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THE EXCITING STARS OF LOW-EXCITATION PLANETARY AND DIFFUSE NEBULAE

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ABSTRACT

Effective temperatures of the exciting stars of 35 low-excitation planetary and five diffuse nebulae have been computed under both of the alternate assumptions that the stars radiate according to Hummer and Mihalas's flux models, and that they behave as blackbodies. The method used employs the ratio of the sum of the nebular forbidden-line fluxes to the flux of Hβ (Stoy's method), with modification for the observed electron temperature and helium content. The present analysis is restricted to nebulae with very weak or absent He II emission, which are therefore expected to be optically thick. Both sets of temperatures correlate very well with measured O^{2+}/O^{+} ratios. The temperatures calculated from the flux models are systematically lower than those calculated under the blackbody assumption, the discrepancy increasing with increasing T. The onset of He II emission is about 46,000 K for the models, and 60,000 K for the blackbodies. The latter is consistent with the predicted He II Lyman continuum emission at 228 Å, whereas the former is not, and it is concluded that the blackbody assumption is superior to that of the set of flux models.

The method provides an independent calibration of effective ultraviolet temperatures on the upper main sequence, in that $T_{\rm eff}$ for θ^1 Ori C (an O6 star) is found to be 37,000 K. Peimbert's finding of a low mass for the planetary in the globular cluster M15 is confirmed. Finally, in accord with earlier studies, it appears likely that cooler planetary nebula nuclei (T < 70,000 K) are as a class less luminous than the maximum luminosity found for the hotter stars.

Subject headings: nebulae: planetary — stars: atmospheres — stars: early-type

I. INTRODUCTION

The first measurements of the temperatures of the exciting stars of gaseous nebulae were made by Zanstra (1931). The method has been extended by Harman and Seaton (1964) to nebulae which are optically thin in the hydrogen Lyman continuum (for which Zanstra temperatures are lower limits), by using He II line fluxes to find the photon flux shortward of λ228 Å, where the nebula should be thick. The Zanstra temperature rests upon the observed ratio of nebular to stellar flux. Stoy (1933), however, developed a method whereby the stellar temperature could be derived from the nebular emission alone. The original method assumes that all the cooling of nebular electrons takes place by the excitation of forbidden lines (a very good approximation—see Fig. 3 in Burbidge, Gould, and Pottasch 1963). Since the cooling rate equals the heating rate, the total forbidden-line flux gives a measure of the rate at which the star heats the nebula, i.e., the energy emitted by the star shortward of the Lyman limit. The flux in the Balmer lines, following Zanstra's (1927) original argument, gives us the stellar photon flux shortward of the Lyman limit. The ratio of energy flux to photon flux is a function of stellar temperature, which we can then measure. In deriving his temperatures, Stoy assumed that the star radiates like a blackbody and took into account the fact that energy is extracted from the starlight in the ionization of hydrogen.

Some important modifications of Stoy's original method are necessary. First, not all of the original energy of the free electrons is available for the excitation of forbidden lines, as the electrons recombine with nonzero energy. This principal modification was first developed by Aller (1956), who also derived a number of values of temperature (T_s) based upon it. In this paper a further modification will be made to take into account the fact that the mean energy of recombination of electrons is less than their mean total energy, since slow electrons are more likely to recombine (see Osterbrock 1974, p. 42).

Second, not all of the forbidden-line radiation is observable. A significant fraction ($\sim$10–20%) of nebular cooling takes place in the excitation of the fine-structure states in the ground terms of ions such as Ne^{+} and O^{2+} (see again Fig. 3 in Burbidge, Gould, and Pottasch 1963). With rare exceptions the resulting emission is not observable, and the emission rates must be calculated.

Third, about 10 percent of the nebula consists of helium, which has a higher ionization potential than does hydrogen. Consequently, more of the stellar energy must be used in ionization than would be used for a pure hydrogen nebula, and less is available for forbidden-line excitation. Also, recombinations to helium can result in photoionization of hydrogen, and this interlocking effect must be included. The above modifications act to increase the calculated temperatures above what would have been calculated by the original method.

Finally, we must consider the fact that the stars do not really radiate like blackbodies. Surface-flux models have been calculated for a grid of stars (Hummer and Mihalas 1970), and the calculations must incorporate these as well as the blackbody assumption.

Stoy's method has some distinct advantages over the Zanstra method. It makes use only of nebular radiation; central star magnitudes do not need to be measured. Consequently, temperatures of the exciting stars of small, "stellar" nebulae, for which the star itself can be nearly unobservable, can be measured. Also, the method makes use only of relative line intensities; absolute fluxes are unnecessary. More importantly, the method is independent of the degree to which the nebula surrounds the star. Both the Stoy and Zanstra methods require that that part of the nebula which does surround the star be optically thick in the Lyman continuum. If the nebula is in fact *thin*, a lower limit to the true temperature will be derived. For an optically thin nebula, however, an optical depth factor will enter into both the numerator and denominator of the ratio of energy flux to photon flux so that the Stoy method should give a closer limit than the Zanstra method, if the latter is based only upon hydrogen lines.

The completeness of absorption, however, is a difficult quantity to estimate. The problem can be avoided in the Zanstra method by using He I or He II lines, for whose Lyman continua the nebulae may be presumed to be thick. The Stoy method cannot be treated so simply, and one must know the actual optical depth at the Lyman limit. Consequently, this application of Stoy's method is restricted to optically thick objects. Presumably, if a star is cool enough that He II lines are absent in the nebular spectrum, the nebula will be optically thick in the Lyman continuum, or very nearly so. This contention is supported by the consistency in the simultaneous solutions for H and He I shown by Harman and Seaton's (1966) Figures 2 and 3, by the fact that these nebulae are generally small and have radii below or near the optically thick-thin dividing line set by Cahn and Kaler (1971), and that they have relatively high densities. Further evidence from the consistency of the temperatures that the nebulae are indeed thick will be given in a later section. The remainder of this paper will then deal only with nebulae for which the He II lines are either weak or absent.

