THE ASTROPHYSICAL JOURNAL, 215:897–913, 1977 August 1

ACCRETION BY MAGNETIC NEUTRON STARS. I. MAGNETOSPHERIC STRUCTURE AND STABILITY

R. F. ELSNER
Department of Physics, University of Illinois at Urbana-Champaign; and
Center for Radiophysics and Space Research, Cornell University

AND

F. K. LAMB†
Department of Physics, University of Illinois at Urbana-Champaign
Received 1976 May 17; revised 1976 December 20

ABSTRACT

This paper is the first of a series in which we present the results of a study of accretion by a slowly rotating, magnetic neutron star when the accretion flow is approximately radial. In this study we examine in detail the physical processes that occur in the neighborhood of the magnetospheric boundary and the manner in which accreting plasma enters the magnetosphere. Here we investigate the conditions necessary for the formation of a magnetospheric cavity and for its stability. We find that although the boundary of the cavity is initially close to instability, magnetospheric models strongly suggest that it is stable until the plasma outside undergoes at least some cooling. Assuming that little plasma enters the magnetosphere when the boundary is stable, we show that there is, typically, time for the plasma outside to form a quasi-static atmosphere around the magnetosphere before instability sets in. We discuss the scale and structure of such static configurations, including the nature of the current layer at the magnetospheric boundary.

We also investigate the structure of the magnetosphere when there is a steady inflow of plasma across the boundary, and describe the possible flow patterns at the boundary and in the interior. Assuming that plasma enters the magnetosphere predominantly via Rayleigh-Taylor instability of the boundary, we discuss the conditions necessary for a steady accretion flow and note that beaming of X-rays from the stellar surface can strongly affect the plasma flow pattern near the boundary. We show that the Alfvén surface, within which the stellar field channels the flow, generally does not coincide with the magnetospheric boundary and could conceivably lie well within it.

A discussion of the applicability of the present study to observed compact X-ray sources shows that it may apply directly to the longer-period pulsating sources, such as A1118−61, 3U 0900−40, A0535+26, and 3U 1728−24, if the accretion flow is sufficiently radial, and to some X-ray burst sources, if these are slowly rotating neutron stars.

Subject headings: stars: accretion — stars: magnetic — stars: neutron — stars: rotation — X-rays: sources

I. INTRODUCTION

Accretion by magnetic neutron stars appears to be a promising model both for many binary X-ray sources (for recent reviews, see Blumenthal and Tucker 1974; Giacconi 1974; Lamb 1975*a*; Lightman, Shapiro, and Rees 1976) and for X-ray and soft γ-ray burst sources (Harwit and Salpeter 1973; Lamb, Lamb, and Pines 1973; Lamb *et al.* 1977). In a previous paper (Lamb, Pethick, and Pines 1973, hereafter LPP) a basic model of the flow of accreting plasma and the emission of X-rays was developed. Similar models have been proposed by Pringle and Rees (1972) and by Davidson and Ostriker (1973), among others.

* Research supported in part by the National Science Foundation through grants GP-41560 and PHY-75-08790 (Illinois); and AST 75-21153 (Cornell).

† Alfred P. Sloan Foundation Research Fellow.

This paper is the first of several in which we present the results of a detailed study of the magnetic neutron star model, in the particular case when the accretion flow is approximately radial and the star is slowly rotating. Although this case is an idealization, we believe that it provides important insights into the physics of accretion by magnetic neutron stars generally. Furthermore, if the slowly pulsing X-ray sources such as A1118−61 (Eyles *et al.* 1975; Ives, Sanford, and Bell-Burnell 1975), 3U 0900−40 (McClintock *et al.* 1976), and A0535+26 (Rosenberg *et al.* 1975) are rotating neutron stars, the results presented here may be directly applicable to them (see Elsner and Lamb 1976*a* and § IV below).

In our investigation we have focused on the physical processes that occur in the neighborhood of the magnetospheric boundary and, in particular, on the manner in which accreting plasma enters the magnetosphere.

These affect important properties of X-ray source models, including the conditions under which a steady accretion flow is possible; the location and dimensions of hot spots at the stellar surface; the character of the X-ray spectrum, which may depend on reprocessing within the magnetosphere as well as on the sizes of surface hot spots; the extent to which the X-rays are beamed; and the time scales of X-ray variability. We have considered particle entry through the polar cusps, magnetic flux reconnection, diffusion of plasma across the magnetopause, and Rayleigh-Taylor instability of the magnetospheric boundary. We conclude that under the conditions typical of the steady, bright binary X-ray sources, plasma enters the magnetosphere of an accreting neutron star via Rayleigh-Taylor instability of the magnetospheric boundary except in circumstances, which we believe to be unusual, when magnetic flux reconnection "opens" the magnetosphere. On the other hand, magnetic flux reconnection may be responsible for the steady background luminosity observed in some bursting X-ray sources (see Lewin 1977).

In the present paper we develop a basic theoretical framework within which to discuss the calculations of subsequent papers. In a second paper (Elsner and Lamb 1976*b*, hereafter Paper II) we discuss the hydromagnetic stability of the magnetospheric boundary in more detail, considering line-tying, finite Larmor radius, and viscosity effects, and describe qualitatively possible plasma flow patterns within the magnetosphere. In a third paper (Elsner and Lamb 1977*a*, hereafter Paper III) we present the results of detailed calculations of the rate at which plasma can enter the magnetosphere when the boundary is stable and use these results to estimate, within the context of the magnetic neutron star model of bursting X-ray sources (Lamb *et al.* 1977), the rate at which plasma "trickles" into the magnetosphere between bursts and the resulting "steady" X-ray luminosity to be expected from these sources. In a fourth paper (Elsner and Lamb 1977*b*, hereafter Paper IV) we present analytical and numerical solutions for the steady flow, heating, and cooling of plasma outside the magnetospheric boundary. Preliminary accounts of some of our results have been given previously in Lamb (1975*a*, *b*), and Elsner and Lamb (1975*a*, *b*, 1976*a*). More recently calculations similar to ours have been reported by Arons and Lea (1976).

The remainder of this paper is organized as follows. In § II we discuss the formation of a magnetosphere, the approach to static equilibrium of the plasma atmosphere that builds up outside the magnetospheric boundary prior to the onset of the Rayleigh-Taylor instability, and the hydromagnetic stability of such configurations. These are of interest both because of their possible relevance to X-ray burst sources and because they provide insight into the initial development of steady accretion flows. In § III we develop further the model of steady accretion flows outlined by LPP, in the particular case when the neutron star is slowly rotating and the accretion flow is approximately radial. We consider in § IV the applicability of the present study to observed X-ray sources and investigate in § V the time scales for heating and cooling of plasma at the magnetospheric boundary, and their implications for accretion flows. Section VI summarizes our results and describes briefly some possible effects that strong X-ray beaming and nonradial flow around the boundary may have on plasma entry into the magnetosphere via the Rayleigh-Taylor instability.

II. STATIC CONFIGURATIONS

In this section we consider the initial interaction of accreting plasma with the stellar magnetic field. As noted in the Introduction, our calculations show that in the absence of cooling the magnetospheric boundary is stable. Assuming that little plasma enters the magnetosphere when the boundary is stable, we show that the shock-heated plasma outside will form an atmosphere there, and that there is time for this configuration to reach static equilibrium. We then discuss the structure of the equilibrium magnetosphere and its shape for various confining pressures. Finally, we consider the hydromagnetic stability of the boundary.

In this discussion, we shall make the following basic assumptions concerning the accretion flow:

1) Accreting matter approaches the magnetosphere at a steady rate $\dot{M}$.

2) The accreting plasma is in spherically symmetric, radial free fall as it approaches the magnetospheric boundary. Thus the mass density ρ, inward radial velocity v_r, and dynamical time scale t_d near the boundary are given by

$$\rho = \rho_{\rm ff} \equiv \dot{M}/v_{\rm ff} 4\pi r^2 = 4.9 \times 10^{-10} r_8^{-3/2} \dot{M}_{17} (M/M_\odot)^{-1/2}\ {\rm g\ cm^{-3}}, \quad (1)$$

$$v_r = v_{\rm ff} \equiv (2GM/r)^{1/2} = 1.6 \times 10^9 r_8^{-1/2} (M/M_\odot)^{1/2}\ {\rm cm\ s^{-1}}, \quad (2)$$

$$t_d = t_{\rm ff} \equiv \tfrac{2}{3} r/v_{\rm ff} = 4.1 \times 10^{-2} r_8^{3/2} (M/M_\odot)^{-1/2}\ {\rm s}, \quad (3)$$

where r is the distance from the neutron star, r_8 is r measured in units of 10^8 cm, and $\dot{M}_{17}$ is the mass accretion rate in units of 10^{17} g s^{-1}.

3) Free fall is halted outside the magnetosphere by a strong, collisionless shock wave, in which the directed kinetic energy of the ions is converted into random thermal motion. The ion temperature T_i' behind the shock wave is then of the order of the free-fall temperature,

$$T_{\rm ff}(r) = GMm_i/k_B r = 1.6 \times 10^{10} A\, r_8^{-1} (M/M_\odot)\ {\rm K}, \quad (4)$$

at the radius r_s of the shock. Here A is the ion mass number. (At present there is some uncertainty about the formation and structure of collisionless shock waves in situations of this kind; see Tidman and Krall 1971; LPP; Shapiro and Salpeter 1975; Arons and Lea 1976. We note, however, that the qualitative features of the equilibrium magnetospheric structure and stability do not depend on the formation of a collisionless shock wave, but only on the heating of the plasma outside the magnetosphere to a temperature $\sim T_{\rm ff}$.)

4) The star rotates sufficiently slowly that the effects of its rotation on the plasma flow can, to a first approximation, be neglected.

a) Formation of a Magnetosphere

Initially, the radial flow of accreting plasma toward a slowly rotating magnetic star will tend to sweep the stellar field inward before it, since the plasma is highly conducting. However, as long as the pressure of the stellar magnetic field, $B^2/8\pi$, increases with decreasing radius more rapidly than that of the accreting plasma, it will eventually halt the inflow, creating a magnetosphere from which the plasma is at first excluded. For example, if the stellar field is dipolar so that $B^2 \sim \mu^2/r^6$, in terms of the stellar dipole moment μ, and if the pressure P of the accreting plasma varies as r^{-n}, then a magnetosphere will form for $n < 6$.

Regardless of the complexity of the magnetic field near the star's surface, at large distances the stellar field B will be dominated by its dipole component, and will vary like r^{-3}. The shape and size of the magnetospheric cavity will therefore be largely determined by the dipole component as long as the stellar magnetic field is confined to a region of dimension much larger than R. On the other hand, higher multipole moments of the undistorted stellar field must be taken into account if the stellar field is confined to a region of dimension comparable to R. Furthermore, confinement of the magnetic field to such a small region will, if the star is highly conducting, induce currents within the star which will cause the distorted stellar field to manifest magnetic multipoles, even if the undistorted field is purely dipolar. (We include in the "stellar" field that field generated by currents within the star, whether intrinsic or induced.)

