O partial pressures and limits the formation of the [C I], C_2, CH, and CN lines to the deep photosphere where dissociation reverses the complete association of C into CO that is found in the upper photosphere. In this way, CO demands that accurate model atmospheres be used in an analysis of these atomic and molecular lines. The influence of N_2 and H_2 is not so important. Fortunately, the dissociation equilibrium constants are well known for the trio.

CO formation links together the C and O abundances so that the [O I], C_2, and CH lines provide loci in the log ε(C) plane whose intersection gives the C and O abundances. Similarly, the CN red system lines provide a family of loci in the log ε(C) versus log ε(N) plane corresponding to different values of the O abundance. A typical set of the loci is shown in Figure 3. Present use of such diagrams was stimulated by their earlier use by Greene (1969).

If high-excitation permitted lines of C I and O I

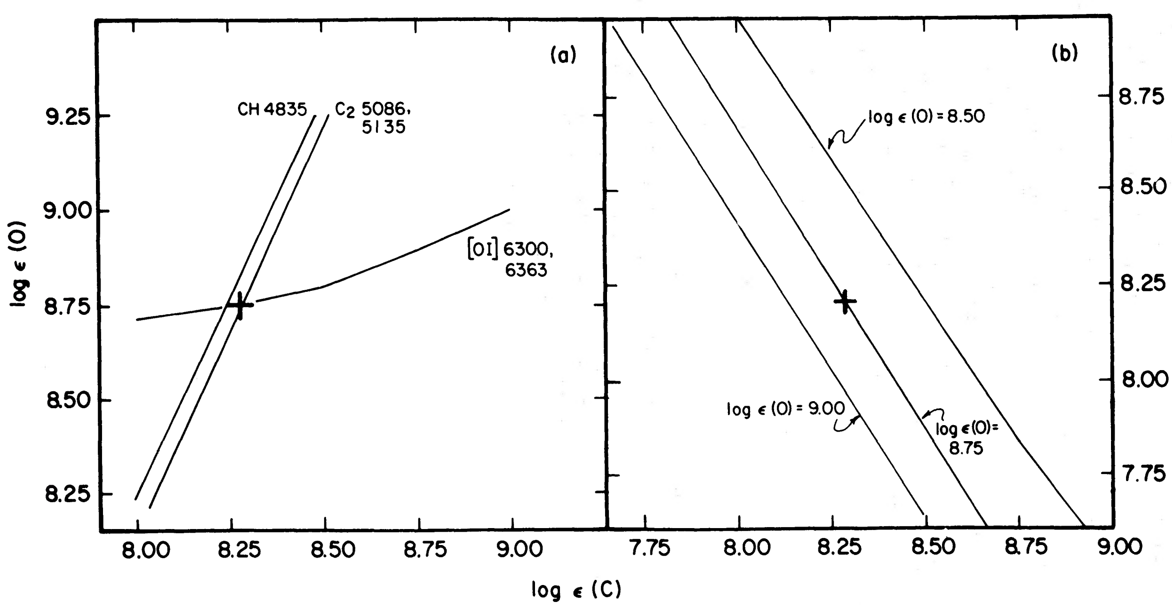

FIG. 3.—Loci of abundance solutions for β Gem in (*a*) the log ε(O) versus log ε(C) plane and (*b*) the log ε(N) versus log ε(C) plane. The cross shows the adopted solution.