The major drawback of this method is that the nebula's spectrum must be rather completely observed. We must have a good accounting of all the important forbidden lines. A large number of objects meet this criterion, however. For these objects for which the data are not as complete as one would like, reasonable estimates of the strengths of the unobserved lines can be made.

II. DEVELOPMENT OF THE METHOD

We begin with an equation similar to that presented by Aller (1956, p. 221, eq. [24]) for a pure hydrogen nebula. The term $\rho(T_s, T_e)$ is the ratio of the energy available for forbidden-line production to the number of ionizing photons multiplied by $\nu(\mathrm{H}\beta) = \nu_\beta$, or

$$\rho(T_s, T_e) = \frac{\int_{\nu_1}^{\infty}[\mathscr{F}(\nu)/\nu](\nu - \nu_e{}^{\mathrm{H}})d\nu}{\int_{\nu_1}^{\infty}\mathscr{F}(\nu)\nu_\beta/h\nu} = \frac{\sum L_F}{\sum L_B\nu_\beta/h\nu_B}\,. \tag{1}$$

Here, ν_1 is the frequency of the Lyman limit, T_s and T_e are the stellar and electron temperatures, respectively; L_F and L_B are the fluxes in the forbidden and Balmer lines, respectively. The right-hand side of equation (1) can clearly be written in terms of relative intensities I_F and I_B, and the denominator can be expressed in terms of $I(\mathrm{H}\beta)$ alone as

$$\sum I_B\frac{\nu_\beta}{h\nu_B} = \frac{\alpha_B(\mathrm{H})I(\mathrm{H}\beta)}{h\alpha(\mathrm{H}\beta)}\,, \tag{2}$$

where $\alpha_B(\mathrm{H})$ and $\alpha(\mathrm{H}\beta)$ are, respectively, the total recombination coefficient for hydrogen (Seaton 1960) and the effective recombination coefficient for Hβ (Brocklehurst 1971). Equation (1) is expressed in terms of the more general stellar flux $\mathscr{F}(\nu)$, rather than in terms of the blackbody adopted by Aller. The quantity $h\nu_e{}^{\mathrm{H}}$ represents the mean energy of recombining electrons, $\bar{E}$. In Aller's development $h(\nu_e{}^{\mathrm{H}} - \nu_1) = (3/2)kT_e$. In fact, $\bar{E}$ is less than $(3/2)kT_e$ because lower energy electrons are more likely to recombine. From the tables of the recombination cooling coefficient presented by Osterbrock (1974) and Seaton (1960), $\bar{E}/(3/2)kT_e = 0.44$ at $T_e = 10{,}000$ K, with only a very small dependence upon T_e.

Actually, to the numerator on the right-hand side of equation (1) should be added the total free-free flux radiated by the nebula. The rate of nebular cooling by free-free processes is, however, very small compared to that produced by forbidden-line excitation. Let H^+ and N_e represent the ion and electron densities, respectively. From Osterbrock (1974) we find that the free-free cooling rate cm^{-3} is about $\frac{1}{2}$ of the energy difference $\mathrm{H}^+N_e\alpha_B(3/2)kT_e - \mathrm{H}^+N_e\alpha_B\bar{E}$. Thus to a very good approximation, we can conveniently absorb the free-free radiation into the left-hand side of equation (1) by redefining $h\nu_e{}^{\mathrm{H}}$ as a fictitious mean electron energy, or

$$h\nu_e{}^{\mathrm{H}} = h\nu_1 + \bar{E} + \tfrac{1}{2}(\tfrac{3}{2}kT_e - \bar{E}) = h\nu_1 + 0.72(\tfrac{3}{2}kT_e)\,. \tag{3}$$

The next step is to add the effect of absorption of photons by neutral helium. We break the Lyman continuum into three frequency regimes where ν_2 and ν_3 are the neutral and ionized helium ionization limits, respectively.

Between ν_1 and ν_2 hydrogen is the sole absorber. Between ν_2 and ν_3, hydrogen and helium compete in absorbing photons. The relative number absorbed by each will depend upon the ratios of neutral H and He and upon the absorption coefficients. From ν_3 to infinity, a photon can ionize H, He, and He^+. Photons with $\nu > \nu_3$ are negligible even at the highest temperatures considered in this study, and this frequency regime will be ignored.

Further complication is introduced by the fact that recombination of electrons with He^+ ions produces radiation that can photoionize hydrogen. This process will make more energy available for forbidden-line production, and will partly offset the absorbing effects of neutral helium. The actual amount of this interlocking is difficult if not impossible to calculate accurately. The subsequent discussion follows Osterbrock (1974). Of the electrons recombining to helium, about one-quarter go to the singlets, and about three-quarters to the triplets. Of those that go to the singlets, one-third (or $\sim 8\%$ of the total) go to $2\,^1S$ and two-thirds ($\sim 17\%$) go to $2\,^1P$. The electrons on $2\,^1P$ go directly to $1\,^1S$ with emission of a photon at $\lambda 584$. When this photon, with frequency ν_{584}, ionizes a hydrogen atom, the resulting free electron will have an energy in excess of $h(\nu_e{}^{\mathrm{H}} - \nu_1)$, so that energy $h(\nu_{584} - \nu_e{}^{\mathrm{H}})$ is added back to the electron gas.

The electrons in $2\,^1S$ will decay to $1\,^1S$ by emission of two photons. Each electron which arrives in $2\,^1S$ will produce 0.56 photons capable of photoionizing hydrogen. Consequently, 0.56×8 percent ≈ 5 percent of all helium recombinations produce such ionizing photons. The mean energy of the hydrogen electrons ejected as a result of the helium two-photon process is $h(\bar{\nu}_{2p} - \nu_1)$ so that $h(\bar{\nu}_{2p} - \nu_e{}^{\mathrm{H}})$ is added to the energy of the electron gas. The ratio $\bar{\nu}_{2p}/\nu_1 \approx 1.2$ as calculated from the data provided by Hummer and Seaton (1964), where it is assumed that the neutral helium two-photon process behaves the same way as the hydrogen and ionized helium counterparts.

