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REDISCUSSION OF ECLIPSING BINARIES. XI. THE RED GIANTS RZ CANCRI AND AR MONOCEROTIS

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ABSTRACT

RZ Cnc and AR Mon are the two known close binaries with both components red giants. Photographs of the visual spectrum obtained at Mount Wilson (RZ Cnc) and Lick (AR Mon) are combined with *B*, *V* photometry by Broglia and Conconi (RZ Cnc) and from Kitt Peak (AR Mon) to derive the properties of these similar systems. The masses are 3.2 and 0.54 $M_\odot$ for RZ Cnc and 2.7 and 0.80 $M_\odot$ for AR Mon. The stellar radii are all in the vicinity of 10 $R_\odot$, with the larger, less massive, cooler components filling their critical Roche lobes. The hotter components of the two eclipsing systems and the cooler component of Capella provide the three directly determined masses of red giants that are free from confining Roche lobes. In AR Mon and Capella they have luminosities and temperatures lying within the considerable range, predicted by models for single stars with their masses, near the bottom of the giant branch of evolutionary tracks. The detached giant of RZ Cnc is underluminous relative to the models by 0.7 mag or more. Successful interpretation of these results may be expected to provide better understanding of the processes occurring in systems evolving with mass exchange and may furnish checks on models of single-star evolution near the start of the giant branch.

Subject headings: stars: eclipsing binaries — stars: individual — stars: late-type

I. INTRODUCTION

RZ Cnc and AR Mon are the two known eclipsing binaries in which the hotter component is a cool giant. These two K-type systems have closely similar periods, $21^d.6$ and $21^d.2$, respectively. A photoelectric light curve of RZ Cnc has been published and analyzed by Broglia and Conconi (1973). Preliminary results for the masses and colors in the system were published some time ago (Popper 1957*a*, 1962). Hiltner (1946) had found Ca II emission in this system and classified the spectra. No modern photometry appears to have been published for AR Mon to supersede the study based on visual and photographic observations by Payne-Gaposchkin (1944). The spectroscopic work of Sahade and Cesco (1944), employing spectrograms of relatively low dispersion, gave, as we shall see, misleading results.

The original impetus for placing these systems on my spectrographic program lay in their potential for supplying fundamental data for cool giant stars for comparison with single-star evolutionary theory. Such data are very few, indeed, the summary presented 10 year ago (Popper 1966) requiring only the addition of 31 Cyg (Wright and Huffman 1967) and AR Mon at the present time. With the realization that large-scale mass exchange is required to interpret these eclipsing systems, their value for providing fundamental data relevant for single-star evolution became questionable, and their priority in the program was reduced. On the other hand, the theory of mass exchange itself has become highly advanced in recent years and has achieved some success, for example, in accounting for the properties of very undermassive stars (cf. the case of AS Eri: Refsdal, Roth, and Weigert 1974; Popper 1973). And work on the properties of the mass-accreting component in close binaries now appears to be making good progress (e.g., Ulrich and Burger 1976). Hence it is opportune to present the results for RZ Cnc and AR Mon.

The early spectrographic observations by Hiltner (1946) and by Sahade and Cesco (1944) at McDonald are of insufficient resolution to separate the lines of the two components of these systems. Observations at 20 Å mm^{-1} (Mount Wilson) and 16 Å mm^{-1} (Lick) show that, while some lines of the two components are resolved, blending in the very crowded photographic region severely limits the confidence that may be placed in the velocities. The visual region of the spectrum has the advantages that the spectrum is much less crowded and that the lines of the cooler component, which, in the case of RZ Cnc, are weak in the photographic region, are enhanced. The spectrographic work discussed in this paper is based primarily on observations in the visual region (λλ5300–6700).

II. SPECTROGRAPHIC MATERIAL

The spectrograms of RZ Cnc listed in Table 1 were obtained at Mount Wilson in the interval 1954–1961. Plates of the Ce series were photographed with the 32 inch (80 cm) camera of the coudé grating spectrograph of the 100 inch (2.5 m) telescope and have, in the second order spectrum, 15.3 Å mm^{-1} in the visual region. Plates of the Xd series were obtained

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TABLE 1

SPECTROGRAMS OF RZ CANCRI

Plate	JD (2,400,000+)	Phase	V_1 (km s^{-1})	Wt	$O - C$ (km s^{-1})	V_2 (km s^{-1})	Wt	$O - C$ (km s^{-1})
Ce 9075.....	34785.736	0.1136	+6	1	+5	+87	1	+5
9164.....	34842.681	0.7447	+28	1	−2	−84	2	+1
9752.....	35201.765	0.3359	−2	½	+1	+118	½	+15
9795.....	35230.735	0.6744	+22	1	−6	−75	1	−1
10300.....	35521.774	0.1217	−12	1	−12	+95	½	+9
10302.....	35523.799	0.2153	−5	1	0	+109	1	−6
10770.....	35760.987	0.1774	−5	2	−2	+99	2	−7
12397.....	36585.917	0.2897	+4	1	+8	+116	1	+2
12402.....	36586.930	0.3365	−3	2	0	+103	2	0
13304.....	37005.688	0.6849	+29	2	+1	−74	2	+3
13310.....	37007.696	0.7777	+30	1	+1	−87	2	−4
13985.....	37236.970	0.3712	+1	2	+1	+91	2	+2
15353.....	37763.707	0.7087	+27	1	−2	−79	1	+3
Xd 2191......	35794.979	0.7449	+25	½	−4	−82	½	+3
2197......	35795.945	0.7896	+33	½	+4	−85	½	−3
6219......	37307.751	0.6415	+35	½	+9	−68	½	−6
6220......	37307.966	0.6515	+25	½	−1	−72	½	−6
6224......	37308.769	0.6886	+24	½	−4	−77	½	+1
6225......	37308.981	0.6984	+40	½	+11	−77	½	+3

