

## ATOMIC NITROGEN AS A PROBE OF PHYSICAL CONDITIONS IN THE INTERSTELLAR MEDIUM

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### ABSTRACT

Important optical line ratios for atomic nitrogen have been calculated from atomic data made available by Berrington, Burke, and Robb. Of particular interest is the density-sensitive ratio

$$R = I(^2D_{3/2} \rightarrow ^4S_{3/2})/I(^2D_{5/2} \rightarrow ^4S_{3/2})$$

which can be expected to yield information about conditions in transition regions of ionization fronts. Problems of interpretation of observations of this and other ratios are discussed, with particular reference to planetary nebulae and Herbig-Haro objects.

*Subject headings:* nebulae: planetary — stars: pre-main-sequence — transition probabilities

### I. INTRODUCTION

It has long been recognized (Seaton and Osterbrock 1957; Saraph and Seaton 1970) that ions of the  $p^3$  configuration have an optical forbidden line intensity ratio,

$$R = I(^2D_{3/2} \rightarrow ^4S_{3/2})/I(^2D_{5/2} \rightarrow ^4S_{3/2}),$$

that is a sensitive function of density but which depends only weakly on temperature.

However, up to the present no calculations of this quantity have appeared in the literature for atomic nitrogen. Apparently this is because (apart from the estimates by Seaton 1958) the appropriate collision strengths have not been available. As a result, there has been little incentive for observers to exploit atomic nitrogen as a probe of conditions in very low excitation regions, and the only published observations of the  $\lambda 5198/5200$  ratio appear to be those of Aller and Walker (1970). However, accurate computations of the collision strengths (Berrington, Burke, and Robb 1975) are now available, and we shall use them here to investigate some astrophysical problems.

### II. CALCULATIONS OF EMISSIVITIES

The level populations are determined by electron collisions and spontaneous emission of radiation. The equation of statistical equilibrium for level  $i$  can be written

$$\sum_{j \neq i} N_j R_{ji} + \sum_{j > i} N_j A_{ji} - N_i \left( \sum_{j \neq i} R_{ij} + \sum_{j < i} A_{ij} \right) = 0, \quad (1)$$

where the  $A_{ij}$ 's are the radiative transition probabilities (taken from Wiese, Smith, and Glennon

1966), and the  $R_{ij}$ 's are the collisional transition rates.

For superelastic collisions  $i > j$ ,  $E_i > E_j$ ,

$$R_{ij} = \frac{8.634 \times 10^{-6} N_e}{\omega_i T_e^{1/2}} \gamma(i, j), \quad (2)$$

where  $\omega_i$  is the statistical weight of level  $i$ ,  $T_e$  is in kelvins, and  $N_e$  in  $\text{cm}^{-3}$ . The temperature-averaged collision strength is

$$\gamma(i, j) = \int_0^\infty \Omega(i, j) \exp(-\xi) d\xi. \quad (3)$$

Here  $\Omega(i, j)$  is the collision strength, and  $\xi$ , the energy parameter, is

$$\xi = m_e V^2 / 2KT_e, \quad (4)$$

where  $m_e$  is the reduced mass of the electron and  $V$  the velocity of the colliding electron. For collisional excitation, where  $i < j$ ,  $E_i < E_j$ , equation (2) must be multiplied by a factor  $\exp[-\Delta E_{ij}/KT_e]$  where  $\Delta E_{ij} = E_j - E_i$ .

For electron-neutral atom interactions, the collision strengths are strong functions of energy, and so the  $\gamma(i, j)$ 's cannot be taken as constant. However, it is convenient to express the accurate  $\gamma(i, j)$ 's [obtained by integrating the  $\Omega(i, j)$  of Berrington, Burke, and Robb] as simple temperature functions.

For collisions from the ground  $^4S$  state, and between the  $^2D$  states,  $\gamma(i, j)$  can best be fitted by an empirical formula of the form

$$\gamma(i, j) = a - b \exp[-T_e/c] \quad (5)$$

in the range  $5000 < T_e < 20,000$  where  $a$ ,  $b$ , and  $c$  are fitting constants.

TABLE 1  
FITTING COEFFICIENTS FOR THE TEMPERATURE-AVERAGED  
COLLISION STRENGTHS OF ATOMIC NITROGEN

| Transition                    | $b$   | $b$   | $c$   | $P$  |
|-------------------------------|-------|-------|-------|------|
| ( $^4S-^2D$ ).....            | 1.208 | 1.303 | 17200 | ...  |
| ( $^4S-^2P$ ).....            | 0.511 | 0.540 | 21500 | ...  |
| ( $^2D_{3/2}-^2D_{5/2}$ ).... | 0.773 | 0.825 | 20300 | ...  |
| ( $^2D_{3/2}-^2P_{1/2}$ ).... | 0.097 | ...   | ...   | 0.69 |
| ( $^2D_{3/2}-^2P_{3/2}$ ).... | 0.147 | ...   | ...   | 0.78 |
| ( $^2D_{5/2}-^2P_{1/2}$ ).... | 0.109 | ...   | ...   | 0.80 |
| ( $^2D_{5/2}-^2P_{3/2}$ ).... | 0.266 | ...   | ...   | 0.72 |
| ( $^2P_{1/2}-^2P_{3/2}$ ).... | 0.071 | ...   | ...   | 1.11 |

