

THE SURFACE GRAVITY AND MASS OF ARCTURUS

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ABSTRACT

We estimate the surface gravity of the K giant Arcturus by comparing synthetic spectra based on model atmospheres with measured wing shapes of the Ca I and Ca II resonance lines calibrated by absolute photometry. Our result, $\log g \approx 1.6 \pm 0.2 \text{ cm s}^{-2}$, is consistent with previous spectroscopic estimates based on weak line ionization ratios, with the exception of a recent determination by Mäcke *et al.* We obtain a stellar mass of $1.1_{-0.7}^{+1.7} M_{\odot}$ for $R = 27 R_{\odot}$. This estimate suggests that Arcturus has *not* suffered substantial mass loss during its post-main-sequence evolution on the "ascending red-giant branch."

Subject headings: stars: atmospheres — stars: individual — stars: late-type

I. INTRODUCTION

Physical theories of stellar atmospheres and interiors ultimately must be tested against real stars. Heretofore, the Sun has served as a well-observed reference against which to measure the success of such theories. Unfortunately, the insights to be gained from this rather mundane, middle-aged, but conveniently nearby G2 V star are limited, especially for understanding evolution toward and away from the zero-age main sequence (ZAMS).

Efforts to establish other nearby stars as "standards," however, are invariably frustrated by the lack of accurate values for one or more of the fundamental stellar parameters: effective temperature, mass, radius, and metallicity.

This problem is especially apparent in the case of the red giant Arcturus (α Boo, K2 III): an effective temperature has been inferred for this evolved star based on broad- and narrow-band spectral indices (Mäcke *et al.* 1975b, hereafter Mäcke *et al.*; Blackwell *et al.* 1975; van Paradijs and Meurs 1974) and a model atmospheres analysis of calibrated scanner data (Johnson *et al.* 1977); a composition has been estimated by differential curves of growth (Griffin and Griffin 1967; Mäcke *et al.* [Sun- α Boo]; van Paradijs and Meurs 1974 [α Tau- α Boo]) and by means of photoelectric photometry in very narrow bands (Gustafsson, Kjaergaard, and Andersen 1974); an angular diameter has been measured directly by Michelson (M), amplitude (A), and speckle (S) interferometry (Pease 1931 [M]; Beavers 1965 [M]; Currie, Knapp, and Liewar 1974 [A]; Gezari, Labeyrie, and Stachnik 1972 [S]; Worden 1976 [S]); and the distance is well known (e.g., Woolley *et al.* 1970). However, Arcturus lacks visible companions, and hence the stellar mass must be determined indirectly.

Unfortunately, recent spectroscopic estimates of the surface gravity of Arcturus (and therefore also the stellar mass, because R is known) differ by up to an order of magnitude (Table 1). This range of inferred masses is uncomfortably large, particularly because the low end of the mass estimates $0.23_{-0.1}^{+0.6} M_{\odot}$ (Mäcke *et al.*) is mildly inconsistent with stellar evolution models which suggest that a star must have a *core* mass of at least 0.4 – $0.6 M_{\odot}$ and a total mass of $\geq 0.7 M_{\odot}$ to reach the position of Arcturus on the ascending red-giant branch in a time less than or comparable with the age of our Galaxy (see, e.g., Iben 1972).

If, however, the "low" mass estimates are correct, and if current stellar evolution theories are not grossly in error, then Arcturus must have shed a substantial portion of its initial ZAMS mass during its post-main-sequence evolution, and has very likely reached its present position in the H-R diagram by some route other than shell hydrogen burning following core hydrogen exhaustion (see, e.g., Mäcke *et al.* for these alternative evolutionary possibilities).

In fact, as Mäcke *et al.* have pointed out, if such mass loss is typical for red giants of Arcturus's luminosity, then we actually have very little understanding of this region of the H-R diagram, because the majority of previous models have been based on constant-mass evolution.

On the other hand, the Mäcke *et al.* mass may be too low. In particular, Mäcke *et al.* estimated the gravity of Arcturus, using the method of weak line ionization ratios (WLIR). In this approach, the surface gravity is determined by requiring that a comparison of measured and computed equivalent widths for the first and second spectra of a particular element (e.g., Fe I, II) yield the same abundance. In principle, if the T - τ model is chosen carefully—for example, to fit

TABLE 1
COMPARISON OF ESTIMATED GRAVITIES AND METALLICITIES FOR ARCTURUS

Log g	[Fe]*	[Ca]*	Reference
1.7–2.0.....	$\left. \begin{array}{l} -0.5 \text{ (i)} \\ -0.6 \text{ (ii)} \end{array} \right\}$	–0.5	Griffin and Griffin 1967
1.7–1.9.....	...	...	Upson 1973
1.9.....	–0.6	...	Gustafsson <i>et al.</i> 1974
1.95 ± 0.2	$\left\{ \begin{array}{l} -0.5 \text{ (i)} \\ -0.6 \text{ (ii)} \end{array} \right.$	–0.33	van Paradijs and Meurs 1974
($T_{\text{eff}} = 4350$)	...	...	
1.6 ± 0.2	...	...	
($T_{\text{eff}} = 4260$)	–0.70 $\pm$ 0.08	–0.45 $\pm$ 0.1	Mäcke <i>et al.</i>
0.90 ± 0.35	...	–0.60 $\pm$ 0.1†	This work
1.6 ± 0.2	...		