Let us assume for the moment that there is no flow of plasma into the magnetosphere. There will then be no emission of radiation from the stellar surface due to accretion and the star will certainly not be a bright X-ray source. The shock-heated plasma will accumulate outside the magnetosphere, forming an increasingly extended atmosphere, and the magnetosphere will assume its equilibrium shape in a time $\sim t_d$. Since the plasma is unexposed to X-rays prior to the onset of instability at the magnetospheric boundary, it cools in a time $t_{\rm cool}$ which is typically much longer than t_d (see § V), and there is therefore a considerable time interval during which the structure of the magnetosphere is determined by a static balance between the plasma and magnetic pressure outside and the pressure of the stellar magnetic field inside.

b) The Scale and Structure of Static Magnetospheres

Consider further the case in which there is no flow of plasma across the magnetospheric boundary and the boundary is in static equilibrium. There are then two fundamental lengths associated with the magnetosphere: (1) the overall scale, $r_m{}^0$, of the magnetosphere, and (2) the thickness, δ_m, of the boundary layer which separates the interior of the magnetosphere from the plasma and magnetic fields outside. If $\delta_m \ll r_m{}^0$, then the gravitational force acting on the plasma in the boundary layer can be neglected, and static pressure balance and continuity of the normal component of the magnetic field imply

$$\left(\frac{B_t{}^2}{8\pi}\right)_{\rm in} + P_{\rm in} = \left(\frac{B_t{}^2}{8\pi}\right)_{\rm out} + P_{\rm out}\,, \qquad (5)$$

across the magnetospheric boundary, S_m. Here $P_{\rm in}$ and $P_{\rm out}$ are the (isotropic) plasma pressures inside and outside the boundary, $(B_t)_{\rm in}$ and $(B_t)_{\rm out}$ are the tangential components of the magnetic field inside and outside, and θ and ϕ are polar angles.

In the absence of plasma flow into the magnetosphere we expect $P_{\rm in}$ to be small compared to $B_{\rm in}{}^2/8\pi$, and $\boldsymbol{B}_{\rm in}$ and $\boldsymbol{B}_{\rm out}$ to be tangential to the boundary. Then the boundary $r_m(\theta, \phi)$ of the magnetosphere is the surface of pressure balance between the pressure of the plasma and embedded magnetic field outside and the pressure of the stellar magnetic field inside; moreover, in this case the nature of the interface can be characterized by the parameter $\beta = 8\pi P_{\rm out}/B_{\rm in}$.

This pressure balance relation can be used to estimate the scale size, $r_m{}^0$, of the magnetosphere. For example, if accretion at the rate $\dot{M}$ created a static, adiabatic atmosphere in which $\boldsymbol{B} = 0$, one would have $P_{\rm out}(r) \sim \rho_{\rm ff}(r)v_{\rm ff}{}^2(r) \propto r^{-5/2}$. Adopting $B_{\rm in} = \mu r^{-3}$ and $P_{\rm out} = \rho_{\rm ff}v_{\rm ff}{}^2$, one finds the characteristic radius

$$r_m{}^0 = r_m{}^{(a)} \equiv 2.7 \times 10^8\, \dot{M}_{17}{}^{-2/7}\mu_{30}{}^{4/7}(M/M_\odot)^{-1/7}\ {\rm cm}\,, \qquad (6)$$

where μ_{30} is the stellar dipole moment in units of 10^{30} gauss cm^3. As a second example, were plasma to cool and accumulate at the magnetospheric boundary, the pressure $P_{\rm out}$ required to support it would become larger than $\rho_{\rm ff}v_{\rm ff}{}^2$ after a time of the order of the cooling time at the boundary. In the limit $P_{\rm out} \gg \rho_{\rm ff}v_{\rm ff}{}^2$ one would have, for a thin shell of plasma of mass M_s, a pressure $P_{\rm out}(r) \simeq GMM_s/4\pi r^4 \propto r^{-4}$ (cf. McCray and Lamb 1976) and a characteristic radius

$$r_m{}^0 = r_m{}^{(s)} \equiv 1.9 \times 10^8 (M_s/10^{17}\ {\rm g})^{-1/2}\mu_{30}(M/M_\odot)^{-1/2}\ {\rm cm}. \qquad (7)$$

However, in the absence of support other than magnetic pressure, the boundary is likely to become Rayleigh-Taylor unstable before such a large quantity of plasma can accumulate, as discussed in § II*d* below.

The thickness of the boundary layer at r_m depends on the physical conditions in the plasma at the boundary. Characteristic values of the most important plasma length scales at the boundary are listed in Table 1. For ion temperatures $T_i \sim 10^9$–10^{10} K, electron temperatures $T_e \sim 10^8$–10^{10} K, and number densities $n \sim 10^{15}$ cm^{-3}, which are typical of conditions near the boundary (see § V), the ion and electron mean free paths are $\sim 10^7$–10^9 cm and 10^5–10^9 cm,

TABLE 1

CHARACTERISTIC PLASMA LENGTH SCALES AT THE MAGNETOSPHERIC BOUNDARY

Parameter	Symbol	Characteristic Value (cm)*
Mean free paths:		
ion-ion	λ_i	$1.8 \times 10^9 (T_i/10^{10}\ \mathrm{K})^2 (Z^4 n_i \ln \Lambda/10^{16}\ \mathrm{cm}^{-3})^{-1}$
electron-electron	λ_e	$1.8 \times 10^5 (T_e/10^8\ \mathrm{K})^2 (n_e \ln \Lambda/10^{16}\ \mathrm{cm}^{-3})^{-1}$
Transverse screening lengths:		
ion	c/ω_{pi}	$0.72\ (n_i/10^{15}\ \mathrm{cm}^{-3})^{-1/2} A^{1/2} Z^{-1}$
electron	c/ω_{pe}	$1.7 \times 10^{-2} (n_e/10^{15}\ \mathrm{cm}^{-3})^{-1/2}$
Larmor radii:		
ion	a_i†	$0.13\ (T_i/10^{10}\ \mathrm{K})^{1/2} A^{1/2} Z^{-1} (B_{\mathrm{in}}/10^6\ \mathrm{G})^{-1}$
electron	a_e‡	$3.1 \times 10^{-4} (T_e/10^8\ \mathrm{K})^{1/2} (B_{\mathrm{in}}/10^6\ \mathrm{G})^{-1}$
Debye length	λ_D§	$2.2 \times 10^{-3}\ (T/10^8\ \mathrm{K})^{1/2}\ (n/10^{15}\ \mathrm{cm}^{-3})^{-1/2}$

* T_i and T_e are the ion and electron temperatures; n_i and n_e, their number densities; A and Z the ion mass and charge numbers; and B_{in} is the magnetic field strength at the magnetospheric boundary. Both ions and electrons are assumed nonrelativistic.

† $a_i \equiv (2k_B T_i/m_i)^{1/2}(m_i c/ZeB)$.

‡ $a_e \equiv (2k_B T_e/m_e)^{1/2}(m_e c/eB)$.

§ $\lambda_D \equiv (k_B T/4\pi n e^2)^{1/2}$.

respectively. Thus if $\delta_m \ll r_m \sim 10^8$ cm, the boundary layer will be collisionless. The other length scales are all much smaller than both the dimensions of the magnetospheric cavity and the collisional mean free paths.

Models of stable, collisionless boundary layers separating regions of different magnetic field strength (cf. Phelps 1969, 1973; Spalding 1971; Willis 1971; Hamasaki *et al.* 1974) show that $\delta_m \gtrsim c/\omega_{pe}$ if $\beta \sim 1$, where ω_{pe} is the electron plasma frequency. If the sheath is broadened by microinstabilities or the presence of trapped particles, as seems probable in the present context, one expects $\delta_m \sim a_i$, the ion Larmor radius. Since $a_i \sim 0.1$ cm (see Table 1), the collisionless approximation is self-consistent and $\delta_m \ll r_m^0$, as we assumed above. In this case the stellar magnetic field threads only the very small quantity of plasma that is in the boundary layer.

c) *Equilibrium Shapes*

The shape of the magnetospheric boundary in static equilibrium depends, in general, on the multipole structure of the star's magnetic field and the variation of confining plasma pressure with position around the boundary. When $r_m^0 \gg R$, so that the stellar field is dipolar ($B^2 \propto r^{-6}$), and the confining pressure is a function only of r, the boundary is convex toward the plasma, the magnetic field increases inward everywhere, and, if gravitational forces were absent, the boundary would be stable (cf. Rosenbluth and Longmire 1957). Since gravitational forces are indeed present, the magnetospheric boundary is driven Rayleigh-Taylor unstable as soon as the weight of the plasma on the boundary becomes sufficiently large, as we show in § II*d*. Furthermore, the boundary forms cusps above the magnetic poles, as pointed out in the context of accreting neutron stars by LPP. (An argument which shows that cusps must form above the poles of a dipole field in the case of static confinement was given in an early paper on the geomagnetosphere by Midgley and Davis 1962; see also Grad and Hu 1966; Spreiter and Summers 1967; Willis 1971 and references therein.)

If in addition the confining pressure has a power-law dependence on radius, $P \propto r^{-n}$, and P_{in} can be neglected, the equilibrium shape of the magnetosphere in three dimensions depends only on the quantity $\nu = 6 - n$. Ram pressure confinement, for example, corresponds to $\nu = 3.5$, whereas confinement by the weight of a thin shell of cold plasma of fixed mass corresponds to $\nu = 2$. The variation of the boundary shape with ν is illustrated by the exact two-dimensional solutions for confining pressure laws corresponding to $\nu = 4$ and $\nu = 2$, which are shown in Figure 1. Shapes similar to these would be obtained by slicing through the corresponding three-dimensional magnetospheres in a plane containing the dipole axis. The solutions shown have been obtained analytically by conformal mapping (the solution for $\nu = 4$ is due to Cole and Huth 1959; the general mapping for arbitrary ν and the solution for $\nu = 2$ are given in the Appendix). Although the shape of the corresponding three-dimensional magnetosphere will actually be slightly different, we expect the character of the variation with ν to be the same. In either two dimensions ($D = 2$) or three dimensions ($D = 3$) the magnitude of B just inside the magnetospheric boundary is given by

$$B_{\mathrm{in}} = \mu_s(\theta)/[r_m(\theta)]^D , \tag{8}$$

where

$$\mu_s(\theta) \equiv \mu[r_m(\theta)/r_m^0]^{\nu/2} . \tag{9}$$

In equation (9), μ is the dipole moment of the undistorted field, r_m^0 is the reference radius defined in § II*b* and the Appendix, and ν is related to D by $\nu = 2D - n$.

Arons and Lea (1976), for the case $\nu = 3.5$, and Midgley and Davis (1962), for the case $\nu = 6$, calculate, using numerical methods, the shape of the magnetospheric boundary in three dimensions (see also Michel 1977). The radius r_e of the magnetospheric boundary at the magnetic equator and the radius r_p of the boundary at the polar cusp, scaled in terms of the appropriate value of r_m^0, are given in Table 2 for these

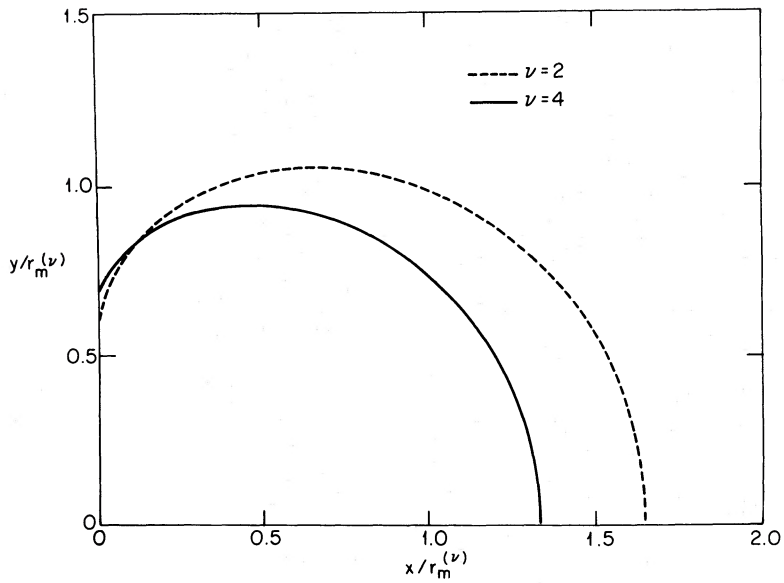

FIG. 1.—The shape of the magnetospheric boundary for the two-dimensional model magnetospheres constructed in the Appendix. The models assume a power law confining pressure, $P \propto r^{-n}$, and are labeled by the index $\nu = 4 - n$. The full shape of the boundary may be obtained from the curve shown in the figure by reflection through both axes.

