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YTTRIUM IN THE PECULIAR A AND B STARS

MARC S. ALLEN
Department of Astronomy, The University of Michigan
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ABSTRACT

Spectroscopic analyses of 18 late B and early A stars have been conducted by use of 2.4 Å mm^{-1} blue plate region spectra from the Dominion Astrophysical Observatory. The physical characteristics of the stellar atmospheres have been determined and abundances calculated for iron, strontium, yttrium, and zirconium with an LTE spectrum synthesis algorithm. Included in the sample were nine mercury-manganese stars; for some of these, previous studies have indicated abundance excesses of yttrium over both strontium and zirconium in violation of the systematics of neutron-addition nucleosynthesis. No such anomalous excess was expected for the nine control stars, which included normal, Am, and Ap spectrum variable stars.

The present work failed to confirm the anomalous yttrium abundance pattern in any star in the sample. The following conclusions were drawn from the detailed abundance results for the Hg-Mn stars: (i) their Sr-Y-Zr abundances are all enhanced relative to the star content; (ii) the relative Sr-Y-Zr abundance patterns are frequently nonsolar in these stars; (iii) but they are not demonstrably nonnuclear; (iv) the prediction of the radiative diffusion theory of element separation for yttrium relative to its neighbors for the appropriate stellar mass range is still to be clearly established, but does not appear promising.

Subject headings: nucleosynthesis — stars: abundances — stars: metallic-line — stars: peculiar A

I. INTRODUCTION

Among the various groups and subgroups of the early main-sequence stars with peculiar spectra are the so-called mercury-manganese (Hg-Mn) stars. Preston (1971) gave good reasons why this group of stars should be considered separately from the other Ap and Bp stars; these reasons include the absence of magnetic fields and spectrum variability, distinctly different abundance anomalies, confinement to a comparatively narrow effective temperature domain (B8–B9), and apparently normal binary incidence (Wolff and Wolff 1974). The Hg-Mn stars characteristically show excesses of phosphorus, manganese, gallium, strontium, yttrium, zirconium, platinum, and mercury. The abundances of these elements seem genuinely variable from star to star in the group. The star 53 Tau, for example, although showing very strongly Mn II lines, has no perceptible Hg II λ3984, a line which is a striking feature in many other manganese stars. Another example is the definite presence of P II but absence of S II in the spectrum of κ Cnc, but just the opposite in φ Her (Cowley 1975). Many interesting aspects of these anomalies are further discussed by Cowley and Aikman (1975*a*) and Cowley (1975). Excellent general reviews of the properties of the peculiar stars have been given by Preston (1974) and Jaschek and Jaschek (1974).

An outstanding spectroscopic peculiarity of the Hg-Mn stars is the great strength of the spectrum of Y II. The abundances of yttrium and the two neighboring elements in the periodic table, strontium and zirconium, show large variations from star to star (summary by Preston 1974; Allen 1975). Many authors (see, e.g., Kodaira 1975; also Guthrie 1971; Ross and Aller 1970; Zimmermann, Aller, and Ross 1970) have found yttrium more abundant than both strontium and zirconium in some of these stars. This is a surprising feature since, inasmuch as yttrium is an odd-*Z* element, this constitutes a departure from an almost universally respected tendency for the abundances of elements heavier than the iron peak to follow a sawtooth pattern with the odd-*Z* elements depleted compared to their even-*Z* neighbors (see Cameron 1973). This tendency has an elementary explanation in terms of the binding energy systematics of the nucleus, the number of stable isotopes at a given *Z*, and neutron-capture cross sections. Indeed, our present understanding of neutron-addition processes (see Burbidge *et al.* 1957) for the nucleosynthesis of the Sr-Y-Zr group of elements seems entirely adequate to explain the solar and chondritic abundance pattern (with yttrium less abundant by about an order of magnitude than each of its neighbors), but cannot give rise to a pattern where yttrium is more abundant than both strontium and zirconium simultaneously (see § III).

It was in part the reported excesses of manganese, phosphorus, yttrium, and mercury in the Hg-Mn stars (but different excesses and deficiencies in the cooler Ap stars) which led to the theory of diffusive separation of elements in stellar atmospheres (Michaud 1970). Theories involving nuclear reactions on stellar surfaces for producing enhanced abundances of heavy elements had experienced time-scale difficulties and/or required the selective acceleration of alpha particles (see Cameron 1971). The case for interior processing was weakened by the apparent youth (Preston 1974) and proximity to the main sequence (Hyland 1967; Snowden 1975) of many early stars with anomalous spectra, and the mixing of reaction products to the observable surface of these early stars also proved to be a formidable theoretical obstacle.

The diffusive separation theory has been worked out in some detail for mercury, including isotopic effects (observed by Cowley and Aikman 1975*b*, White *et al.* 1976), by Michaud, Reeves, and Charland (1974), and for helium by Vauclair, Michaud, and Charland (1974). The prediction for yttrium and its periodic table neighbors is at present more schematic, unfortunately (Michaud *et al.* 1976); the very complex diffusion theory depends critically on the atomic properties of the species under consideration and also on the stellar atmosphere. Nonetheless, with a qualitative understanding of the effective temperature dependence of the diffusion theory and the relatively straightforward predictions of the nuclear theories for the relative abundances of strontium, yttrium, and zirconium, a uniform and systematic analysis of their abundances in the Hg-Mn stars was considered worthwhile.

Succinctly, the goal in the analysis of the nine Hg-Mn stars in the sample was threefold: to see if there were truly a significant departure from the solar abundance pattern present in the Hg-Mn stars; if so, to determine whether such anomalous patterns firmly ruled out a nuclear origin; and finally, to compare the results with the predictions of the diffusion theory insofar as the state of the theory permitted. The spectra of nine additional stars for which no anomalous excess of yttrium with respect to its neighbors was expected were analyzed as controls. These included three cooler Ap stars of the magnetic and spectrum variable type, three Am stars, and three normal stars. The effective temperatures of the three normal stars spanned the range from 7200 to 13,400 K.