For other collisions a simple power law is a better approximation in the same temperature range:

$$\gamma(i, j) = a \left( \frac{T_e}{10^4 \text{ K}} \right)^P. \quad (6)$$

The fitting coefficients are summarized in Table 1. Collision strengths from the ground state to the individual levels of the  $^2D$  and  $^2P$  doublets can be obtained using the identities  $\gamma(^4S-^2D_{3/2})/\gamma(^4S-^2D_{5/2}) = \frac{2}{3}$  and  $\gamma(^4S-^2P_{1/2})/\gamma(^4S-^2P_{3/2}) = \frac{1}{2}$ .

### III. N I RATIO $\lambda 5198/\lambda 5200$ IN PLANETARY NEBULAE

Figure 1 shows the result of statistical equilibrium calculations of the  $I(^2D_{3/2} \rightarrow ^4S_{3/2})/I(^2D_{5/2} \rightarrow ^4S_{3/2})$  intensity ratio for the nitrogen atom. The low-density/low-temperature limit is given by  $\gamma(3, 1)/\gamma(2, 1) = 0.667$  and the high-density/low-temperature limit by  $A(3, 1)\omega(3)/A(2, 1)\omega(2) = 1.566$ . The usual collisional parameter  $x = 10^{-2}N_eT_e^{-1/2}$  is not a very suitable one to use here because of the temperature dependence of collision strengths.

The only published observations of this ratio were made for several planetary nebulae by Aller and Walker (1970), and these are summarized together with the mean line-of-sight local densities in Table 2 (assuming a temperature of  $10^4$  K). For comparison the densities given by the [Cl III]  $\lambda 5537/\lambda 5517$  ratio are also given. There appears to be no correlation between the two values of density, but the [N I] densities are generally an order of magnitude smaller. The nebula NGC 6302 appears to be the most remarkable, since it has very intense [N I] lines, yet the [N I] electron density is a factor of 38 lower than the [Cl III] density.

Note that for three objects the observed ratio exceeds the theoretical high-density limit. As two of these have fairly bright [N I] lines this is unlikely to be caused by observational error, and therefore it would seem that the theoretical transition probabilities are in error. Consequently, it would be useful to recalculate these.

The [N I] lines must come predominantly from very low excitation regions in which hydrogen is only partially ionized. We are therefore limited to two possible zones of emission, the transition zones of photoionization fronts, and regions beyond these fronts that are heated in some way.

We have investigated the first of these possibilities in some detail to determine whether or not it provides an explanation for the [N I] observations.

Ionization fronts were calculated using various initial densities, temperatures, ionization front velocities, and effective stellar radiation field temperatures (Mason 1975). The effects of dynamics, recombinations, charge exchange, and forbidden line cooling in the O I, O II, N I, and N II ions were taken into account. These computations have been used to derive Figure 2, in which the ratio of the final nebular H II density to

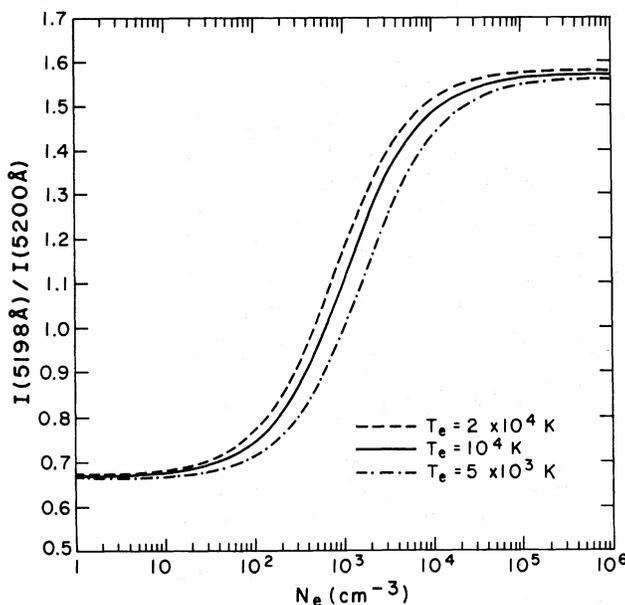


FIG. 1.—The density-sensitive  $\lambda 5198/\lambda 5200$  ratio for the nitrogen atom

