THE ASTROPHYSICAL JOURNAL, 208:819–825, 1976 September 15

CNO ABUNDANCES AND HYDRODYNAMIC MODELS OF THE NOVA OUTBURST. IV. COMPARISON WITH OBSERVATIONS

WARREN M. SPARKS
NASA/Goddard Space Flight Center

SUMNER STARRFIELD
Department of Physics, Arizona State University; and
Los Alamos Scientific Laboratory

AND

JAMES W. TRURAN
Department of Astronomy, University of Illinois
Received 1975 November 19

ABSTRACT

A variety of observations of novae are discussed in light of our theoretical models. We propose that the nearly constant bolometric luminosity of FH Ser observed by Gallagher and Code originates in the nondegenerate hydrogen-burning region at the bottom of the hydrogen-rich envelope which remains after the primary ejection. The shift of the wavelength of peak emission from the visual to shortward of the ultraviolet is caused by the decrease of the photospheric radius of the remnant envelope as the bolometric luminosity stays nearly constant. The oscillations in the light curve of GK Per during the transition stage can be explained by a pulsation of the remnant envelope when it is larger than the semimajor axis of the binary system. The CNO overabundances in novae reported by various observers are strongly suggestive of our nova mechanism. Finally, the implications of the upper limits of ^{13}C and ^{15}N found by Sneden and Lambert in DQ Her are discussed.

Subject headings: hydrodynamics — nucleosynthesis — stars: abundances — stars: interiors — stars: mass loss — stars: novae

I. INTRODUCTION

In a recent series of papers (Starrfield *et al.* 1972; Starrfield, Sparks, and Truran 1974*a*, *b*; hereafter, called Papers I, II, and III, respectively) we have discussed our theoretical models of novae outbursts. Assuming the Kraft model (1963), which has a white dwarf accreting hydrogen-rich material from a binary companion that has overflowed its Roche lobe, we have followed the hydrodynamics of a thermonuclear runaway in the degenerate hydrogen-rich envelope of the white dwarf by means of an implicit hydrodynamic stellar evolution computer code (Kutter and Sparks 1972). The decay of the β^+-unstable nuclei formed during the thermonuclear runaway causes 10^{27}–10^{29} grams of the hydrogen-rich material to be ejected with velocities of 200–2400 km s^{-1}. These values agree with the observations of novae. We have also reproduced the general features in the early portion of the light curve of a fast nova outburst. Upon finding that a less luminous white dwarf requires a longer time to thermonuclear runaway and has a more intense outburst, we have presented a qualitative explanation for the Kukarkin-Parenago relationship (Paper II), as applied to the common and recurrent novae. Lastly, we have explained the oval shape of the nebulosity surrounding DQ Her (Sparks and Starrfield 1973) by taking into account the interaction between the material expanding off the white dwarf and the accretion disk surrounding it.

In this paper we wish to extend the results from our hydrodynamic models and show that they can explain some rather diverse observations of the nova outburst. In § II we will consider the recent ultraviolet observations of FH Ser 1970. In § III a discussion of the oscillations of the light curve of GK Per which occurred during the transition stage will be given. The discussion in these two sections is based upon the behavior of the remnant envelope, i.e., the hydrogen-rich material which is not ejected during the explosive phase of the outburst. We have found that the evolutionary behavior of all of the remnants is similar, and, therefore, we make our comparison with only one remnant model. The elemental and isotopic abundances of carbon, nitrogen, and oxygen produced by a nova outburst will be discussed in § IV.

II. ULTRAVIOLET OBSERVATIONS OF FH SERPENTIS 1970

Recent ultraviolet observations of FH Ser by Gallagher and Code (1974) showed that beginning 4.4 days after maximum the bolometric luminosity remained at approximately $1.6 \times 10^4 L_{\odot}$ for 53 days of observations. They also found that in the same time period the wavelength of peak emission shifted from