* Log (stellar abundance/solar abundance).

† $A_{\text{Ca}}^{\odot} = 2.4 \times 10^{-6}$ (Holweger 1972; Ayres 1976).

optical and infrared continuum fluxes—and if many weak lines are available from both ionization stages of several elements, then the WLIR method should give accurate estimates of log g , at least for stars as hot as Arcturus where local thermodynamic equilibrium (LTE) ionization equilibrium is probably valid (Auman and Woodrow 1975; but see also Ramsey 1977). In practice, however, the WLIR approach is plagued by numerous uncertainties. Perhaps it is not too surprising, then, that independent applications of the WLIR method to the particular case of Arcturus should yield such discordant results (e.g., Table 1).

In § II of this paper we propose an alternative method for estimating stellar surface gravities. Our approach is based on strong line ionization ratios (SLIR). We apply this method to Arcturus in § V, using calibrated profiles of the Ca I and Ca II resonance lines (§ III) and detailed atmospheric models (§ IV), as a comparison to the WLIR-derived gravities of previous work. In § VI we comment on the implications of our results and suggest future applications.

II. METHOD

The wing shapes of the Ca I $\lambda 4227$ and Ca II $\lambda 3968$ (H) and $\lambda 3934$ (K) resonance lines are controlled by radiation damping and pressure broadening in the photospheres of typical late-type stars. These lines are sensitive to gravity, owing to (1) the linear density dependence of the van der Waals damping; (2) the Saha ionization balance; and (3) the variation of photospheric mass column density (m ; g cm $^{-2}$) with log g arising from the quadratic pressure dependence of the dominant continuum opacity source, H $^{-}$ (see, e.g., Ayres, Linsky, and Shine 1975).

Given a particular thermal model for the photosphere of Arcturus, we can determine the calcium abundance necessary to force synthetic profiles of $\lambda 4227$ or H and K to fit calibrated wing shapes. Typically, the calcium abundances determined individually for the Ca I and Ca II lines in this fashion will differ. In fact, we can construct the ratio $\mathcal{R} = A_{\text{Ca}}(\text{Ca II})/A_{\text{Ca}}(\text{Ca I})$ as a function of model surface gravity by the comparison of computed and measured wing shapes. Here $A_{\text{Ca}}(X)$ refers to the calcium

abundance derived from species X . Where $\mathcal{R} \equiv 1$ we obtain both the stellar surface gravity and the calcium abundance.

This method of SLIR is, of course, completely analogous to the WLIR approach used by Griffin and Griffin (1967), van Paradijs and Meurs (1974), Mäcke *et al.*, and others. However, by considering strong lines on the damping portion of the curve of growth, we eliminate two important sources of error inherent in the practical application of the WLIR method: (1) difficulties in measuring accurate equivalent widths of weak features in crowded or noisy spectra; and (2) uncertainties in the nature and modeling of nonthermal Doppler broadening. Furthermore, strong line wing shapes can be used to probe the photospheric thermal structure semiempirically (Holweger 1967; Shine 1973; Ayres 1977).

The fundamental argument against using strong lines is the large uncertainty in the van der Waals damping. However, the γ_{vdw} coefficients can be accurately calibrated against high-quality observations of the solar $\lambda 4227$, H and K wing shapes by using current models of the photosphere of the Sun (Holweger 1972; Ayres 1977).

III. OBSERVATIONS AND ABSOLUTE CALIBRATIONS

Profiles of Ca II H and K and Ca I $\lambda 4227$ were obtained from Griffin's (1968) *Atlas*, which has served as the source for many previous spectroscopic studies of Arcturus (Griffin and Griffin 1967; Mäcke *et al.* 1975a, b; Ayres and Linsky 1975; Kelch and Milkey 1976). The spectral resolution of these data is high and the noise and scattered light levels are low (Griffin 1968). Because we are examining the relatively mild intensity gradients of damping wing shapes, possible problems with the *Atlas* instrumental profile (Mäcke *et al.* 1975a, b) are of little concern.

Each empirical Ca I or Ca II wing shape is represented here by a carefully selected sample of intensity points. These reference intensities are measured in relatively clean regions of the line wings free of major blends, well outside of the Doppler core, but within the impact broadening limit ($\Delta\lambda \lesssim 10 \text{ \AA}$ for Ca II K). A value of A_{Ca} for a given resonance line and model

atmosphere (i.e., $\log g$) is determined by finding the zero of the quantity $\sum (\pi F_{\text{cal}}[A_{\text{Ca}}] - \pi F_{\text{obs}})$, where the sum is taken over the particular set of reference intensities.

a) Absolute Calibrations

The tracings in the *Atlas* are normalized to an estimated continuum level. We determined an absolute flux scale for this pseudocontinuum by means of the narrow-band photometry of Faÿ, Stein, and Warren (1974). This photometry consists of 30 Å bands oversampled at ~ 4 Å intervals. The Faÿ *et al.* fluxes were converted to absolute units at the stellar surface, assuming a limb-darkened angular diameter of $\theta = 0''.023$ (Johnson *et al.* 1977). Using these absolute fluxes and direct planimetry of the *Atlas* tracings, we estimated the true fluxes corresponding to the local 100% levels of Griffin's *Atlas* in the vicinities of H, K, and $\lambda 4227$. Error bars on the measured profiles (see Figs. 2 and 3) indicate the standard deviations obtained by averaging several of the adjacent 30 Å oversampled bands for each line.