with the 16 inch (40 cm) camera of the X spectrograph on the 60 inch (1.5 m) telescope and have 31 Å mm^{-1}, also in the second order visual region. The emulsions employed were baked IIaD (λλ5300–6200) and (simultaneously) 103aE (λλ6200–6700). The average exposure time for an exposure in both series was about 4^{h}. Because of the very long exposure it was not considered practicable to carry out a similar spectrographic program at Mount Wilson on AR Mon, which has nearly the same apparent magnitude as RZ Cnc (as well as nearly the same period and spectral types). The spectrograms of AR Mon listed in Table 2 were obtained in 1967–1969 with the 20 inch (50 mm) camera of the coudé spectrograph of the 120 inch (3 m) Lick reflector and have 16.1 Å mm^{-1} in the second-order visual spectrum. The same spectral region and emulsions were used as for RZ Cnc. Because of an unexpected change in the 103aE sensitizing in the 1960s, these spectrograms extend only to λ6600 rather than to λ6700. Use of the 103aF emulsion is now preferable to 103aE in the region of Hα. The average exposure time for the spectrograms of AR Mon was 110 minutes.

A single 16 Å mm^{-1} spectrogram has been obtained at Lick of each binary in the photographic region as well as several 22 Å mm^{-1} Mount Wilson plates of AR Mon in the photographic region. In the case of RZ Cnc, Hiltner's (1946) result, that the lines of the primary dominate, is confirmed, although a few weaker lines of the secondary can be found in the crowded spectrum. Only the primary (i.e., the component eclipsed at the deeper minimum) shows H and K in emission. In the spectrum of AR Mon the lines of the two components appear roughly equal in strength, both in absorption and in Ca II emission. The ratio of intensities of the Ca II emission in the components of these giant binaries is in the same sense as the ratio of luminosities, in contrast to the systems of shorter period and lower luminosity, in which the secondary of type near K0 is almost always stronger than the primary of type F–G in the Ca II emission (e.g., Popper 1970; see also Young 1975). The photographic absorption spectrum of AR Mon is too crowded and the emission components inadequately resolved, despite the approximate equality of the intensities of the components, for reliable velocity studies. The

TABLE 2

SPECTROGRAMS OF AR MONOCEROTIS

Plate (EC)	JD (2,400,000+)	Phase	V_1	$O - C$	V_2	$O - C$
6146.........	39784.012	0.350	−11	−1	+85	0
6163.........	39785.022	0.3975	−8	−3	+68	+2
6326.........	39846.860	0.313	−7	+6	+100	+5
6342.........	39847.911	0.363	−9	0	+81	+1
6434.........	39897.727	0.712	+35	−2	−74	−1
6435.........	39897.811	0.716	+37	0	−72	+1
6537.........	39960.676	0.680	+35	0	−67	0
6547.........	39961.683	0.726	+37	0	−77	−2
7124.........	40171.963	0.6425	+36	+4	−56	0
7271.........	40226.861	0.231	−16	−1	+97	−4
7289.........	40227.887	0.279	−16	−1	+97	−3

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spectrograms of Sahade and Cesco (1944) are of too low dispersion to see that the lines are the blends of both components.

The spectral type of each system was estimated on the 16 Å mm^{-1} plates of the photographic region, by comparison with standards of luminosity class III, to be between K0 and K2, probably closer to K0. For RZ Cnc this estimate refers to the primary, while for AR Mon it is for the blended spectrum.

Although lines in the visual region of the spectrum are considerably less crowded than at shorter wavelengths, the difficulties of finding lines of each component that are free from blending with lines of the other component are similar to the difficulties with earlier types in the photographic region (Popper 1971), and similar techniques were employed to select lines and to obtain consistent wavelengths. The problems for the two systems differ somewhat. The lines of the two components are of comparable intensity in the visual spectrum of RZ Cnc, whereas the lines of the cooler component are considerably the stronger in the visual spectrum of AR Mon. Hα does not appear to be in emission in either system. The relative intensities of Hα in the two components of RZ Cnc are however, clearly anomalous, the line in the cooler and (according to the photometry) less luminous component being considerably the stronger. Emission may fill in the line of the hotter star.

The lines used for radial velocities are listed in Table 3. The plates of RZ Cnc were measured with a standard Gaertner visual measuring engine, while those of AR Mon were measured with a Grant oscilloscopic comparator. The resulting velocities are listed in Tables 1 and 2. V_1 is the velocity of the component eclipsed at the deeper eclipses. The differing weights assigned in the case of RZm Cnc depend on the dispersion and plate quality. The visual spectrograms of AR Mon are all of the same dispersion, and an exposure integrator was used at Lick, leading to plates of uniform quality, so that weights are not used for AR Mon. The comparison lines used for RZ Cnc are iron arc lines from the third-order photographic region, which overlaps the second-order visual without systematic differences in wavelength. The comparison source for AR Mon was a hollow-cathode Fe-Ne discharge. Shortward of λ5850 the comparison lines are Fe I, while longward only the widely separated Ne I lines are available because they are many orders of magnitude stronger than the Fe I lines in that part of the spectrum.

