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GALACTIC DISTRIBUTION AND EVOLUTION OF PULSARS

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ABSTRACT

The distribution of pulsars with respect to period, z-distance, luminosity, and galactocentric radius has been investigated using data from three extensive pulsar surveys—those carried out at Molonglo, Jodrell Bank, and Arecibo Observatories. It is shown that selection effects only slightly modify the observed period and z-distributions but strongly affect the observed luminosity function and galactic distribution. These latter two distributions are computed from the Jodrell Bank and Arecibo data, using an iterative procedure. The largest uncertainties in our results are the result of uncertainty in the adopted distance scale. Therefore, where relevant, separate calculations have been made for two values of the average interstellar electron density, $\langle n_e \rangle$, 0.02 cm^{-3} and 0.03 cm^{-3}.

The derived luminosity function is closely represented by a power law with index (for logarithmic luminosity intervals) close to -1. For $\langle n_e \rangle = 0.03$ cm^{-3}, the density of potentially observable pulsars is about 90 kpc^{-2} in the local region and increases with decreasing galactocentric radius. These distributions imply that the total number of pulsars in the Galaxy is about 10^5. If only a fraction of all pulsars are observable because of beaming effects, then the total number in the Galaxy is correspondingly greater.

Recent observations of pulsar proper motions show that pulsars are generally high-velocity objects. The observed z-distribution of pulsars then implies that the mean age of observable pulsars does not exceed 2×10^6 years, much less than their mean characteristic age. With this mean age the pulsar birthrate required to maintain the observed galactic distribution is 10^{-4} yr^{-1} kpc^{-2} in the local region and one pulsar birth every 6 years in the Galaxy as a whole. For $\langle n_e \rangle = 0.02$ cm^{-3}, the corresponding rate is one birth every 40 years. These rates exceed most estimates of supernova occurrence rates and may require that all stars with mass greater than $\sim 2.5\ M_\odot$ form pulsars at the end of their evolutionary life.

Subject headings: galaxies: Milky Way — galaxies: structure — luminosity function — pulsars — stars: evolution — stars: stellar statistics

I. INTRODUCTION

There are now 149 known pulsars. This number is sufficiently large to permit an investigation of their distribution with respect to various parameters. The most important intrinsic parameters are the pulsar period, P, and the radio luminosity, L. The other basic parameters for which distributions can be obtained from existing data are related to position within the Galaxy. Since there is no substantial contrary evidence, we assume the galactic distribution to be cylindrically symmetric, and we derive the distributions with respect to galactocentric radius, R, and distance from the galactic plane, z.

The apparent distributions with respect to these parameters are modified from the true distributions by selection effects. We will argue below that selection effects do not significantly modify the observed period and z-distributions, whereas they greatly affect the distributions with respect to luminosity and galactocentric radius. Many different groups have searched for pulsars, using many different methods. However, 110 of the 149 known pulsars were first detected in one of three relatively extensive pulsar surveys: the Molonglo survey (Large and Vaughan 1971), the Jodrell Bank survey (Davies, Lyne, and Seiradakis 1972, 1973, 1977), and the University of Massachusetts–Arecibo survey (Hulse and Taylor 1974, 1975). Selection effects can be computed for each of these three surveys with a fair degree of certainty. All three searches were made at radio frequencies near 400 MHz.

To obtain space densities and luminosities, one must have a method of estimating pulsar distances. Neutral hydrogen absorption measurements (Gómez-Gonzáles and Guélin 1974; Ables and Manchester 1976) show that, especially for the more distant pulsars, dispersion measure is a reasonably accurate distance indicator. For the closer pulsars, those within ~ 1 kpc, H II regions can make a significant contribution to the dispersion measure. However, by using methods such as those of Prentice and ter Haar (1969), we can largely remove the effects of H II regions within this radius.

Arguments based on interstellar scintillation of pulsar signals (e.g., Rickett 1970) and pulsar–supernova remnant associations (e.g., Prentice 1970) have, for some time, suggested that pulsars have large velocities. More recently, direct measurements of pulsar proper motions have confirmed these suggestions and shown that pulsars typically have space velocities in excess of 100 km s^{-1}. This fact, together with the observed z-distribution, places a firm statistical limit of the order of a few million years on the average lifetime of pulsars, much less than the average characteristic age derived from pulse timing measurements.

If pulsars typically have lifetimes of only a few million years, then many must "turn off" at relatively short periods. In addition, unless the braking index, n, defined by

$$d\Omega/dt = -K\Omega^n\,, \tag{1}$$

where K is a positive constant and Ω is the angular pulsation frequency, is much greater than 3 at some stage of a pulsar's life, then many pulsars must be born with relatively long periods.

Knowing the mean active lifetime of pulsars, we can compute the pulsar birthrate necessary to maintain the observed galactic distribution. If only about 20% of all pulsars are observable because of beaming effects, then the computed birthrate of pulsars exceeds estimates of the rate of supernova explosions in the Galaxy derived from observations of supernova remnants. Comparison of the derived pulsar birthrate with computed stellar death rates shows that essentially all stars of mass greater than 2.5 $M_\odot$ may have to become pulsars at the end of their lives.

II. PULSAR DISTRIBUTIONS

In a given survey, the number of detectable pulsars having periods between P and $P + dP$, luminosities between L and $L + dL$, galactocentric radii between R and $R + dR$, and perpendicular distances from the galactic plane between z and $z + dz$ is given by

$$N_1(P, z, R, L)dPdzdRdL = V(P, z, R, L)\rho(P, z, R, L)dPdzdRdL\,, \tag{2}$$

where $V(P, z, R, L)dzdR$ is the volume of the Galaxy between R and $R + dR$ and z and $z + dz$ searched for pulsars of period P and luminosity L, and $\rho(P, z, R, L)dPdL$ is the space density of pulsars having periods between P and $P + dP$ and luminosities between L and $L + dL$ at the galactic position (z, R). Observations show that, apart from a weak correlation between period and luminosity, the distributions of pulsars with respect to these parameters are essentially uncorrelated. Therefore the density ρ can be separated into four independent functions, as follows:

$$\rho(P, z, R, L) = \rho_P(P)\rho_z(z)\rho_R(R)\rho_L(L)\,. \tag{3}$$

We further define a logarithmic luminosity function

$$\Phi(L) = L\rho_L(L)\,, \tag{4}$$

and choose to normalize the functions in equation (3) so that $\Phi(L)$ represents the number of pulsars per unit logarithmic luminosity interval, per unit area projected onto the galactic plane at $R = R_\odot$, where $R_\odot$ is the distance of the Sun from the galactic center, taken to be 10 kpc. Therefore

$$\int_0^\infty \rho_P(P)dP = \int_{-\infty}^\infty \rho_z(z)dz = \rho_R(R_\odot) = 1\,. \tag{5}$$

Equation (2) can be solved by using an iterative procedure (Large 1971) to give the four distributions. In fact, the observed period and z-distributions are not strongly affected by selection, so these distributions can be determined directly from the observed data. The observed luminosity distribution and radial distribution are strongly modified by selection effects and so $\Phi(L)$ and $\rho_R(R)$ are solved for by using the iterative procedure.

a) *Period Distribution*

For the Molonglo survey, pulsars were detected by finding spikes on a chart record, whereas for the Jodrell and Arecibo surveys a computer search for periodicities was carried out. Because of the post-detection time constant of the Molonglo recording system, this search had reduced sensitivity for periods shorter than about 0.3 s (Large 1971). For the Jodrell surveys, the basic period range searched was 0.16 to 4 s; because of the small duty cycle of most pulsars ($\lesssim 5\%$), this survey would also begin to lose sensitivity at about 0.3 s. The basic period range searched at Arecibo was 0.03 to 3.9 s and this survey maintained full sensitivity to about 0.06 s period.