We point out that the application of the Faÿ *et al.* calibration here is in good agreement with a previous calibration of Ca II K based on Willstrop's (1965) 50 Å photometry (Ayres and Linsky 1975). However, the result here for Ca II H is 5%–10% higher than the calibration based on Willstrop's fluxes. The significance of this difference is not clear, because it is comparable with or less than the calibration uncertainties inherent in both approaches.

b) Partial Coherent Scattering Spectrum Synthesis

We adopt here essentially the same approach used to compute wing shapes for the solar H, K, and $\lambda 4227$ lines by Ayres (1977).

The most important parameters that enter the spectrum-synthesis aspect of this problem are the van der Waals coefficients. These are determined semi-empirically against high-quality solar wing shapes and photospheric models as described by Ayres (1977). Table 2 lists the atomic parameters adopted here for the Ca I and Ca II resonance lines, and the solar empirical van der Waals coefficients.

The van der Waals damping is particularly important here because it governs departures from complete redistribution (or noncoherent scattering, hereafter NCS). While such departures are relatively small for

solar profiles of the Ca I and Ca II resonance lines, owing to the high pressures in the photosphere of the Sun (Ayres 1977), partial coherent scattering (PCS) dominates the line-formation problems at the low densities of the outer atmosphere of Arcturus (see also Shine, Milkey, and Mihalas 1975). We therefore use the LTE-PCS approximation described by Ayres (1975) to compute wing shapes.

We point out that some uncertainties remain in the practical application of the PCS approach, because the theoretical redistribution effects have not yet been verified in laboratory experiments at "astrophysically interesting" conditions of high temperature and low density. High temperatures ($T \approx 5 \times 10^3$ K) are required so that the impact-broadened region of the line wings can be resolved, while low perturber densities ($n < 10^{16} \text{ cm}^{-3}$) are necessary to prevent total redistribution of photons by weak (i.e., van der Waals) collisions.

Encouraging results have been obtained in recent laser scattering experiments carried out at high density, but within the impact regime (Driver and Snider 1976), and at low density, although in the statistical (or "quasi-static") regime (Carlsten and Szöke 1976a, b). In particular, Driver and Snider have used tunable dye lasers to probe impurity lines (Na, Ca, etc.) in hot helium produced by a ballistic compressor. Their work with the Na I D lines has shown that the R^{III} redistribution function (in Hummer's 1968 notation), which describes the effects of weak collisions, is accurately represented by complete redistribution. Carlsten and Szöke have observed the "coherent" R^{II} component, as well as the R^{III} collisional redistribution, of laser light scattered in low-temperature ($T \approx 10^3$ K) strontium vapor buffered with argon. Unfortunately, the latter experiment did not resolve the impact region, although extrapolation of the quasi-static results into the impact regime are encouragingly consistent with the quantum-mechanical formalism proposed by Omont, Smith, and Cooper (1972) and others. We expect better experimental tests of PCS in the near future (Driver 1976).

On the other hand, we do find substantial departures from complete redistribution for the Arcturus H, K, and $\lambda 4227$ lines, and even small errors in the PCS formalism could potentially produce large systematic errors in synthetic wing shapes. To account for this possibility, we have applied the SLIR analysis, using pure NCS, as well as PCS with the maximum likely departures from complete redistribution within the

TABLE 2
Ca I, Ca II TRANSITION PARAMETERS

Spectrum	λ (Å)	Transition	$\log gf$	γ_{rad}^*	γ_{vdw}^*
Ca I.....	4226.73	$4s^2 \ ^1S_0 - 4s4p \ ^1P_1$	+0.243	2.18×10^8	2.8×10^{-8}
Ca II.....	3933.66 (K)	$4s \ ^2S_{1/2} - 4p \ ^2P_{3/2}$	+0.117	1.49×10^8	1.7×10^{-8}
Ca II.....	3968.47 (H)	$4s \ ^2S_{1/2} - 4p \ ^2P_{1/2}$	-0.180	1.48×10^8	1.7×10^{-8}

* $\Gamma_{\text{vdw}} = \gamma_{\text{vdw}} n_{\text{H}} (\text{cm}^{-3}) (T[\text{K}]/5000)^{0.3}$; γ_{rad} , Γ_{vdw} = FWHM of Lorentz damping profile in circular frequency units.

context of Omont, Smith, and Cooper (1972):

$$S_v = B_v \quad (\text{LTE-NCS})$$

$$= \sigma_{\max} J_v + (1 - \sigma_{\max}) B_v \quad (\text{LTE-PCS}_{\max}), \quad (1)$$

where

$$\sigma_{\max} = \frac{\gamma_{\text{rad}}}{\Gamma_{\text{vdw}} + \gamma_{\text{rad}}}. \quad (2)$$

Here, S_v is the monochromatic wing source function (the background continuum contribution has been omitted for clarity), J_v is the mean intensity, B_v is the Planck function, γ_{rad} is the radiation damping parameter, and $\Gamma_{\text{vdw}} = \gamma_{\text{vdw}} n_{\text{H}} (T/5000)^{0.3}$ is the van der Waals broadening (see, e.g., Ayres 1975, 1977).

