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THE BINARY PULSAR: PHYSICAL PROCESSES, POSSIBLE COMPANIONS, AND EVOLUTIONARY HISTORIES

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ABSTRACT

A study of some aspects of the binary pulsar, PSR 1913+16, is presented in the light of recently reported observational results. If the companion to the pulsar gives a Newtonian contribution to the observed apsidal motion (a helium star or a rotating white dwarf), then it will probably be observable: the former either optically or through the dispersive effects of a stellar wind, and the latter through a secular change in the observed inclination of the orbit. If alternatively the companion behaves dynamically as a point mass (apsidal motion caused solely by general relativity), observations of the $O(v/c)^2$ frequency shift will furnish a measurement of the masses of the components. In addition, with a tenfold improvement in timing accuracy, the post-Newtonian corrections to the Keplerian ellipse and the effects of aberration could in principle be measured. This would then provide a determination of the orientation of the pulsar spin axis and allow the observation of geodetic spin precession to become a test of general relativity. We also examine the test based on the detection of orbital period changes due to gravitational radiation.

Possible evolutionary histories are discussed. The most probable present system consists of two neutron stars, the end result of an X-ray binary phase followed by a double core star phase. In this scenario the long timing age and short period of PSR 1913+16 are shown to be interrelated if the pulsar was the first neutron star formed. To show that other companions are not ruled out on evolutionary grounds, we construct alternate histories leading to either He star, white dwarf, or black hole companions.

Subject headings: pulsars — stars: binaries — stars: neutron

I. INTRODUCTION

Since the discovery of a pulsar, PSR 1913+16, in a binary orbit (Hulse and Taylor 1975) a variety of physical effects have been proposed, the detection of which could furnish valuable new insight into our understanding of advanced stages of stellar evolution, the correct description of gravity, and the nature of pulsars. In this paper we examine the observable consequences of these effects in more detail, and determine how they might indicate the nature of the companion. In § II we point out how future observations of pulse arrival times may enable us to determine the pulsar's orientation and allow us to test the prediction of precession due to general relativity. This is followed in § III by a discussion of the possible relativistic, tidal, and rotational contributions to the observed apsidal motion and the dissipative effects that might change the orbital period. In § IV we discuss the possible companion stars allowed by existing observations and how future studies of this system might discriminate among them. Possible evolutionary scenarios which could lead to these companions are presented in § V. Finally, in § VI we provide a summary of the principal conclusions of this paper.

II. ORBITAL MOTION OF THE PULSAR

Initial hopes that the system would prove to be "clean" in the sense that it would behave to a good approximation as an accurate clock in a precessing Keplerian orbit have been satisfied. It is not clear yet whether the companion may be treated as a point mass; in this section we assume that this is so. First, we show how an accurate analysis of pulse arrival times and changes in pulse shape can in principal yield the masses and orbital inclination of the binary as well as the orientation of the pulsar spin axis and information on the variation of pulse shape, spectrum, and polarization over the pulsar's surface.

Using the Newtonian approximation, the separation r of the centers of mass of the pulsar and its companion satisfies

$$r = \frac{a(1 - e^2)}{(1 + e \cos \phi)} \tag{2.1}$$

where a is the semimajor axis of the relative orbit, e the eccentricity, and ϕ the true anomaly (measured from periastron). If m_1, m_2 are the masses of the pulsar and its companion, respectively, and $m = m_1 + m_2$, then a is related to the mean orbital angular velocity n (i.e., $2\pi/P$, where P is the orbital period) by

$$Gm = n^2a^3 . \tag{2.2}$$

To date (1975 August) the best values for the directly observed orbital elements, obtained by fitting the pulse arrival times to a formula taking account of the $O(v/c)$ Doppler shift, have been given by Taylor and Hulse (1975) and are (to sufficient accuracy for the purposes of this paper):

$$\begin{aligned} &a_1 \sin i = 7.01 \times 10^{10}\ \mathrm{cm}\,, \quad e = 0.617\,, \quad P = 27{,}907\ \mathrm{s}\,, \\ &\omega = 182^\circ\,, \quad \dot{\omega} = 4^\circ\!.24\ \mathrm{yr}^{-1}\,, \quad a = 1.98\,(m/M_\odot)^{1/3}\,R_\odot\,, \end{aligned} \tag{2.3}$$

where a_1 is the semimajor axis of the pulsar orbit, i the orbit inclination, and ω the longitude of periastron measured from the ascending node. Using these elements, the pulsar mass function is found to be

$$f_1(m_1, m_2) = \frac{m_2{}^3 \sin^3 i}{m^2} = 0.132\ M_\odot . \tag{2.4}$$

The $O(v/c)^2$ corrections to the pulse frequency that arise from the second-order Doppler shift and gravitational redshift have not as yet been detected because within a single orbit they are degenerate with and therefore indistinguishable from a portion of the $O(v/c)$ Doppler shift. However, in the presence of apsidal motion (i.e., precession of the longitude of periastron in the orbital plane) this contribution can be isolated and after ~ 5 years of continuous observations it should furnish a 10 percent measurement of the combination

$$g_1(m_1, m_2) = \frac{(m_1 + 2m_2)m_2}{m^{4/3}}\,, \tag{2.5}$$

provided that there are no complicating effects. At present, these second-order terms contribute a fractional error $\sim 10^{-3}$ in the measurement of $a_1 \sin i$. There is an additional uncertainty in the measurement of $a_1 \sin i$, n, and $\dot{\omega}$ arising from the unobservable part of the Doppler shift contributed by relative motion between the binary and solar-system barycenters. This too is likely to be $\sim 10^{-3}$. For a more complete discussion of the fitting of the observed pulse arrival times see Blandford and Teukolsky (1975) and references therein.

The $O(v/c)^3$ terms, arising from the gravitational time delay across the orbit as well as from post-Newtonian corrections to the orbit, contribute time delays $\lesssim 20\ \mu$s. In addition, several other effects enter at this order. Although these cannot be measured at present (Blandford and Teukolsky 1975), an improvement in accuracy by a factor 10, including removal of interstellar dispersion, may allow such observations.

Let us consider what new information could in principle be obtained by these measurements. First, combining f_1 (eq. [2.4]), g_1 (eq. [2.5]), and the apsidal motion $\dot{\omega}$ (eq. [3.5] below) should yield sufficiently accurate values of m_1, m_2, and $\sin i$ to enable us to calculate and remove the $O(v/c)^3$ time delays. The explicit formulae required have apparently not yet been derived. Second, both the frequency emitted and the frequency received on Earth will be Doppler shifted with respect to a fixed frequency propagating through the interstellar medium. The variable part of both shifts can be corrected for: the former, by observing how the pulse shape changes with frequency at a fixed orbital phase; and the latter, by adjusting the time delay attributed to dispersion. Using the same template to fit the radio signal for all orbital phases will leave an error $\lesssim v/(2\pi\nu c) \sim 10\ \mu$s in the observed arrival times.

Having removed the above contributions, we can now use the Galilean aberration of pulses to measure the orientation of the pulsar's spin axis. We introduce a coordinate system (Fig. 1) with the z-axis along the line of sight to the observer and the x-axis in the direction of the observational ascending node. Let $\boldsymbol{n}$ be a unit vector parallel to z, $\boldsymbol{v}_1$ be the orbital velocity of the pulsar, and $\boldsymbol{\omega}_1$ be its angular velocity. The pulsar's orientation is specified by the angles η and λ. We will assume that the spin angular momentum of both the companion star and the pulsar can be neglected in comparison with the orbital angular momentum $\boldsymbol{L}$.

As a result of aberration, a distant observer at rest with respect to the binary barycenter will receive radiation from the moving pulsar only if the pulse is emitted at a small angle

$$\left(\left|\frac{\boldsymbol{n} \times \boldsymbol{v}_1}{c}\right| \text{ in the pulsar frame}\right)$$

backward from $\boldsymbol{v}_1$. The additional time delay in the observed pulse is

$$\Delta t = \frac{\boldsymbol{v}_1 \times \boldsymbol{n} \cdot \boldsymbol{\omega}_1}{c(\boldsymbol{\omega}_1 \times \boldsymbol{n})^2}\,, \tag{2.6}$$

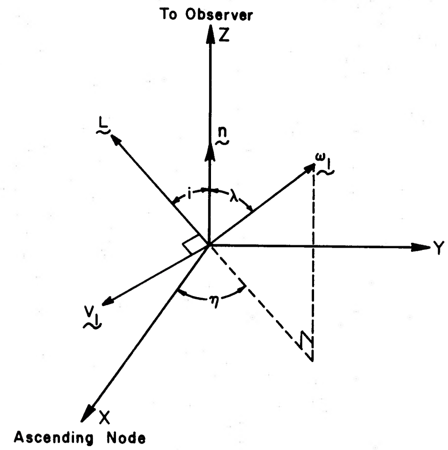

FIG. 1.—Orientation of the pulsar spin axis (ω_1) with respect to the observer direction ($\boldsymbol{n}$) and the observational ascending node (x-axis). The observed pulsar colatitude is λ. We assume here that the companion's spin axis is aligned with the orbital angular momentum vector $\boldsymbol{L}$. The orbital velocity vector of the pulsar is $\boldsymbol{v}_1$.

with periodic part

$$\Delta t = \frac{na_1}{(1 - e^2)^{1/2} c\omega_1 \sin\lambda} [\cos i \cos\eta \cos(\omega + \phi) + \sin\eta \sin(\omega + \phi)] . \tag{2.7}$$

The coefficient in front of the brackets (using eq. [2.3]) is 6.3 csc λ csc i μs.