The phases employed for RZ Cnc (Table 1) are based on the ephemeris of Broglia and Conconi (1973),

$$JD_{min} = 2418702.531 + 21.642998\,E\,.$$

I am not aware of any recent times of minimum of AR Mon that have been published. Unpublished observations of mine (see below) include parts of the ingress of primary minima of 1965 February and March obtained at the Siding Spring Observatory and parts of the egress of primary minimum and of both branches of secondary minimum observed at the Kitt Peak Observatory in the period 1972 December–1973 October (see Table 6). For neither minimum is an epoch of mid-eclipse precisely obtained, but the combined 1965 and 1973 observations of primary show that mid-eclipse occurred at 0.013 phase when the ephemeris given in the *General Catalog of Variable Stars* (*GCVS*) (Kukarkin *et al.* 1969) is employed. The available material is inadequate for obtaining an improved period for AR Mon. The phases in Table 2 were derived with the period given in the *GCVS*, but with the zero point shifted by −0.013 phase:

$$JD_{min} = 2427560.79 + 21.208\,E\,.$$

Broglia and Conconi's observations of RZ Cnc show that its secondary minimum occurs within 0.002 phase of phase 0.5, while my observations of AR Mon show that secondary is separated from primary by 0.500 ± 0.003 phase. Hence we shall assume that both orbits are circular. Circular elements of velocity variations are contained in Table 4,

TABLE 3

LINES USED FOR RADIAL VELOCITIES

λ (Å)	ATOM	RZ CANCRI	AR MONOCEROTIS Cooler	AR MONOCEROTIS Hotter
5328.2	Cr	Yes	Yes	No
5446.8	Fe	No	Yes	+ only
5455.6	Fe	No	Yes	Yes
5497.5	Fe	Yes	No	No
5602.8	Ca	No	Yes	+ only
5615.5	Fe	− only	Yes	Yes
5890.0	Na	− only	Yes	No
5895.9	Na	Yes	Yes	No
6102.7	Ca	Yes	Yes	Yes
6122.2	Ca	No	Yes	− only
6141.7	Fe, Ba^+	No	Yes	No
6191.5	Fe, Ni	No	Yes	+ only
6393.6	Fe	No	Yes	No
6400.1	Fe	Yes	Yes	Yes
6430.8	Fe	Yes	+ only	+ only
6439.1	Ca	Yes	− only	− only
6450.0	Ca, Co	Yes	Yes	Yes
6462.6	Ca, Fe	Yes	Yes	Yes
6609.3	Fe	Yes	No	No
6625.0	Fe, V	Yes	No	No
6663.4	Fe	Yes	No	No
6678.0	Fe	Yes	No	No

TABLE 4

CIRCULAR ELEMENTS

Element	RZ Cnc	AR Mon
P (days)	21.643	21.208
K_1 (km s^{-1})	17.2 ± 1.3(m.e.)	26.5 ± 1.0
γ_1	+12.2 ± 1.2	+11.5 ± 0.9
K_2	101.2 ± 1.2	88.6 ± 0.9
γ_2	+16.1 ± 1.1	+13.0 ± 0.8
ϵ_1*	5.1	2.8
ϵ_2*	4.9	2.6
$a \sin i$ ($R_\odot$)	50.7 ± 0.8	48.3 ± 0.6
$M_1 \sin^3 i$ ($M_\odot$)	3.19 ± 0.13	2.59 ± 0.08
$M_2 \sin^3 i$ ($M_\odot$)	0.54 ± 0.05	0.77 ± 0.04

* Mean error of a velocity of unit weight.

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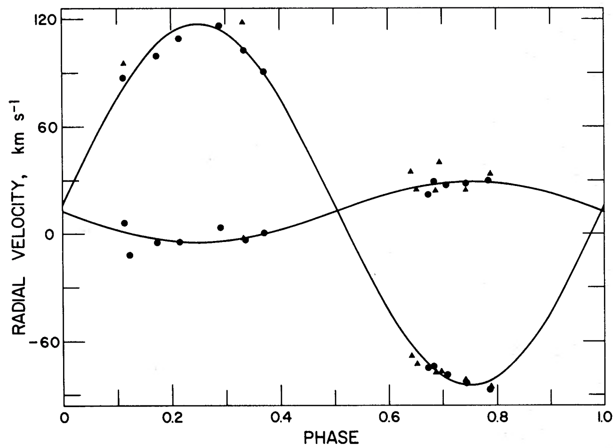

FIG. 1.—Radial velocities of RZ Cnc. *Dots*, weight 1 or 2; *triangles*, weight ½.

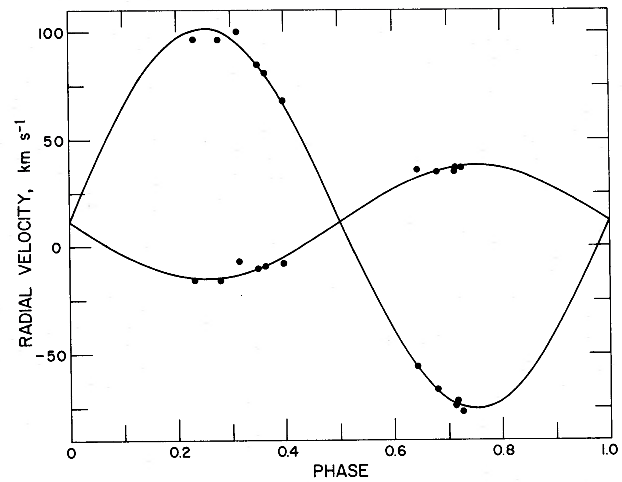

FIG. 2.—Radial velocities of AR Mon

and the representation is shown in Figures 1 and 2. The differences in the values of γ for the two components are in the same direction for the two systems, the difference for RZ Cnc being considerably greater than expected from the mean errors. I have no explanation for this effect. The best systemic velocity to adopt is probably the mean of the two. In the case of AR Mon, Sahade and Cesco (1944) had found $K = 32\ \mathrm{km\ s^{-1}}$, in phase with our K_2, for the single appearing lines in their lower-dispersion spectrograms. Their value is equal to our $(K_2 - K_1)/2$, and is understandable since the two sets of lines are of comparable strength. This result led them to the incorrect conclusion, since repeated in the literature (e.g., Batten 1967), that the hotter component is not visible in the spectrum and must be earlier than K0. Their value of γ, $+14\ \mathrm{km\ s^{-1}}$, is in agreement with ours. Sahade and Cesco found $e = 0.17$, $\omega = 102°$, from the blended lines, leading to a value of $e \cos \omega$ -0.036, much larger than allowed by the photometry.