To preview the results of § V, we derive essentially the same surface gravities and calcium abundances for Arcturus by using maximum PCS and pure NCS, despite large apparent differences in the computed wing shapes. We obtain this result because the NCS-PCS differences are in the same sense for the Ca I and Ca II lines, and therefore tend to cancel in a differential analysis such as is exploited by the SLIR approach.

IV. MODEL ATMOSPHERES

a) Effective Temperature, Angular Diameter, and Stellar Radius

The stellar effective temperature T_{eff} plays a twofold role in the SLIR analysis. First, T_{eff} sets the absolute temperature scale for the photospheric thermal models used to synthesize the Ca I and Ca II wing shapes. Second, the stellar angular diameter θ , which determines the absolute calibration of Griffin's *Atlas* (§ IIIa) and is necessary for converting derived surface gravities into stellar masses, is inextricably coupled to the effective temperature,

$$\theta \propto L_{\odot}^{1/2} T_{\text{eff}}^{-2}, \quad (3)$$

because the apparent stellar luminosity measured at the Earth, $L_{\odot}$, is well known ($\sim \pm 2\%$ for Arcturus; e.g., Blackwell *et al.* 1975).

Hence, T_{eff} can be estimated if θ is known, or vice versa. Unfortunately, the $\sim \pm 15\%$ uncertainty typical of interferometric measurements of the angular diameter of Arcturus (see, e.g., Johnson *et al.* 1977) implies an uncomfortably large (± 300 K) uncertainty in T_{eff} . However, the effective temperature of a star often can be estimated to a higher degree of accuracy by fitting model fluxes to observed continuum slopes based on deblanketed scanner data and well-determined spectral indices. Examples of this "synthetic continuum" approach applied to Arcturus are: Mäcke *et al.*, who derive $T_{\text{eff}} = 4260 \pm 50$ K using empirical T - $\bar{\tau}$ models; Blackwell *et al.* (1975), who determine a "compromise" $T_{\text{eff}} = 4500 (+50, -120)$ K, using scaled Carbon-Gingerich grid models (CGG; Carbon and Gingerich 1969); and van Paradijs and Meurs (1974), who suggest $T_{\text{eff}} = 4350 \pm 50$ K, also using CGG models. A more sophisticated version of

this approach recently was applied to Arcturus by Johnson *et al.* (1977), who compared synthetic stellar energy distributions, including line opacity, with calibrated scanner data, using blanketed flux-constant models. Their result was $T_{\text{eff}} \approx 4250$ K with a suggested uncertainty of ± 100 K. Intuitively, the Johnson *et al.* approach would seem to be more satisfactory than the others cited above, which must rely on (1) uncertain deblanketing corrections, and (2) thermal structures—in the two cases using CGG models—not *ab initio* constructed for the star in question. In particular, we point out the rather close agreement between the Mäcke *et al.* and Johnson *et al.* effective temperatures derived by using thermal models appropriate for the specific case of Arcturus, while the two CGG approaches yield systematically higher effective temperatures. We are therefore inclined to place greatest weight on the Mäcke *et al.* and Johnson *et al.* results, and adopt $T_{\text{eff}} \approx 4250 \pm 100$ K. The error cited here is consistent with the dispersion among the four effective temperature estimates, which we feel is itself a reasonable measure of the true uncertainty of determining T_{eff} by using the synthetic continuum approach.

The corresponding value of θ , based on fitting calibrated scanner data to the $T_{\text{eff}} = 4250$ K synthetic absolute flux curve, is $0''.023 \pm 5\%$. Adopting the parallactic distance cited by Mäcke *et al.*, we derive a stellar radius of $27 R_{\odot} \pm 7\%$.

b) A Grid of Flux-constant Models

A detailed description of the opacity-sampled, LTE atomic and molecular line-blanketed, flux-constant (including mixing length convection with $l/H = 1$) thermal models appropriate for Arcturus has been given by Johnson *et al.* (1977). We consider here a grid of such models constructed for $T_{\text{eff}} = 4250$ K; $\log g = 1.0, 1.5$, and 2.0 cm s^{-2} ; and $\frac{1}{3}$ Lambert's (1968) solar metal abundances. We adopt a 10% helium abundance (by number relative to hydrogen).

The thermal structures of the flux-constant models are compared in Figure 1 to the "empirical" temperature distribution proposed by Mäcke *et al.* The latter was obtained by scaling Holweger's (1967) solar model to $T_{\text{eff}} = 4260$ K and adjusting the resulting thermal structure to fit *Atlas* line shapes.