In deriving equation (2.7), the periodic change in observed pulsar colatitude, λ, due to aberration, which results in a change of pulse shape, has been ignored. This is given by

$$\delta\lambda = \frac{(\boldsymbol{\omega}_1 \times \boldsymbol{n})\cdot(\boldsymbol{n} \times \boldsymbol{v}_1)}{|\boldsymbol{\omega}_1 \times \boldsymbol{n}|c} . \tag{2.8}$$

We then obtain for the part variable on an orbital time scale:

$$\delta\lambda = 0.038 \csc i[\sin\eta \cos i \cos(\omega + \phi) - \cos\eta \sin(\omega + \phi)] \text{ deg} . \tag{2.9}$$

Equation (2.9) predicts that specific pairs of orbital phases should have identical pulse profiles and that by correlating the pulse shapes a measurement of the angle η could be obtained. If we fit the timing residuals at these pairs of orbital phase, using equation (2.7), a measurement of the angle λ would follow. (Note that this requires detecting structure within the pulse shape on time scales $\lesssim 10\ \mu$s.) Successful execution of this procedure would both verify the $O(v/c)^3$ terms and determine the orientation of the pulsar and orbital axes up to a rotation about the line of sight. In addition, any variation found of magnitude greater than that ascribable to aberration would presumably arise from either a propagation effect across the orbit or an electromagnetic interaction of the pulsar with its companion, both potentially important in diagnosing the companion's identity.

Having used this periodic variation to obtain η and λ, one can now measure their secular changes due to geodetic precession. According to general relativity, if the pulsar and orbital angular velocities are not aligned, the pulsar spin axis, $\boldsymbol{\omega}_1$, will precess about $\boldsymbol{L}$ at a rate

$$\langle\Omega_p\rangle = \frac{3}{2}\frac{Gnm_2(1 + m_1/3m)}{a(1 - e^2)c^2} \tag{2.10}$$

(Barker and O'Connell 1975). The change in longitude caused by this precession is unobservable as it is indistinguishable from an error in the period. However, the change in colatitude satisfies

$$\frac{d\lambda}{dt} = -(\sin i \cos\eta)\langle\Omega_p\rangle = -0.53 \cos\eta (m/M_\odot)^{1/3}\left(1 + \frac{m_1}{3m}\right) \text{deg yr}^{-1} , \tag{2.11}$$

where we have used equations (2.3) and (2.4). In addition, the direction of projection of the spin axis on the plane of the sky rotates at a rate given by

$$\frac{d\eta}{dt} = -(\cos i + \cot\lambda \sin i \sin\eta)\langle\Omega_p\rangle$$
$$= -0.53(\cot i + \cot\lambda \sin\eta)(m/M_\odot)^{1/3}(1 + m_1/3m) \text{ deg yr}^{-1} . \tag{2.12}$$

This change in the angle η must be subtracted from any observed rotation of the plane of polarization in order to obtain the "intrinsic" change—that is to say, the change in the plane of polarization seen by observing a different latitude and longitude with the pulsar spin axis fixed in space. Related formulae applicable to a specific model of the pulsar have been given by Hari-Dass and Rakrishnan (1975).

Thus, unless the orbital and pulsar spin axes are aligned, the observed latitude should change by $\sim 3°$ within 5 years, providing for the first time two-dimensional information on the emitted beam shape as well as the variation of spectrum and polarization with latitude. In most pulsar models, radio pulses are emitted in a pencil beam, so PSR 1913+16 might disappear altogether after ~ 20 yr. Alternatively, if the companion is also a pulsar, it could appear at some later time.

After a few months, the total variation due to geodetic precession (eq. [2.11]) will exceed the amplitude of the periodic orbital variation (eq. [2.9]). A comparison of these secular and periodic changes in either the pulse shape or polarization will allow the rate of geodetic precession to be actually measured and compared with the relativistic prediction, equation (2.10). On a less ambitious level, mere detection of secular changes would confirm qualitatively the presence of geodetic precession and that the spin axis of the pulsar was not aligned with the orbital angular momentum. However, even a specific model of the variation of the emitted pulse with latitude will not allow a quantitative measurement of the rate of geodetic precession unless some independent estimate of either η or λ can be obtained.

III. SECULAR CHANGES OF ORBITAL ELEMENTS

We now consider the secular changes of the orbital elements, treating only the pulsar as a point mass. Including the lowest-order relativistic corrections will introduce apsidal motion. In addition, if the companion star is deformed, either tidally or as a result of rapid rotation, a further periastron shift will result. If the companion's spin is not perpendicular to the orbital plane, the observed inclination angle of the orbit will vary with time. Finally, dissipative effects in the system will lead to a change in the orbital period.

To calculate the secular changes induced by an off-axis rotator, we need to develop the formulae for transforming from the binary's invariable plane to the plane of the sky. With this technique we derive the formulae required in order to use apsidal motion as a test of possible companions (§ IV). We also estimate time scales for dissipative processes which might compete with gravitational radiation.

a) *The Binary Geometry*

The geometry can be specified by reference to the plane perpendicular to the total (spin plus orbital) angular momentum vector, i.e., the invariable plane. In Figure 2, we use a stereographic projection onto the invariable plane to label the relevant angles. The total angular momentum vector is aligned parallel to the z-axis. The x-axis is chosen to lie in the plane containing z and the line of sight O. The angle between O and z is called I. The companion star's spin axis, S, makes an angle θ with the z-axis, while the orbital angular momentum vector, L, subtends an angle δ from the z-axis. The plane of the sky intersects the orbital plane at the *observational* ascending node, N. The angle between these two planes is the *observational* inclination angle i. The *observational* longitude of periastron as defined in § II is labeled by ω. In this coordinate system, the orbital plane intersects the invariable

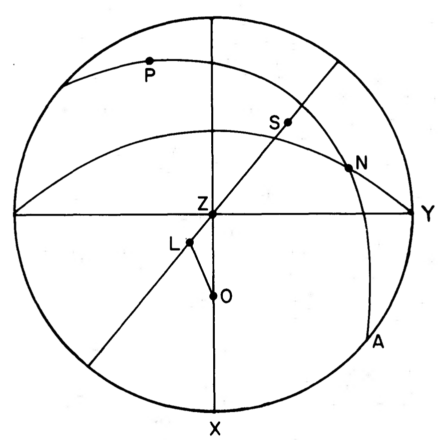

FIG. 2.—Stereographic projection onto the invariable plane of the binary system: Z, total angular momentum vector; O, observer direction; L, orbital angular momentum; S, spin angular momentum of companion; P, periastron; N, observational ascending node; A, dynamical ascending node; NY, plane of the sky; NA, plane of the orbit, OL, observational inclination angle (i); NP, observational longitude of periastron (ω); LZ, dynamical inclination angle (δ); AP, dynamical longitude of periastron (χ); XA, dynamical longitude of the ascending node (Ω); OZ, fixed angle between line of sight and total angular momentum (I); ZS, angle between spin axis and total angular momentum (θ).

plane at ascending node A, with a corresponding dynamical longitude of periastron, χ. The longitude of the ascending node, measured in the invariable plane, is Ω.