II. DATA AND CALCULATIONS

a) *Plate Material*

With the exception of ν Cap, for which no new observations were available and for which the published data of Adelman (1973*a*) were used, all measurements were made on direct intensity tracings of plates acquired by Dr. C. R. Cowley and Mr. G. C. L. Aikman at the Dominion Astrophysical Observatory. The telescope-spectrograph system was the 48 inch (1.2 m) telescope and coudé spectrograph with the image slicer and optimum reflection interchangeable optics assemblies (Richardson 1968). One pair of plates was available for each star, a UV plate covering roughly the interval λ3700–4150 and a blue plate of λ4200–4650. All plates were IIa-O (except for μ Lep, IIIa-J) developed in MWP-2 or D-19 developer. A detailed journal of observations for those stars in the sample which show variability is given by Cowley (1976). The dispersion was 2.4 Å mm^{-1}, and the instrumental profile was 0.074 Å full-width at half-maximum. Originating as they did from only a single plate each, the numerous individual equivalent width measurements were not considered of sufficient reliability to justify reproduction here, but are available on request.

b) *Analytical Procedure*

Lines of Fe I, Fe II, Sr I, Sr II, Y II, and Zr II were identified and measured on the intensity tracings. The number and quality of the measurements depended on the noise level in the tracing and the extent of rotational broadening in the lines; in the sharp-lined stars, the line profiles were assumed to be gaussian in shape and measured for half-width and depth and the equivalent widths calculated by the rule: $W(\lambda) =$ (width at half-depth) $\times$ (depth) $\times$ (1.20). In the broader-lined cases best-fit triangles were drawn in, and the equivalent widths determined in that way. Measurements were not made for very strong lines ($\log W/\lambda \geq -4.3$) for which neither approximation would yield an accurate equivalent width. These lines would be well up on or past the flat part of the curve of growth (COG) and contain no reliable abundance information anyway.

As a first reconnaisance of the effective temperature of a star, either *UBV* photometry from Johnson and Morgan (1953) or Hoffleit (1964) or *uvby* photometry from Lindemann and Hauck (1973) or Crawford *et al.* (1973) was converted to an effective temperature by one of the standard calibrations. For *UBV* measurements, the calibration of either Morton and Adams (1968) or Schild, Peterson, and Oke (1971) was used, whereas in the case of the intermediate band photometry either the calibration of Osmer and Peterson (1974) for hot stars or that of Breger (1974) for the cooler ones was used. All stars in the program were bright so that no reddening corrections were attempted. Small differences between the various scales were disregarded by comparison with the compromises in effective temperature introduced by consideration of other available constraints. Profiles for the hydrogen λ4340.47 line (Hγ) were calculated for each star for a small grid of plausible models near the photometrically indicated effective temperature using the Vidal, Cooper, and Smith (1973) (VCS) theory and were plotted by computer to the same scale as the observed tracing profile so that they could be overlaid. This procedure enabled an accurate assessment of the effects of metal lines in the wing, possible asymmetric defects in the observed profile, and uncertainties in the placement of the continuum level. Because the hydrogen line was calculated in LTE and because our knowledge of the very highest layers of the atmospheres of these stars is so fragmentary, no effort was made to fit the cores, $|\Delta\lambda| \leq 0.5$ Å. Near $T_{\rm eff} =$ 10,000 K, the detailed profile comparison gave information mainly about surface gravity, but in both the cooler and the hotter atmospheres the profile showed sensitivity to temperature as well.

When a satisfactory accommodation between the photoelectric photometry and the Hγ analysis was reached, the model temperature at log $\tau_0 = -1.2$ was used as an excitation temperature to plot a classical curve of growth (COG) for lines of Fe I or Fe II, or both if there were sufficient lines measured for both to make it worthwhile. (Except for the discussion of the *s*-process in § III, τ_0 always refers to the optical depth at 5000 Å.) In addition, the maximum line depth at 4200 Å was estimated by taking the ratio of the Planck function at log $\tau_0 = -4.0$ and that corresponding to the effective temperature. The observed curve of growth was then shifted vertically relative to the general curve of growth of Unsöld (1955) and the microturbulence thus derived. The accuracy of this method depends on many assumptions, particularly the assignment of the boundary temperature. The canonical effects of strong opacity sources in the atmosphere are to cool the outer layers and back-warm the deeper ones, so for those stars where significant UV line opacities may be expected this naïve estimation of the microturbulence may be seriously in error. Other systematic effects which might affect the determination of the microturbulence include strength-dependent errors in the *f*-value scales and consistently incorrect placement of the continuum level.

Tentative assignments of ($T_{\rm eff}$, log *g*, microturbulence) were compared with the results of previous high dispersion work where available, and Fe I and Fe II abundances were then calculated. Ideally, both ionization stages would give identical iron abundances for the correct model atmosphere, assuming no inconsistency between the two absolute oscillator strength scales; Table 1 shows no systematic trend between abundances from the two ionization stages. The practice was to minimize any discrepancy between the abundances to the extent permitted by the photometry and Hγ temperature estimates by adjusting the effective temperature of the model. Finally, the iron abundances were plotted against log W/λ and any strength dependence was removed by revising the microturbulence parameter, which in some (non–Hg-Mn) cases undoubtedly contained a substantial contribution from Zeeman broadening (Wolff 1967 has a good discussion of this point). In no case did the

TABLE 1

MODEL ATMOSPHERES AND IRON ABUNDANCES

Star	Spectral type	Source*	$T_{\rm eff}$ (K)	log *g* (cgs)	micro (km s^{-1})	log Fe I (No. of lines)	log Fe II (No. of lines)
HR 205	B9.5 IIIp(Hg-Mn)	2	12,000	3.7	0.0	7.64 ± 0.34 (15)	7.43 ± 0.30 (20)
53 Tau HR 1339	B9 IV	2	12,000	4.0	3.5	6.88 ± 0.22 (8)	6.90 ± 0.24 (24)
μ Lep HR 1702	B9p Hg, Mn	1	13,400	3.6	1.3	7.65 ± 0.44 (12)	7.26 ± 0.22 (20)
HR 1800	B9p Hg, Si	1	11,500	3.7	1.0	7.40 ± 0.34 (11)	7.27 ± 0.15 (15)
HR 2844	B9p Hg, Mn	1	13,200	3.7	1.2	≤7.73 (3)	7.27 ± 0.31 (10)
κ Cnc HR 3623	B8 IIIp Mn	2	13,700	3.7	0.0	7.79 ± 0.16 (6)	7.56 ± 0.25 (17)
ι Crb HR 5971	AOp? Hg?	1	10,600	3.5	1.3	7.58 ± 0.10 (13)	7.55 ± 0.11 (13)
ϕ Her HR 6023	B9p? Mn?	1	11,800	3.5	0.0	7.71 ± 0.24 (9)	7.67 ± 0.22 (14)
HR 7664	B9p Hg, Mn	1	13,500	3.4	2.5	8.38 ± 0.38 (11)	7.92 ± 0.25 (22)
63 Tau HR 1376	Alm	1	7500	4.0	5.8	7.44 ± 0.14 (17)	7.46 ± 0.22 (8)
o Peg HR 8641	A1 IV	1	9600	3.8	3.2	7.19 ± 0.22 (26)	7.55 ± 0.18 (13)
32 Aqr HR 8410	A5m	1	7900	4.0	4.9	7.34 ± 0.20 (56)	7.67 ± 0.24 (8)
49 Cnc HR 3465	A1p Eu, Cr	1	8800	3.0	5.0	7.25 ± 0.48 (14)	7.29 ± 0.09 (5)
HR 7575	A5p	3	8000	4.0	3.5	7.28 ± 0.30 (39)	7.57 ± 0.25 (16)
γ Equ HR 8097	Cr-Eu-Sr	5	7800	4.0	2.6	7.09 ± 0.32 (49)	7.21 ± 0.28 (6)
28 And HR 114	A7 III	4	7200	3.0	5.1	6.80 ± 0.17 (24)	6.86 ± 0.09 (7)
23 Cas HR 208	B8 III	2	13,400	3.5	3.5	7.60 ± 0.12 (5)	7.44 ± 0.26 (27)
ν Cap HR 7773	B9.5 V	1	10,200	3.7	2.8	7.37 ± 0.26 (32)	7.34 ± 0.19 (17)

* Spectral types were drawn from the following sources: (1) Cowley *et al.* 1969, (2) Cowley 1972, (3) Warren 1973, (4) Cowley and Fraquelli 1974, and (5) Wolff 1967.