three-dimensional model magnetospheres as well as for the two-dimensional model magnetospheres discussed above. Note that the sizes and shapes of the two- and three-dimensional magnetospheres corresponding to the same value of ν need not be the same. Thus comparisons between two- and three-dimensional models must be approached cautiously. Nevertheless, the results given in Table 2 can be summarized by the statement that the steeper the increase of plasma pressure with decreasing radius (i.e., the smaller ν is), the farther the magnetic equator lies from the stellar surface and the more oblate is the overall shape of the cavity.

d) *Stability of the Magnetospheric Boundary*

The magnetohydrodynamic stability of these equilibrium magnetospheres can be determined using the energy principle of Bernstein *et al.* (1958). According to this analysis the magnetospheric boundary is stable to infinitesimal perturbations if

$$\delta W_s = -\tfrac{1}{2}\int_{S_m} dS(\hat{n}_0\cdot\boldsymbol{\xi})^2\hat{n}_0\cdot\nabla(P_0 - B_0^2/8\pi) > 0 \quad (10)$$

for all perturbations $\boldsymbol{\xi}$. Here P_0 is the equilibrium plasma pressure outside the bounding surface S_m, B_0 is the magnitude of the undisturbed vacuum magnetic field inside, and $\hat{n}_0$ is a unit vector normal to the interface which points into the vacuum region. Condition (10) neglects line-tying, viscous, and finite Larmor radius effects, and assumes that convection is absent in the plasma. As discussed by Bernstein *et al.* (1958) (see also Schmidt 1966), one can always find a $\boldsymbol{\xi}$ for which $\delta W_s < 0$ if the inequality

$$\hat{n}_0\cdot\nabla P_0 - \hat{n}_0\cdot\nabla(B_0^2/8\pi) > 0 \quad (11)$$

is satisfied; this is therefore the condition for the onset of instability. A more detailed analysis, to be presented in Paper II, shows that this condition is not significantly altered when line-tying, viscous, and finite Larmor radius effects are taken into account.

Using the equilibrium conditions $\nabla P_0 = \rho_0 \boldsymbol{g}$ and $(\nabla \times \boldsymbol{B}_0) \times \boldsymbol{B}_0 = 0$, where ρ_0 is the unperturbed mass density and $\boldsymbol{g} = -(GM/r^2)\hat{r}$ is the gravitational acceleration, and the relation $\hat{n}\cdot(\boldsymbol{B}_0\cdot\nabla)\boldsymbol{B}_0 = \kappa B_0^2$, where κ is the curvature of the magnetic field lines at

TABLE 2

CHARACTERISTIC RADII OF MODEL MAGNETOSPHERES

Model	Dimension	r_m^0*	ν	r_e/r_m^0†	r_p/r_m^0†
Arons and Lea 1976	3	$r_m^{(a)}$	3.5	1.78	0.91
Midgley and Davis 1962	3	$r_m^{(6)}$	6	1.41	0.90
Cole and Huth 1959	2	$r_m^{(4)}$	4	1.33	0.67
This work	2	$r_m^{(2)}$	2	1.65	0.65

* The scale factor $r_m^{(a)}$ is defined in equation (6) while the scale factors $r_m^{(2)}$ and $r_m^{(4)}$ are defined in the Appendix; the scale factor $r_m^{(6)}$ is given by $r_m^{(6)} = [\mu^2/8\pi P_0]^{1/6}$, where P_0 is the (constant) pressure on the magnetospheric boundary.

† As noted in the text, comparisons of the numerical values of r_e/r_m^0 and r_p/r_m^0 for two- and three-dimensional magnetospheres should be approached cautiously.

the boundary, condition (11) may be conveniently rewritten

$$g_{\rm eff} = \cos \chi GM/r_m^2 - \kappa B_{\rm in}^2/4\pi\rho_{\rm out} > 0 \,. \quad (12)$$

Here χ is the angle between the radius vector and the *outward* normal to the boundary, and we have neglected the force of radiation pressure. The first term in this expression represents the destabilizing effect of the weight of the plasma on the boundary, which tends to drive it Rayleigh-Taylor unstable, while the second term represents the stabilizing effect of the curvature of the field lines at the boundary, which are convex toward the plasma. If the boundary is in static equilibrium, as we have assumed, one has $B_{\rm in}^2/8\pi = [n_i k_B(T_i + ZT_e)]_{\rm out}$, where n_i is the ion number density and T_i and T_e are the ion and electron temperatures. Since κ is necessarily of order r_m^{-1} for $r_m \gg R$, the stabilizing and destabilizing terms in expression (12) are comparable. If one uses the equation for static pressure balance at the boundary, condition (12) for the onset of the Rayleigh-Taylor instability can be rewritten

$$(T_i + ZT_e)_{\rm out} < T_{\rm crit}(\theta) = \alpha(\theta) T_{\rm ff}[r_m(\theta)] \,, \quad (13)$$

where

$$\alpha(\theta) = \cos \chi / 2\kappa r_m \,. \quad (14)$$

Prior to the onset of instability one typically has $T_i = T_e$ in the plasma at S_m (see § V), and hence $(T_i + ZT_e)_{\rm out} = (1 + Z)T_{\rm out}$.

Figure 2 displays the function $\alpha(\theta)$ for the four model magnetospheres discussed earlier in this section. From this figure we conclude that these models have the following important properties in common:

1) To the extent that $T_{\rm out} \propto T_{\rm ff}(r_m)$, the magnetic equator is the least stable point on the boundary.

2) The polar cusps ($\theta = 0°$ and 180°) are absolutely stable to infinitesimal perturbations.[1] Nevertheless, the boundary arbitrarily close to the cusp axes may be driven unstable if the temperature there becomes low enough or, equivalently, if the density there builds up sufficiently.

3) The density of the plasma is sufficient to drive the magnetic equator unstable only if $(1 + Z)T_{\rm out}$ there is at a temperature below $T_{\rm crit}(\pi/2)$ which is typically $\lesssim 0.4\, T_{\rm ff}[r_m(\pi/2)]$. Since, for example, formation of a static atmosphere leads to an initial temperature at the boundary given by

$$(1 + Z)T_{\rm out} \approx T_0 \equiv \frac{\gamma - 1}{\gamma} T_{\rm ff}(r_m) = 0.4\, T_{\rm ff}(r_m) \,, \quad (15)$$

where in the last expression on the right we have assumed $\gamma = 5/3$, and since we expect real magnetospheres to be characterized by an effective $\nu \leq 3.5$, at least some cooling is required before the weight of the plasma is sufficient to drive the boundary unstable. If accreting plasma is available and cooling is sufficiently rapid (see § V), the instability may continue.

4) Since the function $\alpha(\theta)$ is relatively flat except near the cusps, instability may set in over a large fraction of the magnetospheric boundary if $(1 + Z)T_{\rm out} < T_{\rm crit}$ occurs first at the magnetic equator.

The onset of instability for $(1 + Z)T_{\rm out} < T_{\rm crit}$ also bears on the question of whether buildup of static plasma pressure at the boundary can crush the magnetosphere to the surface of the star. If $r_m^0 \sim R$, the magnetosphere is nearly crushed by the initial inflow of plasma, although in this case multipole moments of the stellar field other than the dipole moment are bound to be important in determining the shape of the magnetosphere, if only because of the distortion of the star's magnetic field (recall § II*a*). If on the other hand $r_m^0 \gg R$, the magnetosphere can be crushed only if the pressure of the plasma at the boundary becomes much larger than the free-fall ram pressure. For

[1] The nonzero value of α at the polar cusp for the Midgley and Davis (1962) $\nu = 6$ magnetosphere is presumably an artifact of inaccuracies in their numerical treatment.

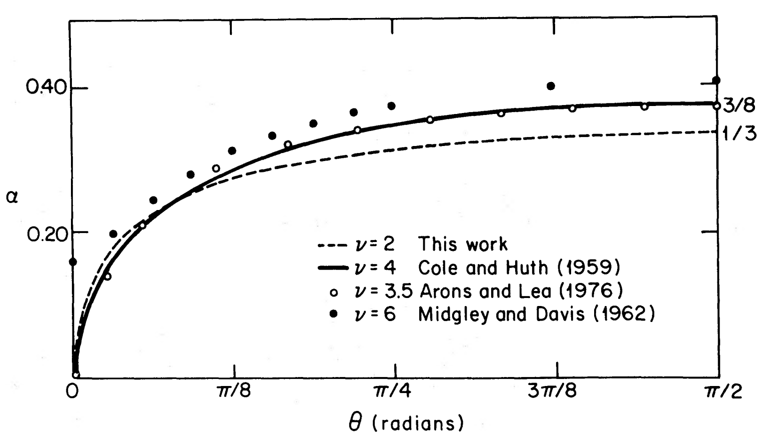

FIG. 2.—The parameter $\alpha = \cos \chi/2\kappa r_m$ as a function of colatitude θ for the four model magnetospheres discussed in the text. The broken line refers to the two-dimensional model constructed in the Appendix whereas the solid line refers to the two-dimensional model of Cole and Huth (1959). The open circles refer to the three-dimensional model of Arons and Lea (1976) and were obtained by reading from Fig. 3 of that work; the estimated reading error is comparable to the size of the circles. The filled circles refer to the three-dimensional model of Midgley and Davis (1962) and were calculated using their equation (9).

example, in order to force the cusps inward to $r \sim R$ would require a pressure there

$$\gtrsim 2 \times 10^8 \dot{M}_{17}^{-1} \mu_{30}^2 (M/M_\odot)^{-1/2} R_6^{-7/2}$$

times the free-fall ram pressure of matter accreting at the rate $\dot{M}$, where R_6 is the stellar radius in units of 10^6 cm. The pressure at the base of a static, adiabatic atmosphere changes little as the size of the atmosphere is increased, provided its gravitating mass is negligible and γ is significantly greater than unity; in such a situation accumulation of plasma hardly changes r_m. However, in any realistic situation the accumulation will end when the accretion radius r_a is reached (in the case of exceedingly low densities) or cooling at the base of the atmosphere becomes sufficient that the matter there falls inward (in the case of moderate or high densities). If $T_{\rm crit}$ is typically ~ 0.1–$0.3\, T_{\rm ff}(r_m)$, as it is for the model magnetospheres shown in Figure 2, the boundary will become Rayleigh-Taylor unstable before the density, and hence the pressure, has increased to more than a few times the free-fall value. Once instability occurs, inflow is likely to be rapid, halting any further buildup of pressure. Thus, it is very unlikely that the magnetosphere can be crushed by the weight of accumulated plasma.

III. STEADY ACCRETION FLOWS

Our analysis so far has been concerned with a static plasma atmosphere surrounding the neutron star magnetosphere. However, the essential feature of magnetic neutron star models of compact X-ray sources is plasma flow into the magnetosphere toward the stellar surface. In this section we assume a steady state and develop further the model outlined by LPP, considering conditions in the plasma approaching the magnetosphere, the scale and structure of the magnetosphere, possible modes of plasma entry into the magnetosphere, and, briefly, plasma flow within the magnetosphere.

In this discussion we shall make the following assumptions in addition to (1)–(4) stated at the beginning of § II:

5) The stellar surface accretes mass at a rate $\dot{M}_s$ equal to $\dot{M}$. The accretion luminosity is then

$$L = GM\dot{M}/R = 1.3 \times 10^{37} \dot{M}_{17} (M/M_\odot) R_6^{-1} \text{ ergs s}^{-1}. \tag{16}$$

6) Essentially all of the accretion luminosity is emitted in the form of X-rays from the surface of the star ($L_x \approx L$).

7) The plasma in the neighborhood of the magnetospheric boundary is exposed to these X-rays.