To transform from the calculated invariable frame angles (δ, χ, Ω) to the observational angles (i, ω) we use spherical trigonometry:

$$\begin{aligned} \cos i &= \cos\delta\cos I + \sin\delta\sin I\sin\Omega\,, \\ \sin\omega &= -\csc i[\sin\chi\,(\sin\delta\cos I - \cos\delta\sin I\sin\Omega) + \cos\chi\sin I\cos\Omega]\,. \end{aligned} \tag{3.1}$$

Of interest are the secular changes in the observational quantities (i, ω) per orbital period, designated Δi, $\Delta\omega$, respectively. Now I is fixed, and we show below that when either the tidal or the rotational contribution to the apsidal motion is important, $\Delta\delta = 0$. (This latter relation may no longer hold when both effects contribute.) By differentiating equation (3.1) we obtain

$$\begin{aligned} \Delta i &= -(\csc i\sin\delta\sin I\cos\Omega)\Delta\Omega\,, \\ \Delta\omega &= \sec\omega\csc i\{\sin I[\cot i\cos\Omega\sin\delta\sin\omega + \sin\chi\cos\Omega\cos\delta + \cos\chi\sin\Omega]\Delta\Omega \\ &\quad - [\cos\chi\,(\sin\delta\cos I - \cos\delta\sin I\sin\Omega) - \sin\chi\cos\Omega\sin I]\Delta\chi\}\,. \end{aligned} \tag{3.2}$$

Thus, changes in the longitude of periastron $(\Delta\chi)$ couple in a complicated way with the regression of the nodes $(\Delta\Omega)$ to produce secular changes in the observational longitude of periastron $(\Delta\omega)$ and in the observational inclination angle (Δi). We will show presently that for the PSR 1913+16 system the angle between the orbital and total angular momentum vectors is small $(\delta \ll 1)$. In this case, one can use the small δ limits of equations (3.1), (3.2):

$$\begin{aligned} i &= I - \delta\sin\Omega\,, \qquad & \Delta i &= -\delta\cos\Omega\Delta\Omega\,, \\ \omega &= \chi + \Omega - \pi/2\,, \qquad & \Delta\omega &= \Delta\chi + \Delta\Omega\,. \end{aligned} \tag{3.3}$$

b) *Apsidal Motion*

We now consider relativistic, tidal, and rotational effects which cause the orbit to deviate from a Keplerian ellipse. General relativity introduces a periastron advance per orbit of

$$\Delta\chi_{\rm GR} = \frac{6\pi Gm}{a(1-e^2)c^2} \tag{3.4}$$

(e.g., Landau and Lifshitz 1962). We will assume that Einstein's theory of gravity is correct. For other theories see, e.g., Will (1975) or Eardley (1975). To first order $\Delta\Omega = 0$, so using equation (3.4), we obtain for the relativistic contribution to the apsidal motion

$$\dot\omega_{\rm GR} = 2.11(m/M_\odot)^{2/3}\ \text{deg yr}^{-1}\,. \tag{3.5}$$

Therefore, if the observed apsidal motion of $4.^\circ 24\ \text{yr}^{-1}$ is wholly relativistic (eqs. [2.3], [3.5]), then

$$m = 2.85\,M_\odot\,, \qquad a = 2.81\,R_\odot\,. \tag{3.6}$$

In addition, the pulsar can raise a tide on its companion, giving it a nonzero quadrupole moment and producing a Newtonian contribution to the apsidal motion. Since the tidal deformation is symmetric above and below the orbital plane, it cannot cause a secular change in the orientation of the orbital plane, i.e., $\Delta\Omega = \Delta\delta = \Delta i = 0$. In the orbital plane, only χ receives a secular change:

$$\begin{aligned} \Delta\chi_{\rm tidal} &= 30\pi k_2(m_1/m_2)(R_2/a)^5 F(e)\,, \\ F(e) &= (1-e^2)^{-5}(1 + \tfrac{3}{2}e^2 + \tfrac{1}{8}e^4) \end{aligned} \tag{3.7}$$

(Cowling 1938), where $F(e) = 17.5$ for the binary pulsar. The number k_2 must be calculated from the mass distribution in the companion (cf. Schwarzschild 1958, chap. 4). Replacing a by m (eq. [2.2]), we have for the tidal contribution to the observed apsidal motion:

$$\dot\omega_{\rm tidal} = 11.2\left(\frac{k_2}{0.01}\right)\left(\frac{m_1}{m_2}\right)\left(\frac{m}{M_\odot}\right)^{-5/3}\left(\frac{R_2}{0.2\,R_\odot}\right)^5 \text{deg yr}^{-1} \tag{3.8}$$

which must be added to the relativistic contribution (eq. [3.5]).

We now consider rotational deformation. We have calculated the size of the secular changes caused by the rotationally induced quadrupole moment of the companion, working in parallel to Kopal (1959), but correcting some minor errors and generalizing to the case of large e and δ (with no tidal effects). It is found as expected, since no dissipation occurs, that $\Delta a = \Delta n = 0$. There is no change in the magnitude of the angular momentum of the spin and orbit, so $\Delta e = 0$. Finally, it turns out that although precession occurs ($\Delta\Omega \neq 0$), it is not accompanied by notation ($\Delta\theta = \Delta\delta = 0$). The secular changes in Ω and χ are given by

$$\Delta\Omega_{\rm rot} = -3\pi\left(\frac{C-A}{m_2 a^2}\right)(1-e^2)^{-2}\frac{\sin(\delta+\theta)\cos(\delta+\theta)}{\sin\delta}, \tag{3.9}$$

$$\Delta\chi_{\rm rot} = 3\pi\left(\frac{C-A}{m_2 a^2}\right)(1-e^2)^{-2}[1-\tfrac{3}{2}\sin^2(\delta+\theta)+\tfrac{1}{2}\sin^2(\delta+\theta)\cot\delta], \tag{3.10}$$

where C and A are the moments of inertia about the spin axis and about an axis perpendicular to the spin axis, respectively. If the companion is uniformly rotating, then $(C-A)/m_2 \equiv \alpha$ can be approximated by $2k_2R_2^5\omega_2^2/(3Gm_2)$ (Cowling 1938).

To conserve the total angular momentum, S and L must be on opposite sides of z, and we have by the law of sines:

$$\sin\delta = \frac{|\boldsymbol{S}|}{|\boldsymbol{L}|}\sin\theta = \left(\frac{C}{\mu a^2}\right)\left(\frac{\omega_2}{n}\right)(1-e^2)^{-1/2}\sin\theta, \tag{3.11}$$

where ω_2 is the companion's angular velocity and μ is the reduced mass of the system ($\mu = m_1m_2/m$). The last expression in equation (3.11) obtains only for the case of uniform rotation. If $|\boldsymbol{S}| \ll |\boldsymbol{L}|$, as it will be for this system (see below), then $\delta \ll 1$, independent of θ, and the small δ approximation (eq. [3.3]) can be used to obtain the *observational* secular changes in i, ω per orbital period:

$$\Delta\omega_{\rm rot} = 3\pi\left(\frac{C-A}{m_2 a^2}\right)(1-e^2)^{-2}(1-\tfrac{3}{2}\sin^2\theta), \tag{3.12}$$

$$\Delta i_{\rm rot} = 3\pi\left(\frac{C-A}{m_2 a^2}\right)(1-e^2)^{-2}\sin\theta\cos\theta\cos\Omega(t). \tag{3.13}$$

Equation (3.12) differs by a factor 2 from Kopal (1959) and Roberts, Masters, and Arnett (1976). The rotational contribution to the observed apsidal motion, using equation (2.3), is

$$\dot\omega_{\rm rot} = 0.84\alpha_6(M/M_\odot)^{-2/3}(1-\tfrac{3}{2}\sin^2\theta)\ \text{deg yr}^{-1}, \tag{3.14}$$

where $\alpha_6 \equiv \alpha/10^6$ km^2. Again the total apsidal advance is obtained by adding $\dot\omega_{\rm rot}$ to $\dot\omega_{\rm GR}$ (eq. [3.5]).

We see that the sign of the contribution to $\Delta\omega$ from rotation can have either sign depending on the orientation of the spin axis. In particular, its contribution can be made very small by having $\theta \approx \sin^{-1}\sqrt{(3/2)} \approx 54^\circ.7$. Furthermore, $\Delta i \approx \Delta\omega_{\rm rot}$ as long as neither θ is close to 0, $\sin^{-1}\sqrt{(3/2)}$, $\pi/2$, nor Ω close to $\pi/2$, $3\pi/2$. As ω increases linearly with time, i oscillates about a mean value I with small amplitude δ. However, the (rotational) apsidal period is longer than the precessional period by a factor $\sim\delta^{-1}$, so the changes in i, ω per orbit are comparable. Therefore, if the apsidal motion caused by an off-axis rotator is observable, so will be the change in orbital inclination.

The *combined* effects of tidal and rotational deformations are simple only if the companion has its spin axis aligned with the orbital angular momentum ($\theta = \delta = 0$). In this case there is no regression of the nodes ($\Delta\Omega = 0$); the secular change in ω is obtained by adding $\dot\omega_{\rm GR}$ (eq. [3.5]), $\dot\omega_{\rm tidal}$ (eq. [3.8]), and $\dot\omega_{\rm rot}$ (eq. [3.14]) with $\theta = 0$. All terms produce an apsidal advance. If the spin axis does make a large angle, θ, with the orbital angular momentum vector, then the dynamical equations cannot be solved analytically without making assumptions about the smallness of e, θ (Kopal 1959) which would be inappropriate for the PSR 1913+16 binary system. This generality is only necessary for the special case of a rapidly rotating, off-axis helium star. White dwarf companions can be treated with the equations already derived.