TABLE 2

ADOPTED ABUNDANCES (log H = 12.00)

Star	Fe (No. of lines)	Sr (No. of lines)	Y (No. of lines)	Zr (No. of lines)
HR 205	7.52 ± 0.34 (35)	5.54 ± 0.42 (2)	5.00 ± 0.62 (11)	5.18 ± 0.26 (8)
53 Tau	6.90 ± 0.24 (32)	3.73 ± 0.16 (2)	4.03 ± 0.58 (10)	4.98 ± 0.24 (28)
μ Lep	7.41 ± 0.44 (32)	4.67 ± 0.18 (3)	4.30 ± 0.51 (5)	≤4.29 (1)
HR 1800	7.33 ± 0.34 (26)	5.28 ± 0.57 (2)	5.06 ± 0.48 (7)	≤4.18 ± 0.54 (2)
HR 2844	7.27 ± 0.31 (10)	5.37 ± 0.54 (2)	5.13 ± 0.55 (4)	≤4.70 (1)
κ Cnc	7.62 ± 0.25 (23)	4.16 ± 0.01 (2)	≤3.95 ± 0.47 (5)	≤4.54 ± 0.22 (3)
ι CrB	7.57 ± 0.11 (26)	4.53 ± 0.01 (2)	3.82 ± 0.56 (9)	4.44 ± 0.13 (21)
ϕ Her	7.69 ± 0.24 (23)	4.04 ± 0.29 (2)	5.08 ± 0.45 (12)	5.19 ± 0.20 (20)
HR 7664	8.07 ± 0.46 (33)	≤3.69 (1)	≤4.40 (1)	≤5.09 (1)
63 Tau	7.45 ± 0.22 (25)	3.78 (1)	3.11 ± 0.12 (3)	3.10 ± 0.39 (8)
o Peg	7.31 ± 0.36 (39)	3.82 ± 0.19 (2)	2.61 ± 0.47 (6)	3.87 ± 0.20 (26)
32 Aqr	7.38 ± 0.33 (64)	4.11 ± 0.07 (3)	3.19 ± 0.25 (4)	3.88 ± 0.29 (7)
49 Cnc	7.26 ± 0.48 (19)	3.99 ± 0.73 (2)	2.13 ± 0.54 (6)	3.49 ± 0.30 (21)
HR 7575	7.36 ± 0.30 (55)	4.84 ± 0.38 (3)	2.08 ± 0.65 (8)	3.45 ± 0.30 (12)
γ Equ	7.10 ± 0.32 (55)	3.95 ± 0.31 (3)	2.44 ± 0.51 (8)	3.38 ± 0.33 (14)
28 And	6.81 ± 0.17 (31)	2.64 ± 0.22 (2)	1.51 ± 0.46 (5)	2.76 ± 0.30 (14)
23 Cas	7.47 ± 0.26 (32)	3.07 ± 0.39 (2)	not present	not present
ν Cap	7.36 ± 0.26 (49)	2.99 ± 0.29 (2)	< 3.32 (1)	not present

final adopted microturbulence differ by more than a km s^{-1} or so from the value derived from the COG study.

The optimal atmospheric parameters are tabulated in Table 1 and abundances based on these values were calculated for strontium, yttrium, and zirconium. In order to minimize the effects of errors in the somewhat uncertain microturbulence determination, only lines which were judged weak enough to be sensitive mainly to abundance were included in the final means. The limiting upper line strength for a star was inferred from the magnitude of the adopted microturbulence or from an inspection of a sample COG for the atmosphere; a careful effort was made to avoid as much as possible the inclusion of lines too far up on the shoulder of the COG. The mean iron abundance was then formed, and the adopted final abundances for all four elements are listed together in Table 2.

The errors appearing with the logarithmic abundances in Table 2 are formal standard deviations of the individual line abundances, except where only two lines are involved, in which case the quoted error is simply the difference between the two averaged values. For a mean iron abundance in Table 2 the error given is the largest of: the Fe I deviation, the Fe II deviation, or the difference between the two abundances. These errors are cited more as an indication of the scatter around the mean abundances than as a quantity of real statistical significance. They take no account of the more serious possible systematic errors in the absolute scales of the various systems of oscillator strengths or of inaccuracies in the adopted atmospheric models. The tendency of the abundances from Y II measurements to show comparatively large formal errors is discussed in § II*d*.

The "normal" abundances to which the calculated ones were referred are the solar ones [on the scale log N (H) = 12.00], except for zirconium, where the chondritic value (=2.95, Cameron 1973) was used. Sources for the other solar abundances are: iron = 7.40, Huber and Parkinson (1972); and 7.39, Smith and Whaling (1973); strontium = 2.82, Lambert and Warner (1968); and yttrium = 2.00, Allen and Cowley (1974). For ν Cap no data were available for yttrium or zirconium (indicating upper limits of 10 mÅ for the equivalent widths of their lines); neither element was spectroscopically present in 23 Cas, but for iron and strontium these normal stars show good agreement with the solar values. Although all four elements seem somewhat depressed in 28 And, the solar relative pattern is preserved, and the possibility that the deficiencies are due to an improper diagnosis of this apparently low-gravity star (with derived $\log g = 3.0$) should not be dismissed.

a) *Computations*

The calculations were carried out at the University of Michigan Computing Center. All numerical procedures were based on very standard physical and mathematical methods so that only a cursory description need be given here.

The model atmospheres used in the analysis were derived by scaling the temperature structures of models computed by Kurucz, Carbon, and Gingerich and tabulated in Gingerich (1969). Of the grid of models for $\log g = 4.0$ and $T_{\text{eff}} = 14{,}000$, 12,000, 10,000, and 8000 K, the temperature variation with optical depth of the model nearest the desired one in effective temperature was scaled by a constant ratio of the (desired effective temperature)/(effective temperature of the published model), and the pressure equation integrated downward with the resulting $T(\tau_o)$ distribution and adopted $\log g$. For the effective temperature regime in which the stars in this sample fall ($7000 \text{ K} \leq T_{\text{eff}} \leq 14{,}000 \text{ K}$), hydrogen is the dominant electron donor so that a precisely correct metal composition is not very important in the pressure and ionization balance calculations. No effort was made to enforce flux constancy, since the initial models were flux constant and standard abundances were assumed for the abundant elements. These approximations were probably justified for the Hg-Mn stars whose spectral peculiarities apparently do not seriously affect their ultraviolet flux distributions and hence their temperature stratifications (Leckrone 1973), but for the cooler Am and magnetic and spectrum variable stars some departures from the normal temperature structures are probably present.