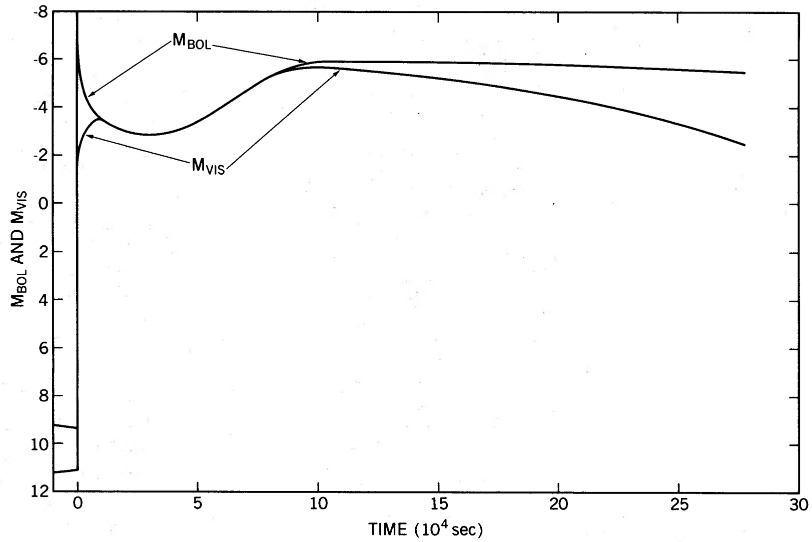

FIG. 1.—The computed bolometric and visual light curves of a hydrodynamic nova model (No. 3 from Paper II)

the visual to shortward of 2000 Å. Infrared observations by Geisel, Kleinmann, and Low (1970) indicated a peak luminosity of $5 \times 10^4 L_\odot$ 90 days after maximum. Therefore, the fast decline in the visual magnitude for FH Ser was caused by the radiative energy shifting to other wavelength regions, not by a decline in the total energy output of the nova. This may, in fact, be true for all novae (Gallagher and Starrfield 1976).

Let us consider a typical light curve of one of our hydrodynamic nova models ($1 M_\odot$ star, $10^{-3} M_\odot$ hydrogen envelope with the carbon and oxygen abundances increased to 15 percent in the bottom hydrogen zone) in Figure 1. This model is called No. 3 in Paper II. The latter part of the light curve has been artificially smoothed to remove the coarse zoning effects which cause large variations in the luminosity that mask the true behavior (Sparks 1969). The initial rise of the bolometric light curve is due to a shock wave initiated by the thermonuclear runaway. The light curve rapidly drops after passage of the shock wave but rises again rapidly as the outer layers are heated by the decay of the β^+-unstable nuclei (^{13}N, ^{14}O, ^{15}O, ^{17}F) formed during the thermonuclear runaway and mixed to the surface by convection (cf. Paper I). (This rapid decline and rise occurs on too short a time scale to appear in Fig. 1.) The effect of this heating is demonstrated by the positive luminosity gradient in the outer layers shown at a time of 2×10^3 s in Figure 2. (All times refer to the time after the initial rise.) At a later time, 1.9×10^4 s, all of the β^+-unstable nuclei have decayed and the outer escaping layers have cooled because of expansion and radiative losses producing the dip in the light curve also shown in Figure 1. At 7.7×10^4 s in Figure 2, we see that the high inner luminosity has progressed outward while the region where the zones are optically thin (to the right of the left-hand bracket) has moved inward. By 2.78×10^5 s this "luminosity wave" has moved through all the zones, giving the final rise in the light curve in Figure 1 (also see Sparks 1969). This luminosity wave is caused by *nondegenerate hydrogen burning at the bottom of the remnant envelope*.

Much of the radiation in the initial sharp peak shown in Figure 1 is in the ultraviolet, and the computed visual light curve has the initial rise, premaximum halt, and final rise structures that are observed (cf. McLaughlin 1939). The cause of the decline in the visual light curve can be seen if we plot radius versus interface number in Figure 3. This plot shows that from 7.7×10^4 s to 2.78×10^5 s the photospheric radius (where $\tau = \frac{2}{3}$ and shown by the left-hand bracket) is decreasing as the ejected material becomes optically thin. The decrease in the photospheric radius and the nearly constant bolometric luminosity (produced by the constant nuclear burning shell source) leads to an increase in the effective temperature and a decrease in the amount of energy appearing in the visual. In order to compare our results with the ultraviolet observations of Gallagher and Code (1974), we removed the outer escaping material and evolved only the remnant envelope. We do this since the velocities of the expanding nebula are so large that the change in the density allowed per time step forces the time step to be very small and this effectively prevents further evolution of the model. This remnant soon evolved to the "equilibrium model" in Figure 3 which has a constant luminosity of $1.7 \times 10^4 L_\odot$ in agreement with their observations.