8) The radiation temperature T_r, defined in § V*a*, of the X-rays is less than the critical temperature $T_{\rm crit}$, defined by equation (13).

a) *Flow toward the Magnetosphere*

As the plasma falls freely toward the magnetospheric boundary at $\sim 10^8$–10^9 cm, the flow will typically be supersonic, as suggested by LPP. That this is so can be seen as follows. Since the plasma is exposed (by assumption) to X-rays from the star, the radiation heating and cooling time scales are short compared to the flow time scale, which is $\sim t_{\rm ff}$ in this region (Buff and McCray 1974). In addition, the electron-ion energy exchange time scale $t_{e\text{-}i}$ is comparable to $t_{\rm ff}$ (cf. § V below). Therefore, the plasma is approximately in local thermal balance, and the electron and ion temperatures, T_e and T_i, are comparable.

In local thermal balance, the plasma temperature T is a function of the parameter

$$\xi_r = L/n_i r^2 = 4.6 \times 10^6 r_8^{-1/2} A R_6^{-1} (M/M_\odot)^{3/2}, \tag{17}$$

where

$$n_i = \rho_{\rm ff}/m_i$$
$$= 2.2 \times 10^{14} A^{-1} r_8^{-3/2} L_{37} (M/M_\odot)^{-3/2} R_6 \text{ cm}^{-3}. \tag{18}$$

For $\xi_r > 10^4$, T is $\sim T_r$, the radiation temperature of the X-rays (see Buff and McCray 1974, Fig. 1), which is typically $\sim 10^7$–10^8 K (Giacconi 1974). Thus the free-fall velocity (eq. [2]) at $r \sim 10^8$ cm is typically much greater than the ion thermal velocity,

$$v_i \equiv (3k_B T_i/m_i)^{1/2}$$
$$= 1.6 \times 10^8 (T_i/10^8 \text{ K})^{1/2} A^{-1/2} \text{ cm s}^{-1}, \tag{19}$$

and the flow is supersonic, although $v_{\rm ff}$ at this radius is typically *less* than the *electron* thermal velocity,

$$v_e \equiv (3k_B T_e/m_e)^{1/2} = 6.7 \times 10^9 (T_e/10^8 \text{ K})^{1/2} \text{ cm s}^{-1}. \tag{20}$$

Moreover, unless $\mathscr{B}^2 \gtrsim 4\pi\rho_{\rm ff} v_{\rm ff}^2$, which is unlikely (LPP, § II), the inflow velocity is also greater than the Alfvén velocity, $v_A' = (\mathscr{B}^2/4\pi\rho_{\rm ff})^{1/2}$, in the embedded magnetic field $\mathscr{B}$, and the flow is therefore super-Alfvénic in this sense.

b) *Scale and Structure of the Magnetosphere*

When there is plasma flow into the magnetosphere, its structure is necessarily more complex than that of the static configurations considered in the previous section. In particular, the static pressure balance equation (5) is no longer valid; instead, momentum balance and continuity of the normal component of the magnetic field across any surface S imply

$$\left(\frac{B_t^2}{8\pi}\right)_{\rm in} + P_{\rm in} + (\rho v_n^2)_{\rm in} = \left(\frac{B_t^2}{8\pi}\right)_{\rm out} + P_{\rm out} + (\rho v_n^2)_{\rm out}, \tag{21}$$

where ρv_n^2 is the dynamic pressure of the plasma in terms of its velocity component v_n normal to the surface. To simplify the discussion of the magnetospheric structure in the presence of flows, let us assume that the magnetic field embedded in the accreting plasma can be neglected.

Far from the star the stellar field will be screened out by currents induced in the infalling plasma. There is therefore a surface S_m, with radius $r_m(\theta, \phi)$, beyond which the stellar magnetic field satisfies $B^2/8\pi \ll P + \rho v_n^2$; outside the magnetospheric boundary S_m the flow of plasma is not appreciably influenced by the stellar field. On the other hand, sufficiently close to the star one expects the screening currents to be negligible, and the large-scale structure of the magnetic field to be essentially unaffected by the presence there of plasma. We suppose this region to be bounded by the surface S_0, with radius $r_0(\theta, \phi)$. Thus the magnetopause, the region which contains the screening currents and in which the magnetic field may take up some of the momentum of the infalling plasma through $\boldsymbol{j} \times \boldsymbol{B}$ forces, has a thickness $\delta_m = r_m - r_0$. Although the magnetic field inside S_0 is unaffected by local currents, it is of course different from the undistorted stellar field because of the screening currents in the magnetopause.

In the presence of plasma flow into the magnetosphere, the magnetopause is likely to be very much broader than the static magnetopause discussed in the previous section, which had a thickness $\delta_m \lesssim a_i$. If the magnetospheric boundary is stable, continuing microturbulence can lead to values of $\delta_m \sim 10^5\, a_i$, while reconnection of strong, large-scale magnetic fields embedded in the accreting plasma to the stellar field can lead to $\delta_m \sim r_m$ (Paper III). If the boundary is Rayleigh-Taylor unstable, the nonlinear development of the instability could conceivably lead to $\delta_m \sim r_m$ (Paper II).

When $\delta_m \ll r_m$, the gravitational force acting on plasma within the boundary layer can be neglected, and hence equation (21) holds approximately with the left-hand side evaluated at r_0 and the right-hand side at r_m. If the magnetic field $\mathscr{B}$ embedded in the accreting plasma at r_m satisfies $\mathscr{B}^2/8\pi \ll \rho v_{\rm ff}^2$, the term $(B_t^2/8\pi)_{\rm out}$ can be neglected. Then in the limiting case

$$(P + \rho v_n^2)_{\rm in} \ll (B_t^2/8\pi)_{\rm in}$$

equation (21) reduces to equation (5) with $P_{\rm out}' = (P + \rho v_n^2)_{\rm out}$ and $B_{\rm out} \approx 0$. The scale r_m^0 of the magnetosphere and the large-scale shape of the magnetospheric boundary are then the same as those of a static configuration with the same exterior pressure $P_{\rm out}'(\boldsymbol{r})$. Obviously r_m^0 and the shape of the magnetospheric boundary depend on the plasma flow pattern. For example, if little plasma accumulates outside the magnetosphere, one has $P_{\rm out}' \approx \rho_{\rm ff} v_{\rm ff}^2 \propto r^{-5/2}$ by momentum balance, and hence $r_m^0 \approx r_m^{(a)}$. Expressing this characteristic radius in terms of the accretion luminosity, one finds (cf. eq. [6])

$$r_m^{(a)} = 2.9 \times 10^8 L_{37}^{-2/7} \mu_{30}^{4/7} (M/M_\odot)^{1/7} R_6^{-2/7}\ \text{cm}. \tag{22}$$

The shape of the boundary should then be close to those of the $\nu = 4$ and $\nu = 3.5$ magnetospheres discussed in § II.

c) *Entry into the Magnetosphere*

By assumption (3) stated in § II, the supersonic flow of plasma toward the magnetospheric boundary is halted by a strong shock wave at $r_s = r_m + \Delta_s$, where Δ_s is the standoff distance from the magnetospheric boundary. In a steady state Δ_s is determined by the requirement that the plasma en route from the shock front to the magnetospheric boundary have time to adjust to the flow boundary conditions at r_m for acceptance of plasma at the accretion rate $\dot{M}$.

If the boundary is Rayleigh-Taylor stable, plasma can enter the magnetosphere via particle entry through the polar cusps, diffusion of plasma across the magnetopause, and magnetic flux reconnection. For each of these entry modes, the inflowing plasma automatically becomes threaded by the stellar field lines and is therefore effectively channeled along them; plasma flows primarily along field lines threading the magnetopause, implying that $(P + \rho v_n^2)_{\rm in} \approx 0$ at $r < r_0$. Were these entry modes able to keep pace with the arrival of plasma at the boundary, with $\rho \sim \rho_{\rm ff}$ and $T \sim T_{\rm ff}$, the standoff distance Δ_s would be small and a self-consistent flow of plasma across a stable magnetospheric boundary would be possible. However, we argue in Paper III that except in circumstances which we believe to be unusual, when magnetic flux reconnection can "open" the magnetosphere, these modes are unable to keep up with the inflow of plasma.

If these modes cannot keep pace, plasma will accumulate between r_s and r_m, causing Δ_s to increase. However, this cannot continue indefinitely: eventually the plasma at the boundary will cool and, according to the analysis presented in § II, when $(1 + Z)T_{\rm out}$ falls below $T_{\rm crit}$ the boundary will become Rayleigh-Taylor unstable. Once hydromagnetic instability sets in, the magnetosphere accepts matter at a much larger rate; if cooling processes are sufficiently effective, the instability should continue and a steady flow may be possible. For the remainder of the present paper we shall assume that plasma enters the magnetosphere predominantly via Rayleigh-Taylor instability of the boundary.

d) *The Alfvén Surface*

The Rayleigh-Taylor instability does not, by itself, cause plasma to be threaded by stellar field lines but rather allows plasma to penetrate the magnetospheric boundary "between stellar field lines." Although the inflow of accreting plasma is initially arrested at the magnetospheric boundary and, following the onset of instability, may continue to be slowed there, the plasma flow within the magnetosphere is unlikely to be channeled by the stellar field until either it is threaded by stellar field lines, after which the plasma flows along the field, or magnetic buoyancy forces become strong enough to appreciably channel the fall of plasma clumps or fingers between field lines. Let us therefore consider the extent of the region around the star in which the plasma flow is dominated by the stellar magnetic field. Following LPP we call the surface bounding this region the Alfvén surface and denote it by S_A. In this discussion, we assume that the transition layer at S_A, in which the flow changes from being unchanneled by the stellar magnetic field to being channeled by it, is thin compared to the radius $r_A(\theta, \phi)$ of the Alfvén surface. When the magnetospheric boundary is stable, the stellar field channels the flow of plasma along stellar field lines threading the magnetopause, and S_A coincides with S_0; if $\delta_m \ll r_m^0$, as is the case when the boundary is stable (unless reconnection of strong, large-scale embedded fields to the stellar field occurs), the surfaces S_m and S_A nearly coincide. On the other hand, when the magnetospheric boundary is Rayleigh-Taylor unstable, the extent to which the stellar field dominates the flow within S_0 depends on the nature of the flow there; the surface S_A obviously cannot lie outside S_0 and could conceivably lie well within S_0. The plasma may or may not be threaded by the magnetic field, and it may be distributed homogeneously or it may be in the form of sinking clumps or narrow streams.

If the magnetic field threads the plasma, then stable, field-aligned flow is possible only if

$$\rho v_{\parallel}^2 < B^2/4\pi , \tag{23}$$

where $v_{\parallel}$ is the flow velocity parallel to the magnetic field (Lamb and Pethick 1974). Otherwise the plasma flow simply carries the magnetic field along and, since $\rho v_{\parallel}^2$ necessarily becomes $\ll B^2/4\pi$ near the stellar surface, becomes unstable at the point where $\rho v_{\parallel}^2 = B^2/4\pi$ (see, for example, Williams 1975). Therefore, if the magnetic field threads the plasma on a time scale much shorter than the inflow time scale, the Alfvén radius r_A is just the radius at which inequality (23) is first satisfied. If, for example, the plasma flow velocity along the field is $\sim v_{ff}$ and the cross-sectional area a of the flow is $\sim 4\pi r^2$, then the equation of continuity, $\dot{M} = \rho v a$, implies that $\rho \sim \rho_{ff}$. Assuming $B \sim \mu r^{-3}$, one finds that the characteristic radius of the Alfvén surface is, for this particular flow, given by

$$\begin{aligned} r_A \sim r_A^{(0)} &\equiv 3.2\times 10^8 \dot{M}_{17}^{-2/7} \mu_{30}^{4/7} (M/M_\odot)^{-1/7}\ \mathrm{cm} \\ &= 3.5\times 10^8 L_{37}^{-2/7} \mu_{30}^{4/7} (M/M_\odot)^{1/7} R_6^{-2/7}\ \mathrm{cm}. \end{aligned} \tag{24}$$

Therefore, if threading of the plasma by stellar field lines occurs quickly, and if this particular flow occurs, then S_A will nearly coincide with S_0. However, we note that *a priori* there is no reason to suppose that the width, $\delta_p = r_m - r_A$, of the zone between the magnetospheric boundary and the Alfvén surface is equal to the width, δ_m, of the magnetopause. In particular, if $v \sim v_{ff}$ but $a \ll 4\pi r^2$, then r_A will be smaller than $r_A^{(0)}$ and δ_p may be larger than δ_m.