The line profiles for Hγ and the metal spectra, the equivalent widths, and the curves of growth were calculated by a program of essentially standard LTE construction; using the Planck function for the source function, the line depths were obtained by integrating the removed flux over a logarithmic optical depth scale (see Cowley 1970, p. 79). Generally 27 depth points were used in the interval $-4.0 \leq \log \tau_0 \leq +1.20$ to give a $\Delta \log \tau_o = 0.2$, except in the case of the hydrogen profiles for $T_{\text{eff}} \leq 10{,}000$ K where $\Delta \log \tau_o = 0.05$ proved necessary. The continuous opacities were calculated at every depth and included those due to hydrogen and helium absorption and scattering and electron scattering; the coding for these was adapted from ATLAS (Kurucz 1970). It is important to realize that while it might be incorrect to neglect other continuous opacity sources for the purpose of calculating the flux-constancy of atmospheres for stars where most of the radiated flux emerges in the near-UV, only the opacities due to hydrogen and electron scattering contribute significantly to the computation of line profiles in the visible spectrum. Provisions were contained in the code for the calculation of blends (as in the case of Sr II λ4305.447 which in the cooler stars was inseparably contaminated by Fe I λ4305.455), and also for hyperfine splitting and depth-dependent microturbulence although neither of these two options was exercised in this study.

The pressure broadening was calculated according to VCS (1973) for the linear Stark broadening of Hγ, to Cowley (1971) for the quadratic Stark broadening of ions and neutrals, and to Unsöld (1955) for van der Waals broadening, which included the effect of neutral helium. Except for hydrogen, the accurate calculation of pressure broadening was not of crucial importance for this abundance study except insofar as it affected the estimation of the microturbulence parameter, since the lines on which the final abundances were based always lay on or near the linear part of the curve of growth.

The sequence followed by the spectrum synthesis code was to calculate a profile and thence an equivalent width from an assumed initial abundance, to compare it with the input equivalent width, and then to adjust the abundance and compute a new equivalent width. This iteration was continued until the input equivalent width was bracketed on the resulting COG, which was then interpolated for the corresponding log (abundance).

d) *Oscillator Strengths*

As in any astronomical abundance analysis, the acquisition of a reliable set of *f*-values proved to be a matter of major concern. Fortunately, in this study only six spectra of four elements were involved so that careful attention could be devoted to each set.

For Fe I the measurements of Huber and Parkinson (1972) (HP) were chosen to set the definitive scale. The use of the hook method on shock-heated gases combines the advantages of insensitivity to line shape and of large dynamic range of the hook method with the accessibility of higher excitation levels possible with shock heating. When additional oscillator strengths were required for weaker lines, the values of Bridges and Kornblith (1974) (BK) were adopted. The HP and BK studies shared 42 lines in common, and differed on the average by $\log \text{HP/BK} = +0.13 \pm 0.09$, so the BK values were augmented by this amount to bring them into accord with the HP absolute scale. In the case of Fe II, oscillator strengths of Warner (1967) (W) were corrected according to the prescription of Smith and Whaling (1973), leaving no perceptible systematic inconsistency with the Fe I absolute oscillator strength scale (see Table 1).

Only five lines of strontium were analyzed. For the resonance lines of Sr II, the Hanle-effect measurements of Gallagher (1967) were used, while the $\lambda\lambda$4161 and 4305 values were taken from Warner (1968). For Sr I λ4607, the oscillator strength was derived from the solar spectrum (Allen 1976*a*), assuming the strontium abundance of Lambert and Warner (1968).

The calculated *f*-values of Krueger *et al.* (1968) (KARC) were used for Y II, except for λ3951 for which the KARC value seemed much too large. As a correction the difference between the KARC and Corliss and Bozman (1962) (CB) values for λ3930 (in the same multiplet) was applied to the CB value for λ3951. The oscillator strengths of KARC have been successful in resolving an inconsistency between the solar and the chondritic yttrium/strontium ratio (Allen and Cowley 1974; Allen 1976*a*), so that one may have confidence in their absolute scale. Unfortunately, abundances calculated with these oscillator strengths show very large scatter as demonstrated by a casual inspection of Table 2. This scatter is also present in the original KARC paper in their analysis of the solar spectrum. As an experiment to see if the scatter in their solar study were due to unsuspected blends or instead intrinsic to the *f*-values themselves, HR 1800 was selected at random for a comparison test. Using 11 Y II lines, an yttrium abundance of 5.22 ± 0.68 was derived for this star. KARC had used their mean solar abundance from all lines to go back and determine *solar f*-values which in many cases differed markedly from the calculated ones. When the resulting solar log *gf* correction was applied to the HR 1800 abundances line by line, the new mean abundance for the star was 5.37 ± 0.35. There was thus little change in the magnitude of the abundance, but the formal logarithmic error was halved, now more in line with typical values for other elements. Since the physical conditions in HR 1800 ($T_{\rm eff}$ = 11,500 K) are very different from those in the Sun, this indicates that the large scatter is present in the oscillator strengths themselves and is not due (at least in most cases) to blending. But inasmuch as the uncorrected KARC scale gave good agreement between the solar and chondritic abundances and the above sample correction produced no important change in the mean, one is encouraged that the use of these oscillator strengths will not lead to systematic error.

The adjustment of the CB Zr II *f*-value scale on the basis of the solar spectrum has been fully discussed elsewhere (Allen 1976*a*), so little needs to be added here. However, there is some dispute over the chondritic abundance itself of zirconium at the present time. If the meteoritic zirconium abundance were to be lowered by a factor of 2 or 4 from the one I used (from Cameron 1973) as suggested by Ganapathy, Papia, and Grossman (1975), my stellar abundances for zirconium would decrease by the same amount.