The character of the energy production has now changed from fast CNO-burning in a degenerate envelope to the normal CNO cycle in nondegenerate material. The burning is occurring in a thin shell. The energy is coming from proton capture rather than the decay of the β^+-unstable nuclei, the shell source temperature has declined from its peak value of $\sim 2 \times 10^8$ K to a value of $\sim 5 \times 10^7$ K, and the

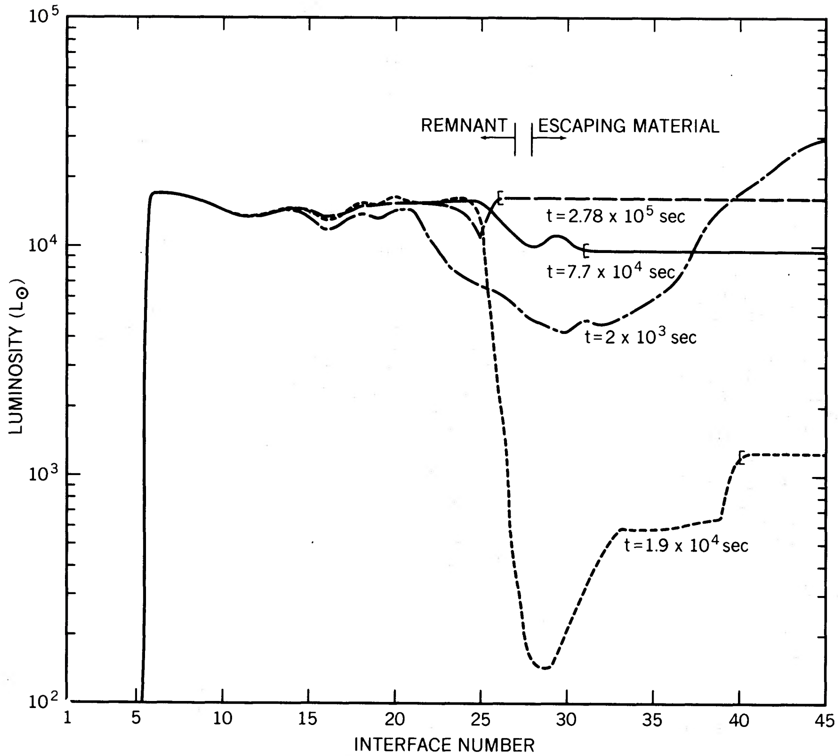

FIG. 2.—Luminosity as a function of interface number and time. The time refers to the time since the initial rise, and the left-hand bracket indicates where the optical depth is $\frac{2}{3}$.

energy generation rate has dropped from 10^{16} ergs g^{-1} s^{-1} to 10^8 ergs g^{-1} s^{-1}. At this energy generation rate, it would take $\sim 4 \times 10^3$ years to convert all the hydrogen to helium in the remnant envelope ($\sim 10^{-3}$ $M_\odot$) if there were no further mass loss. However, such a high luminosity can cause mass loss by means of radiation pressure (see, e.g., Sparks and Kutter 1972; Kutter and Sparks 1974). (A proper study of radiation-pressure mass loss requires the evolution of a finely zoned model which is in progress.) In fact, because the bolometric luminosity of two 50-year-old novae (Gallagher and Holm 1974) is between 10 and