Figure 1 gives histograms of the period distributions for (*a*) the 50 pulsars detected at Arecibo; (*b*) the 99 other known pulsars; and (*c*) all 149 known pulsars. At the long-period end the distributions fall off at about 2.0 s, less than the long-period cutoff for any of the surveys, so the observed period distribution is not significantly modified at long periods by selection effects. Likewise, for short periods the observed cutoff for Arecibo pulsars is similar to that for the other

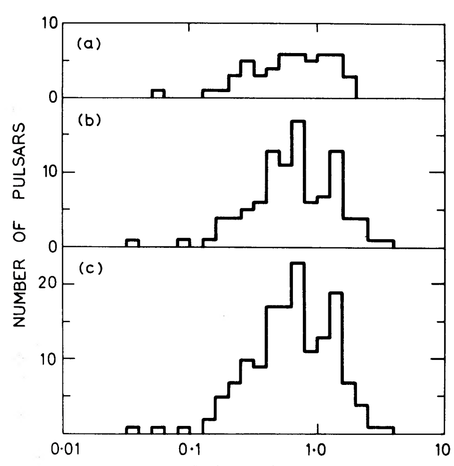

FIG. 1.—Observed period distribution for (*a*) pulsars detected in the Arecibo search, (*b*) all other pulsars, and (*c*) all known pulsars.

searches, despite the fact that the Arecibo search maintained full sensitivity to much shorter periods. To illustrate this, the median period for pulsars detected at Arecibo is equal to that for pulsars detected in other searches, both being 0.65 s. From these arguments one can conclude that the period distribution for all known pulsars (Fig. 1*c*) is not significantly influenced by selection effects and so is a good approximation to the true distribution of pulsar periods. The volume V in equation (2) is therefore effectively independent of period.

b) *Pulsar Distances and z-Distribution*

For most pulsars the dispersion measure is the only available distance indicator. For an electron layer with mean density $\langle n_e \rangle$ at $z = 0$ and an exponential z-dependence with scale height h_e, the distance to a pulsar with dispersion measure DM is given by

$$d = \frac{-h_e}{\sin|b|} \ln\left[1 - \frac{\mathrm{DM}\sin|b|}{h_e \langle n_e \rangle}\right] . \qquad (6)$$

This equation ignores any radial dependence of h_e and $\langle n_e \rangle$, although there is some evidence (Ables and Manchester 1976) for a radial variation in $\langle n_e \rangle$. If it is known that the line of sight to the pulsar passes through an H II region, the contribution of this H II region to the dispersion measure can be estimated and subtracted from DM before computing the distance. For the calculations presented here, this has been done by using the techniques and H II region list of Prentice and ter Haar (1969).

Independent distance estimates based on 1420 MHz hydrogen-line absorption of pulsar signals or pulsar–supernova remnant associations are currently available for 29 pulsars (Ables and Manchester 1976). Based on these results, a value of $\langle n_e \rangle = 0.03$ cm^{-3} was adopted for use in equation (6). There is some indication from the hydrogen absorption data and the Arecibo search results (Hulse and Taylor 1975) that, at least in certain directions, $\langle n_e \rangle$ may be closer to 0.02 cm^{-3}. We have therefore repeated all relevant calculations using $\langle n_e \rangle = 0.02$ cm^{-3} in equation (6). Where independent distance estimates are available they have been adopted in preference to the dispersion-derived distance.

The scale height of the electron distribution, h_e, has been deduced from observations of free-free absorption and interstellar scattering of extragalactic radio sources. Comparison of free-free absorption at 10 MHz for directions in the galactic plane and toward the galactic poles (Bridle and Venugopal 1969) suggests that the equivalent half-thickness of the thermal electron layer is at least 350 pc. Interstellar scattering results (Readhead and Duffett-Smith 1975) give a value of between 430 and 650 pc.

The distribution of the parameter DM sin $|b|$—the "z-component" of dispersion measure—is dependent on the z-distributions of both the electrons and pulsars. Figure 2 shows the observed distributions of this parameter for (*a*) pulsars detected at Molonglo, (*b*) pulsars detected at Arecibo, and (*c*) all known pulsars. Included in the Molonglo data are the 32 pulsars first

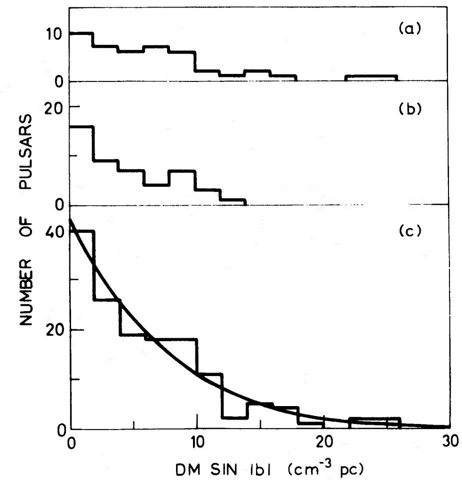

FIG. 2.—Observed distribution of the "z-component" of dispersion measure DM sin $|b|$ for (*a*) pulsars detected at Molonglo, (*b*) pulsars detected at Arecibo, and (*c*) all pulsars with known dispersion measure.

detected there (Large and Vaughan 1971; Vaughan and Large 1972) and 12 pulsars which are within the Molonglo search area and have mean flux densities above the limiting value for this survey (~75 mJy) but which were first detected at another observatory. Three pulsars (PSR 2020+28, 2024+21, and 2028+22) have been deleted from the list of sources detected at Arecibo because they lie outside the area where the search was complete. Hence the total number of pulsars included in Figure 2*a* is 44, in Figure 2*b* 47, and in Figure 2*c* 148. (No dispersion measure is known for PSR 0904+77.)

These figures show that a representative maximum values of DM sin $|b|$ is about 20 cm^{-3} pc. Larger values of DM sin $|b|$ are observed for a few pulsars; these may represent cases where an H II region lies in the path to a relatively nearby source. For example, PSR 0736−40 has DM sin $|b|$ = 25.7 cm^{-3} pc, but lies behind the Gum Nebula. Taking 20 cm^{-3} pc as an upper limit to DM sin $|b|$, this implies that for a mean electron density of 0.03 cm^{-3} the equivalent half-thickness of the electron layer h_e is more than or on the order of 650 pc.