All the electrons which recombine to triplet states ultimately go to $2\,^3S$ which is highly metastable. The electrons can, however, go to $1\,^1S$ by a forbidden transition with emission of a photon at 626 Å, or they can go to $2\,^1S$ by collision, which is followed by two-photon emission. The probability of these transitions occurring is very uncertain, however, since there is strong evidence that $2\,^3S$ can be depopulated by hydrogen Lα photons (see Osterbrock 1964), which will greatly reduce the interlocking between hydrogen photoionizations and the helium triplets. Since there is no firm evidence on the amount of helium triplet interlocking, a limiting case will be assumed in which the helium triplets are ignored.

Equation (2) gives a measure of the number of photons released by the star for a pure hydrogen nebula. If the nebula contains helium, however, it will absorb a fraction of the star's ultraviolet photons. From the preceding assumption that 22 percent of the He recombinations produce hydrogen photoionizations, only 78 percent of the helium recombinations need be counted, so that the denominator of equation (1) should read

$$\frac{\alpha_B(\mathrm{H})I(\mathrm{H}\beta)}{\alpha(\mathrm{H}\beta)h}\left[1 + 0.78\,\frac{\alpha_B(\mathrm{He})}{\alpha_B(\mathrm{H})}\,\frac{\mathrm{He}}{\mathrm{H}}\right],$$

where $\alpha_B(\mathrm{He})$ is the total case B recombination coefficient for helium, and He/H is the helium to hydrogen ratio by number. This term has only a small dependence on T_e.

For a given photon of frequency $\nu_2 < \nu < \nu_3$, we let $M(\nu)$ be the probability that it will be absorbed by H, where $0 < M(\nu) < 1$. We can now include the effects of helium into equation (1) by rewriting it as follows:

$$\begin{aligned}\rho(T_s, T_e, \mathrm{He/H}) &= \Big\{\int_{\nu_1}^{\nu_2} [\mathscr{F}(\nu)/\nu](\nu - \nu_e{}^{\mathrm{H}})d\nu + \int_{\nu_2}^{\nu_3} M(\nu)[\mathscr{F}(\nu)/\nu](\nu - \nu_e{}^{\mathrm{H}})d\nu + \int_{\nu_2}^{\nu_3} [1 - M(\nu)][\mathscr{F}(\nu)/\nu] \\ &\quad \times [0.17(\nu - \nu_e{}^{\mathrm{He}} + \nu_{584} - \nu_e{}^{\mathrm{H}}) + 0.05(\nu - \nu_e{}^{\mathrm{He}} + \nu_{2p} - \nu_e{}^{\mathrm{H}}) + 0.78(\nu - \nu_e{}^{\mathrm{He}})]d\nu\Big\}\Big/ \\ &\quad \int_{\nu_1}^{\infty} \mathscr{F}(\nu)\frac{\nu_\beta}{\nu}\,d\nu = \frac{\sum I_F}{(\alpha_B(\mathrm{H})/\alpha(\mathrm{H}\beta))I(\mathrm{H}\beta)[1 + 0.78(\alpha_B(\mathrm{He})/\alpha_B(\mathrm{H}))\mathrm{He/H}]}\,. \end{aligned} \tag{4}$$

In the left-hand side of equation (4), $h\nu_e{}^{\mathrm{H}}$ is as defined in equation (3), and $h\nu_e{}^{\mathrm{He}}$ is similarly defined as

$$h\nu_e{}^{\mathrm{He}} = h\nu_2 + 0.72\,(\tfrac{3}{2}kT_e)\,. \tag{5}$$

The fraction of photons of frequency ν absorbed by H is

$$M(\nu) = \frac{\mathrm{H}^0 a_\nu(\mathrm{H}^0)}{\mathrm{H}^0 a_\nu(\mathrm{H}^0) + \mathrm{He}^0 a_\nu(\mathrm{He}^0)} = \frac{a_\nu(\mathrm{H}^0)}{a_\nu(\mathrm{H}^0) + a_\nu(\mathrm{He}^0)\mathrm{He}^0/\mathrm{H}^0}\,, \tag{6}$$

where H^0 and He^0 are the densities of neutral H and He, respectively, and the a_ν's are absorption coefficients. The $a_\nu(\mathrm{H}^0)$ are calculated from the formula given by Seaton (1960) and the bound-free Gaunt factors of Karzas and

Latter (1961). The $a_\nu(\mathrm{He}^0)$ are taken from the work of Huang (1948), which are sufficiently close to the improved values calculated by Bell and Kingston (1967). The ratio $\mathrm{He}^0/\mathrm{H}^0$ is found by formulating the balance equations

$$4\pi W\mathrm{H}^0 \int_{\nu_1}^{\nu_3} \frac{\mathscr{F}(\nu)}{h\nu} a_\nu(\mathrm{H})d\nu + 0.22\mathrm{He}^+N_e\alpha_B(\mathrm{He}) = \mathrm{H}^+N_e\alpha_B(\mathrm{H})\,, \tag{7a}$$

$$4\pi W\mathrm{He}^0 \int_{\nu_2}^{\nu_3} \frac{\mathscr{F}(\nu)}{h\nu} a_\nu(\mathrm{He})d\nu = \mathrm{He}^+N_e\alpha_B(\mathrm{He})\,, \tag{7b}$$

where W is the dilution factor. If we divide equation (7b) by (7a) and solve for $\mathrm{He}^0/\mathrm{H}^0$, we obtain

$$\frac{\mathrm{He}^0}{\mathrm{H}^0} = \frac{\alpha_B(\mathrm{He})\mathrm{He/H}}{\alpha_B(\mathrm{H}) - 0.22\alpha_B(\mathrm{He})\mathrm{He/H}} \frac{\int_{\nu_1}^{\nu_3} [\mathscr{F}(\nu)/h\nu]a_\nu(\mathrm{H})d\nu}{\int_{\nu_2}^{\nu_3} [\mathscr{F}(\nu)/h\nu]a_\nu(\mathrm{He})d\nu}\,. \tag{8}$$