TABLE 3

C, N, AND O ABUNDANCES

Star	log ϵ(C)*	log ϵ(N)*	log ϵ(O)*	[C]†	[N]†	[O]†	O/C‡	C/N‡
Sun	8.67	7.99	8.92	...	...	...	1.7	4.8
ϵ Vir	8.36 (0.14)	8.32 (0.14)	8.96 (0.17)	−0.31	+0.33	+0.04	4.0 ± 0.7	1.1 ± 0.1
ϵ Cyg	8.42 (0.15)	8.28 (0.15)	9.00 (0.16)	−0.25	+0.29	+0.08	3.8 ± 0.5	1.4 ± 0.3
β Gem	8.28 (0.13)	8.31 (0.15)	8.75 (0.16)	−0.39	+0.32	−0.17	3.0 ± 0.4	0.9 ± 0.2
γ Tau	8.33 (0.15)	8.39 (0.13)	8.81 (0.15)	−0.34	+0.40	−0.11	3.0 ± 0.6	0.9 ± 0.2
δ Tau	8.33 (0.15)	8.43 (0.13)	8.83 (0.15)	−0.34	+0.44	−0.09	3.2 ± 0.6	0.8 ± 0.2
ϵ Tau	8.36 (0.15)	8.40 (0.13)	8.85 (0.15)	−0.31	+0.41	−0.07	3.1 ± 0.6	0.9 ± 0.2
θ^1 Tau	8.40 (0.15)	8.39 (0.13)	8.91 (0.15)	−0.27	+0.40	−0.01	3.2 ± 0.6	1.0 ± 0.3
γ Cep	8.59 (0.15)	8.32 (0.15)	8.96 (0.17)	−0.08	+0.33	+0.04	2.3 ± 0.5	1.9 ± 0.4
ν^2 CMa	8.69 (0.15)	8.30 (0.15)	8.90 (0.17)	+0.02	+0.31	−0.02	1.6 ± 0.2	2.5 ± 0.4
μ Leo	8.63 (0.16)	8.56 (0.17)	8.96 (0.16)	−0.04	+0.57	+0.04	2.1 ± 0.3	1.2 ± 0.2
α Boo	8.05 (0.06)	7.71 (0.19)	8.65 (0.17)	−0.62	−0.28	−0.27	4.0 ± 0.7	2.2 ± 0.4

* The logarithmic abundances are on the standard scale with log ϵ(H) = 12.0. The estimated uncertainty is given in parentheses.

† $[X] \equiv \log \epsilon(X)_{\rm star} - \log \epsilon(X)_{\odot}$.

‡ O/C and C/N are ratios by number of atoms.

were considered, they would provide C and O abundances, respectively, with minimal interdependence through CO. The reason is obvious: The lines are formed in the deep hot layers where CO formation is unimportant. An analysis of CO lines (see Appendix) yields a unique locus in the log ϵ(O) versus log ϵ(C) plane. If log ϵ(O) > log ϵ(C), CO provides the C abundance with only a very weak dependence on the O abundance. In a carbon-rich star, log ϵ(C) > log ϵ(O); CO monitors the O abundance.

CNO abundances for the 11 program stars are summarized in Table 3. The C/N and O/C ratios are also given because they have a higher accuracy than the elemental abundances with respect to hydrogen. Also, the ratios are of comparable interpretative value.

An error analysis was performed in the manner outlined by Greene (1969); e.g., the internal error in the C abundance for a given line is

$$[\Delta \log \epsilon(\mathrm{C})]^2 = \left[\frac{\partial \log \epsilon(\mathrm{C})}{\partial T_{\rm eff}} \Delta T_{\rm eff}\right]^2 + \left[\frac{\partial \log \epsilon(\mathrm{C})}{\partial \log g} \Delta \log g\right]^2 + \left[\frac{\partial \log \epsilon(\mathrm{C})}{\partial \log \epsilon(Z)} \Delta \log \epsilon(Z)\right]^2 + [\Delta \log W_\lambda]^2 ,$$

where $\epsilon(Z)$ is the total metal abundance and the errors $\Delta T_{\rm eff}$, $\Delta \log g$, and $\Delta \log \epsilon(Z)$ are taken as independent. The Gustafsson *et al.* (1975) model atmosphere grid was used to calculate the derivatives.

Examination of Figure 1 shows that CH is potentially the most accurate source of the C abundance. The C_2 Swan lines are only slightly inferior. The [C I] 8727 Å line is considerably less accurate. The calculations show that the ratios O/C and C/N can be obtained with a superior accuracy; the C_2, [O I] and the C_2, CN combinations give the more accurate ratios. The $T_{\rm eff}$ and log g errors were estimated from the scatter in the values appearing in the literature. When the formal errors were smaller than 100 K in $T_{\rm eff}$ or 0.25 in log g, these minimum values were adopted. An error $\Delta \log \epsilon(Z) = 0.3$ was assumed. The observations are of high quality, and $\Delta \log W_\lambda = 0.05$ is taken.