We consider the Mäcke *et al.* model here because it is based on a presumably more accurate set of metal abundances, as derived in their differential curve-of-growth analysis, than our simple assumption of systematically scaled solar abundances. In particular, the abundance distribution affects the transformation between Rosseland optical depth $\bar{\tau}$ and the mass column density scale m (g cm^{-2}), owing to the effect of metal ionization on the $n_e n_{\text{H}}$ dependence of H^- formation. In fact, small changes in abundance for the important electron donors (Mg, Si, and Fe) have essentially the same effect on the $\bar{\tau} \rightarrow m$ transformation as small changes in the surface gravity. This notion is illustrated clearly in Figure 1, where the Mäcke *et al.* $\log g = 0.9$ model mass scale appears similar to that of the $\log g = 1.0$ grid model, presumably

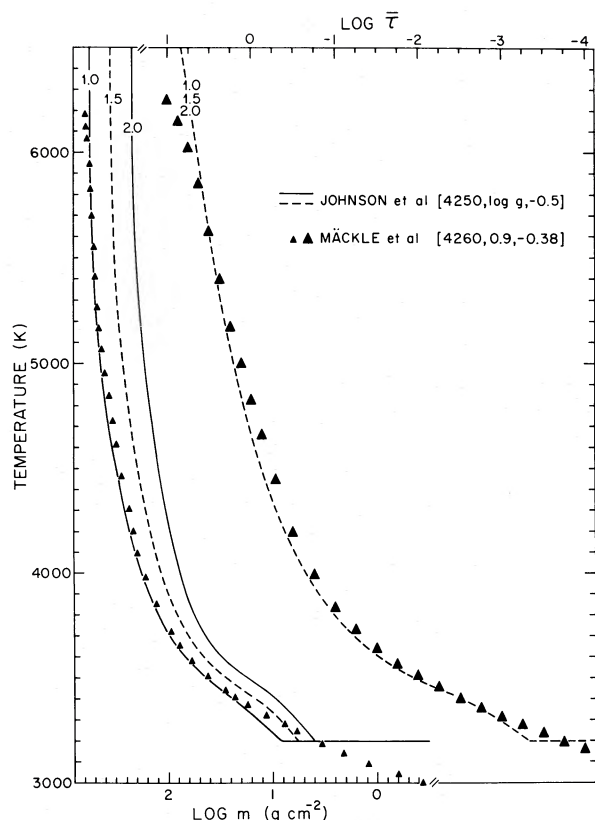


FIG. 1.—Adopted flux-constant thermal models of Johnson *et al.* (1977) and empirical model of Mäcke *et al.* Curves in left-hand side of figure refer to temperature versus mass column density. Curves in middle and right-hand side refer to temperature versus Rosseland optical depth. Flux-constant models are labeled with value of $\log g$. Note: the $T(\bar{\tau})$ structures for the three flux-constant models roughly coincide; for clarity only the $\log g = 1.5$ curve is plotted explicitly. Models are designated by [T_{eff} (K), $\log g$ (cm s^{-2}), and $\log$ metallicity (relative to Sun)].

owing to the ~ 0.1 dex larger electron donor abundances of the former.

We therefore applied the SLIR approach, using the Mäcke *et al.* model as a test of the sensitivity of our method to possible uncertainties in the Arcturus/Sun abundance distributions as well as possible uncertainties arising from differences between “empirical” and flux-constant models. (Note in Fig. 1 that the Mäcke *et al.* model is ~ 150 K hotter than the flux-constant models near $\bar{\tau} \approx 1$, but is substantially cooler at $\bar{\tau} \approx 10$. Despite these apparent differences, both the empirical and the flux-constant models produce very similar continuum fluxes in the 0.4 – $1.0 \mu\text{m}$ region, presumably because the shallower temperature gradient of the Mäcke *et al.* model near $\bar{\tau} = 1$ compensates for its somewhat higher temperatures there.)

We point out that our application of flux-constant models here is relatively independent of the question of whether radiative equilibrium is appropriate for

stellar upper photospheres (e.g., Ayres 1975). The reason is that the flux-constant models all produce blanketed energy distributions in apparent agreement with broad- and narrow-band absolute spectrophotometry at the H^- opacity maximum ($\sim 0.85 \mu\text{m}$) and minimum ($\sim 1.6 \mu\text{m}$) (Johnson *et al.* 1977). Therefore the thermal structure of the photosphere of Arcturus in the layers where the far wings of the Ca I and Ca II resonance lines are formed ($1 \gtrsim \bar{\tau} \gtrsim 10^{-2}$) should be reasonably well determined.

In practice, we truncate the monotonically decreasing upper photosphere temperatures of the flux-constant models at $T = 3200$ K, consistent with the temperature minimum estimated by Ayres and Linsky (1975). Changing this value by ± 100 K has a negligible effect on the Ca I and Ca II far wings.