It is clear from the monotonic decrease in number of pulsars with increasing DM sin $|b|$ that the electron distribution is wider than the pulsar distribution. With some assumptions it is possible to determine the widths of both distributions from the observed distribution of DM sin $|b|$. For an electron density distribution which is solely a function of z, the "z-component" of dispersion may be defined as

$$\Delta(z) \equiv \mathrm{DM}\sin|b| = \int_0^z n_e(z')dz' . \qquad (7)$$

If the number density of pulsars, N_p, is similarly assumed to be a function of z only, then the number of pulsars in equal intervals of Δ is given by

$$N(\Delta) = N_p[z(\Delta)]dz/d\Delta \qquad (8)$$

(cf. Gould 1971). For exponential distributions of electrons and pulsars, i.e.,

$$n_e(z) = n_e(0) \exp(-|z|/h_e) \tag{9}$$

and

$$N_p(z) = N_p(0) \exp(-|z|/h_p), \tag{10}$$

the expected shape of the histogram is

$$N(\Delta) = N_p(0)\left[1 - \frac{\Delta}{h_e n_e(0)}\right]^{(h_e/h_p)-1}, \quad \Delta < h_e n_e(0),$$
$$= 0, \quad \Delta \geq h_e n_e(0). \tag{11}$$

For the Molonglo data (Fig. 2*a*), a least-squares fit of this relation with $n_e(0) = 0.03\ \mathrm{cm}^{-3}$ gave $h_e = 650 \pm 150$ pc and $h_p = 275 \pm 25$ pc ($1\ \sigma$ errors). For the Arecibo data (Fig. 2*b*), the solution for h_e was not significant; under the assumption that $h_e = 1000$ pc, these data have $h_p = 190 \pm 20$ pc. A fit to all of the available data gave $h_e = 1800 \pm 1500$ pc and $h_p = 230 \pm 20$ pc, and this solution is shown on Figure 2*c*. For $n_e(0) = 0.02\ \mathrm{cm}^{-3}$ the corresponding values are $h_e = 2800 \pm 2000$ pc and $h_p = 350 \pm 25$ pc. The quantity $N(\Delta)$ is not very sensitive to changes in the scale height of the electrons. However, it is clear that this scale height is of the order of 1000 pc, much larger than that of most other galactic components. In the following sections of this paper we have taken $h_e = 1000$ pc when computing distances, using equation (6).

The Molonglo survey covered nearly all of the sky south of declination +20°, so this search did not discriminate against pulsars at high galactic latitudes and therefore large values of z. In fact, because of the reduced sky background temperature, the sensitivity of this survey was somewhat greater away from the galactic plane. In contrast, the Arecibo survey was restricted to $|b| \leqslant 4°$. Thus, the values of pulsar scale height derived from the uncorrected distributions of these two surveys probably bracket the true scale height, and we adopt $h_p = 230$ pc as a representative value.

Table 1 gives values of $\langle|z|\rangle$ for several subsets of the known pulsars and the set of all pulsars with known dispersion measure. These groups sample different ranges of distance from the Sun and/or galactocentric radius and yet there is no significant difference between the values of $\langle|z|\rangle$.

These results suggest that the observed z-distribution of pulsars has not been seriously modified by selection effects and is not a strong function of position within the Galaxy. We therefore take

$$\rho_z(z) = \left(\frac{1}{460}\right) \exp\left(-\frac{|z|}{230}\right) \tag{12}$$

as a reasonable approximation to the normalized z-distribution of pulsars. From an analysis of the Jodrell Bank data alone, Davies, Lyne, and Seiradakis (1977) find, for an assumed electron density of $0.025\ \mathrm{cm}^{-3}$, $\langle z^2\rangle^{1/2} = 320$ pc and $\langle|z|\rangle = 270$ pc, whereas, from the Arecibo data alone, Roberts (1976) finds a scale height $h_p \gtrsim 400$ pc for an electron density of $0.02\ \mathrm{cm}^{-3}$. These results are all in reasonable agreement and indicate that the remaining uncertainty in the pulsar z-distribution is dominated by the poorly known value of the local mean electron density.

c) Luminosity Function and Galactic Distribution

It is generally assumed that pulsar pulses are observed when a rotating beam intercepts the Earth. Unfortunately, our knowledge of the shape and orientation of this beam is very limited, and it is not possible to calculate accurate luminosities for pulsars. Instead, we adopt as a pulsar "luminosity" at radio frequencies the simple parameter $L = Sd^2$, where S is the observed mean flux density at 400 MHz in millijanskys (i.e., $10^{-29}\ \mathrm{W\ m^{-2}\ Hz^{-1}}$) and d is the estimated pulsar distance in kiloparsecs. Mean flux density values were obtained from Davies, Lyne, and Seiradakis (1977) for the Jodrell Bank survey, from Hulse and Taylor (1975) for the Arecibo survey, and otherwise from the compilation of Taylor and Manchester (1975). Because of a revised calibration, all flux densities quoted by Hulse and Taylor (1975) were increased by a factor of 2. The distribution plotted in Figure 3 shows that most observed pulsars have luminosities of between 30 and 1000 mJy kpc², although the overall range extends over five orders of magnitude. We show below that the small number of pulsars observed with low luminosities is entirely the result of sensitivity limitations, whereas the falloff at high luminosities reflects a real scarcity of high-luminosity pulsars.

TABLE 1
PARAMETERS OF FOUR SELECTED GROUPS OF PULSARS

Sample	No. of Pulsars	Mean Distance from Sun (kpc)	Mean Galactocentric Radius (kpc)	$\langle\|z\|\rangle$ (pc)
Molonglo............	44	2.5 ± 0.5	9.2 ± 0.3	220 ± 40
Jodrell Bank.......... ($\|l\| \leq 45°$)	23	5.2 ± 0.9	7.1 ± 0.5	260 ± 50
Arecibo.............. (DM $\geq 150\ \mathrm{cm}^{-3}$ pc)	26	7.9 ± 0.6	8.5 ± 0.4	250 ± 30
All known............	148	3.9 ± 0.3	9.0 ± 0.2	230 ± 25