If the magnetic field does not thread the plasma, it can still influence the plasma flow through the action of stresses at the plasma-field boundary. For example, the gradient of magnetic pressure across a plasma blob results in a magnetic buoyancy force, F_m, proportional to $\nabla B^2/8\pi$, that acts to oppose the downward pull on the blob by the gravitational force, F_g, which is proportional to ρg. In Paper II we show that if a plasma blob formed in a dipole field has sufficient weight to begin to sink (i.e., if $F_g > F_m$ initially), its motion through the magnetic field depends on the rate of plasma cooling. If cooling is sufficiently rapid (so that the plasma can be described as having an effective adiabatic index $\gamma < 6/5$), the blob may sink through the magnetosphere to the stellar surface, unless threading halts its fall. If cooling is less rapid (so that $\gamma > 6/5$), then there exists a radius r_b at which F_m balances F_g, and inside which a falling plasma blob is decelerated.

If threading is inefficient, there are at least two types of flow possible within the magnetosphere. First, plasma may sink all the way to the stellar surface without being threaded by stellar field lines or, if cooling is sufficiently rapid, being slowed by magnetic buoyancy forces. If this type of flow occurs, the Alfvén surface coincides with the stellar surface. Second, plasma may sink to a radius $r_t > R$ before being threaded by stellar field lines; if cooling is sufficiently slow that a balance radius $r_b > R$ exists, then we expect $r_t \geq r_b$. Inside r_t the plasma will flow along stellar field lines toward the polar magnetosphere where, if $r_t \gg R$, it will form accretion columns and fall onto the stellar surface near the magnetic poles. If this type of flow occurs, and if the plasma is not channeled by the stellar field outside r_t, then r_A will be equal to r_t and, if $r_t \ll r_m^0$, the Alfvén surface will lie deep inside the magnetosphere. In this case δ_p may be much larger than δ_m.

IV. APPLICABILITY OF THE PRESENT STUDY

Two important assumptions made in the present study are (1) that the neutron star is rotating sufficiently slowly that the effects of rotation on the plasma flow can be neglected, and (2) that the accreting plasma approaches the magnetosphere in approximately spherically symmetric free fall. In this section we discuss briefly the conditions which must be met for these two assumptions to be valid and conclude that while they probably are not valid for the sources Her X-1, SMC X-1, and Cen X-3, they may be valid for 3U 0900−40, A0535+26, A1118−61, and 3U 1728−24, and possibly also for some X-ray burst sources.

a) *Stellar Rotation*

The rotation of the star couples to the motion of the plasma at the Alfvén surface, where the stellar magnetic field first channels the flow (§ III). Thus the effects of the star's rotation on the flow can, to a first approximation, be neglected if the centrifugal force acting on plasma corotating with the star at r_A is small compared to the force of gravity there. This is the case if the angular velocity of the star, Ω_s, satisfies $\Omega_s \ll \Omega_K(r_A) = (GM/r_A^3)^{1/2}$ (see, for example, Lamb and Pethick 1974) or, equivalently,

$$r_A \ll r_c \equiv (GM/\Omega_s^2)^{1/3} = 1.5\times 10^8 P^{2/3} (M/M_\odot)^{1/3}\ \mathrm{cm}.$$

Therefore, the relative importance of stellar rotation can be described by the parameter

$$\omega_s \equiv \Omega_s/\Omega_K(r_A)\,. \tag{25}$$

If r_A has approximately the value $r_A^{(0)}$ given by equation (24), then ω_s is given approximately by

$$\omega_s^{(0)} = 3.6P^{-1}L_{37}^{-3/7}\mu_{30}^{6/7}(M/M_\odot)^{-2/7}R_6^{-3/7}\,. \tag{26}$$

If $r_A \lesssim r_A^{(0)}$, as we expect when the accretion flow outside the magnetosphere is approximately spherically symmetric, radial free fall (recall § III), one finds $\omega_s = \omega_s^{(0)}(r_A/r_A^{(0)})^{3/2} \lesssim \omega_s^{(0)}$. Therefore, for the types of flow considered in the present study, $\omega_s^{(0)} \ll 1$ is a sufficient condition for neglecting the effects of the star's rotation on the accretion flow.

b) Plasma Orbital Motion

The effects on plasma entry into the magnetosphere due to orbital motion of the accreting plasma can, to a first approximation, be neglected if the angular velocity of the plasma, $\Omega_p(r)$, is much less than $\Omega_K(r)$ throughout the region $r_A \lesssim r \lesssim r_m$. Since $\Omega_K(r) \propto r^{-3/2}$ while $\Omega_p(r) \propto r^{-2}$ for free fall or $\propto r^{-3/2}$ for disk flow, we expect Ω_p/Ω_K to be largest at the smallest radius where the orbital motion of the accreting plasma is not yet strongly affected by the stellar magnetic field. Now by definition the stellar magnetic field first channels the flow, bringing the accreting plasma into corotation with the star, at r_A. Therefore, a convenient parameter that indicates the extent to which the accretion flow near the magnetospheric boundary departs from radial inflow is

$$\omega_p \equiv \Omega_p(r_A)/\Omega_K(r_A)\,. \tag{27}$$

If $\omega_p \ll 1$, as may happen if the neutron star accretes plasma (e.g., from a stellar wind) that has insufficient angular momentum to form an accretion disk at $r \gtrsim r_A$ (cf. Davidson and Ostriker 1973; Illarionov and Sunyaev 1975; Shapiro and Lightman 1976), and if in addition $\omega_s \ll 1$, the results of the present study are directly applicable. Unfortunately, the quantity $\Omega_p(r_A)$ is less accessible to observation than is Ω_s. However, accretion of matter with even a small amount of angular momentum will cause Ω_s to change on a time scale short compared to $t_I = (I/\dot{M})(dM/dI)$, the time scale for changes in Ω_s caused by the change in the stellar moment of inertia I (LPP, § VI*b*); one therefore has an observational probe of the size of orbital motion in the observed time scale, $t_s = \Omega_s/|\dot{\Omega}_s|$, for long-term changes in the angular velocity of the star. For a star accreting plasma with significant angular momentum, theory predicts that t_s satisfies (see, for example, Ghosh, Lamb, and Pethick 1977)

$$t_s \gtrsim \frac{MR_g^2\Omega_s}{\dot{M}r_A^2\Omega_p(r_A)}\,, \tag{28}$$

where R_g is the radius of gyration of the neutron star. The inequality results from the fact that the material stresses at r_A may be partially canceled by magnetic and shear stresses. However, for Ω_s sufficiently small compared to $\Omega_p(r_A)$, $\Omega_p(r)$ must go through a maximum at a radius $\sim r_A$; since the shear stress necessarily vanishes at this point, the inequality in this case becomes an approximate equality (cf. LPP, § VI*b*). For a given observed value of t_s, inequality (28) provides a lower bound on $\Omega_p(r_A)$ and hence on ω_p, namely,

$$\omega_p \gtrsim \frac{MR_g^2\Omega_s}{\dot{M}t_s r_A^2\Omega_K(r_A)}\,. \tag{29}$$

Although the precise value of r_A is unknown, the right-hand side of inequality (29) depends only weakly on it ($\propto r_A^{-1/2}$), and hence an approximate lower bound on ω_p can be obtained by setting $r_A = r_A^{(0)}$. One then has $\omega_p \gtrsim \omega_p^{(0)}$, where

$$\omega_p^{(0)} \equiv \frac{MR_g^2}{\dot{M}t_s r_A^{(0)2}}\,\omega_s^{(0)} = 2.4\,P^{-1}\left(\frac{t_s}{10^4\text{ yr}}\right)^{-1} \times \left(\frac{R_g}{10^6\text{ cm}}\right)^2 L_{37}^{-6/7}\mu_{30}^{-2/7}\left(\frac{M}{M_\odot}\right)^{10/7} R_6^{-6/7}\,. \tag{30}$$

Therefore, $\omega_p^{(0)} \ll 1$ is a necessary condition for neglecting the effects of the plasma's orbital motion on the accretion flow.

c) Observed Sources

Although many of the "steady" (i.e., nonbursting) compact X-ray sources in which periodic pulsations have so far not been detected may well be accreting neutron stars (Lamb 1975*b*; Basko and Sunyaev 1976; Elsner and Lamb 1976*a*; Maraschi, Treves, and van den Heuvel 1976), and may even satisfy our model assumptions (1) and (2) above, the fact that no pulsation periods or spin-up time scales are available for these sources means that it is at present impossible to determine whether or not our model assumptions (1) and (2) are satisfied for them. Similarly, even if one adopts the magnetic neutron star hypothesis for some of the X-ray burst sources, unless evidence of stellar rotation is detected in them there is no obvious way of knowing whether or not they satisfy our assumptions. Therefore, in seeking examples of sources which do satisfy our model assumptions we are forced to focus on the regularly pulsating sources.

Table 3 presents estimates of $\omega_s^{(0)}$ and $\omega_p^{(0)}$ for seven such sources. In preparing these estimates we have assumed, for the sake of definiteness, $M = 1.3\,M_\odot$, $R_6 = 1.5$, $R_g = 10^6$ cm (cf. Pandharipande, Pines, and Smith 1976), and $\mu_{30} = 1$; the estimated luminosity that has been used is listed for each source. Table 3 indicates that the present study probably is *not* directly applicable to Her X-1, SMC X-1, and Cen X-3, but could be directly applicable to 3U 0900−40, A0535+26, A1118−61, and 3U 1728−24, if ω_p is small for these sources (Elsner and Lamb 1976*a*). If the sources 3U 0352+30, 3U 1223−62, and 3U 1813−14, which have recently been reported to be pulsating (White, Mason, and Sanford 1975;

TABLE 3

ESTIMATED ACCRETION FLOW PARAMETERS FOR SOME PULSATING X-RAY SOURCES

Parameter	Her X-1	SMC X-1	Cen X-3	3U 0900−40	A0535+26	3U 1728−24	A1118−61
P (s)	1.24[a]	0.716[b]	4.84[a]	283[c]	104[d]	123[e]	405[f]
L (10^{37} ergs s^{-1})	6[g]	60[b]	6[a]	0.1[h]	0.4(?)	2(?)	5(?)
t_s (yr)	3×10^5[i]	...	3×10^3[i]	...	...	...	...
$\omega_s^{(0)}$	1.0	0.7	0.3	0.03	0.04	0.02	0.003
$\omega_p^{(0)}$	0.02	...	0.4	...	...	...	...

[a] Giacconi *et al.* 1974. The luminosity quoted for Cen X-3 is twice the 2–10 keV luminosity assuming a distance of 10 kpc.
[b] Lucke *et al.* 1976.
[c] McClintock *et al.* 1976.
[d] Rosenberg *et al.* 1975.
[e] White *et al.* 1976 and Lewin *et al.* 1976.
[f] Eyles *et al.* 1975 and Ives *et al.* 1975.
[g] Compare McCray and Lamb 1976.
[h] Rappaport, Joss, and McClintock 1976.
[i] Giacconi 1974.