The recombination coefficients $\alpha_B(\mathrm{H})$ and $\alpha_B(\mathrm{He})$ are taken from Seaton (1960) and Osterbrock (1974). Equation (8) is an approximation to the true $\mathrm{He}^0/\mathrm{H}^0$, wherein we assume the nebula to be a geometrically thin shell surrounding the star. The true $\mathrm{He}^0/\mathrm{H}^0$ is a function of distance from the star, and must take into account that W is in fact a function of ν, and may not be taken out of the integrals. The results, however, are quite insensitive to $\mathrm{He}^0/\mathrm{H}^0$, so that this approximation is adequate.

The ratio $\rho(T_s, T_e, \mathrm{He/H})$ was calculated from equation (4) by numerical integration for a grid of T_s, T_e, and He/H under both the assumptions that the stars radiate like blackbodies $[\mathscr{F}(\nu) = B(\nu)]$ and that $\mathscr{F}(\nu)$ is as given by the flux models of Hummer and Mihalas (1970). Hummer and Mihalas present surface fluxes for a grid of T_s, composition, and log g. The calculations of ρ were made by generally adopting the 300 series of chemical composition and log $g = 4.5$, although some other models had to be used for interpolation. The variation in ρ produced by variation in composition and log g is not very large, however—generally less than ± 10 percent from the adopted values.

A sampling of the calculations is presented in Figure 1. The quantity $\rho(T_s)$ is shown for $T_e = 10{,}000$ K and He/H $= 0.10$ both for the blackbody assumption and for the flux models. In order to show the dependence of ρ on T_e and He/H, $\rho(T_s)$ is presented for the blackbody assumption for $T_e = 10{,}000$, 14,000, and 18,000 K and He/H $= 0.10$ and 0.15, and for $T_e = 10{,}000$ K, He/H $= 0$. The dependence of $\rho(T_s)$ on T_e and He/H is the same

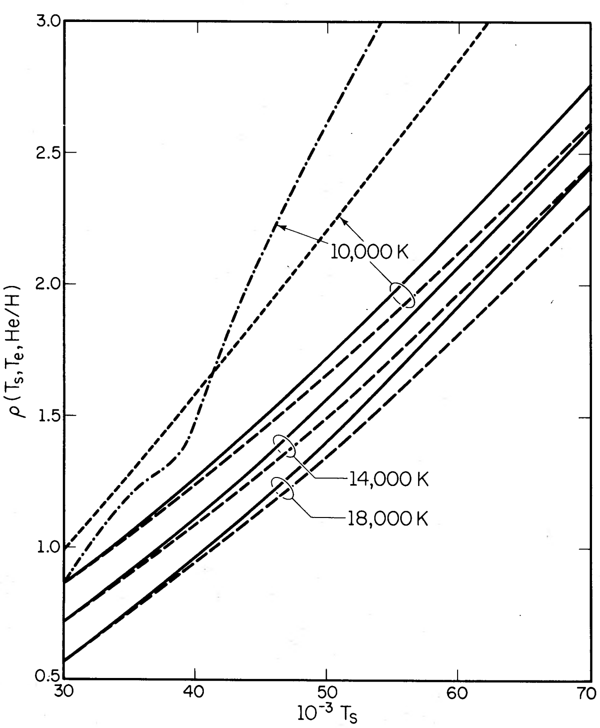

FIG. 1.—The ratio of $\rho(T_s, T_e, \mathrm{He/H})$ plotted against $10^{-3}T_s$. The dot-dash line represents an assumption of Hummer and Mihalas's (1970) models for He/H $= 0.1$, $T_e = 10{,}000$ K. The other curves are for a blackbody assumption for $T_e = 10{,}000$, 14,000, and 18,000. *Solid lines*, He/H $= 0.1$; *dashed lines*, He/H $= 0.15$; *dotted line*, He/H $= 0$.

for the model assumption as for the blackbody assumption. The difference exhibited between the two assumptions will be discussed in the next section.

Note the effect of the inclusion of helium in the calculations. At higher stellar temperatures, its neglect can cause an error of 10,000 K or more in the temperature of the exciting star. At low T_s the effect becomes smaller since more of the stellar radiation is between ν_1 and ν_2 where He has no effect.

III. CALCULATION OF STELLAR TEMPERATURES

In order now to measure the temperature of the exciting star of a nebula, T_s, we must adopt values of T_e and He/H, and then determine the ratio on the right-hand side of equation (4). Interpolation in the grid of $\rho(T_s, T_e, \text{He/H})$ then will determine T_s for each assumption. The denominator of the ratio is simple to compute with the references already given, once He/H is determined.

The numerator of the ratio presents more of a problem because we must allow for unobservable fine-structure forbidden lines, after correction for interstellar extinction. The corrected intensities were taken from Kaler's (1976*a*) catalog, which also gives the appropriate references. The observations of the nebulae are not at all uniform. For some new nebulae all of the optically available spectrum has been observed. For others, estimates had to be made for unobserved lines. For example, if the [N II] lines were not observed, I[N II] was set equal to I[O II]. Also included in the sum was $I(\lambda 10830)$ of He I which is collisionally excited (see Osterbrock 1974); an estimate is made when it is not observed.

Second, we must estimate the degree of fine-structure cooling for these nebulae. The intensities of the fine-structure lines were calculated relative to observable forbidden lines by solving the usual balance equations (see, for example, Aller 1956, pp. 192, 193). The collision cross sections were taken from Saraph, Seaton, and Shemming (1969) and Krueger and Czyzak (1970), and the transition probabilities were taken from the compilation by Garstang (1968).