Examination of the assumption of LTE has been confined to a search for non-LTE effects in the abundance analysis; the use of LTE in model atmosphere construction has not been considered. The statistical equilibrium of the negative hydrogen ion, which dominates the continuum opacity, was investigated by Lambert and Pagel (1968) who showed that LTE is valid. The forbidden atomic and molecular lines are at opposite poles of the non-LTE question. Since the collision rates for transitions within the ground configurations of C I and O I far exceed the radiative rates for these forbidden transitions, LTE is an excellent assumption; Pagel (1971) gives order-of-magnitude estimates for the rates. For the electronic transitions of CH, C_2, and CN, the line formation mechanism is scattering so that non-LTE effects are anticipated.

Line formation is described by the pair of quantities: the number density of molecules in the lower level of the transition, n_l, and the line source function, S_{lu}, where u denotes the upper level. For the CN and CH transitions, the lower level is the ground electronic state. For the C_2 Swan system, the lower electronic state is 600 cm^{-1}, or 0.07 eV above the ground state. These two low electronic states are connected by a very weak intercombination transition. This and the fact that the mean electron energy ($kT \approx 0.33$ eV for $T \approx 4000$ K) exceeds the excitation energy should guarantee that the relative populations are in their LTE ratio. Within ground or quasi-ground states, collisions—even at the densities found in these giants—provide an LTE population distribution within the rovibronic structure (Thompson 1973; Hinkle and Lambert 1975); i.e., n_l is given by the Boltzmann equation for the local kinetic temperature. The non-LTE effects appear in S_{lu}. Excitation in and out of the excited electronic states would appear to be controlled by photon processes (Hinkle and Lambert 1975); i.e., the lines are formed by scattering. In the limit that the molecular lines are weak, S_{lu} reduces to a weighted average of the mean continuum intensity [$J(\lambda)$] where the weights, which take account of the several vibrational bands that can connect a vibrational level in the upper state to the ground state, are calculable from the Franck-Condon factors and the populations of the vibrational levels in the ground state. For the present, the calculations were done for a mean wavelength (λ_{av}); the error inherent in this approximation is very small. The non–LTE line analyses with the line source function $S_{lu} = J(\lambda_{av})$ were carried out for models representative of the temperature extremes of the sample. Corrections to the LTE analysis were found not to exceed 5%.

In the Sun, Hinkle and Lambert (1975) argued that the molecular lines are also formed by scattering. This conclusion may be subject to revision when accurate collision cross sections can replace the very approximate value used in the 1975 analysis. However, scattering is appropriate for the giants because unrealistically large upward revisions to the cross sections would be required to make the collision rates competitive with the radiative rates in such low-density atmospheres. Support for scattering as appropriate to the Sun comes from the center-limb variation of CH (Eugène-Praderie and Pecker 1960), C_2, and MgH lines (Hinkle and Lambert 1975). Again, the calculated corrections for weak lines are small: a 10% reduction in equivalent width for C_2 and MgH, and a slight increase for the CN red system lines relative to the LTE calculation.

These calculations for the Sun and giants indicate that the adoption of LTE should lead to abundance errors of less than 0.05 dex. In the giants, the non–LTE effects are small because the C-containing molecules are restricted to the deeper layers by the effectively complete association of C into CO in the upper layers. The deeper layers are characterized by a near isotropic continuum radiation field in which the mean intensity approaches the local Planck function. This condition is not realized by the strongest lines, such as the band heads of the CN violet system; Mount, Ayres, and Linsky (1975) found in an analysis of the 3883 Å band head that the error due to the LTE assumption was ~0.2 dex. A particularly interesting problem is posed by the weak CH (0, 1) and (1, 2) lines, for which the upper level is excited primarily through the strong lines of the (0, 0) and (1, 1) bands. The weak-line limit is obviously inapplicable. An exact numerical treatment of this case would be of interest. Finally, the vibration-rotation lines of CO and other molecules are formed in LTE (Thompson 1973; Hinkle and Lambert 1975). The CO lines in Arcturus (see Appendix) confirm the abundances provided by the CH and C_2 lines. This constitutes some observational evidence that the non–LTE effects for CH and C_2 are very small.

e) *Comparison with Published Abundances*

In a comparison of published CNO abundances with the new results, a myriad of points require examination as potential sources of an abundance discrepancy. No useful purpose could be served by presenting a detailed accounting for differences between the new and all previously published abundances. Since the new results are the first to show marked changes in the CNO abundances of giants relative to their solar values, some comparison seems desirable.