White *et al.* 1975), prove to be rotating neutron stars, the present study may also be directly applicable to them.

d) *Other Applications*

Our study also applies to accretion onto a magnetic white dwarf provided the mass accretion rate is low enough or the stellar magnetic field is strong enough that the inequality $r_m^0 \gg R$ is satisfied. If instead the condition $r_m^0 \sim R$ holds, then higher magnetic multipole moments can complicate the flow near the stellar surface and plasma may easily crush the magnetic field to the stellar surface along multipole axes. A well-defined magnetospheric cavity does not form under these conditions and our calculations therefore do not apply.

Finally, we note that even in situations where the present calculations are not *directly* applicable, the flow may still resemble in certain respects the flow found here. Thus, if a slowly rotating star is fed by a thin disk that thickens near r_m and the flow there becomes roughly spherically symmetric, it may resemble the postshock flow of the present study. Even if the star is fed by a disk that remains thin at r_m, the Rayleigh-Taylor instability may well play an important role in allowing the relatively cold disk plasma to penetrate the magnetosphere. In the case of accretion by a fast rotator, centrifugal forces as well as viscous and magnetic torques are important at the magnetospheric boundary. Again, although our present calculations do not apply directly, they do represent a first step toward understanding this more complicated problem (see Elsner and Lamb 1976*a* for a discussion of accretion onto fast rotators and ways it is expected to differ from accretion onto slow rotators).

V. HEATING AND COOLING OF PLASMA AT THE MAGNETOSPHERIC BOUNDARY

In the present section we introduce the characteristic time scales of the heating and cooling processes dominant in the region between the shock front and the magnetospheric boundary and discuss their implications for the structure of the postshock layer both before and after the onset of the Rayleigh-Taylor instability at the boundary. The results of simple analytical and more sophisticated numerical calculations of the steady postshock flow are presented in Paper IV; here we introduce only those results from Paper IV relevant to the subject of this paper.

a) *Heating and Cooling Time Scales*

If the electron temperature T_e greatly exceeds the temperature T_r of the radiation field (see below), nonrelativistic electrons cool via inverse Compton scatterings on a time scale (Weymann 1965)

$$t_c = \frac{3m_e c}{8\sigma_T u_r} = 5.8 \times 10^{-3}\left(\frac{r}{10^8\ \mathrm{cm}}\right)^2 L_{37}^{-1}\ \mathrm{s}\,. \qquad (31)$$

The quantities σ_T and u_r are the Thomson cross section and the radiation energy density, respectively. In the last expression on the right-hand side of equation (31), we have used $u_r = L/4\pi r^2 c$, which is valid if the plasma in the postshock region is optically thin. In terms of u_r and the spectral energy density ϵ_ν, the temperature of the radiation field is (Levich and Sunyaev 1971)

$$T_r = k_B^{-1} u_r^{-1} \int_0^\infty d\nu h\nu\epsilon_\nu\,.$$

The Compton cooling time scale t_c is short compared to the dynamical time scale t_d at $r_m^{(a)}$ (cf. eqs. [3] and [22]) if

$$L > 3 \times 10^{36} \mu_{30}^{1/4} (M/M_\odot)^{1/2} R_6^{-1/8}\ \mathrm{ergs\ s^{-1}}\,. \qquad (32)$$

The electrons cool via electron-ion bremsstrahlung on a time scale (Allen 1973, p. 103)

$$t_{\mathrm{br}} \approx 1.4\left(\frac{Z\bar{g}n_e}{10^{15}\ \mathrm{cm}^{-3}}\right)^{-1}\left(\frac{T_e}{10^8\ \mathrm{K}}\right)^{1/2} \mathrm{s}\,, \qquad (33)$$

if $T_e/T_i > m_e/m_i$; here $\bar{g}$ is the average Gaunt factor, of order unity.

The cooling processes we have discussed cool the electrons; if $T_i \gg T_e$ (see below), the ions lose their energy to the electrons on a time scale

$$t_{e-i} = \frac{3m_e m_i}{8(2\pi)^{1/2}Z^2e^4n_e \ln \Lambda}\left(\frac{k_B T_e}{m_e} + \frac{k_B T_i}{m_i}\right)^{3/2}$$
$$\approx 2.5\times 10^{-2}\frac{A}{Z^2}\left(\frac{n_e \ln \Lambda}{10^{16}\text{ cm}^{-3}}\right)^{-1}\left(\frac{T_e}{10^8\text{ K}}\right)^{3/2}\text{ s}. \quad (34)$$

In equation (34) ln Λ is the Coulomb logarithm, and in the second expression we have assumed $T_e/T_i > m_e/m_i$.

In Paper IV (see also Arons and Lea 1976) we show that if a steady flow develops in the postshock zone following the onset of instability at the magnetospheric boundary S_m, the electron temperature throughout most of the zone is typically determined by a local balance between heating due to collisions with ions and Compton cooling, and satisfies $T_i \gg T_e \gg T_r$. The electron temperature is then

$$T_e{}^* \approx 3.2\times 10^8(r_s/10^8\text{ cm})^{-1/5} \times (M/M_\odot)^{-1/5}R_6{}^{2/5}(\ln \Lambda/15)^{2/5}\text{ K}, \quad (35)$$

and the time scale for the electrons to cool the ions by collisions is of order

$$t_{e-i}{}^* \equiv t_{e-i}(\rho', T_e{}^*) = 0.11L_{37}{}^{-1}(r_s/10^8\text{ cm})^{6/5}R_6{}^{-2/5}(\ln\Lambda/15)^{-2/5}\text{ s}, \quad (36)$$

where ρ' is the postshock mass density.

b) *Implications*

Prior to the onset of the Rayleigh-Taylor instability at the magnetospheric boundary, plasma is unable to enter the magnetosphere easily and therefore an increasingly extended atmosphere forms between the boundary and the shock front. Since little plasma accretes onto the neutron star's surface during this phase of the accretion flow, the plasma is not bathed by a strong flux of X-rays, Compton cooling is ineffective, and the plasma there cools either by thermal conduction or by bremsstrahlung. However, a small-scale (scale $\lambda_m \ll [m_e/m_i]^{1/2}r_m$), tangled magnetic field of even moderate strength ($\mathscr{B} \gtrsim 1$ gauss) is sufficient to inhibit conduction on the dynamical time scale (cf. Paper IV). In the discussion which follows we assume that thermal conduction can be neglected. Then $t_{\text{cool}} \sim t_{\text{br}}$, and after a time $\sim t_{\text{ff}}(r_m)$ the atmosphere near S_m settles into approximate hydrostatic equilibrium with $T_i \approx T_e \approx (1 + Z)^{-1}T_0$ (see [15]), provided that $t_{\text{br}} \gg t_{\text{ff}}$ and $t_{\text{br}} \gg t_{e-i}$. Assuming $P_{\text{out}} \approx \rho_{\text{ff}}v_{\text{ff}}{}^2(r)$, and $r = r_m{}^{(a)}$, and using equations (1), (2), and (6), one can show that for an electron-proton plasma the first inequality is satisfied if

$$\dot{M} \ll 10^{19}\mu_{30}{}^{-1/3}(M/M_\odot)^{11/6}\text{ g s}^{-1}, \quad (37)$$

while the second inequality is satisfied if

$$\dot{M} \ll 10^{20}\mu_{30}{}^2(M/M_\odot)^{-4}(\ln\Lambda/15)^{7/2}\text{ g s}^{-1}. \quad (38)$$

In deriving condition (38) we have used expressions (33) and (34), which are valid if the electrons are nonrelativistic; since $k_B T_0/2m_ec^2 \sim 1.4$ for $\dot{M} \sim 10^{20}$ g s^{-1}, this condition is only approximately correct for such high accretion rates.

Following the onset of the Rayleigh-Taylor instability at S_m, accretion onto the neutron star's surface becomes possible and X-ray emission begins. Provided that the plasma at the magnetospheric boundary is then exposed to X-rays, it cools on the time scale $t_{e-i}{}^*$ introduced above. The flow of energy in the postshock zone is then as follows: hot ions ($T_i \sim [1 + Z]^{-1}T_0$) transfer energy to the cooler electrons ($T_e \sim T_e{}^*$) in Coulomb collisions; the electrons, in turn, cool via inverse Compton scatterings with X-rays. In this way, the magnetospheric boundary may remain Rayleigh-Taylor unstable and plasma may continue to enter the magnetosphere.

Whether or not a steady postshock flow is possible depends on the boundary conditions imposed by the nature of the unstable flow. Unfortunately, no reliable calculations of the Rayleigh-Taylor unstable flow in the nonlinear regime are yet available (the present state of knowledge and flow boundary conditions are discussed in Paper II). For the purposes of the present discussion we shall assume that the ions must cool to a temperature $\sim T_{\text{crit}}$ by the time they reach r_m, a simple model which is almost certainly qualitatively correct. This implicitly determines the standoff distance Δ_s. Since $T_i[r_m(\theta)]$ would have the value $0.4\,T_{\text{ff}}[r_m(\theta)]$ (cf. eq. [15]) in the absence of cooling and since $T_{\text{crit}}(\pi/2) < 0.4\,T_{\text{ff}}[r_m(\pi/2)]$ for $\nu \lesssim 3.5$ (recall § V*d*), this model implies that

$$\Delta_s \approx f_{\text{crit}}\Delta_{\text{cool}}, \quad (39)$$

for α not too close to $\frac{3}{8}$, as shown in Paper IV. Here $\Delta_{\text{cool}} = v't_{e-i}{}^*$ is the postshock ion cooling length scale in terms of the postshock flow velocity v', and $f_{\text{crit}} = 1 - (T_{\text{crit}}/T_i')^2$ in terms of the postshock ion temperature T_i'. Since Compton scattering plays a crucial role in rapidly cooling plasma which is exposed to X-rays from the stellar surface, the possibility of a steady flow depends critically on the assumption $T_r < T_{\text{crit}}$; were this not satisfied, the X-rays emitted when accreting matter reaches the stellar surface will tend to restabilize the magnetospheric boundary, which would then cut off the accretion flow into the magnetosphere (cf. Lamb *et al.* 1977).

VI. DISCUSSION

The results presented here together with those of succeeding papers imply that, for a wide range of conditions relevant to compact X-ray sources, the Rayleigh-Taylor instability is the most important plasma entry process. Thus the formation of a bright "steady" X-ray source under the conditions assumed in this study (radial inflow onto a slowly rotating star) generally requires continuing Rayleigh-Taylor instability of the magnetospheric boundary. Similarly, if some of the bursting X-ray sources are neutron stars accreting under these conditions (Lamb *et al.* 1977), the high luminosities observed during bursts probably imply that the boundary is Rayleigh-Taylor unstable during this phase of their activity cycle.

The results of § II*d* show that under realistic conditions at least some cooling is required in order for the weight of the plasma to drive the boundary unstable. We note that this conclusion does not depend on the formation of a strong shock wave, but only on the heating of the plasma outside the magnetosphere to a temperature high enough that inequality (13) is not satisfied. Nevertheless, the magnetospheric boundary is probably initially close to instability at the magnetic equator. In any case, cooling is clearly required in order for plasma to enter the magnetosphere over a large fraction of its surface via this instability.

If the magnetospheric boundary is initially Rayleigh-Taylor stable, as we expect, the plasma outside the magnetosphere cools slowly on the bremsstrahlung or thermal conduction time scale, and the density there, ρ_{out}, increases slowly on the same time scale. If thermal conduction is strongly inhibited by small-scale tangled magnetic fields, inequality (13) is violated for some value of θ after a time of the order of the bremsstrahlung time scale and the boundary at that colatitude becomes locally Rayleigh-Taylor unstable. The exact value of θ for which the boundary is *first* driven unstable depends on the balance between two competing factors: First, since the function $\alpha(\theta)$ has its maximum value at $\theta = \pi/2$ whereas $\alpha(0) = \alpha(\pi) = 0$ (cf. Fig. 2), the plasma must lose the least amount of thermal energy at the magnetic equator in order to drive the boundary locally unstable, while it must lose essentially all of its energy in order to drive the boundary locally unstable close to the polar cusps. Second, the local bremsstrahlung cooling time scale is initially proportional to $r_m(\theta)$ for an adiabatic atmosphere laid down by freely falling plasma, and it is therefore shorter near the polar cusps than it is at the magnetic equator. Thus if $\alpha(\pi/2)$ is comparable to (but still smaller than) 0.4, so that only a little thermal energy must be radiated away in order to drive the equator unstable, the boundary will first be driven unstable near $\theta = \pi/2$, while if $\alpha(\pi/2) \ll 0.4$, the boundary will first be driven unstable nearer the polar cusps than the magnetic equator.