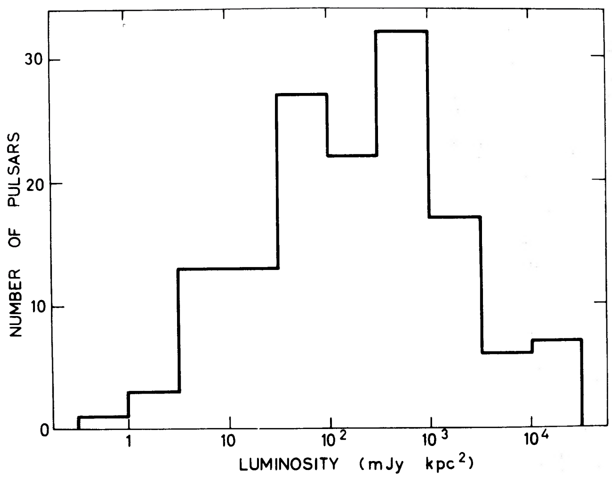

FIG. 3.—The observed luminosity distribution of pulsars

The observed distribution of pulsars within 1.5 kpc of the Sun is shown projected on to the galactic plane in Figure 4. This figure shows that (*a*) the density of known pulsars decreases with increasing distance from the Sun, and (*b*) there are more known pulsars in the hemisphere toward the galactic center than there are in the hemisphere toward the anticenter. Effect (*a*) is a result of selection, as all searches are insensitive below a certain flux density level. There are 20 known pulsars within 0.5 kpc of the Sun, implying a local density of 25 kpc^{-2}. Because of incompleteness of the surveys, especially for flux densities below 50 mJy, this value represents a lower limit to the local pulsar density. For $\langle n_e \rangle = 0.02\ \mathrm{cm}^{-3}$ the corresponding density is 18 kpc^{-2}. The higher density of pulsars in directions toward the galactic center cannot be attributed to selection effects; it indicates that the density of pulsars is greater within the solar circle than outside it. Similar conclusions were reached by Lyne (1974) on the basis of the longitude dependence of pulsar densities derived from the Jodrell Bank sample, and by Hulse and Taylor (1975) on the basis of the lack of high-dispersion pulsars in the Arecibo sample.

We have shown above that neither the observed period distribution nor the observed z-distribution of pulsars is seriously affected by selection. Equation (2) can therefore be integrated over these variables to give

$$N(R, L)dRdL = A(R, L)\rho_R(R)dR\Phi(L)dL/L\,, \quad (13)$$

where

$$A(R, L)dR = \int_0^\infty dP \int_{-\infty}^\infty dz \rho_P(P)\rho_z(z)V(P, z, R, L)dR \quad (14)$$

represents the volume of the column (extended in the z-direction) between R and $R + dR$, weighted by the function $\rho_z(z)$, or, effectively, the area of the galactic disk between R and $R + dR$ searched for pulsars of luminosity L. The true density distributions $\rho_R(R)$ and

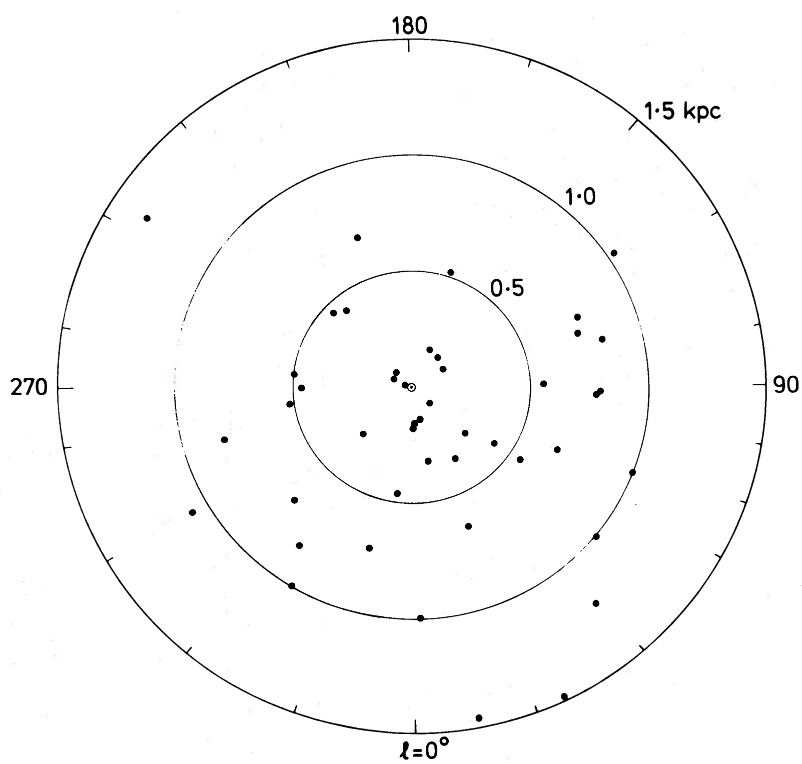

FIG. 4.—Observed distribution of pulsars within 1.5 kpc of the Sun projected on to the galactic plane

$\Phi(L)$ can then be obtained from the observed distributions in R and L by using

$$\rho_R(R) = \frac{\int N(R, L)dL}{\int A(R, L)\Phi(L)dL/L} \qquad (15)$$

and

$$\Phi(L) = \frac{L \int N(R, L)dR}{\int A(R, L)\rho_R(R)dR} . \qquad (16)$$

Because of the lack of selection against any period range where $\rho_P(P)$ is distinctly nonzero, the luminosity function $\Phi(L)$ will not be affected by the observed weak correlation between period and luminosity.

In the Jodrell Bank and Arecibo surveys the sensitivities were high enough so that areas at galactocentric radii significantly different from that of the Sun were effectively searched, whereas the Molonglo survey detected primarily "local" pulsars (see Table 1). Furthermore, the limiting mean flux density of these two surveys is somewhat better defined than that of the Molonglo survey because of the use of periodicity analysis rather than single-pulse detection. Therefore, to compute $\rho_R(R)$ and $\Phi(L)$, we have used only the data from the Jodrell Bank and Arecibo surveys, a total of 90 pulsars.

i) *Selection Effects*

The area function $A(R, L)$ for each survey was computed by using a Monte Carlo approach. A sample of points was generated, distributed randomly within a square 40 kpc × 40 kpc centered on the galactic center and with an exponential distribution in z of scale height 230 pc (350 pc for $\langle n_e \rangle = 0.02\ \mathrm{cm}^{-3}$). The galactic latitude and longitude of each point were computed, and for those points which fell within the area of sky covered by one of the surveys (Table 2), the expected dispersion measure was computed assuming an exponential electron distribution of scale height 1000 pc. The minimum detectable luminosity for a pulsar at the location of the sample point was then obtained using the relation

$$L_{\min} = S_{\min} d^2 , \qquad (17)$$

where d is the distance of the point from the Sun and

$$S_{\min} = \beta S_0(1 + T_{\mathrm{sky}}/T_R)(1 + \mathrm{DM}/\mathrm{DM}_0)^{1/2} . \qquad (18)$$

In this relation β is a factor representing the reduction in sensitivity resulting from displacement of the source from the beam center; T_{sky} is the sky background temperature at 400 MHz, approximated by

$$T_{\mathrm{sky}} = 30 + 270[1 + (l/40)^2]^{-1}[1 + (b/3)^2]^{-1}\ \mathrm{K} \qquad (19)$$

for l, b in degrees, and $-180° < l \le 180°$; DM is the dispersion measure for a pulsar at the sample point; and S_0, T_R and DM_0 are parameters given for each survey in Table 2. Parameters for the Jodrell Bank survey are from Davies, Lyne, and Seiradakis (1977), and those for the Arecibo survey are from Hulse and Taylor (1975). In the Jodrell Bank survey, part A, the sky was sampled on a square grid of 0°.707 centers using a beam of half-power width 0°.7, whereas for parts B and C the grid was at 1°.0 centers (Davies, Lyne, and Seiradakis 1977). The effect of this survey pattern on the volume sampled was estimated by randomly choosing a point within a 0°.707 square for each sample point in part A and within a 1°.0 square for points in parts B and C, and computing the beam factor β accordingly. For the Arecibo survey two effects contribute to the beam factor. The first results from the fact that scans were made at constant declination, with adjacent scans overlapping at the half-power points. A displacement from the beam center was randomly chosen within the half-power width for each sample point and the gain factor computed accordingly. The second factor contributing to β results from the zenith-angle dependence of the gain of the Arecibo antenna, which falls to a relative value of 0.4 at a zenith angle of 20°.