V. RESULTS

a) Modeling of the Ca I, II Resonance Lines

Representative results of the wing shape synthesis for Arcturus are illustrated in Figures 2 (Ca II H and K)

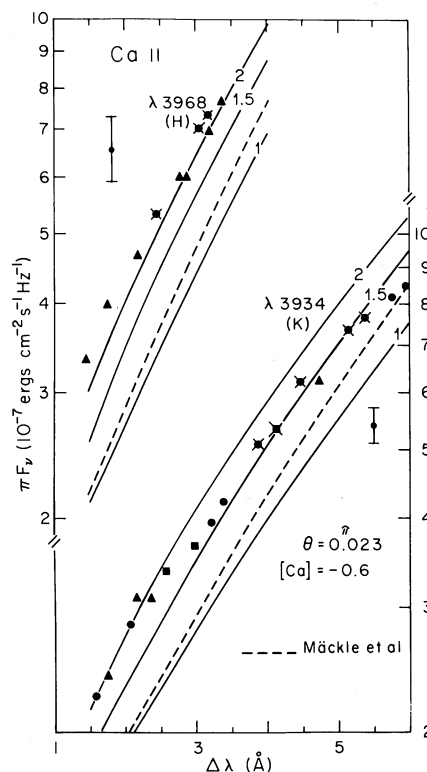


FIG. 2.—Partial coherent scattering synthesized profiles of Ca II H and K for the flux-constant and empirical thermal models, and fixed calcium abundance $[\text{Ca}] = -0.6$. The wing shapes computed for the flux-constant models are designated by value of $\log g$. Filled circles and triangles, measured long- and short-wavelength wing points, respectively. Filled squares, overlapping points. “Crossed” symbols refer to adopted abundance calibration intensities. The error bars refer to uncertainties estimated for the Ca II calibrations relative to the flux curve of Johnson *et al.* (1977).

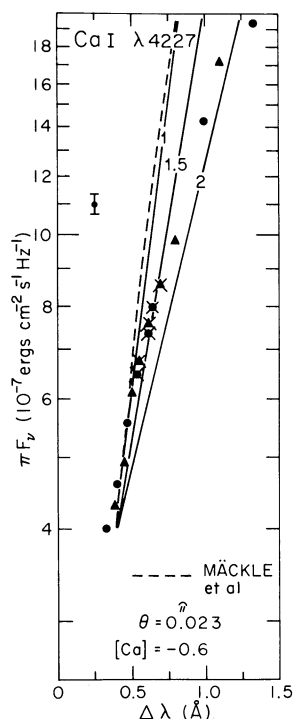


FIG. 3.—Same as Fig. 2 for Ca I $\lambda 4227$. Notice “opposite” behaviors of the Ca I and Ca II wing shapes as function of gravity: $\lambda 4227$ broadens with increasing $\log g$, while H and K broaden with decreasing $\log g$.

and 3 (Ca I $\lambda 4227$). The filled circles and triangles designated by crosses in these figures are the long- and short-wavelength wing points adopted for the fitting procedure to determine A_{Ca} for each model. The wing shapes longward of the reference intensities were judged to be too strongly influenced by blends, especially in the case of $\lambda 4227$.

The “opposite” nature of the gravity dependences in the Ca I and Ca II damping wings, which provide the essential sensitivity of the SLIR method, are clearly demonstrated by the spectrum synthesis here.

The final results of the SLIR analysis are summarized in Figure 4, which illustrates the variation of $\mathcal{R} \equiv A_{Ca}(\text{Ca II})/A_{Ca}(\text{Ca I})$ as a function of $\log g$ obtained for the grid of flux-constant models and the various scattering assumptions. The shaded rectangle refers to the full PCS/PCS comparison of Ca II H and K with Ca II $\lambda 4227$: the upper boundary corresponds to [K/4227] and the lower boundary to [H/4227]. The differences between H and K presumably arise from errors in the relative calibration of the *Atlas* tracings, although line haze effects cannot be ruled out. We point out that an intrinsic error of the same magnitude is present owing to the $\sim \pm 10\%$ uncertainty in the relative oscillator strengths of the Ca I and Ca II resonance lines (Ayres 1977).

The long-dash curve was obtained by using NCS for $\lambda 4227$ and PCS for Ca II K, while the dot-dash curve is based on NCS for both lines. It is encouraging

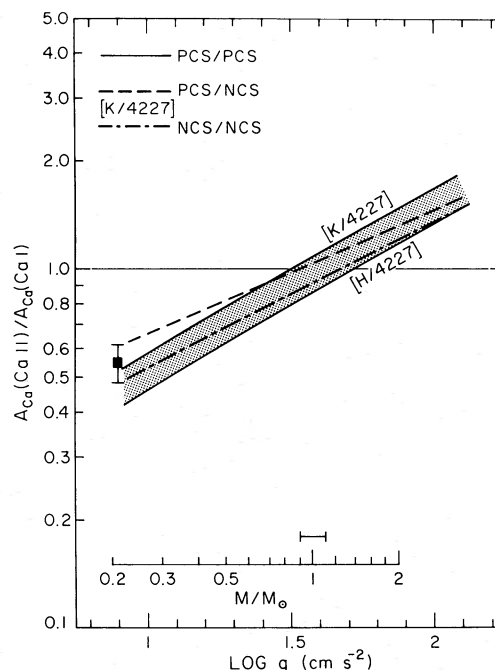


FIG. 4.—Derived SLIR abundance ratios $\mathcal{R}$ as function of $\log g$. See text for details. The $M/M_{\odot}$ scale is for $R = 27 R_{\odot}$. The error bar immediately above this scale refers to the minimum uncertainty in M owing to the minimum uncertainty in R .

that the error associated with the maximum likely uncertainty in the line-formation mechanism is of the same order ($\sim \pm 0.1$ dex in $\log g$) as that produced by the small uncertainties in the relative gf -values and calibrations.