III. PHOTOMETRIC MATERIAL

Broglia and Conconi (1973) have published photoelectric photometry in the *U*, *B*, *V* system for RZ Cnc and have reviewed the previous literature. My *U*, *B*, *V* observations, on which the published colors of the components were based (Popper 1957*a*) were obtained at the Lowell Observatory in 1955. The equipment and procedures have been described (Popper 1957*b*). The observations are less numerous than those of Broglia and Conconi and do not include the descending branch of primary minimum. The same comparison star employed by Broglia and Conconi was used. My results are generally in quite good agreement with theirs, as shown in Table 5. The Lowell results at primary minimum are placed in parentheses since the observations do not pass through zero phase. The Lowell ultraviolet observations are very few in number. Broglia and Conconi give evidence for instabilities in the light curve, increasing with time in the period 1959–1965, particularly during secondary eclipse. There is also a variation with time of the form of the light curve outside eclipse, as shown by the trend of the coefficients A_1 and A_2 (Table 5), which are defined by the expression,

$$l = l_0(1 + A_1 \cos \theta + A_2 \cos 2\theta) ,$$

TABLE 5

PHOTOMETRY OF RZ CANCRI

Item	Broglia and Conconi			Popper		
	V	*B*	*U*	*V*	*B*	*U*
Maximum	8.67	9.87	10.87	8.69	9.87	10.96:
Primary minimum	10.03	11.45	13.02	(9.96)*	(11.36)*	(12.89:)*
Secondary minimum	9.21	10.32	11.20	9.24	10.34	11.30:
A_1:						
1955	...	...	...	−0.016 ± 0.003	−0.028 ± 0.003	...
1959–1960	−0.005 ± 0.001	−0.006 ± 0.002	−0.028 ± 0.003	...	...	...
1961–1965	+0.033 ± 0.002	+0.044 ± 0.004	+0.043 ± 0.009	...	...	...
A_2:						
1955	...	...	...	−0.102 ± 0.003	−0.098 ± 0.002	...
1959–1960	−0.082 ± 0.002	−0.076 ± 0.002	−0.076 ± 0.004	...	...	...
1961–1965	−0.079 ± 0.003	−0.063 ± 0.005	−0.052 ± 0.013	...	...	...
Comparison star	8.485	9.75	11.18	8.50	9.76	11.18

* 0.007 phase.

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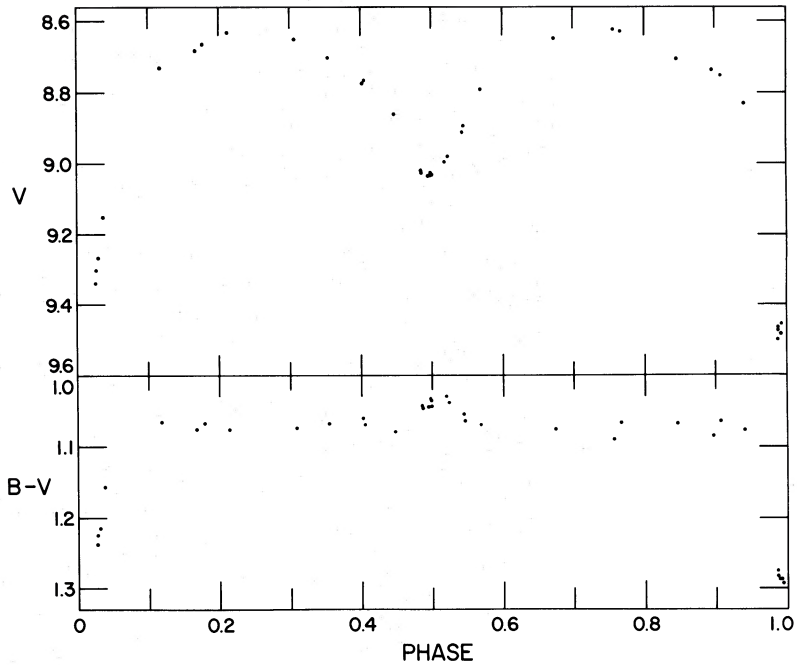

FIG. 3.—Photometric observations of AR Mon (KPNO 1972–1973)

TABLE 6

MAGNITUDES OF AR MONOCEROTIS

JD_{helioc} (2,441,000+)	Phase	V	B	U
604.991.........	0.2126	8.625	9.701	10.44
635.968.........	0.6732	8.644	9.719	10.42
640.957.........	0.9084	8.754	9.817	10.58
653.913.........	0.5193	8.995	10.025	...
653.979.........	0.5225	8.979	10.018	...
654.958.........	0.5686	8.791	9.860	10.55
661.922.........	0.8970	8.737	9.822	10.57
663.860.........	0.9884	9.464	10.740	11.92
663.902.........	0.9904	9.458	10.746	11.95
663.965.........	0.9933	9.455	10.749	11.91
685.868.........	0.0261	9.340	10.579	...
685.909.........	0.0281	9.303	10.529	...
685.956.........	0.0302	9.263	10.479	...
693.854.........	0.4027	8.766	9.835	10.54
716.808.........	0.4850	9.021	10.064	10.68
716.818.........	0.4855	9.023	10.068	10.69
722.795.........	0.7673	8.626	9.693	10.44
743.758.........	0.7557	8.622	9.712	...
745.691.........	0.8469	8.703	9.770	10.49
748.670.........	0.9873	9.499	10.783	11.95
749.731.........	0.0374	9.147	10.305	11.20
752.709.........	0.1778	8.663	9.731	10.43
772.651.........	0.1181	8.732	9.799	10.50
773.658.........	0.1656	8.682	9.758	10.49
776.654.........	0.3068	8.649	9.722	10.46
777.647.........	0.3536	8.702	9.770	...
778.644.........	0.4007	8.776	9.836	10.59
779.649.........	0.4481	8.860	9.939	...
780.663.........	0.4959	9.035	10.079	10.70
780.699.........	0.4976	9.034	10.066	10.64:
780.705.........	0.4979	9.026	10.068	10.65:
780.712.........	0.4982	9.030	10.064	...
781.633.........	0.5416	8.907	9.962	10.61
781.687.........	0.5442	8.892	9.955	...
980.959.........	0.9403	8.830	9.907	10.68
981.998.........	0.9892	9.476	10.764	11.95

where l is the light outside eclipse and θ is the phase angle measured from mid-primary.