A 20 × 15 array of cells, each of dimensions 1 kpc × 1 semidecade of luminosity, was used to represent the area function $A(R, L)$. For each sample point which fell within the regions surveyed, elements of $A(R, L)$ at the appropriate R and at all $L \ge L_{\min}$ were incremented. Where the two surveys overlapped, parameters for the more sensitive survey were used. When this process had been repeated a sufficient number of times to obtain adequate statistics, the $A(R, L)$ array contained (after normalization) the total area of the galactic disk searched for each interval of galactocentric radius and luminosity. The desired density distributions $\rho_R(R)$ and $\Phi(L)$ were then obtained from the observed distribution $N(R, L)$ by numerically integrating equations (15) and (16) starting

TABLE 2

PARAMETERS OF PULSAR SURVEYS

Survey	Receiver Noise Temperature, T_R (K)	Minimum Detectable Flux Density, S_0 (mJy)	Dispersion Cutoff, DM_0 (cm^{-3} pc)	No. of Pulsars Detected
Jodrell Bank*.............	110	13	250	51
Arecibo†.................	130	1.5	1280	47

* Part A: l = 354° to 115°, $|b| < 7°$. Part B: l = 4° to 115°, $7° < |b| < 9°.5$. Part C: l = 115° to 240°, $|b| < 4°$.

† Part A: l = 42° to 60°, $|b| < 4°$. Part B: l = 187° to 191°, $|b| < 3°$ (the actual region surveyed was irregular in shape but covered 24 square degrees as indicated).

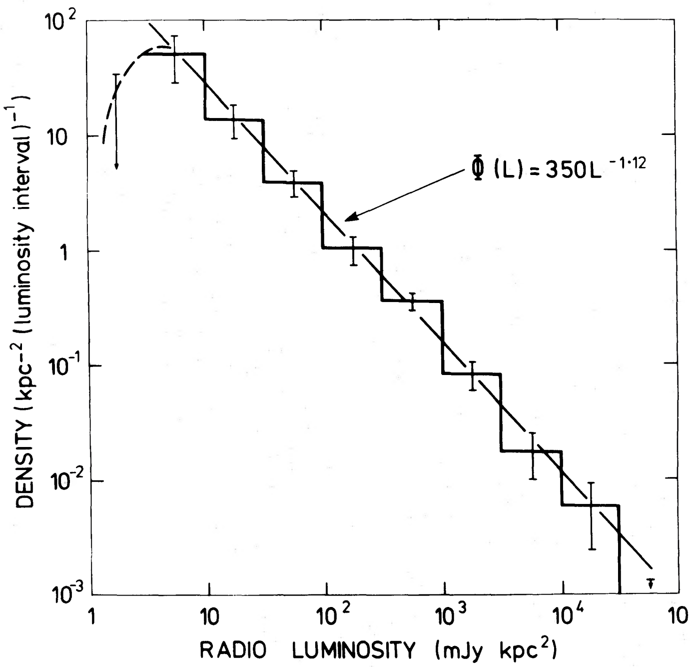

FIG. 5.—Luminosity function $\Phi(L)$, that is, density of pulsars per square kiloparsec projected on to the galactic plane at $R = R_\odot$ per semidecade of luminosity, based on data from the Jodrell Bank and Arecibo surveys and assuming a mean interstellar electron density $\langle n_e \rangle = 0.03\ cm^{-3}$. The straight line is a least-squares fit of a power law to the data (omitting the two upper limits) and has a slope of -1.12 ± 0.05.

with initial guesses and iterating a few times until a stable solution was obtained. Figures 5 and 6 show the results of these calculations for the case $\langle n_e \rangle = 0.03\ cm^{-3}$.

ii) *Luminosity Function*

The luminosity function shown in Figure 5 is clearly close to power law over the observed range. A weighted least-squares fit of the function

$$\Phi(L) = kL^{-\gamma} \quad (20)$$

to the datum points (excluding the upper limits) yielded the results $k = 350 \pm 50$ pulsars per square kiloparsec per semidecade luminosity interval and $\gamma = 1.12 \pm 0.03$ (1 σ errors). The observed upper limit at the low-luminosity end suggests that the luminosity function turns over at a luminosity of the order of 3 mJy kpc^2. Pulsars with luminosity of 3 mJy kpc^2 would have been detected by the Arecibo survey provided they were closer than about 1.5 kpc. For an assumed bandwidth of 400 MHz and a conical radiation beam of width 20°, this limiting luminosity is equivalent to 4×10^{26} ergs s^{-1}.

If the luminosity function is assumed to be power law at luminosities above $L_0 = 3$ mJy kpc^2 and zero at lower luminosities, then the total density of potentially observable pulsars in the solar neighborhood is

$$D(10) = \int_{L_0}^{\infty} \Phi(L) dL/L = 90 \pm 15\ kpc^{-2}. \quad (21)$$

For a scale height in z, $h_p = 230$ pc, the corresponding volume density of pulsars in the solar neighborhood is about 200 kpc^{-3}.

For an assumed mean electron density of 0.02 cm^{-3}, the derived luminosity function is again well represented by a power law with $k = 250 \pm 100$ and $\gamma = 0.95 \pm 0.05$. Because of the generally greater distance of the pulsars, the limiting luminosity appears to be higher, about 10 mJy kpc^2. If we adopt this value for L_0, the local area density of pulsars is $D(10) = 29 \pm 12$ kpc^{-1} and the corresponding volume density is $40 \pm 17\ kpc^{-3}$.