It should be emphasized that we have made a crucial assumption in the above analysis: that an initially off-axis companion will react to an external torque in the same fashion as a *rigid* body. In fact, there are three possibilities: (i) The body is fluid and its spin angular momentum axis executes a rather complicated motion, resolvable as a precession *with* nutation, which does not differ greatly from the rigid-body motion predicted above. (ii) The differential torque acting on the star causes a dissipative mixing and an alignment of the spin axis on a rapid time scale (possibly a precessional time scale). (iii) Magnetic fields or crystallized cores rigidify a rotating white dwarf companion allowing at least *uniform* off-axis rotation. The present high eccentricity of its orbit makes it seem unlikely that a companion with a spin angular momentum comparable with the orbital angular momentum could have been aligned as a result of dissipative processes in the past.

c) Dissipative Effects

The above effects do not change the orbital energy, period, or semimajor axis. We now consider four classes of dissipative effects. The dominant one seems to be loss of energy by gravitational radiation. This causes the orbital period to decrease on a characteristic time scale of (Wagoner 1975*a*)

$$\tau_P \equiv \frac{P}{\dot{P}} = -3.8 \times 10^8 \left[\frac{m_1 m_2}{(1.42\, M_\odot)^2}\right]^{-1} \text{yr}\,, \tag{3.15}$$

where we have assumed $m \approx 2.85\, M_\odot$ (eq. [3.6]). To measure this time scale to ~ 10 percent will require ~ 10 yr of careful monitoring of the pulse arrival times (Blandford and Teukolsky 1975). (The associated change in eccentricity will be undetectable.) By this time, the values of m_1, m_2 will *probably* be known from a measurement of $g_1(m_1, m_2)$, equation (2.4), and so this period change constitutes a specific high-order prediction of general relativity. Other theories of gravity give different values of τ_P (e.g., Eardley 1975).

Competing with this will be other dissipative effects. The simplest such process to consider is mass loss from either the pulsar or its companion. If either star loses mass at a rate $\dot{m}_{1,2} > 0$ so that there is no velocity change in the instantaneous rest frame of the star—if the mass flow is spherically symmetric—then it can be shown (e.g., Jeans 1928) that the rate of change of orbital period satisfies

$$\tau_P = -2m/\dot{m}\,. \tag{3.16}$$

As the pulsar slows down, it will lose mass in the form of relativistic particles and electromagnetic fields. From the observed upper limit on the pulsar's spin down rate (Taylor and Hulse 1975),

$$|\nu/\dot{\nu}| \gtrsim 2 \times 10^7 \text{ yr}\,, \tag{3.17}$$

we obtain

$$\dot{m}_1 = 4\pi^2 I\nu\dot{\nu}c^{-2} \lesssim 10^{13} I_{45} \text{ g s}^{-1}\,, \tag{3.18}$$

where $I = 10^{45} I_{45}$ g cm^2 is the pulsar moment of inertia. Using equation (3.16) and assuming that the sum of the masses is given by equation (3.6), we find that the time scale for the period to change due to this effect is $\gtrsim 10^{13}$ yr. The only possible companions that might be able to support a mass loss at a rate $\gtrsim 10^{-8}\, M_\odot$ yr^{-1} are a rapidly rotating neutron star (for instance, if the companion were identical to the Crab pulsar, the period would change on a time scale $\sim 2 \times 10^8$ yr) or a helium star (cf. § IV).

Next we consider tidal effects. Adapting the formulae of Alexander (1973), the time scales for the orbital period and spin angular momentum to change due to tidal lag are

$$\tau_P \approx \tfrac{3}{2}(1 - e^2)^6 \left(\frac{m_2^2}{m_1 m}\right)\left(\frac{n}{\omega_2}\right)\left(\frac{a}{R_2}\right)^8 \left(\frac{m_2}{\langle\mu\rangle R_2}\right)\,,$$

$$\tau_S \approx \frac{S}{L}\tau_P \approx (1 - e^2)^{11/2}\left(\frac{m_2}{m_1}\right)^2 \left(\frac{I_2}{m_2 R_2^2}\right)\left(\frac{a}{R_2}\right)^6 \left(\frac{m_2}{\langle\mu\rangle R_2}\right)\,,$$

$$\langle\mu\rangle \equiv \frac{9}{R_2^9}\int_0^{R_2} \mu(r) r^8 dr\,, \tag{3.19}$$

with μ the viscosity and I_2 the companion's moment of inertia. For tidal effects to be significant, one must have $\langle\mu\rangle \gtrsim 10^{13}$ g cm^{-1} s^{-1} (Alexander 1973), which can only be achieved (*a*) in the presence of turbulent outer envelopes due to convection (Zahn 1966) or tidally driven shear (Press, Wiita, and Smarr 1975) or (*b*) with magnetic viscosity (Sutantyo 1974). We consider two possible binary systems with parameters chosen to satisfy the apsidal motion (§ IV) and to minimize the tidal time scales. First, let the companion be a 1.00 $M_\odot$ helium star. We find

$$\tau_P \approx \frac{4 \times 10^{10}}{\langle\mu\rangle_{13}} \text{ yr}\,, \qquad \tau_S \approx \frac{2 \times 10^9}{\langle\mu\rangle_{13}} \text{ yr}\,, \tag{3.20}$$

where $\langle\mu\rangle \equiv 10^{13}\langle\mu\rangle_{13}$ g cm^{-1} s^{-1}. Second, we take a 0.5 $M_\odot$ white dwarf companion, spinning rapidly ($\omega_2 = 0.1$ rad s^{-1}):

$$\tau_P \approx \frac{10^{16}}{\langle\mu\rangle_{13}} \text{ yr}\,, \qquad \tau_S \approx \frac{7 \times 10^{14}}{\langle\mu\rangle_{13}} \text{ yr}\,. \tag{3.21}$$

The exact value taken by $\langle\mu\rangle_{13}$ depends on the particular viscous mechanism operating. For tidally driven shear we find $\langle\mu\rangle_{13} = 0.7$ for the first case and 0.02 for the second case. Magnetic viscosity in a white dwarf (with field strengths $\sim 10^{3-6}$ gauss and electrical conductivity $\sim 10^{19}$ cgs) could make $\langle\mu\rangle_{13} \approx 10^5$ and the time scale for the period to change $\lesssim 3 \times 10^9$ yr (Wagoner 1975*b*).

Finally, proper motion or acceleration of the barycenter (Brumberg *et al.* 1975; Blandford and Teukolsky 1975) can give a contribution to the period change. Such an effect should also contribute to the change of pulsar period and so barring an unlikely cancellation, the value of $\tau_\nu = |\nu/\dot{\nu}|$ (eq. [3.17]) will set an upper limit on the magnitude of this contribution. For instance, if PSR 1913+16 were a member of a triple system with a third body of approximately stellar mass in a wide orbit of semimajor axis $\sim a'$, the acceleration of the binary would be $\sim GM_\odot a'^{-2}$ and the pulsar period would change on a time scale $\sim ca'^2 G^{-1} M_\odot^{-1}$. At present $\tau_\nu \gtrsim 2 \times 10^7$ years, so we can already rule out the presence of a stellar-mass third body within ~ 100 AU.

IV. THE COMPANION STAR

What is the identity of the companion star? This question, interesting in itself, is also crucial if the masses of the components of the binary system are to be measured. Utilizing the equations derived in the last section, we will first map out the precise ranges of masses, rotation rates, and tilt angles allowed by current observations. We then show that the only two companions capable of contributing major nonrelativistic additions to the apsidal motion—He main-sequence stars and rapidly rotating white dwarfs—may both be observable. If these future observations are negative, then it becomes rather implausible that the total mass of the system is significantly different from 2.85 $M_\odot$.

It is convenient to parametrize possible binary systems by the masses m_1 of the pulsar and m_2 of the companion. We can then display allowed systems in the (m_1, m_2)-plane (see Fig. 3). The mass function, equation (2.3), gives us the values of (m_1, m_2) excluded by the condition $\sin i \ngtr 1$. If the apsidal motion is due solely to general relativity, equation (3.7), permitted solutions lie along the heavy straight line $m_1 + m_2 = 2.85$. However, the possibility of tidal or rotational contributions to the apsidal motion, as discussed in § III, will yield alternative solutions. We note at this stage that the constraints imposed by the absence of an observed eclipse or by the imposition of the Roche geometry at periastron (e.g., Flannery and van den Heuvel 1975) do not rule out any of the following solutions.

a) Helium Main-Sequence Stars

First, we consider helium stars. As has already been pointed out (Masters and Roberts 1975; Webbink 1975) stars of significantly larger radius than a He star, such as H main-sequence stars, will produce far too large a contribution to the apsidal motion. Those He stars which are allowed by the observed $\dot{\omega} = 4\overset{\circ}{.}24\ \text{yr}^{-1}$ can be calculated by using the sum of equations (3.5) and (3.9), following Roberts, Masters, and Arnett (1976) and using the values of k_2 calculated therein. The curved line which results (see Fig. 3), shows that if the pulsar mass $m_1 >$ 0.1 $M_\odot$, then the mass of the He star lies between 0.6 $M_\odot$ and 1.2 $M_\odot$, forcing us to a "low mass" system