A number of ions were examined, and it was found that only O^{2+}, Ne^{2+}, and Ne^{+} added a significant amount to the sum $\sum I_F$. The intensities of the fine-structure lines of O^{2+} and Ne^{2+} are readily calculated in terms of the nebular line intensities of these ions. Singly ionized neon has no readily observable lines, so we must estimate an ionic abundance. Since all these nebulae have absent or very weak He II lines, Ne^{3+} will not be present, and the total Ne density is simply $Ne^{+} + Ne^{2+}$. Kaler (1973) found that Ne/O = 0.41 for planetary nebulae in general. Then $Ne^{+}/O = 0.41 - Ne^{2+}/O$. The latter ratio was calculated for each nebula from the [Ne III], [O II], and [O III] forbidden lines, so that Ne^{+}/O could be computed. The strength of the Ne^{+} line can then be calculated readily in terms of the [O III] or [Ne III] forbidden lines. Generally the fine-structure lines constitute about 10–20 percent of $\sum I_F$. Unobserved forbidden lines in the ultraviolet are ignored, as most of these arise from highly ionized atoms which do not exist in these nebulae with cooler central stars (see Flower 1968).

For the above calculations, and for interpolation in the grids, we must adopt electron temperatures T_e, electron densities N_e, and values of He/H. For planetaries, T_e was taken from the compilation by Cahn (1976) or from Kaler, Aller, and Czyzak (1976), and N_e was calculated from forbidden-line ratios given by the above reference, Kaler's (1976*a*) catalog, or Aller and Epps (1976), or else was taken from Kaler (1970). If T_e was unmeasured, 10,000 K was adopted, except for Hb 12 (14,000 K). For the diffuse nebulae, the data are from Kaler (1970). Generally, He/H was taken as 0.1, except for NGC 6803 (He/H = 0.13; Lee *et al.* 1974), and Me 2-2 (He/H = 0.15; Kaler 1974).

The results of the calculations are given in Tables 1*a* (planetaries) and 1*b* (diffuse nebulae). Table 1*a* presents the nebula's common name, the Perek-Kohoutek (1967) number (PK), values of $10^{-3}T_s$ for the assumptions of Hummer and Mihalas's (1970) models and the blackbody approximation (BB), and an estimate of the quality of the results where A is highest and C lowest. A denotes nebulae for which the spectrum is well observed, and all important lines measured. For nebulae denoted by B some estimates (e.g., [N II] lines in the infrared) had to be made. C means that fairly large estimates had to be made, that the spectrum is not well observed, or that the observations are anomalous. Table 1*b* gives the same information, except for the Perek-Kohoutek number. The same models were used for the stars of the diffuse nebulae as for the planetary nuclei, since the results are not very sensitive to log g.

Errors in the temperatures are difficult to estimate, but the following appear realistic. For the blackbody assumption: A, ± 2000 K; B, ± 4000 K; C, $\pm 10,000$ K. For the model assumption, A, ± 1500 K; B, ± 2500 K; C, ± 6000 K. The errors are different because of the different slopes of $\rho(T_s)$ for the different assumptions.

IV. EVALUATION OF THE MODELS

From Table 1 or Figure 1 we see that the temperatures derived from the two assumptions (models and blackbodies) are quite discordant at the upper end of the temperature range. The function $\rho(T_s, \text{models}) \approx \rho(T_s, \text{BB})$ for low temperatures, but rises much more steeply, such that at the high end of the range T_s(BB) is over 15,000 K greater than T_s(model) for a given value of ρ. The reason for this behavior is that in the upper temperature range the photon flux between ν_2 and ν_3, relative to that between ν_1 and ν_2, is greater for the models than for the blackbodies. In addition, the slope of $\mathscr{F}(\nu)$ between ν_2 and ν_3 is such that there are more high-energy photons near ν_3 (relative to those near ν_2) for the models than for the blackbodies, all of which results in a greater production of forbidden lines.

TABLE 1

TEMPERATURES OF EXCITING STARS

NEBULA	PK	$10^{-3}T_s$ Model	$10^{-3}T_s$ BB	QUAL.	NEBULA	PK	$10^{-3}T_s$ Model	$10^{-3}T_s$ BB	QUAL.	NEBULA	PK	$10^{-3}T_s$ Model	$10^{-3}T_s$ BB	QUAL.
a. Planetary Nebulae														
NGC 40	120+9°1	32	32	A	NGC 6891	54−12°1	42	49	B	Hu 2-1	51+9°1	34	37	B
NGC 6210*	43+37°1	45	58	A	IC 418	215+24°1	33	34	A	J320	190−17°1	45	57	B
NGC 6369	2+5°1	49	65	C	IC 2149	166+10°1	34	37	B	M1-58	22−3°1	42	50	B
NGC 6543	96+29°1	39	42	A	IC 3568*	123+34°1	47	61	B	M1-59	23−2°1	37	40	B
NGC 6567	11−0°2	44	56	B	IC 4406*	319+15°1	53	73	C	M1-64	64+15°1	45	57	B
NGC 6572	34+11°1	46	60	A	IC 4593	25+40°1	38	42	A	M1-73	51−3°1	33	36	B
NGC 6629	9−5°1	40	45	B	IC 4634	0+12°1	44	55	B	M2-23	2−2°4	40	44	C
NGC 6790*	37−6°1	49	66	B	IC 4732	10−6°1	48	64	B	Mel-1*	52−2°1	49	65	B
NGC 6803*	46−4°1	47	62	A	IC 4846*	27−9°1	47	61	B	Me2−2	100−8°1	38	42	B
NGC 6807	42−6°1	46	60	B	IC 4997	58−10°1	42	49	A	PS-1	65−27°1	28	23	B
NGC 6826	83+12°1	40	45	A	BD +30°3639	64+5°1	31	30	A	Vyl-1	118−8°1	42	52	C
NGC 6833	82+11°1	41	49	B	HB 12	111−2°1	34	38	C					
b. Diffuse Nebulae														
NGC 604		33	35	B	IC 1470		34	37	C	η Car		31	31	B
NGC 1976		34	37	A	30 Dor		38	42	B					

* λ4686 He II present in nebular spectrum.