Although a hint of the N abundance increase in giants was provided by Greene (1969), van Paradijs and Kester (1976, hereafter VPK) summarize the available literature and the results of their own abundance analysis with the statement that the relative CNO abundances are solar. We have attempted to analyze their procedures.

Two differences with respect to present procedures are obvious immediately. VPK based their calculations on the Harvard-Smithsonian Reference Atmosphere for the Sun (Gingerich *et al.* 1971) and on the Carbon-Gingerich (1969) model giant atmospheres. Our use of the Holweger-Müller (1974) solar atmosphere and of the atomic and molecular line-blanketed Gustafsson *et al.* (1975) models represent justifiable improvements. Upon substitution of our choice of models for the VPK choice, our line list yields corrections to their abundances of [C] ≈ -0.08, [N] ≈ -0.14, and [O] ≈ 0.02 for the typical giant.

Two stars, ϵ Cyg and α Boo, are common to the two analyses. After adjustments are made for the use of different model atmospheres and different equivalent widths, the corrected VPK oxygen abundances for these two stars are within 0.06 dex of the present results. Since VPK round off their abundances to ± 0.05 dex, the agreement for oxygen is satisfactory.

After a similar adjustment, the VPK product of the C and N abundances, which is provided by the CH and CN red system lines, shows good agreement with the present results. A major conflict occurs in the distribution of the abundance product between C and N: The C abundance given by VPK is higher and the N abundance correspondingly lower than present results. VPK use the CH, $\Delta v = 0$, sequence near 4200 Å to derive the C abundance.

VPK published the CH line list. Many lines are severely saturated and ill suited to an abundance analysis. When their weak to medium-strong lines ($W_\lambda < 70$ mÅ) are run through the present analysis procedures, the C abundance for ϵ Cyg is within 0.05 dex of the C abundance obtained from the C_2 lines and the CH (0, 1) line. Similar calculations for CH lines in α Boo are summarized in the Appendix.

If the entire CH line list is analyzed, the C abundance shows a dependence on equivalent width. This suggests that the microturbulent velocity should be increased. Alternative possibilities include errors in the equivalent width and the possible intervention of non–LTE effects. The C abundance of VPK appears to be based on the entire line list including the strong lines. In their analysis, observed equivalent widths are desaturated and are used to provide by extrapolation the desaturated equivalent widths of a fictitious line of zero rotational energy. If their extrapolated equivalent widths for the fictitious lines are run through the present procedures, the C abundance for both ϵ Cyg and α Boo is $\sim$0.15 dex higher than the abundance provided by direct analysis of the weaker lines. This result suggests that their procedure of desaturation and extrapolation to zero rotational energy contains a systematic error through an apparently infelicitous use of the strong lines. A 0.15 dex error in the C abundance corresponds to a factor of 2 error in the resultant C/N ratio when CH and CN lines are combined.

An interesting sidelight on the use of the CH, $\Delta v = 0$, lines is shed by the CN violet ($\Delta v = -1$) lines also used by VPK. Both CN red and violet system lines were available for four stars. Nitrogen abundance differences were 0.25, −0.05, −0.30, and −0.15 dex. Although, as VPK note, the mean difference is only 0.06 ± 0.23 dex, the disturbing fact is that the errors are so large for two groups of lines from the same molecule—a comparison to which the larger sources of error cannot contribute. Perhaps the CN violet system lines are blended with unidentified metal lines. However, the N abundance differences are not obviously correlated with the metal abundance. A contributing factor to the errors is likely to be the uncertainties in measuring equivalent widths for the violet system lines in the crowded spectrum around 4200 Å. If this is correct, similar uncertainties should plague the CH, $\Delta v = 0$, sequence lines.

This discussion has attempted to reconcile the VPK claim for solar CNO relative abundances in giants with the new results which show a marked reduction in the C/N ratio in giants. Improved model atmospheres for the Sun and the giants together with an apparent error in their CH analysis explain the apparent discrepancy.