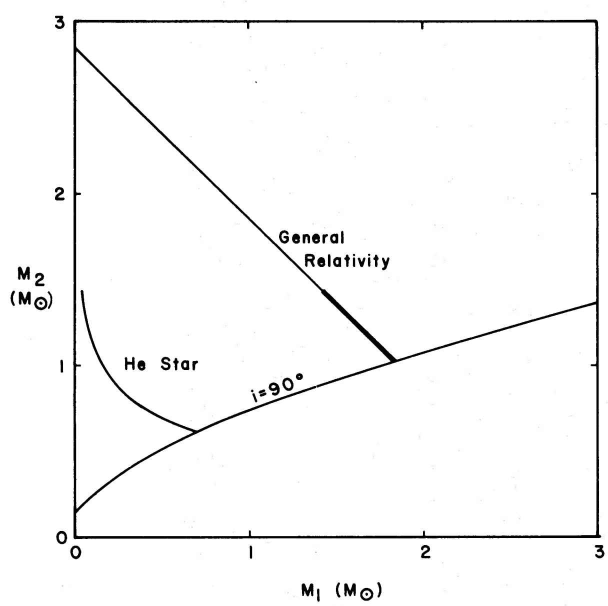

FIG. 3.—Possible binary systems allowed by apsidal motion plotted in the (m_1, m_2)-plane. The observed pulsar has mass m_1; and the companion, mass m_2. The region below $i = 90°$ is excluded by the mass function. Systems for which the apsidal motion is due solely to general relativity (companions: neutron stars, black holes, or slowly rotating white dwarfs) lie along the straight line $m_1 + m_2 = 2.85$. The slowly rotating white dwarfs lie on the thickened part of this line ($m_2 < 1.44\ M_\odot$). A slowly rotating helium main-sequence companion star would in addition cause tidal contributions to the apsidal motion. Such systems are plotted along the curved line (He star).

($m \sim 1.4\ M_\odot$)—in contrast to the "high mass" system ($m \sim 2.8\ M_\odot$) when relativity dominates. Rotation of the He star, with an aligned spin axis, further reduces the total mass. (In the following section we see that this separation into high and low mass limits is convenient for discussing the evolutionary history.)

As pointed out by Roberts, Masters, and Arnett (1976), measurement of the combination $g_1(m_1, m_2)$, equation (2.4), will not necessarily rule out a He main-sequence star. Fortunately, there are two further possible ways by which a He star might be detected: by its optical properties or by its stellar wind.

First, Davidsen *et al.* (1975) have presented evidence that the line of sight in the direction of the pulsar may break out of the reddening layer well before 5 kpc, the pulsar's putative distance. The total extinction in the V-band would then be 3–4 mag rather than $\sim$10 mag which was at first suggested (Hulse and Taylor 1975). If this is substantiated, a 1 $M_\odot$ He star would have $m_v \sim 20$ at a distance of 5 kpc which should certainly be observable. Because of its high effective temperature, such a star, although reddened, should be substantially bluer than its neighbors (at a comparable distance). Failure to observe such a star would not altogether rule out this possibility, because of the uncertainties inevitably present in a determination of the reddening, but would provide strong evidence against it.

Second, there is the question of mass loss from a helium star. Let us take for a definite example the 0.7 $M_\odot$ and 1.0 $M_\odot$ He main-sequence models in Paczynski (1971*a*). By a slight extension of the Lucy and Solomon (1970) stellar wind calculations we find mass loss rates of 6×10^{-14} and $4 \times 10^{-11}\ M_\odot\ \text{yr}^{-1}$, respectively. As an upper limit one might take the Lamers, van den Heuvel, and Petterson (1975) formula for mass loss from early-type stars which yield 10^{-11} and $10^{-9}\ M_\odot\ \text{yr}^{-1}$, respectively. Helium stars do not necessarily lose mass by the detailed mechanisms considered by the above authors, but it is likely that these numbers bracket the actual situation. We note that only the most extreme case would imply a period change on a time scale $\lesssim 3 \times 10^8$ yr (eq. 3.16), so this probably will not compete with gravitational radiation (§ III).

However, the existence of thermal electrons within the orbit will cause a variable dispersion measure. At present the upper limit on this variation is $\Delta\text{DM} < 10\ \text{cm}^{-3}$ pc (DM $= 167\ \text{cm}^{-3}$ pc), but one should be able to push the limit to $\lesssim 0.01\ \text{cm}^{-3}$ pc. The corresponding upper limit on the thermal electron density within the orbit would be $\sim 10^6\ \text{cm}^{-3}$. The absence of free-free absorption and orbital variation in the pulsar scintillation will probably set an inferior limit. If the momentum flux (per steradian) from the He star exceeds that from the pulsar (i.e., $\dot{M}_{-10} V_8 > L_{34}$), then the line of sight will almost certainly intersect the stellar wind in which the electron density is $n_e \approx 3 \times 10^6 \dot{M}_{-10} V_8^{-1}\ \text{cm}^{-3}$. This would probably be detectable as a variable dispersion measure. (Here $10^{-10}\dot{M}_{-10}\ M_\odot\ \text{yr}^{-1} = \dot{M}$, the wind velocity is $10^8 V_8\ \text{cm s}^{-1}$, and $10^{34} L_{34}$ ergs s^{-1} is the pulsar luminosity.) On the other hand, a null measurement of variable dispersion could indicate that the He star mass loss is sufficiently low that most of the ionized gas is swept from the system by the pulsar.

b) *White Dwarf Companions*

Next let us consider degenerate white dwarf companions (DD). If these are nonrotating, then they lie along the high-mass line $m_1 + m_2 = 2.85\ M_\odot$. Because of the Chandrasekhar mass limit on the one hand and the mass function limit on the other, we see that possible masses range between 1.02 and 1.41 $M_\odot$ with corresponding pulsar masses between 1.83 and 1.44 $M_\odot$ and inclination angles between 46°.5 and 90°, respectively.

Alternatively we may have a rotating degenerate dwarf in the system. If it is slowly rotating ($|\boldsymbol{S}| < 10^{50}$ g cm s^{-1} and $m_2 < 1.4\ M_\odot$), then viscosity effects will tend toward uniform rotation (Durisen 1973). However, there are secularly stable differentially rotating dwarfs with $|\boldsymbol{S}| \lesssim 3.5 \times 10^{50}$ g cm^2 s^{-1}, $m_2 < 2.5\ M_\odot$. To specify rotating white dwarfs it is convenient to use their mass m_2 and their "deformation" α_6 (eq. [3.14]), rather than m_2 and S. Models of differentially rotating degenerate dwarfs (Ostriker, private communication; Durisen 1973; their $n' = 0$ sequence) enable us to evaluate $\alpha_6(m_2, S)$ (see Fig. 4). The region of interest is bounded on the left by the "$T/W = 0.137$" line above which secular instability arises, and on the right by central densities of $\sim 10^{9.5}$ g cm^{-3}, beyond which instabilities develop from either inverse β-decay or from pycnonuclear reactions (e.g., Ostriker 1972). Lines of constant α_6 and the line of maximum uniform rotation are displayed. These models were constructed using the Chandrasekhar equation of state with $\mu_e = 2$ (e.g., Ostriker and Bodenheimer 1968).

We find that $|\boldsymbol{S}|/|\boldsymbol{L}| < 0.33$ for all (m_1, m_2) with $m_1 > 0.1\ M_\odot$, so we can use the small δ limit for the rotational apsidal advance (eq. [3.13]), together with the relativistic term (eq. [3.5]). If we equate this sum to the measured apsidal motion, the result is a quadratic equation in

$$\zeta \equiv (m/M_\odot)^{-2/3}\,,$$
$$\alpha_6[1 - (3/2)\sin^2\theta]\zeta^2 - 5.05\zeta + 2.52 = 0\,. \tag{4.1}$$

Thus, for a given angle of tilt (θ), the lines of constant α_6 in the (m_2, S_2)-plane (Fig. 4) map onto lines of constant m in the (m_1, m_2)-plane. The three heavy lines in the (m_2, S_2)-plane which bound the various regions of allowed white dwarfs are also shown in Figures 5 and 6 for the case of aligned ($\theta = 0$) and extremely nonaligned ($\theta = 90°$) rotation.