My Kitt Peak observations of AR Mon, obtained in 1972–1973, are shown in Figure 3 and listed in Table 6. The comparison star employed was HD 57276 (9.1, K5), for which the values $V = 8.57$, $B - V = +1.29$, $U - B = +1.38$, corresponding to K2–K3 III, were obtained from numerous comparisons with U, B, V standards. The third decimal in the V and B magnitudes of Table 6 is valid only for deriving variations of brightness. Details of the Kitt Peak photometric program will be published separetely. My photometric observations of AR Mon obtained at Siding Spring Observatory in 1965 are fewer in number and poorer in quality, and have been employed solely to obtain the phase of mid-primary, as described above. From the photometry of Table 6 we may derive the quantities listed in Table 7.

In their photometric solution for RZ Cnc, Broglia and Conconi (1973) have applied the Russell model (i.e., similar ellipsoids) to their 1959–1960 observations and have obtained values for the relative radii, r_h, r_c (h: hotter; c: cooler), and inclination i in good agree-

TABLE 7

PHOTOMETRY OF AR MONOCEROTIS

Item	V	B	U
Maximum light........	8.625	9.70	10.42
Primary minimum......	9.47	10.76	11.94
Secondary minimum....	9.03	10.07	10.69
A_1..................	0.004	0.002	−0.013
	±0.003	±0.005	±0.010
A_2..................	−0.096	−0.091	−0.098
	±0.005	±0.005	±0.012

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ment with those obtained by Gaposchkin (1949) from photographic observations. Because of the large mass ratios, 5.9 and 3.3 for RZ Cnc and AR Mon, respectively, and because the more massive star is the smaller in each system, the two components differ greatly in shape, so that the Russell model is a poor approximation. The more massive star is nearly spherical in both cases, and the light variation between eclipses is dominated by the ellipticity of the cooler star. In order to obtain physically more realistic models for these systems, the program of Wood (1971*a*, *b*, 1972, and private communications) was employed, in which the stars are assumed to be triaxial Roche-model ellipsoids in synchronous rotation, the axial ratios depending on the mass ratios and sizes of the stars. In employing Wood's program to obtain a "best" fit to the observations both inside and outside eclipses, the geometric elements were assumed to be the same in both V and B regions of the spectrum. Limb-darkening coefficients were adopted to be 0.65 in V and 0.80 in B for the hotter star and 0.70 in V and 0.85 in B for the cooler in both systems. The work of Gustafsson *et al.* (1975) indicates that the temperature structure of the photospheres of cool stars, and hence the darkening law, is not very sensitive to surface gravity in the range $\log g = 1.0$–3.0. The geometric and light elements are relatively insensitive to the choices of the darkening coefficients. The observed shape of the light curve between minima determines, for a given mass ratio and radius of the cooler component, the relation between the gravity-darkening coefficient and the limb-darkening coefficient for that component (e.g., Merrill 1970). The gravity-darkening coefficients for the cooler components (Table 8) have been derived from this relation. The coefficients for the hotter, nearly spherical components have little effect on the solution and are adopted arbitrarily to be somewhat smaller than those for the cooler components in the two systems. With values of A_2 as large as those in Tables 5 and 7, the fit to the observations outside eclipse plays an important part in the determinations of the photometric elements, which are given in Table 8. Iteration is required in the solutions since the ratios of the axes of the ellipsoids, employed in the solutions, depend on the sizes of the stars, which are derived in the solutions. The initial approximation was one in which all the light variation between eclipses is assumed to be caused by distortion of the secondary. The elements are adjusted by trial and error to achieve a "best" fit to both V and B observations.

The residuals relative to the solutions of Table 8 are shown in Figures 4 (RZ Cnc) and 5 (AR Mon). The effect of unequal maxima, found in many binaries, is present in RZ Cnc with a difference of about 0.05 mag. The solutions fitting the maximum near phase 0.75 are considerably more satisfactory than those forced through the points near phase 0.25. In view of this effect, of the instability in the light curve of RZ Cnc found by Broglia and Conconi, and of the small number of observations of AR Mon, further refinements of the solutions to improve the distributions of the residuals do not appear warranted. The quantities ϵ in Table 8 are the rms residuals.