The question is: Which of these two assumptions results in the better set of temperatures? One would almost automatically assume that the set derived from the flux models would be better than that derived from the blackbody assumption, but that does not seem to be the case. Leaving aside for the moment the question of the validity of the Stoy method itself, let us look at the nebular ionization level associated with the stars in Table 1. The nebulae which show the presence of He II λ4686 are denoted by an asterisk. There is a clear critical temperature for the onset of nebular He II radiation. For the blackbody assumption, $T_s \geqslant 60{,}000$ K; whereas for the models, $T_s \geqslant 46{,}000$ K. The nebulae just above the critical temperature have values of $I(\lambda 4686)$ from one to five percent of $I(\mathrm{H}\beta)$. From Seaton's (1960) total recombination coefficients for H^+ and He^{2+} and Brocklehurst's (1971) effective recombination coefficients for $\mathrm{H}\beta$ and He II Pα, we find that the recombination rate for He^{2+} is $\geqslant 5 \times 10^{-3}$ that for H^+ for these nebulae. If the nebula is optically thick, we can relate the former to the rate of stellar photon production

$$\mathscr{N}(\mathrm{He}^+) = \int_{\nu_3}^{\infty} (\mathscr{F}\nu/h\nu)d\nu\,,$$

and the latter to

$$\mathscr{N}(\mathrm{H}) + \mathscr{N}(\mathrm{He}) = \int_{\nu_1}^{\nu_3} [\mathscr{F}(\nu)/h\nu]d\nu\,,$$

and thus we would expect that $R = \mathscr{N}(\mathrm{He}^+)/[\mathscr{N}(\mathrm{H}) + \mathscr{N}(\mathrm{He})]$ also should be $\sim 5 \times 10^{-3}$ or greater. Hummer and Mihalas (1970) tabulate $\mathscr{N}(\mathrm{H})$, $\mathscr{N}(\mathrm{He})$, and $\mathscr{N}(\mathrm{He}^+)$ for all the models and for blackbodies. The models, for the critical temperature of 46,000 K, show that $R < 10^{-5}$, more than two orders of magnitude too small. For the blackbodies at 60,000 K, however, $R \approx 3 \times 10^{-3}$, which is consistent with the required value. We could bring the critical temperature of 46,000 K for the models into agreement with the stellar photon production ratio R, if we assume that the nebula is optically thin for $\nu_1 < \nu < \nu_3$, and that the rate of nebular absorption in this frequency region is $\sim 10^{-3}[\mathscr{N}(\mathrm{H}) + \mathscr{N}(\mathrm{He})]$. This assumption, however, is not at all justified by the Zanstra temperatures (T_{Z}) for these stars (see Harman and Seaton 1966), as the smooth transition between T_{Z} from H for stars of nebulae without He II and T_{Z} from He II for the higher temperature stars indicates that the nebulae of Table 1 are in fact optically thick. In order to have consistency in the Zanstra temperatures, the number of photons between ν_1 and ν_3 absorbed can be no less than $\sim 10^{-1}$ of those produced.

The Zanstra temperatures computed by Harman and Seaton (1966) provide another, though related, argument. Their values of T_{Z}, based upon blackbodies, also show that the critical temperature for He II line production is 60,000 K, consistent with that found from the Stoy method for blackbodies. Now if we assume the models, we find that for the nebulae with He II lines, the critical stellar temperature would have to be $\sim$85,000–90,000 K, whereas from the Stoy method the critical temperature is 46,000 K. The blackbody assumption gives a smooth transition between the nebulae with He II and those without, whereas the model assumption shows a strong discontinuity. In addition, if we correct Harman and Seaton's T_{Z} for nebulae without He II to what they would be for the models, many nebulae without He II lie above the model-critical temperature of 46,000 K.

A large number of values of T_{Z} have been recalculated by using the improved magnitudes of Shao and Liller (1976) and Kaler (1976*b*). The above general conclusions are supported by the newer work, which will be discussed at a later time.

One other possibility must be considered, that the method itself is in error. The interlocking between H and He is already set to give the maximum temperature, so that the only room for error would be in $\sum I_n$, the sum of the observed forbidden-line intensities. In order to bring the model-critical temperature for He II production up to 60,000 K, we would have to increase $\sum I_n$ by 60 percent. This figure seems much too high in view of the detailed observations that have been made of nebulae; there is no indication as to where in the spectrum the missing radiation might be found. In order to increase ρ by 60 percent we would have to increase the fine-structure flux by a factor of between 5 and 10, which cannot be justified with present data.

The conclusion is that the blackbody assumption gives more realistic temperatures than do Hummer and Mihalas's (1970) models. The Stoy method is sensitive to the shape of the flux curve $\mathscr{F}(\nu)$; and line blanketing, suggested by Hummer and Mihalas, might be important.

V. DISCUSSION

The temperatures presented in Table 1 can be used to discuss a number of interesting points. Following the discussion in § IV above, the values of T_s derived under the blackbody assumption will be adopted.

The calculations provide an independent calibration of the effective ultraviolet temperature of the upper main sequence, i.e., 37,000 K for the O6 star θ^1 Ori C, which excites the Orion Nebula NGC 1976. This figure is fairly close to the value of 40,000 K suggested by Morton (1969) for an O6 star, as derived from the Zanstra method. Note here the high temperature of 42,000 K of the exciting star of the 30 Doradus Nebula: it must be one of the hottest on the main sequence.