From an analysis of the Jodrell Bank survey data, Davies, Lyne, and Seiradakis (1977) also find a power-law luminosity function. Their calculations, which are based on a mean electron density $\langle n_e \rangle = 0.025\ cm^{-3}$, give a slope $\gamma = 0.96$, in good agreement with the values derived above. From the Arecibo survey data alone, Roberts (1976) obtains a marginally flatter luminosity function with $\gamma = 0.7 \pm 0.3$. It is likely that Roberts effectively overestimated the volume sampled at low luminosities. Analysis of the Arecibo data by using the procedures described above gives $\gamma = 1.0 \pm 0.1$.

iii) *Galactic Distribution*

The derived galactocentric distribution for $\langle n_e \rangle = 0.03\ cm^{-3}$, plotted in Figure 6 in units of pulsars per square kiloparsec, shows a large increase in density inside the solar circle ($R = 10$ kpc). For $R > 14$ kpc, the density drops effectively to zero, with only upper

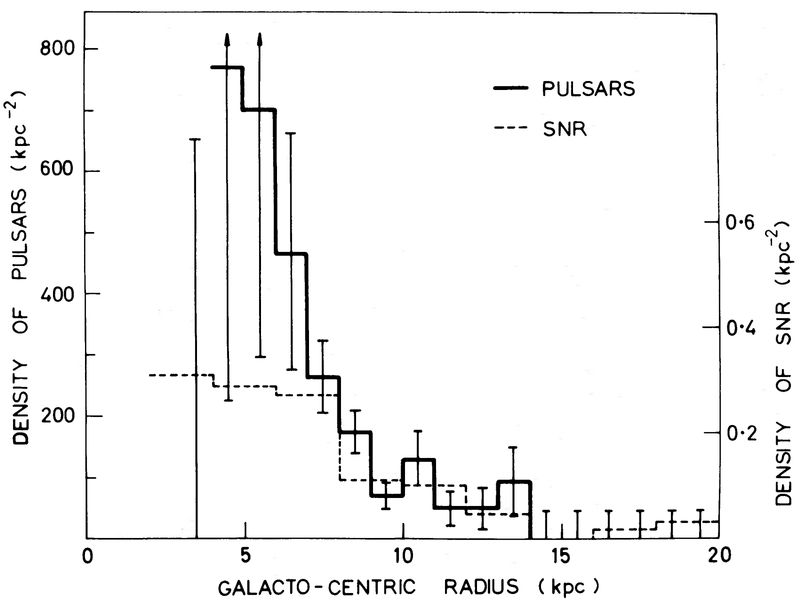

FIG. 6.—Density of pulsars projected on to the galactic plane as a function of galactocentric radius. The dashed line represents the density of supernova remnants (Clark and Caswell 1976).

limits being obtained. For $R < 7$ kpc the statistics become very poor, but the results indicate that the density may exceed 500 kpc^{-2}. On the other hand, densities lower than 200 kpc^{-2} would still be consistent with the derived values.

Also plotted in Figure 6 is the galactocentric distribution of known supernova remnants as derived by Clark and Caswell (1976). For $R \gtrsim 7$ kpc the two distributions are similar, a result consistent with the idea that pulsars are born in supernova events. (We shall show below, however, that the rate of pulsar formation is apparently much greater than the rate of supernova remnant production.) The radial distribution of pulsars is also very similar to that of other galactic constituents, such as carbon monoxide molecules, ionized hydrogen, and background γ-radiation (Burton 1976).

The total number of potentially observable pulsars in the Galaxy is given by the integral

$$N_G = 2\pi \int_0^\infty RD(R)dR\,, \qquad (22)$$

where $D(R) = D(10)\rho_R(R)$. Integration of the data in Figure 6 gives $N_G = (1.3 \pm 0.4) \times 10^5$, where the quoted error reflects only the statistical uncertainties in $D(R)$. A number of systematic uncertainties deriving from assumptions made in the analysis also affect the value of N_G. First, N_G is underestimated because the densities $D(R)$ were assumed to be zero for $R < 4$ kpc and $R > 14$ kpc. The effect of this is probably small, however, since for the interior part the area is small (only 8% of the area within 14 kpc) and for the outer part densities are probably low, so that the total contribution to N_G is not large. Any incompleteness in the surveys beyond that allowed for in the computation of $A(R, L)$ would also result in N_G being underestimated. The fact that pulsars with luminosity less than 3 mJy kpc^2 are known to exist (Fig. 3) suggests that the assumed cutoff to the luminosity function may be too abrupt, again leading to an underestimate of N_G. However, the greatest uncertainty in N_G comes from the poorly known value of the local mean electron density. For $\langle n_e \rangle = 0.02$ cm^{-3} the distribution in galactocentric radius is similar to that shown in Figure 6, except that number densities are lower and the increase in density in the inner regions is not as marked. For example, $D(5\text{ kpc}) = 90 \pm 45$ kpc^{-2}, about a factor of 3 greater than $D(10\text{ kpc}) = 29 \pm 12$ kpc^{-2}. The derived total number of observable pulsars in the Galaxy is $(2.1 \pm 0.9) \times 10^4$, about a factor of 6 smaller than the value based on $\langle n_e \rangle = 0.03$ cm^{-3}.

If pulsars are rotating objects which emit a narrow pencil beam of radiation, then only a fraction of the total number of pulsars will be observable from any one point in space. For a beam of half-width θ, making an angle α with the rotation axis, a fraction $f = \sin\alpha \sin\theta$ of 4π sr is swept across by the beam as the star rotates. Therefore, for a random distribution of angles, α, the mean fraction is $\langle f \rangle = (2/\pi)\sin\theta$. If we assume that emission beamwidths in latitude and longitude are comparable, then $\theta = 10°$, so $\langle f \rangle \approx 0.1$. Observations of interpulses suggest that pulsars may generally radiate two beams, so for $\theta \approx 10°$ about 20% of all pulsars will be observable from any one direction. Taking this factor into account, the total number of active pulsars in the Galaxy is $(6.5 \pm 2.0) \times 10^5$ for $\langle n_e \rangle = 0.03$ cm^{-3} and $(1.0 \pm 0.4) \times 10^5$ for $\langle n_e \rangle = 0.02$ cm^{-3}.

The space density and total number of observable pulsars in the Galaxy have previously been estimated by several authors. Large (1971) obtained a value of

$N_G = 5 \times 10^5$ based on a mean electron density of $0.05\ \text{cm}^{-3}$. For $\langle n_e \rangle = 0.03\ \text{cm}^{-3}$ this value would be reduced to about 2×10^5, in good agreement with the value of 1.3×10^5 obtained above. From their analysis of the Jodrell Bank data, Davies, Lyne, and Seiradakis (1977) obtain a local space density of $70\ \text{kpc}^{-3}$ and a total number of observable pulsars $N_G \approx 3 \times 10^5$. These values, based on $\langle n_e \rangle = 0.025$, are somewhat larger than the corresponding values derived above. This difference is primarily due to the lower value ($\sim$0.65 mJy kpc^2) adopted for the cutoff of the luminosity function. Roberts (1976) obtains the much lower value of $8\ \text{kpc}^{-3}$ for the local space density of observable pulsars. This value, based on a mean electron density $\langle n_e \rangle = 0.02$, is less than half of the lower limit derived above directly from the known pulsars within 0.5 kpc of the Sun (cf. Davies, Lyne, and Seiradakis 1976) and is a factor of 5 lower than the value of $40 \pm 17\ \text{kpc}^{-3}$ obtained from our complete analysis. A more recent computation by Roberts (private communication), which incorporates the revised flux density scale for the Arecibo pulsars, yields a density of $23\,(+33, -15)\ \text{kpc}^{-3}$, in satisfactory agreement with our results.