If the conditions necessary for continued Rayleigh-Taylor instability of the magnetospheric boundary are met (a radiation temperature that is sufficiently low and a luminosity that is sufficiently high for effective Compton cooling, plus an adequate supply of matter), a steady accretion flow into the magnetosphere may be possible. However, once plasma flows across the magnetospheric boundary, inequality (13) is no longer a rigorous condition for stability of the boundary, as it was derived under the assumption that the plasma atmosphere outside the magnetosphere was static. Also, the boundary layer between the interior and exterior of the magnetosphere may broaden to a thickness $\delta_m \sim r_m$ after instability sets in, or significant amounts of plasma may build up in the outer portions of the magnetosphere. The plasma flow itself may alter the gross shape and stability of the boundary; indeed, it is not certain at present that the nonlinear development of the Rayleigh-Taylor instability is such as to permit a steady flow. Nevertheless, the force that drives and keeps the magnetospheric boundary Rayleigh-Taylor unstable is the weight of the plasma at the boundary, and the time scale for this weight to build up is sensitive to the rate of cooling of the plasma outside the magnetosphere.

Once instability sets in and X-ray emission from the stellar surface begins, those portions of the boundary illuminated by X-rays will, under typical conditions, be driven and remain Rayleigh-Taylor unstable due to the very short Compton cooling time scale there. If X-rays from the stellar surface illuminate the magnetospheric boundary uniformly, and if $c_s t_{e-i}^* < r_m$, where t_{e-i}^* is the ion cooling time scale (cf. eq. [36]), we expect approximately radial flow into the magnetosphere over virtually the entire boundary since even were the plasma flow velocity comparable to the local sound speed c_s, it could only flow around a small fraction of the boundary before cooling to a temperature that would satisfy inequality (13). In Paper IV we show that the condition $c_s t_{e-i}^* < r_m$, which is approximately equivalent to the condition $\Delta_s < r_m$, is satisfied if

$$L > L_{\text{crit}} \equiv 2 \times 10^{38}(\gamma_s - 1)^{105/64}(\gamma_s + 1)^{-91/32}\mu_{30}{}^{-3/16} \times (M/M_\odot)^{29/16} R_6{}^{-11/32}\left(\frac{\ln \Lambda}{15}\right)^{-7/16} \text{ ergs s}^{-1}, \quad (40)$$

where γ_s is the effective adiabatic index across the shock. If $\gamma_s = 5/3$, for example, inequality (40) is satisfied for $L \sim 10^{37}$ ergs s^{-1}. However, if L is much less than L_{crit}, a substantial plasma "atmosphere" must exist outside the magnetosphere and significant nonradial motion of the plasma near r_m may be expected, even if the illumination of the boundary is more or less uniform.

On the other hand, if the X-rays from the stellar surface are narrowly beamed or if large portions of the boundary lie in the shadows of plasma streams within the magnetosphere (see Paper II), the pattern of flow across the boundary will be strongly affected and there is likely to be significant nonradial motion, since plasma is most likely to enter the magnetosphere over those portions of the boundary that are illuminated. If $\alpha(\pi/2) \ll 0.4$, for example, plasma first enters the magnetosphere near the polar cusps. This plasma accretes onto the surface of the neutron star near its magnetic poles, and the resulting X-rays emitted from the polar caps may preferentially illuminate the polar magnetosphere. If this happens, plasma continues to enter the magnetosphere mainly in its polar regions. On the other hand, the X-rays may be emitted from the polar caps in a fan-shaped beam and may then preferentially illuminate the equatorial magnetosphere. If this happens, the boundary near $\theta = \pi/2$ will rapidly be driven Rayleigh-Taylor unstable, allowing plasma to enter there as well. Thus, under some conditions accretion onto the stellar surface of plasma entering the magnetosphere over a portion of the boundary may produce a comparatively narrow X-ray beam that illuminates previously stable portions of the boundary. If so, there would result a systematic variation in the zones of plasma entry and in the direction in which X-rays are beamed. Such a variation would be observed at Earth as a systematic time variation in the X-ray flux.

If significant nonradial flow occurs, the plasma pressure confining the magnetosphere is no longer a function of radius alone, and the stability and shape of the boundary may differ from that described in §§ II and III. One property of the magnetosphere that may lead to flows around the boundary is that, in the polar regions, the boundary lies deeper in the gravitational well of the neutron star than it does in the equatorial plane. Thus, for sources in which $\Delta_s \gtrsim r_m$, plasma may have time to drain away from the magnetic equator into the polar cusps before it enters the magnetosphere, provided that there is sufficient cooling of the plasma to reduce the back pressures tending to halt such flow. If this type of flow occurs, a substantial fraction of the plasma may enter the polar magnetosphere by means of Rayleigh-Taylor instability of the boundary there.

We note that conditions in the flow outside the magnetospheric boundary can be such that a steady flow of plasma across the boundary is not possible. Lamb *et al.* (1977) examine various models in which unsteady flows may occur and apply these models to the bursting X-ray sources. Further development of their models may provide important insights into the conditions under which the magnetosphere of an accreting neutron star can accept plasma at a steady rate.

Finally, we emphasize that under the conditions considered here (radial accretion onto a slowly rotating neutron star), the Alfvén surface S_A, within which the stellar magnetic field channels the flow, need not coincide with the magnetospheric boundary S_m, and could conceivably lie well within it (§ III*d*). On the other hand, if one accepts the magnetic neutron star model of pulsating X-ray sources, the existence of strongly modulated pulsations in some sources (Giacconi 1974) and the order-of-magnitude agreement of observed spin-up time scales with theoretical values assuming $r_A \sim 10^8$ cm (Elsner and Lamb 1976*a*) is evidence that, at least under some conditions, one can have $r_A \gg R$. Clearly an important objective of future studies will be to develop more accurate estimates of r_A.

VII. CONCLUDING REMARKS

In the preceding sections we have described the formation of a magnetosphere around an accreting magnetic neutron star, and have analyzed the structure of such magnetospheres, both in static equilibrium and in the presence of plasma flow across the boundary. We have shown that the stability of the boundary is sensitive to heating and cooling of plasma outside the magnetosphere. Thus if accreting plasma enters the magnetosphere predominantly via Rayleigh-Taylor instability of the boundary, which we argue is usually the case, the zones of entry and the pattern of the flow can be greatly altered if the X-rays emitted from the stellar surface are strongly beamed. We have shown further that in the presence of plasma flow into the magnetosphere there are in general *two* important surfaces around the star: the magnetospheric boundary, S_m, outside which the stellar field is screened out, and the Alfvén surface, S_A, inside which the matter flow is channeled by the magnetospheric field. We have also discussed the implications of these results for compact X-ray sources.

The calculations presented in this and subsequent papers apply to accretion by a slowly rotating neutron star whenever the flow outside the magnetosphere approximates spherically symmetric radial inflow. Thus, our results apply directly to accretion onto such a star from a spherically symmetric cloud of gas around it, or from the stellar wind of a binary companion when the angular momentum of the accreted matter is too small to cause the formation of an accretion disk. These results may therefore be directly applicable to some observed "steady" X-ray sources, as discussed in § IV. For example, in Elsner and Lamb (1976*a*) we applied the results of this study to the long-period X-ray sources 3U 0900−40 and A0535+26 and suggested a possible explanation for the strong energy dependence of the pulse profiles observed in these sources. These results may also be directly relevant to some X-ray burst sources (see Lamb *et al.* 1977).

In Paper II we shall present the results of our analysis of the Rayleigh-Taylor instability of the magnetospheric boundary and its consequences for steady accretion flows, using as a framework for our discussion the magnetospheric structure presented here. There we give the results of a normal-mode analysis of the onset of instability and a qualitative analysis of possible accretion flows within the magnetosphere.

In Paper III we shall consider the processes by which plasma outside may enter the magnetosphere when the boundary is Rayleigh-Taylor stable. These processes are particle entry through the polar cusps, diffusion of plasma across the magnetopause, and reconnection at the magnetopause of tangled magnetic fields embedded in the accreting plasma to the stellar magnetic field. In this investigation we make use of the results presented in this paper for the structure and stability of the magnetosphere together with specific models for each of these entry processes to show that in the absence of Rayleigh-Taylor instability, the magnetosphere is usually unable to accept matter at a rate equal to that at which it arrives at the boundary for conditions characteristic of bright binary X-ray sources. The only exceptions are situations, which we believe to be unusual, when magnetic flux reconnection can "open" the magnetosphere. We discuss the implications of these results for both "steady" and bursting X-ray sources. In Paper IV we shall present analytical and numerical solutions for the flow and cooling of plasma between the standoff shock wave and the magnetospheric boundary, and discuss the implications for the production and beaming of X-rays.

It is a pleasure to thank Professors S. Ichimaru, D. Q. Lamb, R. McCray, L. Mestel, F. C. Michel, C. J. Pethick, D. Pines, J. E. Pringle, Drs. I. Easson and D. Roberts, and P. Ghosh and R. Masters for helpful and stimulating discussions during the course of this work. F. K. L. would like to acknowledge the warm hospitality of the Institute of Astronomy, Cambridge, during part of this work.

APPENDIX

THE SHAPE OF THE MAGNETOSPHERIC BOUNDARY

The basic shape of the magnetospheric cavity created by a dipolar stellar magnetic field, as well as an idea of the way in which the shape differs for different scalings of confining pressure with radius, can be obtained from the equilibrium shapes and magnetic field structures of two-dimensional model magnetospheres. Field geometries similar to the ones in such models would be obtained by slicing through the corresponding three-dimensional magnetospheres in a plane containing the dipole axis. In the present Appendix we present exact, analytical solutions for two such models. In these models we assume that (1) the magnetospheric cavity is created by a line dipole, and (2) the magnetic field is confined by a cylindrically symmetric external pressure which has a power-law dependence on radius. The conformal mapping technique that we use here has been used previously by Cole and Huth (1959), Dungey (1961), and Hurley (1961*a*, *b*) in problems related to the shape of the geomagnetosphere.