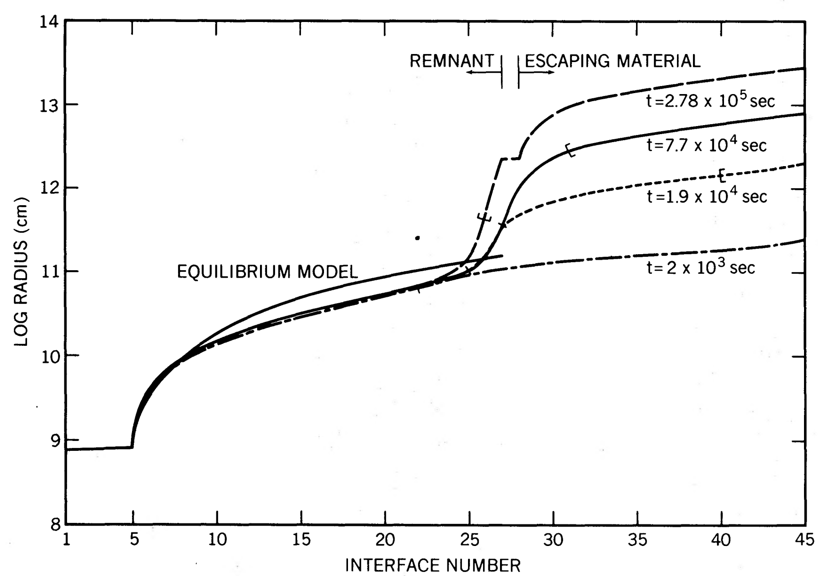

FIG. 3.—Radius as a function of interface number and time. The time refers to the time since the initial rise, and the left-hand bracket indicates where the optical depth is $\frac{2}{3}$. The "equilibrium model" is the remnant after it has reached nearly hydrostatic and thermal equilibrium.

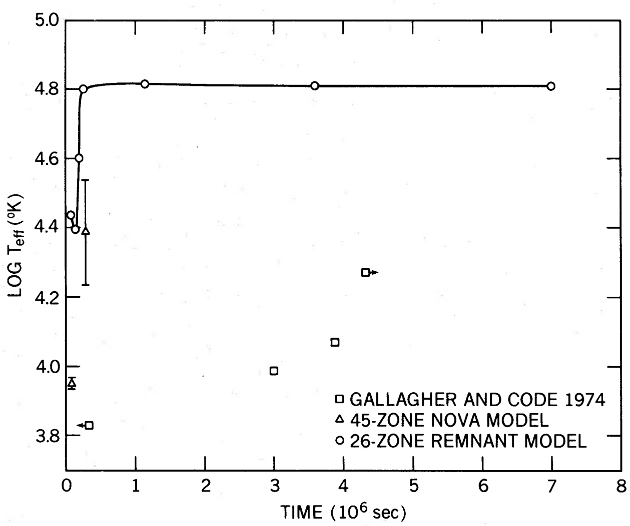

FIG. 4.—Effective temperature versus time. The squares represent observed values from Gallagher and Code (1974) assuming Wein's law. The triangles and the circles represent the calculated values from the nova model and the remnant model, respectively.

$100\,L_\odot$ and a 7-year-old nova is less than $500\,L_\odot$ (Gallagher 1974), it appears that the observed remnant envelope does not evolve on a nuclear time scale. In addition, it is important to note that the remnant hydrogen envelope, extending to a large radius, must be either expelled, burned, or cooled in order for the white dwarf to return to conditions of a small radius and low luminosity necessary before another outburst can occur. As has already been discussed (Paper I), the hydrogen layers of the white dwarf must be reasonably degenerate in order for a thermonuclear runaway to produce an outburst.

Gallagher and Code (1974) also found that the wavelength of peak emission shifted shortward with time. Using Wein's law, we have derived effective temperatures for their observations and have plotted both our theoretical and their observed values in Figure 4. The rather large error bars for the last point of the 45-zone nova model are caused by the normal difference procedures which assume uniform temperature zones. The removal of the outer escaping material in order to evolve the remnant envelope causes an increase in the initial effective temperature so that the remnant calculation represents an upper limit on the effective temperature as a function of time. The final effective temperature for the remnant envelope ($\log T_e = 4.8$) can also be compared with the values for V603 Aql ($\log T_e = 4.40$) and RR Pic ($\log T_e > 4.54$) found by Gallagher and Holm (1974). They also point out that if they had compared their observations with spherical atmospheres, they would have found even higher effective temperatures. As can be seen in Figure 4, the effective temperature of both of the theoretical models increases faster than the observed effective temperature. This discrepancy is due to the fact that the computer code (Kutter and Sparks 1972) is one-dimensional and cannot take into account the existence of the binary companion. In an actual event, a large part of the remnant envelope will expand past the Roche lobe (the red companion already fills its lobe) to form a common envelope about both stars (cf. § III). The binary system will have a larger angular velocity than the material in the envelope. This will produce gravitational stirring (Ostriker 1976; Paczynski 1976) which will be an additional energy source for the envelope. In fact, if all of the kinetic energy lost by the rotating binary system as it reacquires the common envelope is converted to radiation energy, it will be equal to the very high luminosity from the hydrogen shell source for a period of ~ 100 days. This will keep the outer radius larger and the effective temperature lower for a longer time than is shown in our models. Also the angular momentum picked up by the common envelope from the rotating binary system will reduce its effective gravity and further slow the rate of decrease of the radius.