III. INTERNAL MIXING AND CNO ABUNDANCES

The CNO abundances were obtained in order to test further the proposal originated by Iben (1964) that a convective envelope mixes material to the surface of a giant from a zone which underwent partial processing through the CNO cycle during the main-sequence lifetime. Observations of the $^{12}C/^{13}C$ ratio suggest that the mixing is in accord with theoretical expectations, at least for giants that are on the first ascent of the giant branch (Dearborn, Lambert, and Tomkin 1975). The important new result presented in this paper is that agreement between theory and observations extends to the CNO abundances.

After the convective envelope has changed the atmospheric composition, the $^{12}C/^{13}C$ ratio is reduced to $\sim$20–30 for initial values in the range 40–90. With an allowance for observational error, stars with $^{12}C/^{13}C \gtrsim 20$ are deemed consistent with the theory; the following discussion refers to these stars. Stars with $^{12}C/^{13}C \ll 20$ are currently unexplained; α Boo and ϵ Cyg are the only stars in the present sample to fall in this category. The new CNO abundances provide constraints on proposals to explain ^{13}C enhancements above the levels achieved by the convective envelope. Additional stars with low $^{12}C/^{13}C$ ratios are now being observed in order to define the general characteristics of this interesting group.

The observed CNO abundances for the current sample (α Boo and ϵ Cyg are excluded from this discussion) show the following abundance characteristics:

i) The oxygen abundances are solar to within the observational uncertainty; [O] ranges from −0.18 to +0.04.

ii) The carbon abundances are decreased and the nitrogen abundances increased relative to the solar values; the mean values are [C] = −0.23 and [N] = +0.35 dex, respectively. A more significant quantity is the C/N (and C/O, N/O) ratio, for which the accuracy is higher; the unweighted mean is C/N = 1.2 as compared with 4.8 for the Sun. The difference in these ratios is considerably in excess of the observational errors.

Inspection of the theoretical calculations for giants reveals that the CNO abundance differences are consistent with the predictions. A comparison (Fig. 4) is presented in a plot of the C/N ratio versus the $^{12}C/^{13}C$ ratios. Theoretical results for a 2 $M_\odot$ star (Dearborn, Eggleton, and Schramm 1976; Dearborn 1976) are plotted, with straight lines connecting the initial and final (after mixing) abundances; the trajectories that are traced out as mixing develops are unavailable at the present time. The predictions are not very sensitive to the adopted stellar mass. Results for two initial C/N ratios are shown: C/N = 5.5 and C/N = 3.0. These predictions for final abundances quite clearly converge on the observed points; the CNO and $^{12}C/^{13}C$ observations confirm the existence and extent of the convective mixing phase.

In a discussion of the relation between observed CNO abundances in giants and proposals for effecting changes in the atmospheric composition, one point should not be overlooked: The mixing of nuclear-processed material into the atmosphere cannot erase completely the initial set of CNO abundances. However, the current lack of information on these initial abundances appears not to be a constraint on the quantitative interpretation of the results.

The arrows in Figure 4, which represent the evolutionary calculations, converge on the location $^{12}C/^{14}N \approx 1$ and $^{12}C/^{13}C \approx 20$ for a range in the initial abundances approximating the canonical uncertainty of 0.3 dex. With the exception of several classes of peculiar stars and metal-deficient stars, the meager information available in the literature is consistent with a small spread in initial abundances. Weak internal evidence adds further support. Since CNO-cycle processing converts C into N with unit efficiency, the initial and final total abundances (C + N) are equal; little or no O is processed at the relevant temperatures. Then, the observed C + N abundance for the giants is a measure of the initial total. This total shows a small spread and a hint of a correlation with the metal (Fe) abundance. However, the correlation depends greatly on the two SMR stars μ Leo and ν^2 CMa, for which peculiar initial CNO abundances