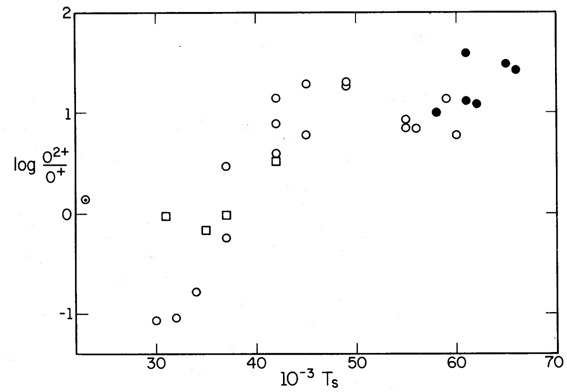

FIG. 2.—The log of the ion ratio O^{2+}/O^{+} plotted against exciting star temperature $10^{-3}T_s$. The circles and boxes represent planetary and diffuse nebulae, respectively. The filled circles indicate the nebulae which show He II lines. The point representing Ps–1 has a dot in the circle.

We would expect that the ionization level in a nebula would be dependent upon the temperature of the exciting star. As an ionization index, the O^{2+}/O^{+} ratio was calculated for those nebulae in Table 1 for which electron densities can be determined reliably from forbidden lines. The calculations were made by employing the references given in § III of this paper. The values of log (O^{2+}/O^{+}) are plotted against $10^{-3}T_s$ in Figure 2. The planetaries are represented by circles (those with nebular He II emission are filled); the diffuse nebulae, by boxes. The correlation between the two quantities is very good. Starting at $T_s \approx 30{,}000$ K, O^{2+}/O^{+} rises rapidly by a factor of ~ 100 until a temperature of about 43,000 K is reached, at which point the rise proceeds at a much slower rate. At temperatures below 43,000 K, the O^{2+}/O^{+} ratio would in fact provide a good index of stellar temperature. The existence of such a good correlation suggests that the assumptions on which this method is based, particularly that the nebulae are optically thick, are good.

The one significantly discrepant point in Figure 2 (indicated by a dot in the circle) is the planetary Ps–1 = K648, the nebula in the globular cluster M15. T_s is extremely low, only 23,000 K, while the O^{2+}/O^{+} ratio is rather high, more in line with what would be expected at $T_s = 35{,}000$ K. The nebula is reasonably well observed (Peimbert 1973; O'Dell, Peimbert, and Kinman 1964), and T_s and O^{2+}/O^{+} should be reliable. Peimbert (1973) shows that the ionized mass of Ps–1 is considerably less than that of galactic planetaries in general, only 0.018 $M_{\odot}$. Either Ps–1 is surrounded by a large neutral shell, or the mass is indeed low. The data presented here suggest that the latter is the case. If the mass is very low, the nebula will be optically thin, T_s will be a lower limit, and O^{2+}/O^{+} would be expected to be large for the temperature of the star, as the O^{+} shell will be truncated. The low mass is supported by Peimbert's (1973) value of He/H = 0.09 which is consistent with the lack of a neutral shell (that is, there is probably no neutral helium present). Evidently the planetary in M15 is a truly low mass object and is different from the typical galactic planetary.

Harman and Seaton (1964) (see also Seaton 1966) have suggested that in the early course of their evolution the nuclei of planetary nebulae increase in luminosity as the temperature increases. This apparent increase has been supported by the work of O'Dell (1968) and Webster (1969). With the large number of optically thick planetary nebulae with relatively cool central stars treated in this paper it should be possible to see whether the cooler central stars do indeed have lower luminosities than their hotter counterparts.

We can determine the luminosity of the central star of a planetary nebula, $\mathscr{L}$, from the Hβ flux, $\mathscr{F}(\mathrm{H}\beta)$, since from Zanstra's original argument the number of Balmer photons equals the number of stellar ultraviolet photons. (The final numbers are so crude because of poor distances that the effects of helium are generally ignored.) Thus we can write that

$$\mathscr{L} = 4\pi D^2 \frac{\alpha_B(\mathrm{H})}{\alpha(\mathrm{H}\beta)} \frac{\mathscr{F}(\mathrm{H}\beta)}{\xi h\nu} \int_0^\infty B(\nu)d\nu \Big/ \int_{\nu_1}^\infty \frac{B(\nu)}{h\nu}\,d\nu\,, \tag{9}$$

where D is the distance of the nebula, ξ is the fraction of the sky covered by the nebula as seen from the star (taken from Harman and Seaton 1966), and the other quantities are defined above.

The calculation of luminosities would be simple were it not for the problem of distances to these nebulae. There simply is no straightforward method which gives reliable individual distances. The one planetary whose distance is reasonably well known, Ps–1 (M15) appears to be different from the typical galactic planetary and must be excluded from this study. The commonly used constant-mass, or photometric, method (see, for example, Seaton 1966) will not work for the optically thick objects considered here, as the ionized mass is less than the true mass, and the distances and stellar luminosities will be overestimated.

A variety of distance methods have been applied to the nebulae of Table 1. First, distances were taken from Cudworth (1974). These distances assume that the nebulae all have the same absolute magnitude, which was found by the method of statistical parallaxes. Clearly all that can be derived here is a mean luminosity.

Second, Harman and Seaton (1964) and Seaton (1966) used the electron density derived from the [O II] lines and the Hβ flux (corrected for extinction) to determine distance. Distances and luminosities are derived here by using both [O II] and [Cl III] to find the electron density. We have that

$$D_{\rm pc} = 7.60 \times 10^{22} \frac{\mathscr{F}(\mathrm{H}\beta)}{\epsilon N_e^2(\theta'')^3}\,, \tag{10}$$

where N_e is the true electron density found from forbidden lines, θ'' is the angular radius in seconds of arc, $\mathscr{F}(\mathrm{H}\beta)$ is the corrected Hβ flux, ϵ is the filling factor, the fraction of the spherical volume defined by the mean radius of the nebula that is actually filled with radiating matter, and He/H is assumed to be 0.1. In calculating the distances the [O II] and [Cl III] line data were taken from Kaler's (1976*a*) catalog, or Aller and Epps (1976), and N_e was derived from the tables of Saraph and Seaton (1970) and Krueger, Aller, and Czyzak (1970). The observed $\mathscr{F}(\mathrm{H}\beta)$ were taken from Cahn and Kaler (1971), Perek (1971), or Kaler (1976*b*), the extinction corrections from Cahn (1976) or Kaler (1976*a*), and the radii θ'' from Cahn and Kaler (1971).