III. EVOLUTION OF PULSARS

a) *Pulsar Ages*

For most pulsars the only available age indicator is the so-called characteristic age, $\tau = \frac{1}{2}P(dP/dt)^{-1}$, based on the observed rate of period increase. For the Crab Nebula pulsar the value is $\tau = 1240$ years, close to the known age of about 920 years. Likewise, the characteristic age of the Vela pulsar, $\tau = 1.1 \times 10^4$ years, is consistent with age estimates of the Vela supernova remnant (Clark and Caswell 1976). These facts have been used to infer that characteristic ages are good indicators of the actual age of pulsars; that is, that pulsars are born with a very short period and subsequently slow down with a braking index $n \approx 3$. However, there are strong counterarguments which suggest that, at least for the longer-period pulsars, the characteristic age is often an overestimate of the true age.

A number of observations suggest that pulsars are born with (or acquire soon after birth) a large spatial velocity. The observed z-direction of pulsars has a scale height of the order of 250 pc compared with the 60–80 pc scale height for O–B stars commonly thought to be their progenitors. As first pointed out by Gunn and Ostriker (1970), these observations can be reconciled if pulsars are high-velocity objects which move significantly away from the plane during their lifetime. Observations of pulsar scintillation (e.g., Rickett 1970; Ewing *et al.* 1970; Galt and Lyne 1972) imply that the diffraction pattern formed by irregularities in the interstellar medium typically moves across the Earth at velocities of a hundred kilometers per second or more. Since this velocity is greater than that of the Earth and of probable velocities of diffracting irregularities, the simplest explanation is that the pulsars are moving with velocities of this order.

Another independent indication of high velocities is obtained for those pulsars associated with supernova remnants. From an analysis of filamentary velocities, Trimble (1971) showed that the expansion center of the Crab Nebula was displaced from the pulsar, implying a transverse velocity for the pulsar of about $100\ \text{km s}^{-1}$ since birth. The Vela pulsar is similarly displaced from the center of the supernova remnant. In this case a higher velocity of about $500\ \text{km s}^{-1}$ is indicated.

More recently, proper motion has been directly measured for several radio pulsars (Manchester, Taylor, and Van 1974; Anderson, Lyne, and Peckham 1975; Backer and Sramek 1976). These results, summarized in Table 3, show that velocities in excess of $100\ \text{km s}^{-1}$ are common. The mean transverse velocity for this sample of pulsars is $180\ \text{km s}^{-1}$. If we assume that the velocity vectors are randomly oriented, then the mean z-component of velocity is $\langle |v_z| \rangle \approx 180/\sqrt{2} \approx 130\ \text{km s}^{-1}$. For these objects $\langle |z| \rangle \approx 175$ pc, so if they were born close to the galactic plane their mean age must be $\langle |z| \rangle / \langle |v_z| \rangle \approx 1.3 \times 10^6$ years, much less than their mean characteristic age of 5×10^6 years. This result is unaffected by the uncertainty in the mean electron density, because both $\langle |v_z| \rangle$ and $\langle |z| \rangle$ are proportional to distance. Similar arguments can be made for the much larger sample of all pulsars with known characteristic ages, provided it is assumed that the mean velocity obtained above is representative for the larger sample. While there may be some bias toward pulsars with large

TABLE 3

PULSAR PROPER MOTIONS AND TRANSVERSE VELOCITIES

PSR	Characteristic Age, τ (10^6 yr)	Proper Motion, μ (0\".001 yr^{-1})	Distance, d (pc)	Transverse Velocity (km s^{-1})	z (pc)	References
0531+21	0.0012	12 ± 3	2000	110	−200	1
1237+25	2.3	102 ± 18	370	190	+370	2
0834+06	3.0	52 ± 14	480	120	+210	2
1929+10	3.1	159 ± 25	110	80	−7	3
1133+16	5.0	365 ± 36	180	310	+160	2, 3, 4
0823+26	5.0	135 ± 10	790	510	+420	2
0329+54	5.5	14 ± 4	2500	170	−54	2
2016+28	5.9	20 ± 8	1000	95	−69	2
0950+08	17.3	$\lesssim 100$	100	$\lesssim 50$	+71	3

REFERENCES.—1, Trimble 1971; 2, Anderson *et al.* 1975; 3, Backer and Sramek 1976; 4, Manchester *et al.* 1974.

proper motion in the sample of Table 3, any such bias is thought to be small and hence unlikely to affect the conclusion seriously. For the sample of pulsars with known characteristic ages, $\langle|z|\rangle = 260$ pc, implying a mean "kinematic" age of 2×10^6 years, much less than the mean characteristic age of about 4×10^7 years.

In Figure 7 the pulsars have been divided into five groups of increasing characteristic age and $\langle|z|\rangle$ plotted against $\langle\tau\rangle$ for each group. If the characteristic age were a reliable indicator of true age, then $\langle|z|\rangle$ would be a monotonically increasing function of $\langle\tau\rangle$. Clearly this is not the case. The full line in Figure 7 is computed on the assumptions that the parent population is exponentially distributed in z with a scale height of 80 pc and that the pulsars have an isotropic velocity distribution with a mean amplitude $\langle|v|\rangle = \sqrt{3}\langle|v_z|\rangle = 260$ km s^{-1} (cf. Gunn and Ostriker 1970). The dashed curves bracket these values by a factor of 2 in each direction. This figure demonstrates that if pulsars typically have space velocities of the order of 200–300 km s^{-1}, then their mean age cannot exceed a few million years. If we adopt 2×10^6 years for the present mean age of observable pulsars, then these pulsars must, on the average, have an active lifetime of twice this value—i.e., 4×10^6 years.

Several possible reasons for the turnoff of pulse emission have been proposed. Decay of the pulsar magnetic field on the time scale of a few million years could produce the observed distributions (Gunn and Ostriker 1970; Lyne, Ritchings, and Smith 1975). Likewise, magnetic alignment on a similar time scale (e.g., Jones 1976) would result in a turnoff of the pulsed emission. In the models of Sturrock (1971) and Ruderman and Sutherland (1975), emission ceases when the accelerating potentials are no longer sufficient to produce a cascade of electron-positron pairs.

Besides implying turnoff of the pulsed emission at a relatively short period for most pulsars, a short lifetime for pulsars has other important implications. An actual age less than the characteristic age for a pulsar implies either that the pulsar was born with a relatively long period, or that the braking index has been substantially greater than 3 at some stage during the lifetime of the pulsar. For example, multipole radiation may be important during the early stages of a pulsar's life, leading to a rapid increase in period. The fact that the Crab pulsar has a braking index which is less than 3 (Groth 1975) suggests that this effect is unlikely to be important for most pulsars. Magnetic decay provides an alternative mechanism. Significant decay near the end of a pulsar's life would reduce the period derivative and increase both the characteristic age and the braking index. Determination of the braking index of pulsars whose characteristic age is greater than a few million years would provide a test of these two alternatives. A braking index greater than 3 would indicate magnetic decay, whereas a braking index less than or equal to 3 would indicate that the pulsar was born with a period not much less than its present value. Unfortunately, with present techniques, a datum span of many years is required to put significant upper limits on the braking index of pulsars with large characteristic ages.