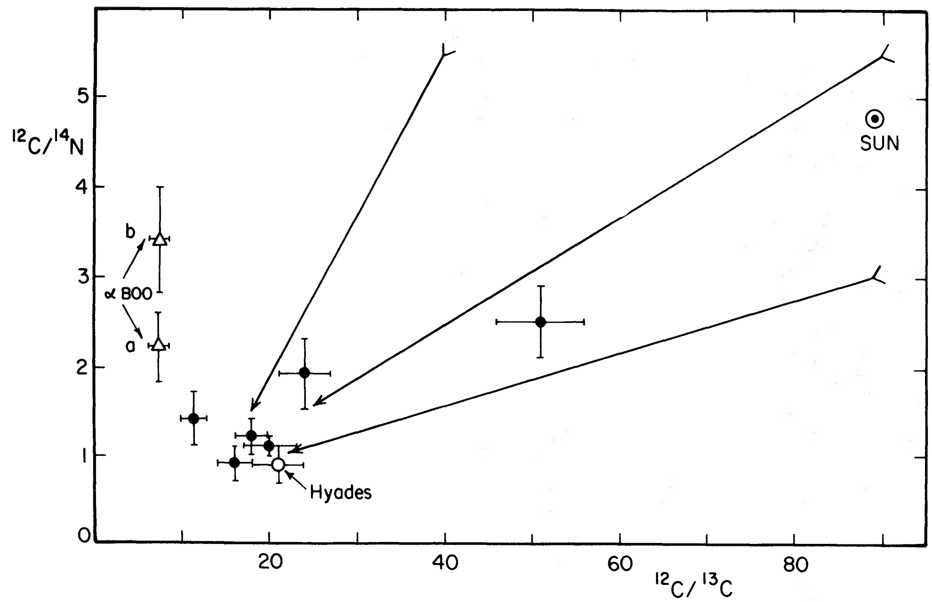

FIG. 4.—Observed and predicted $^{12}C/^{14}N$ and $^{12}C/^{13}C$ abundance ratios. A mean value (*open circle*) is shown for the four Hyades giants. The two solutions for α Boo are for (*a*) T_{eff} = 4250 K and log g = 1.7, and (*b*) T_{eff} = 4250 K and log g = 0.9. Arrows connect assumed initial abundances (at the base) to the final abundances for the convectively mixed red giant (at the tip).

are likely. Of course, differences in the initial C/N ratios are not uncovered by inspection of the C + N sum.

The Hyades cluster stars are recognized to be metal-rich relative to the Sun. An attempt is being made to obtain accurate CNO abundances for main-sequence cluster members; a rough comparison of equivalent widths shows CNO overabundances, but the ratios appear approximately solar. Sparse CNO abundance data for SMR stars (Spinrad and Luebke 1970; Greenstein and Oinas 1968) suggest nonsolar abundances; quantitative information is needed. The μ Leo results and the convective envelope hypothesis suggest an initial C enhancement with respect to the Sun. The SMR subgiant ν^2 CMa occupies an interesting location in Figure 4. Since the high $^{12}C/^{13}C$ ratio is incompatible with a convectively mixed star, ν^2 CMa is either unmixed or in the mixing phase.

The category of peculiar main-sequence stars includes the N-rich OB stars (Walborn 1970) and the peculiar (Ap) and metallic-line (Am) stars. The OB stars evolve into supergiants outside the luminosity range of the current sample. If the diffusion hypothesis (Michaud 1970; Watson 1970, 1971; Smith 1971) is valid, the abundance anomalies in the Ap and Am stars are erased in the giant phase.

In summary, this CNO abundance analysis, together with the earlier determinations of the $^{12}C/^{13}C$ ratio, shows that there is a group of G and K giants which have undergone the convective mixing predicted by the stellar evolution calculations. Arguments have been presented which show that this new result is not subject to serious reservations arising from the lack of a precise knowledge of the initial CNO abundances of these giants.

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APPENDIX

THE [C I] LINE AT 8727 Å

Initial hopes that the electric quadrupole C I transition at 8727 Å would parallel the [O I] lines in contributing to the CNO abundance analysis have evaporated. This Appendix outlines the problem.

Identification (Lambert and Swings 1967) of the [C I] line in the solar spectrum provided a useful addition to the C abundance indicators. Lambert (1977) groups this line with the C_2 Swan and CH *A–X* systems as a source of an accurate C abundance. With the *f*-value log $gf = -8.21$ based upon the average of two theoretical calculations (Nussbaumer 1971; Nicolaides and Sinanoğlu 1973), the 8727 Å line provides a solar C abundance that is consistent with that derived from the C_2 and CH lines (and other lower-quality indicators).