The principal difference between the Russell-model results of Broglia and Conconi (1973) for RZ Cnc and those of Table 8 is in the light ratios. Their values of 0.59 in V and 0.44 in B for the "rectified" stars

TABLE 8

PHOTOMETRIC SOLUTIONS

QUANTITY	RZ CANCRI		AR MONOCEROTIS	
	V	B	V	B
x_h (adopted)	0.65	0.80	0.65	0.80
x_c (adopted)	0.70	0.85	0.70	0.85
y_h (adopted)	1.2	1.3	1.2	1.5
y_c	1.32	1.46	1.30	1.66
l_c/l_h ($\theta = \pm\pi/2$)	0.778	0.590	1.02	0.75
l_c/l_h ($\theta = 0, \pi$)	0.511	0.358	0.66	0.41
α_0 (pri. min.)	0.976	0.979	0.79	0.81
ϵ_1 (mag)	± 0.022	± 0.030	± 0.021	± 0.031
	Pri (=h)	Sec (=c)	Pri (=h)	Sec (=c)
V(mag) ($\theta = \pm\pi/2$)	9.32	9.60	9.39	9.36
V($\theta = 0, \pi$)	9.33	10.06	9.40	9.85
$B - V$ ($\theta = \pm\pi/2$)	+1.08	+1.38	+0.92	+1.26
	(+1.10)*	(+1.30)*	(+0.98)	(+1.18)*
$B - V$ ($\theta = 0, \pi$)	+1.08	+1.47	+0.92	+1.44
	(+1.10)*	(+1.47)*	(+0.98)*	(+1.43)*
a	0.201	0.258	0.22	0.325
b	0.200	0.231	0.22	0.29
c	0.199	0.220	0.22	0.27
i	87°5		81°	
θ	25°1		29°5	

* See text for explanation of adjusted values of $B - V$.

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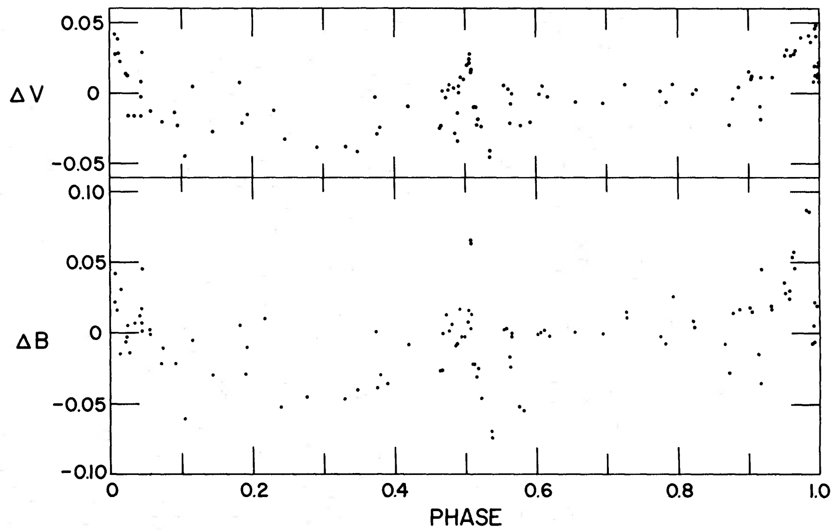

FIG. 4.—Photometric residuals for RZ Cnc. The observations are those of Broglia and Conconi (1973). The residuals are relative to the elements of Table 8.

should be compared with the values 0.78 and 0.59 at quadrature in Table 8. It was not possible to obtain a satisfactory solution for RZ Cnc on the basis of the assumption, adopted by Broglia and Conconi, that the eclipses are total and annular.

The colors, $B - V$, of the components of RZ Cnc correspond to spectral types K1 III and K3–4 III, one subclass earlier than assigned by Hiltner (1946) and in agreement with the K0–2 estimate of this paper for the primary. The colors deduced for AR Mon correspond to G8 III and K2–3 III (at quadrature). In this case the estimated types by Sahade and Cesco and by me of approximately K0 are based on the blended lines of the components of nearly equal strength in the photographic region.

The ultraviolet observations have not been used for the photometric solutions because of their generally lower precision than in the other two bands, particularly for such red stars as these.

The size of the secondary in RZ Cnc conforms very closely to the value $b_c = 0.233$ for the Roche lobe corresponding to the given mass ratio, 0.17, whereas in the case of AR Mon the Roche-lobe value of b_c is 0.27, compared with the value 0.29 obtained in the photometric solution. Attempts to fit the photometry with $b_c = 0.27$ lead to systematic deviations. But since the number of observations is rather small and they show considerable scatter, these deviations should not be taken as strong evidence that the secondary of AR Mon exceeds its limiting Roche lobe in size.

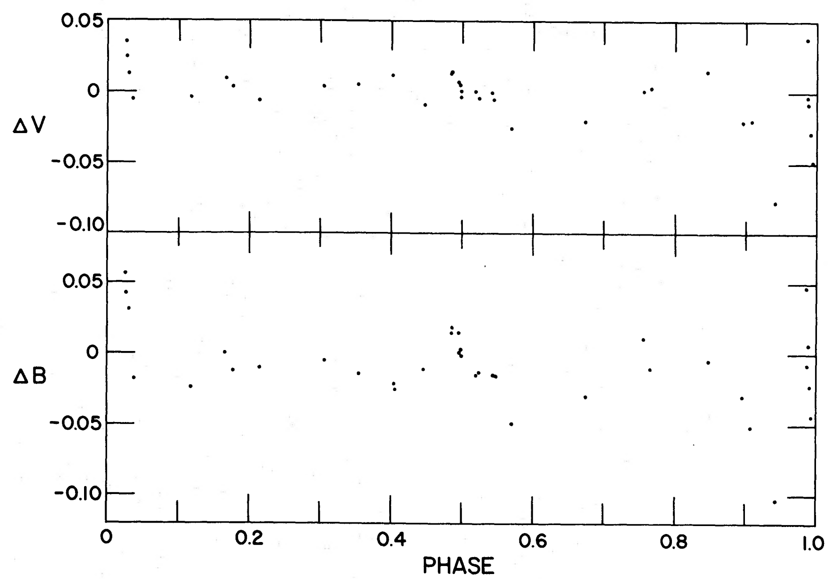

FIG. 5.—Photometric residuals for AR Mon. The observations are those of Table 6 and Figure 3. The residuals are relative to the elements of Table 8.