There are a variety of problems with this method. The filling factor ϵ is essentially unknown for each nebula. There may well be clumping below the limit of resolution. Kaler (unpublished) shows that $\epsilon \approx 0.5$ for optically thin nebulae near the limiting radius required for the nebula to be optically thick. It is presumed that this value holds for the optically thick nebulae as well. The major difficulty now is one of propagation of errors. Since $\mathscr{L}$ goes as D^2, $D \sim \epsilon^{-2}$, N_e^{-4}, and θ^{-6}, so that the luminosities are very sensitive to the exact values of these quantities.

A further difficulty involves the problem of density gradients. Lee *et al.* (1974) and Kaler *et al.* (1976) have amply demonstrated that strong density gradients can exist in planetary nebulae, so that neither [O II] nor [Cl III] may give the appropriate "mean" density that should be used in equation (10). The [O II] densities are probably too small, and the resulting distances and luminosities will be too large. Aller and Epps (1976) show that the density derived from [O II] is generally smaller than that derived from [Cl III].

Finally, the method of "dust distances," described by Cahn and Kaler (1971), was used to derive the luminosities. The distances used here were taken from the recent work of Cahn (1976 and unpublished) which employs an improved dust model of the galaxy and the newest radio and optical data to derive extinction. The major difficulty with this procedure is that the interstellar dust distribution is so clumpy that adoption of a mean dust distribution will result in large errors.

Except for Cudworth's, the distance determinations described above yield an enormous range of luminosity, over several orders of magnitude, for the stars of Table 1. The distances are so crude that no evolutionary track is discernible if the stars are plotted on a (log $\mathscr{L}$, log T_s)-diagram. Possibly median luminosities ($\bar{\mathscr{L}}$) may have some meaning, and these have been found for all the distance methods employed. The results of this study are summarized in Figure 3, in which log $\mathscr{L}/\mathscr{L}_\odot$ is plotted against log T_s. The crosshatched area shows the rough boundaries of the nuclei of optically thin nebulae which are at or near maximum luminosity. These objects exhibit moderate or strong He II emission. Temperatures and luminosities for these stars were derived by using the Harman-Seaton (1966) procedure, Cahn's and Kaler's (1971) distances modified for observed extinction, and Shao's and Liller's (1976) and Kaler's (1976*b*) central star magnitudes (see § IV).

The correct values of T_s were used in equation (9) for the calculation of the luminosities of the stars of Table 1. However, for the purpose of display, they are plotted as a group in Figure 3 at the mean temperature of $\approx$50,000 K. The medians of the distributions of luminosities are presented for the dust, [O II] and [Cl III] distances by horizontal bars. The lengths of the vertical bars represent the spread in log $\mathscr{L}/\mathscr{L}_\odot$ for each distance method. The numbers in parentheses show the number of stars for which luminosities were calculated. Note that the [O II] lines give the highest luminosities; these values are almost certainly overestimates because of gradients. The bar labeled "B.D." in Figure 3 represents the median of the "best determined" luminosities, those for which the [O II] and [Cl III] lines give luminosities within an order of magnitude of one another. The B.D. and [Cl III] values are probably the most reliable.

The mean value of $\mathscr{L}/\mathscr{L}_\odot$ found from Cudworth's (1974) distances is labeled "Cud" in Figure 3. Cudworth also indicated that the scale factor for the constant-mass method should be increased by a factor of 1.45. If that is true, the crosshatched area should be increased by 0.3 in the log for proper comparison.

The vertical spread in Figure 3 is so large due to poor distances that it is not possible to discern the intrinsic spread. It is not possible to draw any kind of evolutionary path through these points. About all we can say at present is that the mean or median luminosity of the nuclei of these low-excitation planetaries appears significantly less than the maximum luminosity that is found for the nuclei of the optically thin, high-excitation planetaries.

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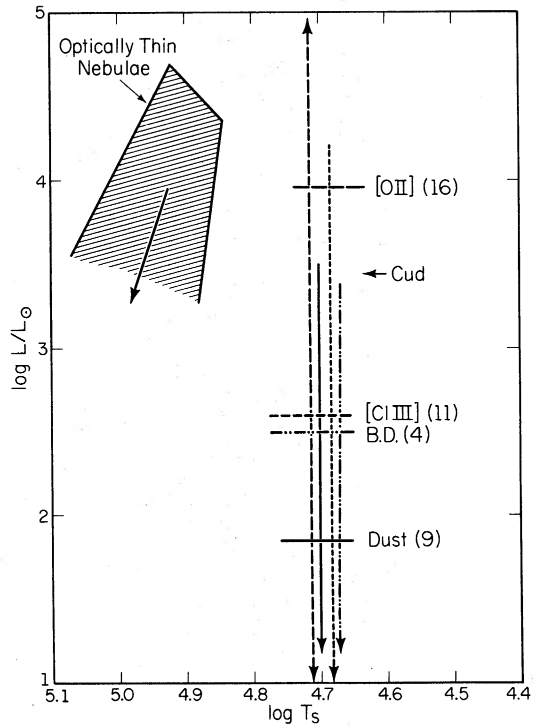

FIG. 3.—Luminosities of exciting stars of planetary nebulae. The crosshatched area represents the regime of high-temperature stars near maximum luminosity. The horizontal bars represent the median luminosities, and the vertical bars the ranges of the luminosities. Luminosities are found from the distance methods as indicated: [O II] (———), [Cl III] (----), dust (—), best determined (—··—··).

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