The filled square in Figure 4 refers to the mean $\mathcal{R}$ value derived by using the Mäcke *et al.* empirical thermal model (PCS/PCS: [K/4227], upper error bar; [H/4227], lower error bar). As noted in § IV, the difference between the Mäcke *et al.* mean $\mathcal{R}$ value and that obtained by extrapolating the flux-constant model curves to $\log g = 0.9$ is caused by the differences between the absolute thermal structures on a T - τ scale as well as by the 0.1 dex larger electron donor abundances adopted by Mäcke *et al.* However, extrapolating their SLIR value to $\mathcal{R} = 1$ produces a surface gravity only 0.1 dex smaller than that obtained for the flux-constant models.

b) Inferred Calcium Abundance

Considering solely the PCS/PCS SLIR synthesis, and averaging the values derived from [H/4227] and [K/4227], we estimate a stellar calcium abundance relative to the solar abundance obtained by Holweger (1972) and Ayres (1977) of $[Ca] \approx 0.6 \pm 0.1$. This particular value was adopted to synthesize the wing shapes illustrated in Figures 2 and 3. The relative abundance obtained here is somewhat smaller than that proposed in previous work (i.e., $[Ca] \geq -0.5$; Griffin and Griffin 1967; van Paradijs and Meurs

1974; Mäcke *et al.*), but is certainly consistent within the uncertainties of the individual analyses.

We point out that the Mäcke *et al.* thermal model apparently is inconsistent with the observed Ca I and Ca II wing shapes for their inferred calcium abundance $[Ca] \approx -0.45$, and indeed for *any* value of the calcium abundance, unless the relative calibrations of H, K, and $\lambda 4227$ are grossly in error. In particular, if A_{Ca} is increased so that the Mäcke *et al.* synthesized profile fits $\lambda 4227$, the K wing profile computed for the same abundance will fall even farther below the measured wing shape. Conversely, decreasing A_{Ca} to fit the Ca II wing shape will intolerably worsen the agreement with $\lambda 4227$.

The apparent inconsistency of the Mäcke *et al.* model presumably arises from their underestimate of the surface gravity of Arcturus. If so, their derived abundance distributions may also contain systematic errors.

c) Possible Uncertainties

Possible sources of error in the SLIR analysis include uncertainties in the relative calibrations, the van der Waals damping, the wing shape reference points owing to "line haze," the stellar effective temperature, and the possible failure of the implicit assumptions in the modeling scheme for real stellar atmospheres.

The first source of error is expected to be small because the H, K, and $\lambda 4227$ lines are accurately tied to one another and to the model continuum through the Faÿ *et al.* absolute photometry. In test calculations, we determined that a $\pm 20\%$ relative error between the Ca I–Ca II calibrations and the model continuum produced less than a ± 0.1 dex change in $\log g$.

We tested the second possible source of uncertainty by using the 1σ upper and lower limits on γ_{vdw} for K and $\lambda 4227$ as determined by Ayres (1977). Encouragingly, we obtained essentially the same SLIR curves as for the nominal values of γ_{vdw} , although the individual calcium abundances derived for $\mathcal{R} = 1$ did reflect the ± 0.1 dex uncertainty of the absolute van der Waals damping.

The third source of uncertainty, that due to possible line haze effects, is more difficult to assess. If all three resonance lines are equally affected by "veiled lines," the effect on the derived abundance and surface gravity could be similar to that produced by changing the relative calibration between the wing shapes and model continuum, and therefore would be small (e.g., above). On the other hand, if weak lines disturb one wing shape more heavily than the others, larger systematic errors could be introduced. We have chosen reference points from wing regions as free of apparent weak blends as possible to minimize potential problems with line haze. There is, unfortunately, no guarantee that we have been completely successful.

The fourth major source of uncertainty involves the *absolute* thermal structures of the flux-constant and empirical models. Although the effective temperature of Arcturus appears to be well established by the work of Mäcke *et al.* and Johnson *et al.*, relatively small

systematic errors in the absolute temperature scales could potentially produce large systematic errors in deriving a value of $\log g$. In particular, transitions such as Ca I $\lambda 4227$ that arise from the ground states of minority species are very temperature-sensitive. To test this possible source of error, we applied the SLIR approach with versions of the $T_{\text{eff}} = 4250$ K, $\log g = 1.5$ flux-constant model scaled to $T_{\text{eff}} = 4150$ and 4350 K. In this fashion, we determined that a ± 100 K error in T_{eff} would produce a ± 0.25 dex error in the derived surface gravity and ± 0.2 dex in the calcium abundance. A similar effect on WLIR-derived surface gravities for Arcturus has been noted by van Paradijs and Meurs (1974; see also Table 1). These results suggest that both methods should be applied only to stars for which accurate estimates of T_{eff} are available.