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The light ratios obtained in the solutions for $\theta = \pi/2$ are in qualitative agreement for both systems with the appearance of the spectrograms if one takes into consideration the increase in the intrinsic strengths of the spectrum lines with advancing spectral type. In the visual region, the lines of the components are of nearly equal strength for RZ Cnc, while those of the cooler component are considerably the stronger in the case of AR Mon.

Comparison of the observed magnitudes in the U band (Tables 5 and 7) with those expected for systems composed of luminosity class III stars with the properties given in Table 8 leads to the conclusion that there may be 0.1–0.15 mag of excess ultraviolet radiation in each system. Given the faintness of the stars in the U band and the range of observed values of $U - B$ for a given value of $B - V$ among red giants (Iriarte *et al.* 1965), the small ultraviolet excess is not firmly established.

From the effective temperatures and bolometric corrections for giant stars, tabulated for different values of $B - V$ by Johnson (1966), we may compute the expected ratios of the radiant fluxes of the components in the V band. These predicted ratios may be compared with those obtained from the light ratios and ratios of the projected areas. For both systems at both quadrature and conjunction the ratios predicted from the colors are larger than those obtained from the solutions of the light curves given in Table 8, although the discrepancy in surface-brightness ratio is small (0.05 mag) for RZ Cnc at conjunction. It averages 0.3 mag in the other cases. The values of $B - V$ shown in parentheses in Table 8 are values that would remove these discrepancies, which are presumably a consequence of uncertainties in the relation between $B - V$ and radiant flux. These changes in $B - V$ do not affect previous conclusions materially. For the sake of consistency the values in parentheses in Table 8 will be used as the basis for the temperatures employed in computing the absolute magnitudes given in the next section.

IV. DISCUSSION

From the information in Tables 4 and 8 the properties of the systems, listed in Table 9, may be derived. The radiative properties (luminosity, effective temperature) are those of the stars at quadrature, which are intermediate in projected area and in surface temperature between the properties seen at conjunction and those seen pole-on. More sophisticated averages over the surfaces are not warranted. The luminosities are calculated from the radii and effective temperatures, the latter being based on the values of $B - V$ shown in parentheses in Table 8 and Johnson's (1966) temperature scale. The $\pm$ values given in Table 9 are mean errors for the masses and estimated uncertainties for the remaining properties. The stars are smaller than "typical" stars of luminosity class III, but are much larger than typical subgiants.

The most straightforward approach available at the present time to the interpretation of these two semidetached giant systems with large mass ratios is probably in terms of evolution with mass exchange through the inner Lagrangian point of each system. This model was developed, of course, to explain the properties of "typical" Algol systems, from which RZ Cnc and AR Mon differ most obviously in the properties of their more massive components. If we consider, however, the total mass and total specific orbital angular momentum of Algol systems, RZ Cnc and AR Mon are not outstanding, although AR Mon does have somewhat greater angular momentum than other systems of similar total mass. In carrying out this comparison I have been guided by the result for AS Eri (Popper 1973) and have assumed that the masses of the present primaries of the most thoroughly studied semidetached systems approximate those of detached main-sequence stars of the same spectral types. Since there are no strong selection effects that would be unfavorable toward the discovery of other eclipsing binaries similar to our red-giant systems, it is reasonable to hypothesize that RZ Cnc and AR Mon differ from the other semidetached systems by being in a shorter-lived state of their evolutionary histories.

With respect to the interpretation of the secondary components in the framework of mass exchange, Ziółkowski (1970) has computed several tracks of the secondaries (original primaries) for mass exchange in case "B", in which the originally more massive star reaches the critical Roche lobe after the completion of core hydrogen burning. One of these tracks passes, during the rapid ascent of its Hayashi track, close to the position (mass, radius, luminosity) of the secondary of RZ Cnc. The secondary of AR Mon lies between two of Ziolkowski's computed tracks. Until a more complete grid of such tracks is available for a variety

TABLE 9

PROPERTIES OF THE COMPONENTS

Property	RZ Cancri Hotter	RZ Cancri Cooler	AR Monocerotis Hotter	AR Monocerotis Cooler
Mass ($M_\odot$)	3.20 ± 0.15	0.54 ± 0.05	2.69 ± 0.10	0.80 ± 0.05
Radius ($R_\odot$)	10.2 ± 0.5	12.2 ± 0.7	10.8 ± 1.0	14.2 ± 1.5
log g (cgs)	2.9 ± 0.2	2.0 ± 0.3	2.8 ± 0.4	2.0 ± 0.6
M_{bol}	+1.1 ± 0.02	+1.3 ± 0.2	+0.6 ± 0.3	+0.6 ± 0.4
M_v	+1.5	+1.9	+0.8	+1.0
log T_e	3.66	3.62	3.68	3.65

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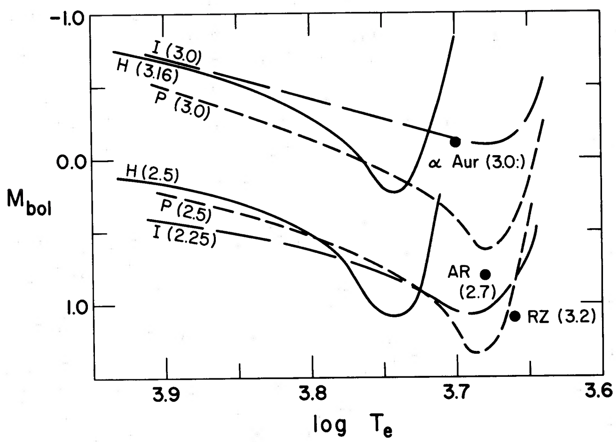

FIG. 6.—Evolutionary tracks and the detached giant components of Capella, RZ Cnc, and AR Mon. The full tracks (H) are based on Hejlesen (1975); the long-dashed tracks (I) on Iben (1967); and the short-dashed tracks (P) on Paczynski (1970) or Plavec (unpublished). Numbers in parentheses are masses in solar units. All tracks are for $X = 0.70$, $Z = 0.03$.

of initial conditions, no further conclusions about the interpretation of the secondaries are justified. It may be recalled that a comparison of the secondary of AS Eri with available crude models indicated excellent agreement (Popper 1973), whereas a more detailed discussion (Refsdal, Roth, and Weigert 1974) showed that the apparent agreement was misleading, that mass loss from the system appears required, and that numerous uncertainties in the interpretation of AS Eri remain.