III. DISCUSSION

In the interest of brevity, detailed star by star discussions of the abundance analyses beyond the general procedural outline in § II*b* above have been omitted here (but are provided by Allen 1976*b*). I am grateful to C. R. Cowley for permission to reproduce in Table 3 some of his unpublished statistical identification results. The reader's attention is directed to the close correlation between the significance level of an identification and the corresponding derived abundance, once allowance is made for the effective temperature of the star. Of particular interest are the large values of the significance parameter *s* for Zr II in those stars where detailed calculations have shown

TABLE 3

MONTE CARLO IDENTIFICATION TRIALS*

Star		Fe I	Fe II	Y	Zr
HR 205.....	f =	0.000	0.000	0.000	0.000
	s =	5.29	13.78	9.54	7.80
	H/N =	10/56	21/48	9/23	7/20
53 Tau......	f =	0.180	0.000	0.060	0.000
	s =	1.26	11.34	2.05	12.66
	H/N =	6/60	24/48	4/24	16/21
μ Lep.......	f =	0.765	0.000	0.005	1.000
	s =	−0.41	16.10	4.42	−0.84
	H/N =	1/57	22/43	5/23	0/21
HR 1800....	f =	0.000	0.000	0.000	0.115
	s =	8.04	16.55	15.17	1.83
	H/N =	18/58	33/48	20/23	3/20
HR 2844....	f =	0.680	0.000†	0.000	1.000
	s =	−0.12	5.53	5.92	−0.82
	H/N =	1/52	8/31	5/24	0/22
κ Cnc.......	f =	0.025	0.000	0.595	1.000
	s =	2.43	22.28	0.10	−0.83
	H/N =	6/27	29/38	1/19	0/17
ι CrB.......	f =	0.000	0.000	0.000	0.000
	s =	13.11	20.21	10.98	10.05
	H/N =	35/61	41/48	18/24	15/21
φ Her......	f =	0.000	0.000	0.000	0.000
	s =	8.65	13.13	14.95	10.44
	H/N =	25/60	35/46	22/24	18/21
HR 7664....	f =	0.000	0.000	1.000	1.000
	s =	7.78	23.74	−0.88	−0.76
	H/N =	15/51	40/47	0/22	0/18
63 Tau......	f =	0.000	0.000	0.000	0.005
	s =	15.84	11.38	5.29	3.14
	H/N =	49/59	35/48	14/24	9/21
ο Peg.......	f =	0.000	0.000	0.000	0.000
	s =	17.50	21.10	5.10	12.58
	H/N =	45/60	42/48	10/24	20/21
32 Aqr......	f =	0.000	0.000	0.000	0.000
	s =	14.57	8.45	4.96	7.25
	H/N =	49/55	33/48	13/23	15/20
49 Cnc......	f =	0.000	0.000	0.475	0.085
	s =	8.93	13.07	0.27	1.80
	H/N =	27/61	36/48	3/24	5/21
HR 7575....	f =	0.000	0.000	0.380	0.180
	s =	8.06	9.18	0.47	1.10
	H/N =	47/64	47/53	9/24	9/21
γ Equ.......	f =	0.000	0.000	0.000	0.000
	s =	12.88	8.36	6.67	5.80
	H/N =	55/61	36/49	20/24	17/22
28 And.....	f =	0.000	0.000	0.000	0.000
	s =	18.94	11.86	4.88	5.43
	H/N =	48/61	29/48	10/24	10/21
23 Cas......	f =	1.000	0.000	1.000	1.000
	s =	−1.28	15.78	−0.67	−0.86
	H/N =	0/60	18/43	0/24	0/22
ν Cap.......		not avail.	not avail.	not avail.	not. avail.

* Unpublished results of C. R. Cowley. The quantity *f* is the probability of chance occurrence of the obtained number of coincidences with nonsense wavelengths, *s* is the significance of the identification, *H*/*N* is number of hits *H* on *N* laboratory wavelengths. Results for strontium are not shown because the Sr II spectrum has only 4 observable lines, which limits the usefulness of the statistical method; the presence or absence of Sr II is unambiguously established by the resonance lines. For further discussion of the statistical method, see Hartoog *et al.* (1973).

† The Fe II result for HR 2844 used a wavelength tolerance of 150 mÅ, instead of 60 mÅ as for all other tests.

this element to be very abundant (HR 205, 53 Tau, ι CrB, and φ Her), and the paucity of coincidences with lines of Fe II for the low-iron stars HR 2844 and 53 Tau compared to κ Cnc and especially HR 7664, in spite of the latter two stars' substantially higher effective temperatures. The Y II identification is very secure by this technique except in the hottest stars and also 53 Tau, whose yttrium overabundance is indeed somewhat less pronounced than in most of the other Hg-Mn stars in the sample.

Table 2 shows the collected abundances for iron, strontium, yttrium, and zirconium for all 18 stars. Iron exhibits no order-of-magnitude excesses or deficiencies in any star. Except for the excess in HR 7664 and the deficiencies in 53 Tau, 28 And, and possibly γ Equ, each star exhibits virtually a solar iron content within the errors. Beyond this general overall similarity of the iron abundances, it appears that the range of iron abundances (Table 2) for the Hg-Mn stars is much broader than for the other categories: 1.17 dex as compared to 0.14 for the Am stars, 0.26 for the magnetic stars, and 0.11 for two normal stars (or 0.66 including the uncertain case of 28 And). The rms deviation for the iron abundances in the Hg-Mn stars is 0.32, whereas, omitting 28 And, it is 0.12 for all the other stars combined. This indicates that (i) the iron abundances of the magnetic stars and the Am stars in the sample do not vary much from those of the normal stars, and (ii) that there is a contrasting real intrinsic scatter in the iron contents of the Hg-Mn stars. These results are very compatible with the finding of Searle, Lungershausen, and Sargent (1966) that the manganese stars showed in many cases abnormal Ti/Fe, Cr/Fe, and Mn/Fe ratios, while the same ratios for stars of the Si, Eu-Cr, Eu-Cr-Sr, Sr, and Am groups were normal.

Leaving iron and coming to the Sr-Y-Zr group, two generalizations are immediately possible: all of these elements are greatly enhanced in the Hg-Mn stars, and their relative abundances are distinctly different from star to star. This same degree of diversity is not present in the cooler Ap and the Am stars; these two groups show very similar patterns. Although the error bars in Figure 1 are disappointingly large, there appears to be no general tendency for these elements to mimic the solar relative abundance pattern in the Hg-Mn stars. Hence we may anticipate *ab initio* that whatever nuclear or nonnuclear process is responsible for the observed abundances of the three elements in these stars will differ from the corresponding process for the Sun. The standard *r*- and *s*-process mixture derived from the "cosmic" abundances cannot account for the relative abundances of strontium, yttrium, and zirconium in the Hg-Mn stars.