Finally, at 90 days after maximum visual light Geisel, Kleinmann, and Low (1970) observed that the infrared radiation, due to dust at about 900 K, peaked at $5 \times 10^4\,L_\odot$. They suggested "the presence of a previously undetected source of energy comparable to or greater than the energy released in optical emission." A hydrodynamic nova model including grain formation is impossible at this time. However, if grain formation in the nova ejecta takes place on a time scale of days as they suggest, then it is reasonable to assume that the grains are reradiating the far-ultraviolet flux from the white dwarf in the infrared.

III. OSCILLATIONS OF THE LIGHT CURVE OF GK PERSEI

During the transition stage (see McLaughlin 1939) some novae show oscillations in their light curves. GK Per is the best example of this phenomenon, showing quasi-periodic oscillations of about 4 days and an amplitude of about 1.5 mag (Campbell 1903). These began 23 days after maximum when the nova had declined about 4 mag. It is the purpose of this section to demonstrate that a pulsation of the remnant envelope could be responsible for these oscillations when it is larger than the semimajor axis of the binary system.

We begin by determining the semimajor axis from Kepler's law

$$A^3 = 43.5(M_{\rm WD} + M_{\rm RED})P^2\,, \qquad (1)$$

where A is the semimajor axis in 10^{10} cm, P is the period in hours, and $M_{\rm WD}$ and $M_{\rm RED}$ are the masses of the white dwarf and the red binary companion, respectively, in solar units. Two solutions for the spectroscopic orbit of GK Per are in the literature. Kraft (1964) found a highly eccentric orbit with a period of 1.904 days while Paczynski (1965) found a solution with a circular orbit and a period of 0.685 days. Following Mumford (1967) and Gallagher and Oinas (1974), we adopt Paczynski's solution which gives $M_{\rm WD}/M_{\rm RED} = 5$. We assume that $M_{\rm WD} = 1.2\,M_\odot$

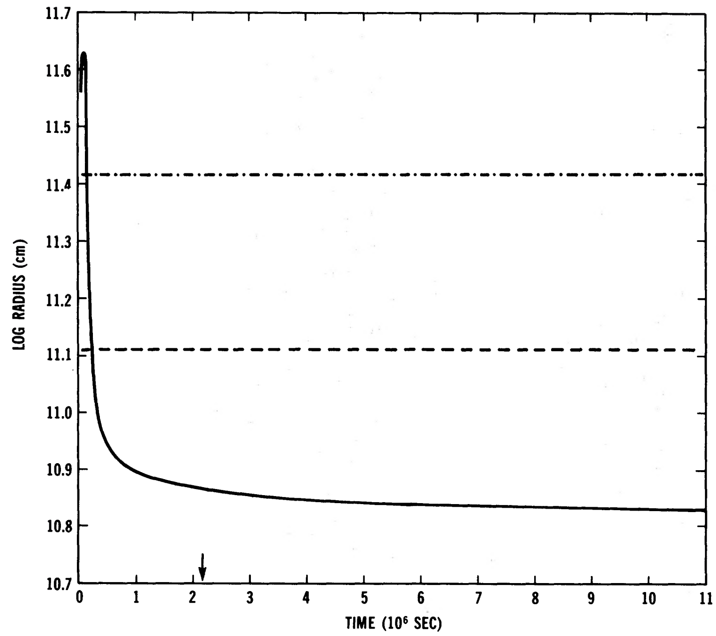

FIG. 5.—The outer radius of the calculated remnant model as a function of time. The dot-dashed line represents the semimajor axis of GK Per, the dashed line represents the equivalent Roche lobe radius of the white dwarf component, and the arrow represents the beginning of the transition stage.

on both theoretical (Paper III) and observational (Warner 1973) grounds (since it enters as the cube root, its value is not critical to this discussion). Thus, from equation (1) we find $A = 2.6 \times 10^{11}$ cm. The equivalent Roche lobe radius for the white dwarf is then 1.3×10^{11} cm (Kopal 1959, p. 136).

Figure 5 shows the radius of our remnant model as a function of time where we have also drawn lines giving the values of the Roche lobe radius for the white dwarf and the semimajor axis of GK Per. We see that the radius of the remnant envelope grows to a size that is larger than the binary but, later on, shrinks within the Roche lobe of the white dwarf. Our other remnant model calculations as well as Nariai's (1974) analytical calculations show a similar behavior. As with our earlier work (Paper II), the time scale for the radius decrease is much shorter than is observed.