With respect to the interpretation of the present primaries of the two giant systems, we may ask whether we can understand the manner in which they have achieved their properties as a consequence of evolution of the mass-gaining components. We have noted that the presently more massive component of AS Eri appears to have adjusted its structure, following accretion of most of its mass, to that of a normal main-sequence star of its present mass. Could subsequent evolution of a similar star with or without further accretion, in a system with the total mass and orbital angular momentum of RZ Cnc or AR Mon, lead to a star with the properties of the present primary of RZ Cnc or AR Mon? In Figure 6 the properties of these primaries are shown in conjunction with evolutionary tracks computed for single stars of similar masses. The cooler component of Capella is also shown (Wright 1954). Those tracks labeled "I" are based upon Iben's (1967) work; those labeled "H" are based upon the models of Hejlesen (1975; Hejlesen *et al.* 1972); and those labeled "P" are scaled from Paczynski's (1970) work or are taken from computations by Plavec and Polidan (unpublished) obtained with Paczynski's program. The tracks shown are for a composition $X = 0.70$, $Z = 0.03$ and are relatively insensitive to moderate changes ($\Delta X = \pm 0.05$, $\Delta Z = \pm 0.01$) of composition.

The three stars shown in Figure 6 are the only cool giant stars—i.e., with radii falling in the range of luminosity class III stars—not confined by Roche lobes and with masses determined directly. Because of the considerable differences between the tracks near the bottom of the giant branch computed by different investigators, it is difficult to draw firm conclusions from a comparison of theory and observation. All three stars have effective temperatures near those predicted for the bottom of the giant branch by Iben and Paczynski. The detached giant components of Capella and AR Mon fall within the considerable range (0.5 mag) of luminosities for a star of given mass at this phase of evolution, whereas RZ Cnc is underluminous relative to the models by a factor of 2 or more. More information is required in order to decide whether the underluminosity of the primary of RZ Cnc can be attributed to its having gained mass, or whether modification of the single-star mode of evolution is called for. According to Ulrich and Burger (1976), a star accreting mass will adjust quickly to its equilibrium configuration if the rate of accretion is lower than that causing an appreciable fractional mass increase in its Kelvin-Helmholtz contraction time, which is about 10^6 yr for the present primary of RZ Cnc. We do not know whether the primary is still gaining mass, and if it is, at what rate. If the single-star models are completely trustworthy, the discrepancy in luminosity for this star could provide clues to the mass transfer. Additional uncertainty results from our lack of knowledge of possible loss of mass and of angular momentum from the system during its development. Adequate models of these binaries would have to provide consistent time scales for the two components to reach their present states. It is reasonable to speculate that the scarcity of systems like RZ Cnc and AR Mon compared to typical Algol systems is a consequence of the more rapid expansion of their present primaries through the giant phases than through the main-sequence phases. Further speculation is outside the realm of this paper.

Koch (1970) has noted that the components of RZ Cnc lie close to the tracks computed for pre–main-sequence contraction of single stars of the given masses, in contrast to the discrepancy in luminosity shown in Figure 6. In the discussion of AS Eri (Popper 1973) reasons for preferring the hypothesis of post–main-sequence evolution have been given briefly. In the cases of the more massive components of RZ Cmc and AR Mon we may add the doubt (Larson 1969) that contracting stars of mass greater than 2 $M_{\odot}$ go through a phase corresponding to the bottom of the giant branch. But no hypothesis may be summarily dismissed in the absence of more concrete evidence than is available for these systems.

V. CONCLUSION

Spectroscopic and photometric observations have been combined to determine the properties of the two giant systems RZ Cnc and AR Mon, which are summarized in Table 9. More photometry is required of AR Mon, for which the observations are barely

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adequate for a first analysis. Both systems should be monitored for instabilities of the kind noted by Broglia and Conconi (1973) for RZ Cnc, although they are not sufficiently great to call into question the basic properties of the components. Spectroscopically the stars resemble normal giants of Population I, except perhaps for the strength of Ca II emission. They have the best determined masses of any giants, the only other direct determination of comparable quality being of the cooler component of Capella. The secondaries of RZ Cnc and AR Mon appear to fill their critical Roche lobes, while the primaries are significantly smaller. The cooler component of Capella and the detached star in AR Mon have temperatures and luminosities within the considerable range of available models for single stars near the start of the giant branch. The detached component of RZ Cnc is underluminous relative to the models by a factor of 2 or more. More detailed models of evolutionary tracks of the components in binaries undergoing mass exchange are required in order to interpret these results. It needs to be determined, in particular, whether the detached components are currently gaining mass at a rate so high that they are unable to adjust to equilibrium configurations. The properties of the components filling their Roche lobes are determined with sufficient reliability to provide tests of models of mass exchange. The time scales predicted for the two components in each system would have to be comparable in a satisfactory model.

Observations used in this investigation were obtained at the Kitt Peak, Lick, Lowell, Mount Wilson, and Siding Spring observatories. It has been a privilege to use these facilities. H. L. Burger carried out the photometric solutions, while P. J. Dumont reduced the Kitt Peak photometry. Discussion of various aspects of this work with M. Plavec, R. S. Polidan, and H. L. Burger have been helpful. The National Science Foundation has provided financial support for this research over the last six years.

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