Clearly the correctness of the derived Sr-Y-Zr abundance patterns depends on the consistency of the oscillator strength scales of these elements. Although only 28 And of the three normal stars allowed the calculation of abundances for all three elements, their relative abundances in this star were solar. The other cases, 23 Cas and ν Cap, show that the strontium and iron scales are consistent. The close connection of the

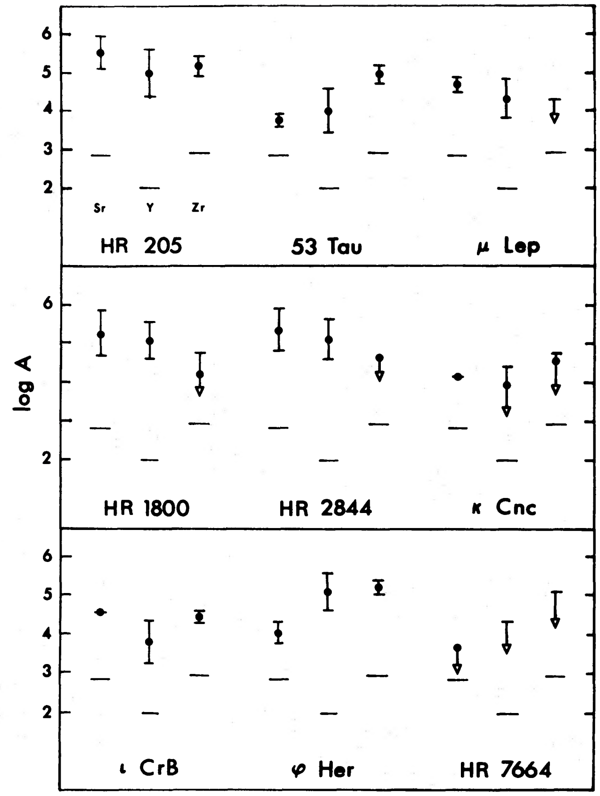

FIG. 1.—Abundances in the Hg-Mn stars (log H = 12.00). Horizontal bars represent solar values.

yttrium and zirconium oscillator strengths to the solar (and therefore "normal") abundances then gives assurance of the accuracy of their absolute scales (see § II*d*). These considerations lead to confidence in the derived Sr-Y-Zr patterns even in the absence of additional standard stars showing measurable Y II and Zr II spectra.

We may then ask if it is possible to reproduce the observed abundance patterns by changing the parameters of a hypothetical nucleosynthetic event from those inferred from the solar system; if not, we would be forced to abandon a nuclear explanation without further discussion. If it were possible, the next step would be to attempt to identify a physical environment where nuclear processing with the required outcome might be expected to take place. The work of van den Heuvel (1968*a*, *b*, *c*, *d*) and Guthrie (1971 and references therein) has proceeded along these lines in the context of a shedding or exploding companion star, but their ideas ran into difficulties with capture inefficiency for the processed debris (Cameron 1971) and inadequate understanding of the astrophysical details of dynamic explosive nucleosynthesis.

According to the classical theory of Burbidge *et al.* (1957), the solar system abundances of strontium, yttrium, and zirconium are entirely due to the slow and rapid neutron addition processes. The *p*-process contributions are very small (Seeger, Fowler, and

Clayton 1965) and will be ignored here. It is possible to carry out a very simplified *s*-process calculation for the relative abundances of the three elements of interest for various different exponential exposure distributions of the form $\rho(\tau) = Ge^{-\tau/\tau_0}$ using the exact *s*-process chain solution of Clayton and Ward (1974). [In this immediate discussion, τ departs from its usual meaning of optical depth, and signifies instead the neutron irradiation defined as $\tau = \int n(n)V(therm)dt$, where $n(n)$ is the neutron number density, $V(therm)$ is the thermal neutron velocity, and the integral is over time; τ has the units cm^{-2} or mb^{-1}.] As can be seen in Figure 2–5 of Seeger, Fowler, and Clayton (1965), the general shape of the $N\sigma$ (number $\times$ neutron capture cross section) versus A curve is only weakly dependent on the precise type of exposure distribution (exponential or power-law) and on the thermal energy of the irradiating neutrons. Qualitatively, yttrium is located exactly on the steep shelf in the $N\sigma$ curve caused by the neutron shell closing ($N = 50$) in its only stable isotope. For low neutron exposures, the very small neutron capture cross sections $\sigma(^{88}\mathrm{Sr}) = 6.9$ mb, $\sigma(^{89}\mathrm{Y}) = 21$ mb, and $\sigma(^{90}\mathrm{Zr}) = 12$ mb (Allen, Gibbons, and Macklin 1971) effectively choke the flow toward higher atomic weights, and the drop in $N\sigma$ at these nuclides is very steep; for greater exposures the shelf is much shallower.

Figure 3 shows the elemental relative abundances of strontium, yttrium, and zirconium calculated according to Clayton and Ward (1974) for a variety of neutron exposures, where the contributions from all stable *s*-process isotopes have been summed for each element. If the exposure is large, the fact that zirconium has four stable *s*-process isotopes cancels the effect of lower individual isotopic abundance, and this element is produced in quantities comparable to strontium but considerably greater than yttrium. This is the case for the best fit solar distribution which is characterized by $\tau_0 = 0.27\ \mathrm{mb}^{-1}$ (Clayton and Ward 1974). For very low neutron exposures, however, the flow into zirconium is so severely depressed that even when all of its isotopes are added together this element may still be less abundant than yttrium with its single isotope (see Figure 3). The abundance pattern resulting from this "starved" *s*-process is Sr > Y > Zr; it is possible in this case to make Y $\sim$ Zr or even Y > Zr, but strontium will always exceed yttrium by at least half an order of magnitude. Note that few other consequences of such low neutron flux processing will be observable; elements on the lighter side of strontium are spectroscopically inaccessible, and the drastic reduction in available seed nuclei for elements heavier than zirconium will prevent any substantial production of these elements.

Having seen how Sr > Y > Zr might be obtained in the *s*-process, it is clear that the opposite pattern, Sr < Y < Zr, cannot occur. But possibly it might in the *r*-process. The *r*-process calculations of Seeger, Fowler, and Clayton (1965) assumed a static environment with fixed neutron density and temperature and they were forced to smooth the resulting very sharp distribution of nuclei in atomic weight (some A were completely bypassed) by not only averaging results for different temperatures but also averaging yields for neighboring A. Schramm (1973) showed that in a realistic dynamically expanding *r*-process the yield's very sharp variation with A is smoothed out by the continuous decline in neutron density. In the first approximation, therefore, for adjacent elements the final relative *r*-process yield will be roughly proportional to the number of stable *r*-accessible isotopes for each. In the case of interest, strontium has one, yttrium has one, and zirconium has five. So on this crude basis it is reasonable to conclude that the pattern Sr $\sim$ Y < Zr is a possible one for *r*-process nucleosynthesis. But in no case could one obtain Y $\gg$ Sr. Other observational consequences of such an *r*-process event are not so easily evaluated as for the *s*-process case.

Considering together these predictions from the theory of neutron addition, possible patterns include: Y < Sr $\sim$ Zr; Sr > Y $\sim$ Zr; and Zr > Sr $\sim$ Y. The pattern which is conspicuously absent is the one where Y > Sr and Y > Zr simultaneously. Yet it is this very one, however, which had been previously reported present in the Hg-Mn stars (see, e.g., references in § I) and for which no evidence has been found in any of the nine stars of this group in this study. It is true that yttrium is high in ϕ Her, HR 1800, and HR 2844, but Figure 1 reveals no evidence that the only certainly nonnuclear abundance pattern is present in these stars.