In the G- and K-giant program stars, the CO molecule interferes and the [C I] line shows considerable sensitivity to both $T_{\rm eff}$ and log g (see Fig. 1). Other lines show weaker dependences on those quantities; hence the [C I] line was not considered the primary source of the C abundance. Observations were obtained for several program stars with the Reticon detector. Since the line is predicted to be weak, the influence of possible blending lines is potentially serious. An Fe I line from the intercombination multiplet RMT 999 was noted by Lambert and Swings; the $\lambda = 8727.133$ Å calculated from the Fe I energy levels (Reader and Sugar 1975) places the line on top of the [C I] line at $\lambda = 8727.126$ Å (Moore 1970). In the Sun, this blend is insignificant. Fe I lines are stronger in the G and K giants, and assessment of the Fe I contribution to the [C I] line was attempted.

The absolute f-value of the Fe I line has not been measured. Kurucz and Peytremann (1975) report log $gf = -4.05$ from a semiempirical calculation. In order to minimize the dependence on the absolute scale of the Fe I f-values and the model atmospheres, other members of the multiplet at 8466.5 and 8482.0 Å were observed in a sample of the program stars. Their relative intensities are consistent with the calculated gf-values. Scaled to the 8727 Å Fe I line, these lines predict a small contribution to the feature assigned to [C I]; e.g., β Gem shows an observed $W_\lambda = 9.9$ mÅ, and the Fe I predicted contribution is $W_\lambda = 2.0$ mÅ. However, the 8727 Å Fe I line is the weakest member of the intercombination multiplet; hence the possibility exists that its calculated gf-value is the most susceptible to the approximations in the intermediate coupling calculation. After examination of a photographic iron arc spectrum, Richter (1976) estimates that the gf-value is at least a factor of 8 smaller than that for the 8680 Å line, another member of the multiplet. When the calculated gf-values are used, this estimate translates to log $gf \leq -3.4$ for the 8727 Å line. To obtain a C abundance consistent with that provided by the C_2 and CH lines, the Fe I contribution must be approximately tripled; i.e., log $gf \approx -3.6$ is required, which is consistent with this upper limit. Clearly, an accurate measurement of the Fe I gf-value is of paramount importance to the use of the 8727 Å feature as a C abundance indicator.

Careful comparison of predicted and observed wavelengths for ^{12}CN and ^{13}CN lines shows that the ^{13}CN line 3–1 $Q_1(34)$ falls at 8727.34 Å, or 0.22 Å to the red of the [C I] line. This weak line is seen on Reticon spectra as a partially resolved line. On lower-resolution spectra, the ^{13}CN line would merge with the stronger [C I]–Fe I blend. An unclassified FeH band spans the 8727 Å region (Carroll, McCormack, and O'Connor 1976). The line list (Carroll 1974) shows two FeH lines within the [C I]–Fe I blend. However, other nearby FeH lines are either very weak or absent, so that $W_\lambda \lesssim 2$ mÅ is likely for the typical star.

The problem raised by the 8727 Å line is seen when the equivalent C abundance is compared with that obtained from the C_2 lines. Figure 5 shows the abundance loci for α Boo. Results for two surface gravities are given: log $g = 0.9$ (Mäckle *et al.* 1975) and log $g = 1.7$. The latter is the "traditional" value and is fairly typical of the values adopted for the other giants. In the log ϵ(O) versus log ϵ(C) plane, the [C I] locus is displaced from the C_2 Swan locus by $\sim$0.3 dex; the isotopic wavelength shift for the [C I] line is most probably negligible, and a small