If coordinates x and y are chosen so that the dipole is at the origin and the y-axis is parallel to the dipole axis, the undistorted magnetic field is

$$\boldsymbol{B}(x, y) = (\hat{x} \sin 2\theta + \hat{y} \cos 2\theta)\mu'(r')^{-2}\,, \tag{A1}$$

where $\hat{x}$ and $\hat{y}$ are unit vectors, $\theta = \tan^{-1}(y/x)$ is the polar angle measured from the y-axis, μ' is the dipole moment, and $r' = (x^2 + y^2)^{1/2}$. The external pressure can be parameterized by the expression

$$P(r') = P_0(r'/r_0)^{-n}\,. \tag{A2}$$

Since $B^2 \approx (r')^{-4}$, the two-dimensional analog of the quantity ν defined in § II is $\nu = 4 - n$. Similarly, the two-dimensional analog of the reference radius $r_m{}^0$ is

$$r_m{}^{(\nu)} = \left[\frac{(\mu')^2}{8\pi P_0 r_0{}^n}\right]^{1/\nu}. \tag{A3}$$

The equilibrium magnetic field structure can be obtained simply by conformal mapping if there exists a transformation which maps the (x, y)-plane onto the (α, β)-plane, say, in such a way that the magnetospheric boundary in the (x, y)-plane, defined by the curve $r_m(\theta)$, is mapped onto a circle in the (α, β)-plane. For arbitrary $n < 4$, the required transformation is

$$\alpha = [u \cos(n\theta/2) + v \sin(n\theta/2)](r')^{-n/2}\,, \tag{A4}$$

$$\beta = [-u \sin(n\theta/2) + v \cos(n\theta/2)](r')^{-n/2}\,, \tag{A5}$$

where $u = B_y/B^2$ and $v = -B_x/B^2$. The coordinates x and y satisfy Laplace's equation on the (α, β)-plane. The boundary conditions at the magnetospheric boundary $r_m(\theta)$ are (*a*) pressure balance, $B(r_m)^2/8\pi = P(r_m)$, and (*b*) tangency of the magnetic field at the boundary, i.e., $(B_y/B_x)_m = (dy/dx)_m$, where the subscript m indicates that the equation holds only along the curve $r_m(\theta)$. In the (α, β)-plane condition (*b*) becomes

$$\frac{\partial \ln r'}{\partial \alpha} \sin[(1 - n/2)\theta] - \frac{\partial \ln r'}{\partial \beta} \cos[(1 - n/2)\theta] = 0\,, \tag{A6}$$

$$\frac{\partial \theta}{\partial \beta} \sin[(1 - n/2)\theta] + \frac{\partial \theta}{\partial \alpha}[(1 - n/2)\theta] = 0\,. \tag{A7}$$

At $r' = 0$ the magnetic field $\boldsymbol{B}$ must approach the undistorted dipolar field, which is given by equation (A1).

The solution for $\nu = 4$ has been given by Cole and Huth (1959): the magnetospheric boundary is the surface

$$x = r_m{}^{(4)}[\sin(\phi/2) - \tfrac{1}{3} \sin(3\phi/2)]\,, \tag{A8}$$

$$y = r_m{}^{(4)}[\cos(\phi/2) - \tfrac{1}{3} \cos(3\phi/2)]\,, \tag{A9}$$

where $0 \leq \phi \leq 4\pi$, and the polar cusp is located at $\phi = 0$. For $\nu = 2$ the magnetospheric boundary is the surface

$$x = r_m^{(2)} \exp\left[-\tfrac{1}{2}\cos(2\psi)\right] \sin\left[\psi - \tfrac{1}{2}\sin(2\psi)\right], \tag{A10}$$

$$y = r_m^{(2)} \exp\left[-\tfrac{1}{2}\cos(2\psi)\right] \cos\left[\psi - \tfrac{1}{2}\sin(2\psi)\right], \tag{A11}$$

where $-\pi \leq \psi \leq \pi$, and the polar cusp is located at $\psi = 0$.

Two interesting quantities are the radius at which the polar cusps close, $r_p \equiv r_m(0)$, and the radius of the magnetic equator, $r_e \equiv r_m(\pi/2)$. For $\nu = 4$, $r_p = 2r_m^{(4)}/3$ and $r_e = 4r_m^{(4)}/3$, while for $\nu = 2$, $r_p = r_m^{(2)}/e^{1/2}$ and $r_e = r_m^{(2)}e^{1/2}$. The full curves $r_m(\theta)$ for the two solutions are shown in Figure 1.

REFERENCES

Allen, C. W. 1973, *Astrophysical Quantities* (London: Athlone Press).
Arons, J., and Lea, S. M. 1976, *Ap. J.*, **207**, 914.
Basko, M. M., and Sunyaev, R. A. 1976, *M.N.R.A.S.*, **175**, 395.
Bernstein, I. B., Frieman, E. A., Kruskal, M. D., and Kulsrud, R. 1958, *Proc. Roy. Soc. A.*, **244**, 17.
Blumenthal, G., and Tucker, W. 1974, *Ann. Rev. Astr. Ap.*, **12**, 23.
Buff, J., and McCray, R. 1974, *Ap. J.*, **189**, 147.
Cole, J. D., and Huth, J. H. 1959, *Phys. Fluids*, **2**, 624.
Davidson, K., and Ostriker, J. P. 1973, *Ap. J.*, **179**, 585.
Dungey, J. W. 1961, *J. Geophys. Res.*, **66**, 1043.
Elsner, R. F., and Lamb, F. K. 1975*a*, *Bull. AAS*, **7**, 545.
———. 1975*b*, *Bull. AAS*, **7**, 241.
———. 1976*a*, *Nature*, **262**, 356.
———. 1976*b*, preprint (Paper II).
———. 1977*a*, in preparation (Paper III).
———. 1977*b*, in preparation (Paper IV).
Eyles, C. J., Skinner, G. K., Willmore, A. P., and Rosenberg, F. D. 1975, *Nature*, **254**, 577.
Ghosh, P., Lamb, F. K., and Pethick, C. J. 1977, *Ap. J.*, **216**, in press.
Giacconi, R. 1974, in *Astrophysics and Gravitation, Proceedings of the 16th Solvay Congress* (Brussels: L'Université de Bruxelles), p. 27.
Grad, H., and Hu, P. N. 1966, *Phys. Fluids*, **9**, 1610.
Hamasaki, S., Davidson, R. C., Krall, N. A., and Liewer, P. C. 1974, *Nucl. Fusion*, **14**, 27.
Harwit, M., and Salpeter, E. E. 1973, *Ap. J.* (*Letters*), **186**, L37.
Hurley, J. 1961*a*, *Phys. Fluids*, **4**, 109.
———. 1961*b*, *Phys. Fluids*, **4**, 854.
Illarionov, A. F., and Sunyaev, R. A. 1975, *Astr. Ap.*, **39**, 185.
Ives, J. C., Sanford, P. W., and Bell-Burnell, S. J. 1975, *Nature*, **254**, 578.
Lamb, D. Q., Lamb, F. K., and Pines, D. 1973, *Nature Phys. Sci.*, **246**, 52.
Lamb, F. K. 1975*a*, Proc. 7th Texas Symposium on Relativistic Astrophysics, *Ann. NY Acad. Sci.*, **262**, 331.
———. 1975*b*, in *Proc. of the International Conference on X-rays in Space*, ed. D. Venkatesan (Calgary: University of Calgary), p. 613.
Lamb, F. K., Fabian, A. C., Pringle, J. E., and Lamb, D. Q. 1977, *Ap. J.*, in press.
Lamb, F. K., and Pethick, C. J. 1974, in *Astrophysics and Gravitation, Proceedings of the 16th Solvay Congress* (Brussels: L'Université de Bruxelles), p. 135.
Lamb, F. K., Pethick, C. J., and Pines, D. 1973, *Ap. J.*, **184**, 271 (LPP).
Levich, E. V., and Sunyaev, R. A. 1971, *Soviet Astr.—AJ*, **15**, 363.
Lewin, W. H. G. 1977, *M.N.R.A.S.*, **179**, 43.
Lewin, W. H. G., Hoffman, J., and Doty, J. 1976, presented at AAS High Energy Astrophysics Division Meeting, January 27–29, Cambridge, MA.
Lightman, A. P., Shapiro, S. L., and Rees, M. 1976, Proc. Enrico Fermi Summer School on the Physics and Astrophysics of Neutron Stars and Black Holes, Varenna, Italy, in press.
Lucke, R., Yentis, D., Friedman, H., Fritz, G., and Shulman, S. 1976, *Ap. J.* (*Letters*), **206**, L25.
Maraschi, L., Treves, A., and van den Heuvel, E. P. J. 1976, presented at AAS High Energy Astrophysics Division Meeting, January 27–29, Cambridge, MA.
McClintock, J. E., *et al.* 1976, *Ap. J.* (*Letters*), **206**, L99.
McCray, R., and Lamb, F. K. 1976, *Ap. J.* (*Letters*), **204**, L115.
Michel, F. C. 1977, *Ap. J.*, in press.
Midgley, J., and Davis, L. 1962, *J. Geophys. Res.*, **67**, 499.
Pandharipande, V. R., Pines, D., and Smith, R. A. 1976, *Ap. J.*, **208**, 550.
Phelps, A. D. R. 1969, D.Phil. thesis, Oxford University.
———. 1973, *Planet. Space Sci.*, **21**, 1497.
Pringle, J. E., and Rees, M. 1972, *Astr. Ap.*, **21**, 1.
Rappaport, S., Joss, P. C., and McClintock, J. E. 1976, *Ap. J.* (*Letters*), **206**, L103.
Rosenberg, F. D., Eyles, C. J., Skinner, G. K., and Willmore, A. P. 1975, *Nature*, **256**, 628.
Rosenbluth, M. N., and Longmire, C. L. 1957, *Ann. Phys.*, **1**, 120.
Schmidt, G. 1966, *Physics of High Temperature Plasmas* (New York: Academic Press), chap. 5.
Shapiro, S. L., and Lightman, A. P. 1976, *Ap. J.*, **204**, 555.
Shapiro, S. L., and Salpeter, E. E. 1975, *Ap. J.*, **198**, 671.
Spalding, I. J. 1971, in *Advances in Plasma Physics*, ed. A. Simon and W. B. Thompson (New York: Interscience), Vol. **4**, p. 79.
Spitzer, L. 1962, *Physics of Fully Ionized Gases* (New York: Interscience).
Spreiter, J. R., and Summers, A. L. 1967, *Planet. Space Sci.*, **15**, 787.
Tidman, D. A., and Krall, N. A. 1971, *Shock Waves in Collisionless Plasmas* (New York: Wiley-Interscience).
Weymann, R. 1965, *Phys. Fluids*, **8**, 2112.
White, N. E., Huckle, H. E., Mason, K. O., Charles, P. A., Pollard, G., Culhane, J. L., and Sanford, P. W. 1975, *IAU Circ.*, No. 2870.
White, N. E., Mason, K. O., and Sanford, P. W. 1975, *IAU Circ.*, No. 2854.
White, N. E., Huckle, H. E., Mason, K. O., Charles, P. A., Pollard, G., Culhane, J. L., and Sanford, P. W. 1976, presented at AAS High Energy Astrophysics Division Meeting, January 27–29, Cambridge, MA.
Williams, D. J. 1975, *M.N.R.A.S.*, **171**, 537.
Willis, D. M. 1971, *Rev. Geophys. Space Phys.*, **9**, 953.

Note added in proof.—The period derivatives of the pulsating sources 3U 0900−40, SMC X-1, 3U 1728−24 = GX 1+4, 3U 1223−62 = GX 301-2, and A0535+26 have recently been measured (Henry and Schreier 1977, *Ap. J. Letters*, **212**, L13; Rappaport and Joss 1977, *Nature*, **266**, 683; Primini, Rappaport, and Joss, preprint). The corresponding value of $\omega_p^{(0)}$ (see § IV and Table 3) is of order unity in each case except that of 3U 0900−40 for which one finds $\omega_p^{(0)} \approx 0.06$. The present study, which assumes that the accreting plasma approaches the magnetosphere in approximately spherically symmetric free fall, is therefore directly applicable to 3U 0900−40 but probably not to the other four sources, where the angular velocity of the accreting plasma is apparently significant. However, even in these four sources the Rayleigh-Taylor instability may well play an important role in allowing plasma to penetrate the magnetosphere (cf. § IV*d*). The applicability of the present study to the pulsating sources A1118−61, 3U 0352+30, 3U 1813−14 = GX 17+2, 3U 1258−61 = GX 304-1 (McClintock *et al.*, preprint), 3U 1626−67 (Markert *et al.*, *IAU Circ.*, No. 3054), and 3U 1538−52 (Becker *et al.*, *IAU Circ.*, No. 3039) is as yet unknown.

R. F. Elsner: Center for Radiophysics and Space Research, Space Sciences Building, Cornell University, Ithaca, NY 14853

F. K. Lamb: Department of Physics, University of Illinois, Urbana, IL 61801