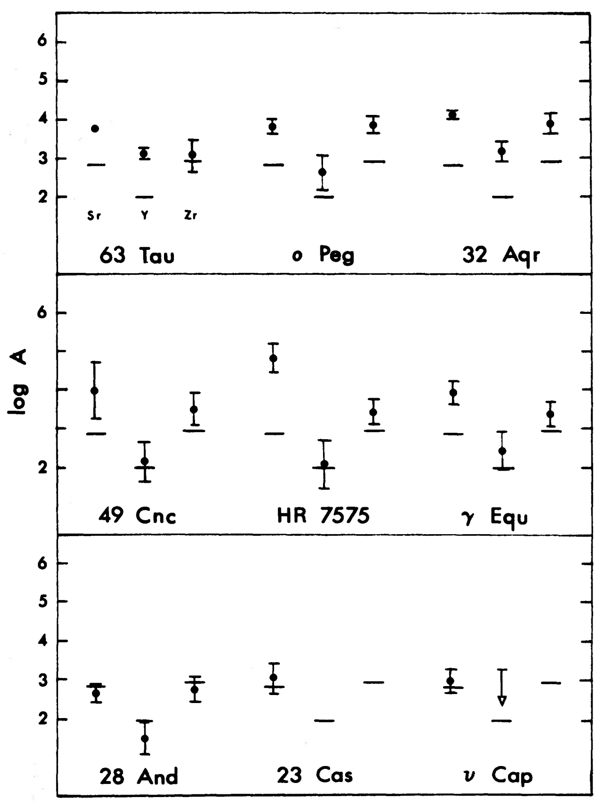

FIG. 2.—Abundances in the control group

A brief look at the results for the nine non–Hg-Mn stars (Figure 2) indicates a general agreement with the *s*-process predictions. If the *s*-process were responsible for the enhancements in the Am stars, for example, *o* Peg and 32 Aqr would indicate an approximately solar neutron exposure whereas 63 Tau would indicate a somewhat weaker one. Other determined abundances exhibit nuclear patterns as well except possibly that the Y/Sr ratio for the magnetic stars may be somewhat low. There is a tantalizing suggestion that the magnetic stars may differ systematically from the Am stars in having normal yttrium contents and smaller zirconium enhancements than the Am stars, in contrast with the sharing of large strontium overabundances between the two groups.

Of course this does not mean that because the pattern is not provably nonnuclear in the Hg-Mn stars that it is therefore of nuclear origin, and a comparison with other theories is in order. The absence of detectable magnetic fields obviates the need for a discussion of the magnetic accretion model (Havnes and Conti 1971; Havnes 1974). Next we come to a comparison of the observational results with the diffusion theory of Michaud and others.

The microturbulence parameters established for the stars in the analysis clearly show the Hg-Mn stars having significantly more quiescent atmospheres than the other stars. In some cases (HR 205, κ Cnc, and ϕ Her), there was no evidence for any microturbulence at all, and except for 53 Tau and HR 7664 it was ≤ 1.5 km s^{-1}, which is roughly the accuracy of the determination. There is increasing evidence that the microturbulence parameter really does represent some material motion with an optically thin scale size (Canfield 1971; Gray 1973). If this interpretation were correct, the finding of a negligible microturbulence would be favorable to the theory of diffusive separation. The atmospheric stability requirement imposed by this theory is very severe ($\sim 10^{-3}$ cm/s according to Michaud 1970). No correlation exists for the Hg-Mn stars between the yttrium abundances and the microturbulent velocities, but with the exception again of 53 Tau the derived microturbulences are close enough to zero that they may represent random errors in the determination of zero velocities. This would agree with the findings of Smith and Parsons (1975) in their Fourier study of line profiles in ι CrB, ϕ Her, and ν Her. Searle, Lungershausen, and Sargent (1966) also found systematically low microturbulences for the Hg-Mn stars.

As a group, the Hg-Mn stars possess very small projected rotational velocities, averaging $\langle v \sin i \rangle = 29$ km s^{-1} (Abt, Chaffee, and Suffolk 1972) and having a broad frequency distribution maximum between 20 and 40 km s^{-1} (Wolff and Wolff 1974). Slow rotation would also favor the diffusive separation of elements by suppressing meridional circulation. On the other hand, these stars do not have the large magnetic fields of the magnetic stars to help stabilize their atmospheres. There is also evidence for line profile assymetries in some of the manganese stars, including ι CrB, χ Lup, and HR 4072 (Smith and Parsons 1975, 1976). If these asymmetries are due to substantial radial mass outflow as suggested by these authors and by Karp (1976), the extreme atmospheric quiescence required for diffusive element separation may be ruled out.

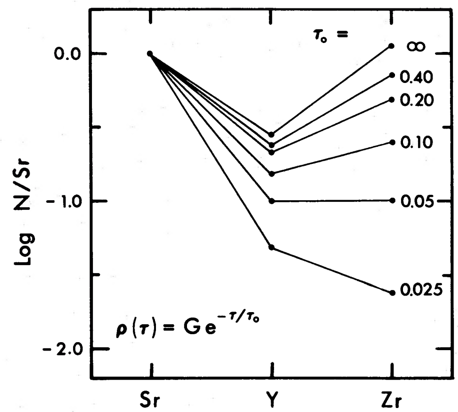

FIG. 3.—Yield from exponential exposure *s*-process (relative to strontium). See § III.

The most recent diffusion work germane to the present problem is that of Michaud *et al.* (1976). Previous analysis of the diffusive behavior of the yttrium group is contained in Michaud (1970), which showed the temperature dependence of the radiation force transmitted through photoionization (Michaud 1970, Fig. 3*d*). This graph showed peaks in the force (for $\tau_o = 0.1$, τ_o signifying the optical depth at 5000 Å) at $T_{\rm eff} = 9200$, 10,600, and 11,600 K for strontium, yttrium, and zirconium, respectively. Figures 4–6 show the abundances of strontium, yttrium, and zirconium plotted against $T_{\rm eff}$ for all the stars in the sample. No correlation is apparent in Figure 4 for strontium, and all three normal stars have normal

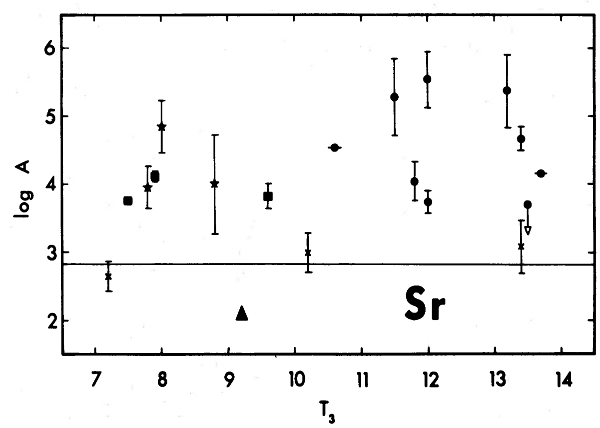

FIG. 4.—Derived abundances of strontium plotted against stellar effective temperatures. Solid circles represent Hg-Mn stars, solid squares Am stars, solid stars Ap stars, and crosses the normal stars. The bold triangle shown at the bottom indicates the maximum in the radiative force from photoionization as given by Michaud 1970.