The start of the transition stage for GK Per, which is the time that the oscillations begin, is indicated by the arrow in Figure 5. If we assume that the pulsation period of the model is equal to the sound travel time across the remnant (Cox and Giuli 1968), then the pulsation period at this time is only a few hours. However, it can be increased to 4 days (using a homology transformation) if the radius of the remnant is increased to about 10^{12} cm. The observed bolometric luminosity and effective temperature of FH Ser found in § II and of other novae (Bath and Shaviv 1976) also indicate radii of this order during the transition stage.

This value is close to the peak radius of the remnant (cf. Fig. 5) and suggests that it is the entire remnant envelope which becomes pulsationally unstable. However, we must extend the time period over which the radius is large in order to agree with the observations. Again, as in § II, this time period is extended when we consider the effects of gravitational stirring by the binary system. It is not our purpose to propose a specific mechanism to drive the pulsation but merely to point out that a pulsation of the remnant at a radius of 10^{12} cm has the proper period. In fact, the pulsations are probably excited by the initial outburst and may die out because there is no driving mechanism.

IV. ELEMENTAL AND ISOTOPIC ABUNDANCES OF CNO

In Papers I and II we have demonstrated that an enhancement of CNO nuclei in the material undergoing a thermonuclear runaway is necessary for ejection. Comparing our predicted abundances in the ejecta with solar abundances (Cameron 1968), we found that carbon was 4 times solar, nitrogen 60 times solar, and oxygen 3 times solar. The isotropic ratios of CNO nuclei in the ejecta were also distinctive. Observational evidence bearing upon these features is reviewed in this section.

As noted in Paper I, Pottasch (1959) observed that carbon was solar, nitrogen 45 times solar, and oxygen 5 times solar for five novae, while Mustel' and Boyarchuk (1959) and Mustel' and Baranova (1965) found that the CNO nuclei were ~ 100 times solar for DQ Her. In addition, Antipova (1969) argued that a CN enhancement was necessary to explain the appearance of the CN absorption bands in DQ Her. For the recent slow nova HR Del 1967, different observers have found that carbon and nitrogen were 10 times solar (Ruusalepp and Luud 1971), oxygen was greater than 3 times solar (Sanyal and Robbins 1975), nitrogen was 5–10 times solar (Anderson and Gallagher 1975), and carbon was approximately 10 times solar while nitrogen and oxygen were approximately 100 times solar (Antipova 1974). Ruusalepp and Luud (1971) also studied the emission spectrum of novae and planetary nebulae and found that oxygen and nitrogen are more abundant in novae than in planetaries. Finally, Ruusalepp and Luud (1971) found that the fast novae (and therefore the brighter novae) are more nitrogen-enhanced than slow novae. This agrees with our theoretical studies, since our models with larger CNO enhancements were brighter, faster, and necessarily more energetic. The range in values determined for HR Del measures the difficulties associated with such observational efforts. Nevertheless, we feel that *the large number of observations of CNO overabundances in novae provide strong support for our theoretical models*.

Another of our theoretical predictions is that the concentrations of ^{13}C, ^{15}N, and ^{17}O will be greatly enhanced in nova ejecta. An average over our models reveals: 74 percent of the carbon is ^{13}C, 61 percent of the nitrogen is ^{15}N, and 21 percent of the oxygen is ^{17}O. These correspond to isotopic enhancements relative to solar material by factors of 66, 160, and 570, respectively. Such isotopic anomalies are, in fact, an unambiguous signature of CNO hydrogen burning at high temperatures on hydrodynamic time scales, and provide a potentially powerful test of our theoretical models. Unfortunately, there are major difficulties associated with obtaining isotopic abundances in nova ejecta. Sneden and Lambert (1975) have recently found that $^{13}\mathrm{C/C} \leqslant 40$ percent or $^{15}\mathrm{N/N} \leqslant 33$ percent or $^{13}\mathrm{C/C} \leqslant 25$ percent and $^{15}\mathrm{N/N} \leqslant 25$ percent for the very slow nova DQ Her. They obtained these values from analyses of the CN molecular bands near 4215 Å which appeared in the spectrum near maximum light.