For a mean active lifetime of 4×10^6 years, the local density of observable pulsars computed above, $D(10) = 90 \pm 15$ kpc^{-2}, implies a pulsar birthrate of 2.2×10^{-5} yr^{-1} kpc^{-2}. If we further assume that only 20% of all pulsars are seen because of beaming effects, then the required local birthrate is about 10^{-4} yr^{-1} kpc^{-2}, and the corresponding rate for the Galaxy as a whole is one pulsar birth every 6 years. If $\langle n_e\rangle = 0.02$ cm^{-3} rather than 0.03 cm^{-3}, then the required birthrate in the local region (including the beaming

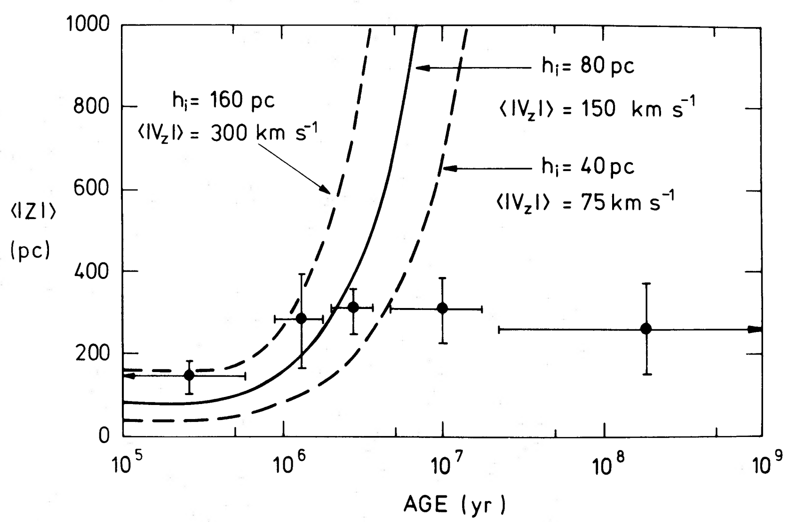

FIG. 7.—Mean absolute value of the z-distance for five groups of pulsars divided according to characteristic age. The solid line represents the expected value of $\langle|z|\rangle$ for a population of pulsars of initial scale height 80 pc moving in random directions with a mean velocity of 260 km s^{-1}, corresponding to $\langle|v_z|\rangle = 150$ km s^{-1}. The dashed curves are for velocities and scale heights different by a factor of 2, as indicated on the figure.

factor) drops to about 3×10^{-5} yr^{-1} kpc^{-2}, or one every 40 years in the Galaxy. If very many pulsars with ages $\leqslant 4 \times 10^6$ years have luminosities less than 3 mJy kpc^2 (10 mJy kpc^2 for $\langle n_e \rangle = 0.02$ cm^{-3}), these birthrates would have to be increased accordingly.

It is generally assumed that pulsars are formed in supernova events. If all supernovae form pulsars, then the occurrence rate of supernovae should equal the pulsar birthrate. Using observations of supernovae in external galaxies, Tammann (1974) suggests that the expected time between supernova events in our Galaxy should be about 25 years and may be as short as 10 years if our Galaxy is of type Sbc. These intervals are in reasonable agreement with the above estimates of the pulsar birthrate, given the uncertainties involved. Supernova rates derived from radio observations of remnants are, however, much smaller. Ilovaisky and Lequeux (1972*a*) find a mean time between events of 50 ± 25 years, whereas Clark and Caswell (1976) obtain a best estimate of 150 years, again with about a 50% uncertainty. It is possible that some supernovae do not form supernova remnants as conventionally defined, or that some supernova remnants remain undetected, especially in the inner regions of the Galaxy (cf. Fig. 6).

Models of stellar evolution have so far been unable to define accurately the range of stellar masses likely to leave neutron star remnants. For example, some authors (e.g., Barkat, Reiss, and Rakavy 1974) suggest that stars with mass between about 7 and 10 $M_\odot$ will form neutron stars, whereas other authors (see Paczyński 1973) suggest that the lower mass limit is nearer 4 $M_\odot$. Ostriker, Richstone, and Thuan (1974) have investigated the death rates of stars in the mass ranges $4 < M/M_\odot < 8$ and $M/M_\odot > 8$ and find rates of 3.6×10^{-5} yr^{-1} kpc^{-2} and 1.1×10^{-5} yr^{-1} kpc^{-2}, respectively. Both of these rates are considerably less than the best estimate of the pulsar birthrate derived above, 10^{-4} yr^{-1} kpc^{-2}. For $\langle n_e \rangle = 0.02$ cm^{-3} the derived birthrate, 3×10^{-5} yr^{-1} kpc^{-2}, is comparable with the death rate of all stars with $M > 4\ M_\odot$. From the data of Ostriker, Richstone, and Thuan, all stars with mass greater than about 2.5 $M_\odot$ would have to form neutron stars at the end of their evolution to achieve a birthrate of 10^{-4} yr^{-1} kpc^{-2}.

Figure 7 shows that the scale height for even the youngest pulsars is $\langle |z| \rangle \approx 150$ pc, much greater than the values 60–80 pc applicable to OB stars, supernova remnants, and other extreme Population I objects (cf. Blaauw 1965; Ilovaisky and Lequeux 1972*b*). Even stars of mass $\approx 2\ M_\odot$ have $\langle |z| \rangle < 100$ pc. The relatively large scale height of young pulsars, together with the fact that theoretical models suggest that neutron stars will not result from collapse of stars with $M \leqslant 4\ M_\odot$, leads us to suggest that many pulsars may be the descendants of older objects. One possible class of progenitors would be white dwarfs, of mass close to the Chandrasekhar limit, which become unstable because of accretion or other reasons (Canal and Schatzman 1976) and produce neutron stars without producing a normal supernova remnant.

Let us now summarize briefly. We have considered quantitatively the selection effects which have determined the sample of known pulsars, and derived approximations to the true distributions of pulsars with respect to period, luminosity, distance from the galactic plane, and distance from the galactic center. From the measured proper motions of nine nearby pulsars we have shown that the mean age of observable pulsars cannot exceed approximately 2×10^6 years. This implies that the pulsar birthrate may be as high as 10^{-4} yr^{-1} kpc^{-2} in the local region, or one pulsar birth every 6 years in the Galaxy, and is too large to be consistent with either the death rate of massive stars or the rate of occurrence of the galactic supernovae which produce long-lived remnants. These results suggest that many pulsars may be formed in events which do not involve the explosive detonation of a high-mass star.

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