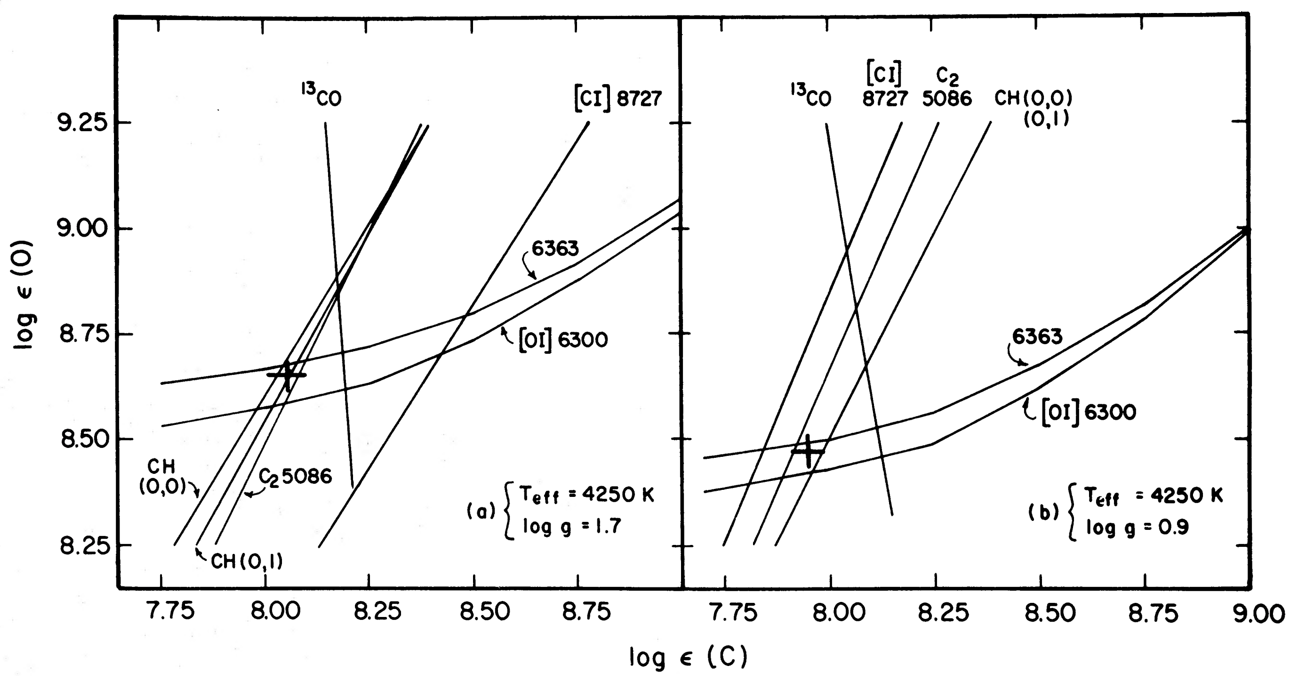

FIG. 5.—Loci of abundance solutions for C and O indicators in α Boo for two model atmospheres: (*a*) $T_{\rm eff} = 4250$ K and log $g = 1.7$, and (*b*) $T_{\rm eff} = 4250$ K and log $g = 0.9$.

correction was applied so that the [C I] and C_2 loci refer to ^{12}C alone. Similar displacements between the loci were found for all stars for which 8727 Å spectra were available. Observations of the CH A–X system (Lambert and Dearborn 1972 and unpublished observations) and of ^{13}CO vibration-rotation 2 μm lines (Hinkle, Lambert, and Snell 1976) were available for α Boo; the ^{12}C abundance was calculated from weak ^{13}CO lines and from the ratio $^{12}C/^{13}C = 7$. Both sets of lines are well calibrated by laboratory measurements, and the solar lines match the revised abundances (Lambert 1977). With the "traditional" surface gravity, the splendid agreement between the C_2, CH, and CO loci (see Fig. 5*a*) supports the conclusion that the [C I] line is blended; current uncertainty over the *f*-value does allow the blending Fe I line to account for the discrepancy. However, adoption of the low surface gravity introduces a new dimension to the problem (see Fig. 5*b*), as all four loci cross in the log ϵ(O) versus log ϵ(C) plane. In this interpretation, the [C I] line yields a consistent abundance. To achieve this consistency, a low surface gravity must be adopted for *all* the observed stars. The evidence for and against this radical hypothesis must be marshaled elsewhere. Insofar as the abundance question is concerned, the important observation is that the hypothesis of low surface gravity may resolve the [C I]–C_2 inconsistency, but it does not change the sense of the abundance changes relative to the solar values. Application of Occam's razor would probably rule in favor of the higher surface gravity and a blend (a stronger Fe I line, a FeH contribution, etc.) within the 8727 Å feature. An accurate measurement of the relative *f*-values for the Fe I line and other members of the multiplet should assist in a resolution of this problem.

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