In their concluding remarks Sneden and Lambert interpret their upper limits as approximate estimates of the isotopic abundances and on this basis find them in disagreement with our models. The point of disagreement arises from an investigation of the effects of accretion on the progress of the outburst (Starrfield, Sparks, and Truran 1974*c*). In this study, we included the effects of only the recently accreted material in the calculations and neglected all of the fine details of the accretion process (e.g., the emitted spectrum and the accretion shock structure). As a result of the infall, a temperature inversion was produced in the white dwarf envelope which effectively prevented the CNO-enhanced material from reaching the surface layers. Since there is no mixing during the later stages of the expansion, we found that the surface layers showed no evidence of our ejection mechanism—enhanced ^{13}C, ^{15}N, and ^{17}O. This is in apparent disagreement with the findings of Sneden and Lambert if we treat their numbers, as they did, as firm values.

In an actual event, the temperature inversion would probably not be realized when accretion occurs over a long time period, and CNO-enhanced material would probably reach the surface by maximum light. It might appear that there is now a discrepancy in the opposite sense—our calculations predict too much of an isotopic enhancement. We believe that this discrepancy is not very serious for the following reasons. Very accurate descriptions of the accretion process and the mixing process (i.e., time-dependent convection and overshooting) will be necessary to determine the exact amount of processed CNO material seen at maximum light. In addition, the CNO-enhanced material coming off the white dwarf will collide and mix with material in the ring surrounding the white dwarf, thus reducing the enhancement. This possibility is especially applicable for DQ Her since the inclination of the binary orbit (and thus the ring) is near 90° (Walker 1956; Mustel' and Boyarchuk 1970). In fact, the collision with the ring in this particular orientation has been utilized to explain the oval shape of the nebulosity now surrounding DQ Her (Sparks and Starrfield 1973). Finally, DQ Her was a very slow nova and our models seem most applicable to the very fast novae such as CP Pup, V603 Aql, or Nova Cygni 1975. We should keep in mind that while the observed upper limits on the isotopic abundances are a factor of 2 or 3 lower than our theoretical values (based on models without accretion), ^{13}C/C is a factor of 36 larger and ^{15}N/N is a factor of 89 larger than solar system values. Also, normal CNO burning will produce only $^{13}\mathrm{C/C} \approx 0.20$ and $^{15}\mathrm{N/N} = 2\text{–}4 \times 10^{-5}$ (Caughlan and Fowler 1962).

It is clear from the abundance results presented in this section that novae show strong evidence of the presence of the enhanced concentrations of CNO nuclei that we have found necessary to produce a nova outburst. Unfortunately, this does not constitute proof that these nuclei have been processed through a high-temperature CNO hydrogen-burning region. Such proof awaits a definitive study of CNO isotopic abundances in the nova ejecta.

V. SUMMARY

We have used the results of our hydrodynamic nova models to explain a number of diverse observations. The nearly constant bolometric luminosity of FH Ser observed by Gallagher and Code (1974) originates within the nondegenerate hydrogen-burning shell source at the bottom of the hydrogen-rich envelope which remains after the primary ejection. The shift of the wavelength of peak emission to the ultraviolet is due to the decrease of the photospheric radius of the remnant envelope as the bolometric luminosity stays nearly constant. The oscillations in the light curve of GK Per during the transition stage can be explained by a pulsation of the remnant envelope when it extends past the semimajor axis of the binary. The CNO overabundances in novae reported by various observers are strongly suggestive that our nova mechanism is actually operating in real novae. Nevertheless, we shall have to wait for a definitive study of the isotopic abundances before we can be certain that high-temperature CNO hydrogen burning has occurred during the outburst.

The authors gratefully acknowledge useful discussions with Drs. Edward Devinney, Jay Gallagher, David Lambert, and Jeremiah Ostriker. We also thank an anonymous referee for many pertinent comments. J. W. Truran and S. Starrfield thank the National Science Foundation for support under their respective grants MPS 73-05117 and AST73-05271-A01. In addition, S. Starrfield is pleased to thank Dr. Peter Carruthers for the hospitality of the Los Alamos Scientific Laboratory and a gerneous allotment of computer time. He also thanks Arizona State University for a Faculty Grant-in-aid and a generous allotment of computer time.