We see that when $\theta = 0°$ (Fig. 5), most of the region below $m = 2.85\ M_\odot$ is allowed and, in particular, a fairly large region can be filled by rapid but *uniformly* rotating DD's. Notice that any solution (m_1, m_2) involving a He

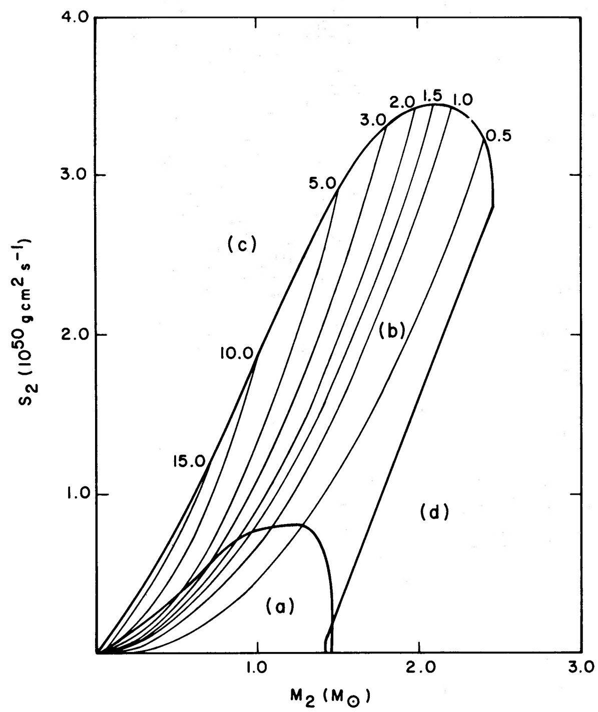

FIG. 4.—The spin angular momentum S_2 calculated from models of rotating degenerate dwarfs of mass M_2. Region (*a*) contains uniformly rotating DD. Region (*b*) contains the allowed differentially rotating DDs. As one crosses from region (*b*) into region (*c*), the DD becomes secularly unstable; while crossing from region (*b*) into region (*d*) the DD becomes unstable to inverse β-decay. Lines of constant α_6 (units of 10^6 km^2) are drawn and labeled along the top curve.

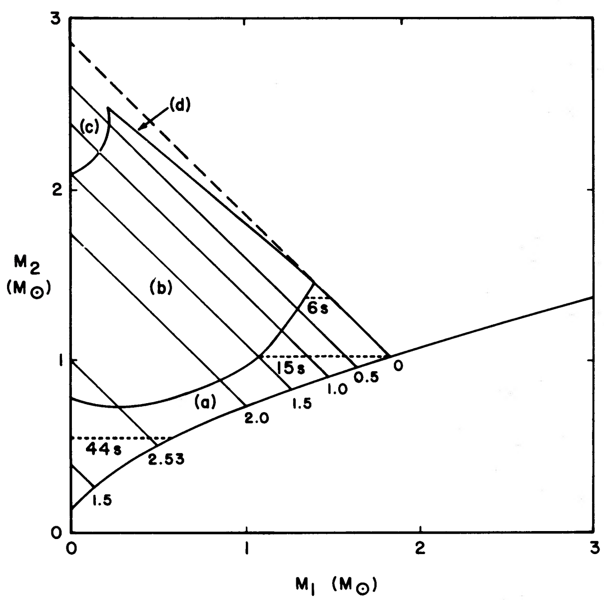

FIG. 5

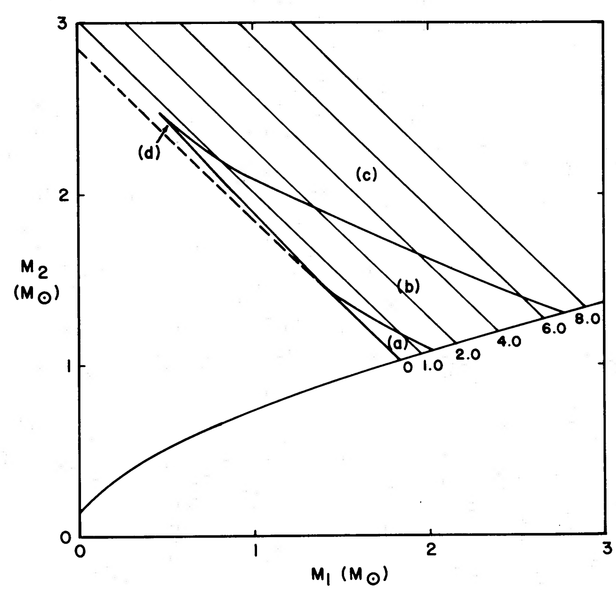

FIG. 6

FIG. 5.—Possible binary systems with aligned ($\theta = 0°$) rapidly rotating white dwarf companions. The axes, $i = 90°$, and general relativity lines are as in Fig. 3. Regions (*a*), (*b*), (*c*), (*d*) are as in Fig. 4. The lines of constant α_6 are indicated with the values given just below the $i = 90°$ line. Note that $m(\alpha_6)$ is double-valued. The three dotted lines give the approximate rotational periods for selected uniformly rotating white dwarfs.

FIG. 6.—Possible binary systems with perpendicular ($\theta = 90°$) rapidly rotating white dwarfs. The regions are as in Figs. 3 and 4. Again lines of constant α_6 are shown.

star companion can just as well be solved with a rotating DD. The region *above* $m = 2.85\ M_\odot$ can also be partially filled if $54° < \theta < 90°$ (see Fig. 6), so that the total mass of the system could be as high as $4.1\ M_\odot$ with the pulsar's mass being as large as $2.8\ M_\odot$ (cf. Roberts, Masters, and Arnett 1976). Note that solutions with $0 \leq \alpha_6 \lesssim 0.5$ are still dominated by general relativity ($m \sim 2.8\ M_\odot$), even though the white dwarf companions can be quite massive ($m_2 < 2.5\ M_\odot$), rapidly rotating, aligned or nonaligned.

To rule out observationally a rotationally dominated off-axis white dwarf companion, we take advantage of the predicted secular change induced in the orbital inclination angle (Δi, eq. [3.12]). From the quoted error in the measurement of $a_1 \sin i$ (Taylor and Hulse 1975), we can already estimate that $\Delta i \sim \theta \Delta\omega_{\rm rot} \lesssim 10^{-6}$, unless $\theta \approx \sin^{-1}\sqrt{\frac{2}{3}}$ or $\frac{1}{2}\pi$ or unless $\Omega \approx \frac{1}{2}\pi$ or $3\pi/2$. This must be compared with the measured value $\Delta\omega \sim 6 \times 10^{-5}$. Thus only if $\theta \lesssim 1°$ (again excluding special angles) could the rotational contribution to the apsidal motion be comparable with the relativistic contribution. Blandford and Teukolsky (1975) have shown that this limit can be improved by a factor 10 after a few years of monitoring pulse arrival times. If changes in $\sin i$ are still not detected, it would seem implausible that the spin axis would be so precisely aligned ($\theta \lesssim 0.^\circ 1$) after the supernova explosion which created the high eccentricity. (The explosion would have had to be very symmetric to avoid tipping the orbital plane.) Therefore, we would regard the failure to detect a nonzero Δi as a strong indication that rotational contributions to apsidal motion can be ignored.

c) *Neutron Stars and Black Holes*

Finally, we turn to neutron star or black hole companions. Perhaps the most important single new discovery that could be made is the detection of a second pulsar. This would indicate that the binary behaves dynamically as two point masses, and since the component masses would be known, the detection of the $O(v/c)^2$ terms would be promoted to an observational test. It is also possible that a pulsar companion could manifest itself indirectly through some electrodynamic interaction with the observed pulsar. For instance, an orbital variation $\gtrsim 0.1$ rad m^{-2} in the rotation measure could probably be detected. The observation of such a variation would imply that the component of the field associated with the companion parallel to the line of sight satisfy $\langle B_\parallel n_e \rangle \gtrsim 3$ gauss cm^{-3}.

A black hole companion is of course even more difficult to detect. However, should the orbital period change on a time scale different from that associated with gravitational radiation (eq. [3.18]), then it would probably be fair to say that a black hole companion was incompatible with the correctness of general relativity theory.