Fifth, we point out that LTE may be inadequate for the photospheric Ca I/Ca II ionization equilibrium in cool, low-density stellar atmospheres such as that of Arcturus (Ramsey 1977). A complete non-LTE, multilevel, multi-ion treatment of calcium in Arcturus comparable to Auer and Heasley's (1976) work for the Sun would therefore be desirable.

Finally, we must always worry that our plane-parallel, hydrostatic, single-component, LTE thermal models do not accurately represent the physical properties of real stellar atmospheres. This is especially true for low-gravity stars such as Arcturus that may possess much more inhomogeneous and dynamic photospheres than the Sun. The truth of the matter is that we at present have very few practical alternatives to the "standard" atmospheric assumptions cited above. In particular, numerical methods for solving radiative transfer problems in dynamic and inhomogeneous atmospheres are currently tractable only in the simplest situations. Furthermore, we still have very little morphological understanding of the role of inhomogeneities in the *solar* atmosphere, let alone for a star whose *diameter* we can barely resolve. Of course, the understanding of such problems, at least from an observational point of view, would be greatly aided by orbiting interferometers and large space telescopes. For the present, however, we can only hope that our basic assumptions are not too grossly in error.

d) Surface Gravity and Mass of Arcturus

The surface gravity we obtain for Arcturus based on the flux-constant models and what we feel are reasonable estimates for the uncertainties cited above is $\log g \approx 1.6 \pm 0.2 \text{ cm s}^{-2}$. The mass of Arcturus is then $1.1_{-0.7}^{+1.7} M_{\odot}$ for $R = 27 R_{\odot}$.

VI. DISCUSSION

a) Mass Loss on the Ascending Red-Giant Branch

Our estimate of the surface gravity of Arcturus based on strong line wing shapes is reasonably consistent with previous spectroscopic estimates (e.g., Table 1) based on weak line strengths, with the exception of the recent work of Mäcke *et al.*, who derive a factor of ~ 5 smaller value. We obtain a stellar mass consistent with evolution tracks appropriate to a

$\sim 1 M_{\odot}$ star with mild Population II abundances (see, e.g., Mäcke *et al.*, Fig. 5; Iben 1972, Fig. 6). We therefore need not, as Mäcke *et al.* must, invoke substantial mass loss ($\Delta M \approx M_{\text{ZAMS}}$) in the previous evolutionary history of Arcturus.

This result should be comforting to theorists of stellar interiors, who are hard-pressed enough as it is to understand why the Sun is not making copious amounts of neutrinos (Bahcall and Davis 1976), without also being confronted by unexpectedly early mass loss in the archetypical red giant. In particular, constant mass (i.e., $\dot{M} \equiv 0$) models are probably adequate for studying post-main-sequence evolution on the red-giant branch, at least up to the luminosity of Arcturus ($\sim 230 L_{\odot}$).

In fact, empirical evidence suggests that Arcturus is currently shedding mass at a rate of $\dot{M} \leq 10^{-8} M_{\odot} \text{ yr}^{-1}$ (Reimers 1975a; Chiu *et al.* 1977). This mass loss rate implies a potential total mass loss at the time of helium flash of only $\Delta M \leq 0.2 M_{\odot}$ (Reimers 1975b). Therefore, the current mass of Arcturus is probably very nearly its initial ZAMS mass, and hence the age of Arcturus is probably $\geq 10^{10}$ yr (Iben 1972).

Of course, the whole question of mass loss on the red-giant branch, especially for cool stars more luminous than Arcturus, is not settled on observational, much less theoretical, grounds. This question is important because mass loss is a direct route for returning nuclear-processed material back into the interstellar medium, thereby influencing future generations of stars. We are hopeful that improved observations,

particularly in the vacuum ultraviolet, and temporal monitoring of mass loss in stars such as α Boo, α Tau (K5 III), and α Ori (M2 I) will provide a hard basis of fact upon which to build increasingly sophisticated models of stellar mass loss and evolution.

b) For the Future

It would be useful to apply the SLIR method to other stars, particularly those in binary systems for which direct mass estimates are available. Such applications would require high-dispersion, well-calibrated profiles of H, K, and $\lambda 4227$ with low noise and scattered light characteristics, as well as detailed thermal models comparable with those constructed for Arcturus by Johnson *et al.* (1977).

In addition, the SLIR approach potentially could be applied to ultraviolet observations of the Mg I $\lambda 2852$ and Mg II $\lambda 2803$ (*h*) and $\lambda 2796$ (*k*) resonance lines, although these important features are accessible only from above the Earth's atmosphere (e.g., Kondo *et al.* 1972; Kohl and Parkinson 1976) and are even more severely blanketed than the calcium lines.

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