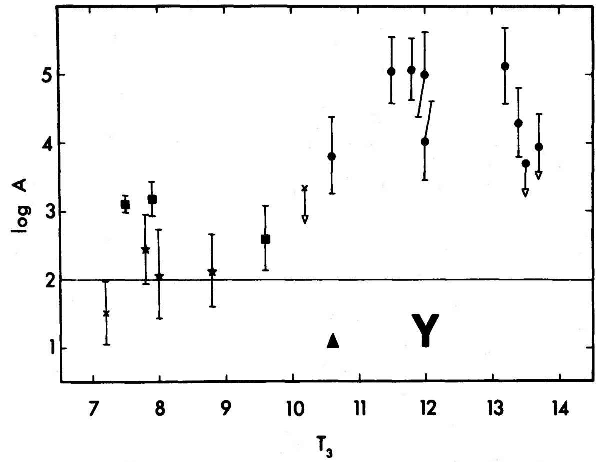

FIG. 5.—Same as Fig. 4, for yttrium

contents of this element irrespective of temperature. Yttrium does suggest a peak in Figure 5, but located closer to 12,000–13,000 than to 10,600 K as predicted by Michaud (1970). In the case of zirconium, what would otherwise constitute an encouraging agreement with the diffusion prediction is marred by the low zirconium content of HR 1800. Both the Monte Carlo results and inspection of the tracings support the numerical outcome that the Zr II spectrum is weak in this star. On the whole, these three figures cannot engender much enthusiasm for the photoionization-driven diffusion of these elements, although the outlook is not unrelievedly bleak in the cases of yttrium and zirconium. Additional uncertainties entering the prediction arise from the translation of radiative force excesses to mass fluxes and the location and mechanism of final deposition of diffused material.

In the later paper (Michaud *et al.* 1976) the contribution to the radiative force from transitions into the continuum is shown to be much smaller than the force through unsaturated lines for the heavier elements, at least in the deeper layers of the atmosphere. The theoretical predictions are very dependent on the mass of the star and on its effective temperature. Of the models presented, the case most nearly representative of the Hg-Mn stars would be M/M(solar) = 3.3 (Allen 1973). Unfortunately the detailed element by element predictions which are given for less massive stars are not presented for this greater mass, presumably because of the increased uncertainty in the calculations carried further out toward the boundary of the atmosphere. Insofar as for the more massive stars (without hydrogen convection zones) the mass cm^{-2} with which the upwardly diffusing material must be mixed is that of the visible atmosphere alone, the overabundances for "unabundant" elements can be enormous, and established in a very brief time (Cowley and Day 1976; Michaud *et al.* 1976). Michaud *et al.* present a graph (their Fig. 6) showing (radiative acceleration)/(gravitational thermal diffusion acceleration) = $F(R)/F(GT)$ where for yttrium the ratio is only of order unity as opposed to roughly 100 for yttrium's neighbors. This result, although calculated quite deep in the atmosphere at $T =$ 30,000 K, might indicate that yttrium would be greatly less abundant than strontium or zirconium for M/M(solar) = 3.3 if the effects of diffusion in these deeper layers were felt in the higher visible layers.

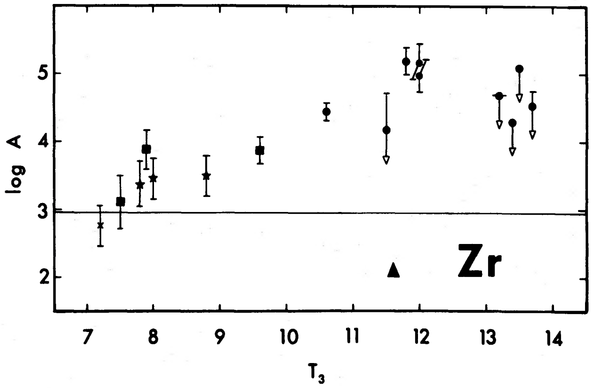

FIG. 6.—Same as Fig. 4, for zirconium

The reason for this surprising result may be simply seen by considering a typical model atmosphere for $T_{\rm eff}$ = 12,000 K and log g = 4.0, roughly representative of a typical Hg-Mn star. One may justifiably assume since the ionization potential of Y II is only 12.24 eV, less than that of hydrogen, that at any depth at or below the optical continuum forming region yttrium will be at least doubly ionized. It will not be four-times ionized at any depth pertinent to the discussion since Y IV has a noble gas configuration and an ionization potential of 61.8 eV (Allen 1973). Using the Saha equation (and equating partition functions) with the sample (12,000, 4.0) model we find for relative ionic populations n(IV)/n(III) $\sim$ 15 and 440 at τ_o = 10 and 100, respectively. No appreciable radiation force is to be expected on the krypton-like noble gas electronic configuration, so any yttrium present in the zone $10 \leq \tau_o < 100+$? can be expected, in the absence of laminar or turbulent flows, to *sink* in the Hg-Mn stars. Yttrium atoms reaching the bottom of this noble gas zone from below must pass through it to reach the observable part of the atmosphere and will probably be unable to do so. This very definitely contradicts the observational findings which show yttrium very abundant and certainly nowhere more than an order of magnitude below strontium or zirconium. The value of 100 for $F(R)/F(GT)$ for zirconium ought to lead to very large enhancements of Zr II in the line forming regions; the suggested huge Zr/Y ratios can definitely be ruled out by the derived abundances in my sample of Hg-Mn stars.

Summarizing, the following conclusions may be drawn concerning the Sr-Y-Zr group in the Hg-Mn stars: (i) they are all enhanced compared to the solar content; (ii) their relative pattern is frequently nonsolar in these stars; (iii) but the relative pattern is not demonstrably nonnuclear; (iv) the prediction of the radiative diffusion theory for yttrium compared to its neighbors for the appropriate stellar mass range is still to be clearly established, but does not appear promising. Inasmuch as the determination of Y/Sr and Y/Zr abundance ratios for these stars is only weakly model-dependent, this apparent weakness of the current version of the diffusion theory must be viewed as a serious shortcoming.

I would like to thank my advisor, Dr. C. R. Cowley, for introducing me to the study of the hot spectroscopically peculiar stars, for the extensive use of his plate material, and for much guidance during the course of the work. Dr. R. L. Sears read the manuscript at several important junctures and provided valuable commentary. I am also grateful for a Rackham Dissertation Grant which provided funds for a trip to the Dominion Astrophysical Observatory in British Columbia.

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MARC S. ALLEN: Institute for Astronomy, 2680 Woodlawn Drive, Honolulu, HI 96822