V. EVOLUTIONARY POSSIBILITIES

Where did the present system come from? In this section we investigate this question starting from the analysis of the pulsar and companion in §§ III and IV. We show that the most likely progenitor was an X-ray binary which shrank to the required presupernova configuration by becoming a double core star. This scenario gives a natural explanation for both the short period and the long timing age of PSR 1913+16. Indeed, it seems likely that the observed pulsar was the first of two neutron stars formed in this system. However, the above is not the only plausible evolution, so we exhibit two alternative evolutions which yield high- and low-mass systems.

a) *The Ideal Supernova*

It seems fairly certain that the observed high eccentricity of the PSR 1913+16 binary system resulted from a supernova explosion (see Harrison and Tademaru 1975 for an alternative explanation). Let us consider the simplest such explosion to see what we can learn about the presupernova configuration. We assume that there were two stars in circular relative orbit (radius a_i), one of which (mass $m_{\rm pr}$) exploded to form a neutron star (mass m_n), not necessarily the observed pulsar. If the explosion was instantaneous, spherically symmetric, and caused no mass loss to the companion (mass m_c), then one may deduce the parameters of the prior system in terms of the final values of m_n, m_c, e, a (Boersma 1961; Gott 1972). The relevant formulae are displayed in Table 1, together with

TABLE 1

PRESUPERNOVA CONFIGURATIONS FOR TYPICAL HIGH- AND LOW-MASS SYSTEMS

QUANTITY	FORMULA	HIGH MASS $m_n = 1.42\ M_\odot$, $m_c = 1.42\ M_\odot$	LOW MASS $m_n = 0.60\ M_\odot$, $m_c = 0.65\ M_\odot$
$m\ (M_\odot)$..........	$m_n + m_c$	2.85	1.25
P_i (hr)...........	$[(1-e)^3/(1+e)]^{1/2}P$	1.44	1.44
$a_i\ (R_\odot)$...........	$(1-e)a$	1.08	0.82
$m_i\ (M_\odot)$..........	$(1+e)m$	4.61	2.02
$m_{\rm pr}\ (M_\odot)$.........	$(1+e)m_n + em_c$	3.17	1.37
$\Delta m\ (M_\odot)$.........	em	1.76	0.77
$V_{\rm sp}$ (km s^{-1})......	$(Gm/a)^{1/2}(m_c/m)e(1-e^2)^{-1/2}$	171	136

representative numerical values. We refer to the above assumptions as describing an Ideal Supernova (ISN). Our results may be modified by two effects: (i) As suggested by Flannery and van den Heuvel (1975), the explosion might have been asymmetric. For example, a small backward kick of ~ 90 km s^{-1} during the explosion of a 4 $M_\odot$ progenitor would also leave two 1.4 $M_\odot$ neutron stars, in contrast to the 3.17 $M_\odot$ progenitor required for the high-mass scenario in Table 1. (ii) If the last supernova explosion occurred more than 3×10^8 yr ago, gravitational radiation will have had time to change the postexplosion orbital elements substantially (cf. Wagoner 1975*a*). However, we show below that it is improbable that the explosion occurred more than a few million years ago.

We saw in the last section that we can consider the present binary system in two limits as being high-mass ($m \sim 2.8\ M_\odot$, apsidal motion dominated by general relativity) or low-mass ($m \sim 1.4\ M_\odot$, apsidal motion dominated by tides or rotation). From the formula given in Table 1, we see that high-mass systems tend to have progenitors with mass $m_{\rm pr} \sim 2$–$3.5\ M_\odot$ whereas low-mass systems have $m_{\rm pr} \sim 1$–$2\ M_\odot$. This suggests that there are two distinct pulsar formation processes to consider. The first would occur when a massive 12–20 $M_\odot$ main-sequence star develops a 3–5 $M_\odot$ He core, then loses its hydrogen envelope. The core continues through the advanced stages of nuclear evolution, losing some of its expanding outer layers and developing a 1.4 $M_\odot$ iron center. This central region then collapses (Arnett and Schramm 1973), forming a 1.4 $M_\odot$ neutron star and a Type II supernova. Since for this high-mass case $m \approx 2.85\ M_\odot$, we choose the numerical example in Table 1 to have $m_1 = m_2 = 1.42\ M_\odot$. The second process would involve the stripped-off 1.4 $M_\odot$ degenerate core of a 3–8 $M_\odot$ main-sequence star accreting enough material from its companion to go over the Chandrasekhar mass limit and implode. This would most likely cause a Type I supernova (e.g., Whelan and Iben 1973; Wheeler 1974) and produce a low-mass ($< 0.8\ M_\odot$) pulsar. For the low-mass case the present companion must be a helium star or a rapidly rotating degenerate dwarf (§ IV). We choose the numerical example in Table 1 by demanding that the current system lie on the He star curve in Figure 3 and that the neutron star have a 1.4 $M_\odot$ progenitor [i.e., $m_{\rm pr} = 1.4 = (1 + e)m_n + em_c$]. We now examine the evolutionary histories in more detail.

b) High-Mass Systems

For systems whose apsidal motion is dominated by general relativity, we have found that a possible companion to the pulsar could be a neutron star, black hole, slowly rotating white dwarf, or a massive rapidly rotating DD with $\alpha_6 < 0.5$. We will first discuss how an X-ray binary can lead to the first two possibilities and then examine an alternative scheme for producing white dwarf companions.

Following Flannery and van den Heuvel (1975), we consider a binary system initially containing a 16 $M_\odot$ and a 4 $M_\odot$ main-sequence star. After case B mass exchange (Kippenhahn and Weigert 1967) there remains a 4 $M_\odot$ He star orbiting a 16 $M_\odot$ main-sequence star (a Wolf-Rayet system; see Paczynski 1967). The He star finally explodes, forming a 1.4 $M_\odot$ neutron star; then the system rapidly recircularizes (eq. [3.19]). The next phase is an X-ray binary (van den Heuvel and Heise 1972). When the 16 $M_\odot$ star leaves the main sequence, it will expand and lose its envelope, ending in a close binary with a $\sim 4\ M_\odot$ He core orbiting a 1.4 $M_\odot$ neutron star (van den Heuvel and De Loore 1973).

Although we believe this to be the most likely evolution whose end result is two 1.4 $M_\odot$ neutron stars, the actual mechanism bringing about the essential loss of orbital angular momentum per unit mass may be different from those described by van den Heuvel and De Loore (1973). When the primary of the X-ray binary expands, its rate of mass loss exceeds by orders of magnitude the Eddington-limited accretion rate of the neutron star ($\sim 10^{-8}\ M_\odot$ yr^{-1}). As a consequence, both the neutron star and the He core of the primary will soon be surrounded by a common envelope forming a double-core star. According to the double-core hypothesis (see, e.g., Ostriker 1975 or Paczynski 1975), the frictional force on the neutron star acts as an effective torque for converting the orbital angular momentum into spin angular momentum of the envelope. As the neutron star spirals in, the radius of the star increases due to the input of frictional luminosity. This allows the envelope to act as a more effective sink of angular momentum and also reduces its local density so that the inward spiraling is slowed. This self-regulation continues until the neutron-star–helium-core separation has decreased to the order of twice the radius of the He core; i.e., the dense core fills its Roche lobe. At this point the great rise in local density causes the frictional luminosity to rise so fast that it can no longer be radiated at the surface and the envelope is finally driven off, halting the inward spiral. Since a 4 $M_\odot$ core has a radius $R_{\rm He} \approx 0.5\ R_\odot$ (Paczynski 1971*a*), the resulting orbital separation will be $a \sim 1\ R_\odot$. The outer layers of the He core will expand further before carbon ignition (cf. Webbink 1975), and there will probably be some additional mass loss from the system. If so, the high-mass presupernova configuration given in Table 1 can quite plausibly be attained.

Within the above evolutionary context we reconsider two seemingly incompatible facts about PSR 1913+16: to wit, its long timing age, $\frac{1}{2}\tau_v > 10^7$ yr, and its short period, 59 ms. Assuming that the estimate of the distance to the pulsar (~ 5 kpc) based on the observed dispersion measure is approximately correct, then the height above the plane must be ~ 200 pc. If the physical age of the pulsar were the same as the timing age and the system originated in the galactic plane, then to have reached a height of only 200 pc in $\sim 10^7$ yr implies that the system must be moving perpendicularly to the plane at $\lesssim 20$ km s^{-1}. Since the last supernova imparted a space velocity of ~ 170 km s^{-1}, we conclude that the binary would have had to recoil at an angle $\lesssim 7°$ from the plane. Alternatively, if the perpendicular velocity is ~ 170 km s^{-1} and the system has passed several times through the galactic plane (with an oscillation period of $\sim 10^8$ yr), then the probability of its being so close to the plane now is very small (Oort 1965). Even if the pulsar were formed in the first of two supernova explosions, the lifetime of the 3.16 $M_\odot$ He core is only $\sim 2 \times 10^6$ yr (Paczynski 1971*a*), which is substantially less than 10^7 yr. We therefore conclude that it is more likely that the pulsar is at most a few million years old and that it must have formed with nearly its present period. A similar situation occurs for PSR 1133+16 (Manchester, Taylor, and Van 1974), and appears to be statistically true for many other pulsars (see Taylor and Manchester 1975 or Ostriker and Gunn 1969, who found pulsar detectability decreased drastically after $\sim 10^{6.5}$ yr). If a more sensitive search revealed a supernova remnant, this would imply an upper limit of $\lesssim 10^5$ yr on the age of the system.