REFERENCES

Anderson, C. M., and Gallagher, J. S. 1975, private communications.
Antipova, L. I. 1969, *Soviet Astr.—AJ*, **13**, 288.
———. 1974, *Highlights in Astronomy*, **3**, 501.
Bath, G. T., and Shaviv, G. 1976, preprint.
Cameron, A. G. W. 1968, *Origin and Distribution of the Elements*, ed. L. H. Ahren (Oxford: Pergamon Press).
Campbell, L. 1903, *Harvard Ann.*, **48**, 90.
Caughlan, G. R., and Fowler, W. A. 1962, *Ap. J.*, **136**, 453.
Cox, J. P., and Giuli, R. T. 1968, *Principles of Stellar Structure* (New York: Gordon & Breach), chap. 27.
Gallagher, J. S. 1974, *Bull. AAS*, **6**, 211.
Gallagher, J. S., and Code, A. D. 1974, *Ap. J.*, **189**, 303.
Gallagher, J. S., and Holm, A. V. 1974, *Ap. J. (Letters)*, **189**, L123.
Gallagher, J. S., and Oinas, V. 1974, *Pub. A.S.P.*, **86**, 952.
Gallagher, J. S., and Starrfield, S. 1976, in preparation.
Geisel, S. L., Kleinmann, D. E., and Low, F. J. 1970, *Ap. J. (Letters)*, **161**, L101.
Kopal, Z. 1959, *Close Binary Systems* (New York: Wiley).
Kraft, R. P. 1963, *Adv. Astr. and Ap.*, **2**, 43.
———. 1964, *Ap. J.*, **139**, 457.
Kutter, G. S., and Sparks, W. M. 1972, *Ap. J.*, **175**, 407.
———. 1974, *Ap. J.*, **192**, 447.
McLaughlin, D. B. 1939, *Popular Astr.*, **47**, 410, 481, 583.
Mumford, G. S. 1967, *Pub. A.S.P.*, **79**, 283.
Mustel', E. R., and Baranova, L. I. 1965, *Soviet Astr.—AJ*, **9**, 31.
Mustel', E. R., and Boyarchuk, A. A. 1970, *Ap. and Space Sci.*, **6**, 183.
Mustel', E. R., and Boyarchuk, M. E. 1959, *Soviet Astr.—AJ*, **3**, 744.
Nariai, K. 1974, *Pub. Astr. Soc. Japan*, **26**, 57.
Ostriker, J. P. 1976, in *Proc. IAU Symposium No. 73, Close Binary Systems*, ed. S. Mitton and J. Whelan (Dordrecht: Reidel), in press.
Paczynski, B. 1965, *Acta. Astr.*, **15**, 197.
———. 1976, in *Proc. IAU Symposium No. 73, Close Binary Systems*, ed. S. Mitton and J. Whelan (Dordrecht: Reidel), in press.
Pottasch, S. 1959, *Ann. d'Ap.*, **22**, 412.
Ruusalepp, M., and Luud, L. 1971, *Tartu Obs. Pub.*, **39**, 89.
Sanyal, A., and Robbins, R. R. 1975, private communication.
Sneden, C., and Lambert, D. L. 1975, *M.N.R.A.S.*, **170**, 533.
Sparks, W. M. 1969, *Ap. J.*, **156**, 569.
Sparks, W. M., and Kutter, G. S. 1972, *Ap. J.*, **175**, 707.
Sparks, W. M., and Starrfield, S. 1973, *M.N.R.A.S.*, **164**, 1P.
Starrfield, S., Sparks, W. M., and Truran, J. W. 1974*a*, *Ap. J. Suppl.*, **28**, 247 (Paper II).
———. 1974*b*, *Ap. J.*, **192**, 647 (Paper III).
———. 1974*c*, *Mém. Soc. Roy. Sci. Liège*, **8**, 413.
Starrfield, S., Truran, J. W., Sparks, W. M., and Kutter, G. S. 1972, *Ap. J.*, **176**, 169 (Paper I).
Walker, M. F. 1956, *Ap. J.*, **123**, 68.
Warner, B. 1973, *M.N.R.A.S.*, **162**, 189.

WARREN M. SPARKS: Code 671, NASA-Goddard Space Flight Center, Greenbelt, MD 20771

SUMNER STARRFIELD: Department of Physics, Arizona State University, Tempe, AZ 85281

JAMES W. TRURAN: Department of Astronomy, University of Illinois, Urbana, IL 61801