Although the timing age does not give us the physical age, it can be used to estimate the surface magnetic field strength of the pulsar. Assuming the spin-down is caused by magnetic stresses acting at the light cylinder, we calculate:

$$B_s^2 \sim \frac{I_1 c^3}{\tau_\nu R_1^6 (2\pi\nu)^2} \tag{5.1}$$

(e.g., Ruderman 1972), where B_s is the surface component of the dipole field evaluated at the polar cap, I_1 is the pulsar moment of inertia ($\sim 6 \times 10^{44}$ g cm^2), and R_1 is the neutron star radius ($\sim 8 \times 10^5$ cm). Application of equation (4.2) for a pulsar of mass $\sim 1.4\, M_\odot$ then gives an upper limit on the magnetic field strength $B_s \lesssim 10^{11}$ gauss. Most other pulsars for which slowing-down times are available yield values $B_s \sim 10^{12}$ gauss. (In fact, the value of $\nu^2 \tau_\nu$ is already larger than that known for any other pulsar except PSR 1952+29.) The corresponding luminosity (rate of loss of stellar rotation energy) L is then given by

$$L = 4\pi^2 I_1 \nu^2 \tau_\nu^{-1} \lesssim 10^{34} \text{ ergs s}^{-1} . \tag{5.2}$$

The inferred low magnetic field on the pulsar suggests a natural explanation for its fast period, if the pulsar were created during the *first* supernova explosion. In that case, the neutron star will have been accreting matter both when it was the X-ray source and while it was spiraling in. Since these stages last for $\sim 10^6$ yr and the Eddington-limited accretion rate is $\dot{M} \approx 10^{-8}\, M_\odot$ yr^{-1}, the pulsar will not have changed its mass appreciably. However, the additional torque exerted by the accreting matter will tend to drive the pulsar to an equilibrium period. Using equations (19) and (20) of Davidson and Ostriker (1973) and assuming the accretion rate given above, we find that a surface magnetic field strength of $\sim 10^{11}$ gauss yields an equilibrium period of 60 ms in $\lesssim 2 \times 10^5$ yr. In addition, the low inferred surface magnetic field strength might be attributable to the pulsar having this peculiar history. Thus, in the above X-ray binary evolution we find that the pulsar timing age and period are closely interrelated.

If, alternatively, the first neutron star is able to accept a significant amount of mass during the mass transfer phase, it is possible that it could be pushed over the maximum mass limit for a stable neutron star, and this would provide a possible way for creating a black-hole companion. What is the ultimate fate of a system containing two neutron stars or a neutron star and a black hole? We have evolved the present orbit forward in time, varying the energy and angular momentum in accordance with the formulae derived by Peters (1964). We find that within a finite time ($\sim 3.1 \times 10^8$ yr), gravitational radiation will have reduced the size of the orbit to a few tens of kilometers. Tidal forces will then destroy the neutron stars, and only a small fraction of the mass could possibly escape from such a deep potential well. The endpoint of such an evolution would then seem to be a ($\sim 2.8\, M_\odot$) rapidly rotating neutron star (if such can exist) or a black hole.

We now consider a completely different evolution whose end result is a white dwarf companion. Let two main-sequence stars of masses 8 and 6 $M_\odot$ be orbiting with a separation of 3 AU. The primary evolves until on the second time up the giant branch it reaches its Roche lobe. It then spills over, and a type C mass transfer occurs (Paczynski 1971*b*). Let us assume that the total mass and angular momentum are conserved (see below); then the end result is a system much farther apart ($a \sim 25$ AU) containing a massive $\sim 1.3\, M_\odot$ carbon-oxygen white dwarf primary and a $\sim 12.7\, M_\odot$ main-sequence secondary (see, e.g., Lauterborn 1970). The secondary expands on *its* second time up the giant branch and envelops the white dwarf to form a double-core star. The white dwarf spirals in, as the neutron star did in the X-ray binary scenario, toward the secondary's core which is like an evolved 3.5–4 $M_\odot$ He star with a C-O core (Paczynski 1971*c*). This yields a presupernova configuration much as before except that the companion is a white dwarf instead of a neutron star. If, in the formation of the white dwarf from the 8 $M_\odot$ star, the angular momentum of shells is conserved, then the resulting white dwarf can become differentially rotating (Ostriker and Bodenheimer 1968) and more massive than 1.3 $M_\odot$. If the supernova explosion is slightly asymmetric with respect to the orbital plane, the plane will be tipped and an off-axis rotating DD will result. In any of these three cases we can end with a total mass $\sim 2.8\, M_\odot$.

c) Low-Mass Systems

One of the difficulties of the last scenario discussed is that we require the white dwarf to spiral in from ~ 25 AU to $\sim 1\, R_\odot$ during the lifetime of the $\sim 14\, M_\odot$ star. This problem is a direct consequence of our demand for conservation of mass and angular momentum during the type C mass transfer.

Let us reconsider what the progeny of our 8 $M_\odot$ and 6 $M_\odot$ stars would have been if severe mass and angular momentum loss occurs. In this case the 1.3 $M_\odot$ white dwarf core will spiral *inward* toward the 6 $M_\odot$ star, and most of the envelope of the 8 $M_\odot$ star will be lost from the system. If the white dwarf is left $\lesssim 50\, R_\odot$ from the

6 $M_\odot$ star, then the latter will expand and envelop the white dwarf *before* helium is ignited in the core. The white dwarf then spirals inward, forming a double core star, blowing off most of the envelope and eventually settling into a close orbit, $a \sim 2\ R_{He} \sim 0.25\ R_\odot$, around the $\sim 0.6\ M_\odot$ He core. In consequence of this He core being much smaller than the 4 $M_\odot$ He core in the high-mass evolution, the final separation after a double core phase would seem to be too small by a factor $\sim 1/3$ to give the inferred presupernova configuration (Table 1). Invoking an asymmetric explosion does not alter this conclusion.

Nevertheless, let us consider what would happen if the above difficulties could be overcome. It is likely that a small hydrogen envelope is left on the He star (particularly since pure low-mass He stars are not observed in close binaries despite their $\sim 2 \times 10^7$ yr lifetime). If so, the He star will eventually expand and overflow its Roche lobe, transferring mass to the white dwarf. From the results of Sienkiewicz (1975), we see that $\sim 0.1\ M_\odot$ could be accreted in the time available without significant expansion of the white dwarf. After exceeding the Chandrasekhar mass limit, the white dwarf would implode to form a $\sim 0.6\ M_\odot$ neutron star.

Alternatively, as the He star itself would probably not expand significantly to overflow its Roche lobe, transfer mass, and trigger the implosion of its companion, until toward the end of its main-sequence life (Paczynski 1971*a*), the He star might by now have evolved nonviolently into a white dwarf. In fact, since the He star would have had time to come into corotation (eq. [3.19]) with period ~ 1.4 hr, its spin angular momentum would be $|\boldsymbol{S}| \sim 0.7 \times 10^{50}$ g cm^2 s^{-1}. If this is conserved in the contraction to an aligned rotating DD, then from Figures 4 and 5 we see that this is just the value necessary to give the rotational contribution required to yield the presently observed apsidal motion.

As mentioned above, there are potential problems in both of these schemes, and they are obviously more contrived than the high-mass evolution proposed to produce a pair of neutron stars. However, they do demonstrate that low-mass helium-star and white-dwarf companions are not ruled out on evolutionary grounds.

VI. SUMMARY

We have seen that the only two companions which would cause a significant nonrelativistic contribution to the apsidal motion—a helium main-sequence star or a rapidly rotating white dwarf—are likely to be observable. The He star may be optically detectable because of the low reddening toward PSR 1913+16, and its stellar wind may cause a variable dispersion measure. If no change in the inclination of the orbit is seen in the next few years, we can conclude that a white dwarf, which rotationally dominates the apsidal motion, must have its spin axis aligned to within 0°1 of the orbital axis. If none of these effects is seen, it becomes implausible that the total mass is significantly different from $\sim 2.85\ M_\odot$, derived on the basis of a purely relativistic apsidal motion.

In this case the pulsar and its companion would act dynamically as point masses. A tenfold improvement in timing accuracy would then, in principle, allow the post-Newtonian corrections to the orbit to be verified and the pulsar orientation to be determined using aberration corrections. Measurement of the effects of geodetic precession would then become a quantitative test. It should be possible to obtain a two-dimensional surface map of the pulsar emission.

From an evolutionary history point of view, the most probable present system consists of two neutron stars, the end result of an X-ray binary followed by a double-core star phase. The total mass given above suggests that both neutron stars have masses $\sim 1.4\ M_\odot$. Confirmation of this would constitute an important test of the theory of late stages of stellar evolution. A calculation of the changes induced by the last supernova implies a high space velocity, ~ 170 km s^{-1}, for the binary. This, coupled with its relative proximity to the galactic plane, suggests that the explosion occurred $\sim 10^6$ years ago when the pulsar's period was about the same as it is now. The long timing age of the pulsar ($> 10^7$ yr) indicates a low surface magnetic field, which could have led to the observed fast period of the pulsar, had it been the accreting neutron star during the X-ray binary phase.

To show that companions other than neutron stars are not ruled out on evolutionary grounds, we have constructed histories leading to either He star, white dwarf, or black hole companions. Difficulties with these evolutions were discussed. Finally, we have examined dissipative processes, such as tidal effects or mass loss, which in this system might compete with gravitational radiation. It seems likely that even if a He star or white dwarf were present, gravitational radiation would still be the dominant cause of orbital period change.

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