TABLE 2  
DENSITY DERIVED FROM THE [N I] RATIO COMPARED WITH THAT GIVEN BY THE [Cl III] RATIO  
FOR SEVERAL PLANETARY NEBULAE TAKEN FROM OBSERVATIONS BY ALLER AND WALKER (1970)

| Nebula           | $R[\text{N I}]$ | $N_e (T_e = 10^4 \text{ K})$ | $R[\text{Cl III}]$ | $N_e (T_e = 10^4 \text{ K})$ |
|------------------|-----------------|------------------------------|--------------------|------------------------------|
| 119-6° 1.....    | 1.25            | $1.9 \times 10^3$            | 0.73               | $1.1 \times 10^3$            |
| IC 2003.....     | 1.17            | $1.3 \times 10^3$            | 1.20               | $7.6 \times 10^3$            |
| IC 418.....      | 1.86*           | $> 10^5$                     | 2.25               | $4.2 \times 10^4$            |
| NGC 2440.....    | 1.34            | $3.0 \times 10^3$            | 1.09               | $9.4 \times 10^3$            |
| NGC 6309.....    | 0.85            | $2.8 \times 10^2$            | 1.34               | $1.2 \times 10^4$            |
| NGC 6302.....    | 1.27            | $2.1 \times 10^3$            | 3.29               | $7.9 \times 10^4$            |
| NGC 6720 Ia..... | 1.06            | $8.3 \times 10^2$            | ...                | ...                          |
| NGC 6720 Ib..... | 0.84            | $2.5 \times 10^2$            | ...                | ...                          |
| NGC 6720 R.....  | 0.90            | $3.5 \times 10^2$            | 0.77               | $1.6 \times 10^3$            |
| NGC 6741.....    | 1.95            | $> 10^5$                     | 1.51               | $1.2 \times 10^4$            |
| 86-8° 1.....     | 1.92            | $> 10^5$                     | 1.22               | $7.9 \times 10^3$            |
| IC 5217.....     | 1.47*           | $7.8 \times 10^3$            | 1.26               | $9.3 \times 10^3$            |
| NGC 7009 A.....  | 1.27*           | $2.1 \times 10^3$            | 1.12               | $9.6 \times 10^3$            |
| NGC 7009 B.....  | 1.00*           | $5.2 \times 10^2$            | 1.25               | $8.9 \times 10^3$            |

\* Values of  $R[\text{N I}]$  which are particularly uncertain because of faintness of the lines.

the initial undisturbed H I density is plotted against the ratio of density given by the [N I] ratio to the final nebular electron density. This latter ratio can be roughly interpreted as the ratio of density given by the [N I] lines to density given by the [Cl III] lines, since the latter are produced away from the transition region.

The observed [N I] ratio depends on the line of sight taken through the nebula. We have considered two cases. In the first we look at right angles to the front. The density given here is approximately the density at which the  $\lambda 5198$  and  $\lambda 5200$  lines have their greatest emissivity. The result is shown as solid lines in Figure 2. In the second case we look in the direction of motion of the front. In this case the emissivity in each line must be integrated along the line of sight and the weighted mean ratio calculated. The result is shown as dashed lines on the figure, and (as might have been expected) the density contrast between the N I emitting region and the nebula proper is reduced in comparison with the first case.

The curves divide into two branches depending on whether the front is *R*-type (supersonic with respect to upstream gas) or *D*-type (subsonic with respect to downstream gas). For the *R*-branch, the value of the temperature ahead of the ionization-front makes very little difference because dynamical effects are small, and for both the *D*- and *R*-branches the effective stellar temperature has a small effect on these curves because charge exchange dominates the nitrogen ionization balance. (The effective wavelength of the incident photons is chosen as  $\frac{2}{3}$  the Lyman limit for the purpose of the diagram.)

It is clear that the ratio of [N I] density to nebular density can only be of order 1:2, and that it would be difficult to use this ratio to distinguish observationally between unevolved and evolved ionization fronts.

This conclusion remains essentially the same if the planetary nebulae contain small dense neutral globules within them, such as described by Dyson (1968). These will produce two regions of low excitation: first, close to the ionization front surrounding them; second, in a

comet-tail-like region shadowed from the ionizing flux of the central object (such as discussed by Van Blerkom and Arny 1972). The first of these possibilities suffers the objections given above with the further disadvantage that the density is higher nearer the globule giving an even higher [N I] density. The shadow region behind the globule has likewise higher density than the rest of the nebula because of its lower equilibrium temperature—the only heat input being in the diffuse radiation field instead of being composed of both diffuse and direct components as in the rest of the nebula.

We therefore conclude that for the majority of nebulae listed in Table 2, the [N I] emission is not produced predominantly in the transition zones of ionization fronts, but in a heated and partially ionized region further out. One way of producing such a region occurs naturally as a phase in the evolution of the ionization front. Tenorio-Tagle (1975) has shown that as the *R*-type front slows down close to the speed of sound, a shock is formed in the transition zone and propagates forward faster than the ionization-front. This shock is initially capable of collisionally heating the gas to about  $2 \times 10^4$  K, and so a relatively thick, partially ionized zone can be created beyond the point at which the ionizing photons are used up. The conditions in the shock-heated region are similar to those discussed in the following section. If the mean temperature in this zone is  $10^4$  to  $1.3 \times 10^4$  K then the observations could be explained.

Another way in which the shock could be formed would be by a collision of a cloud of gas, produced in a mass loss stage of the central object, with a preexisting shell of gas either of interstellar origin or remaining from an earlier mass-loss stage.

Finally, it is possible that the central objects of these nebulae are a source of relativistic particles. This could be expected to partially ionize material beyond the photoionization front.

More detailed observations might be expected to cast light on these points. In particular, evidence that the

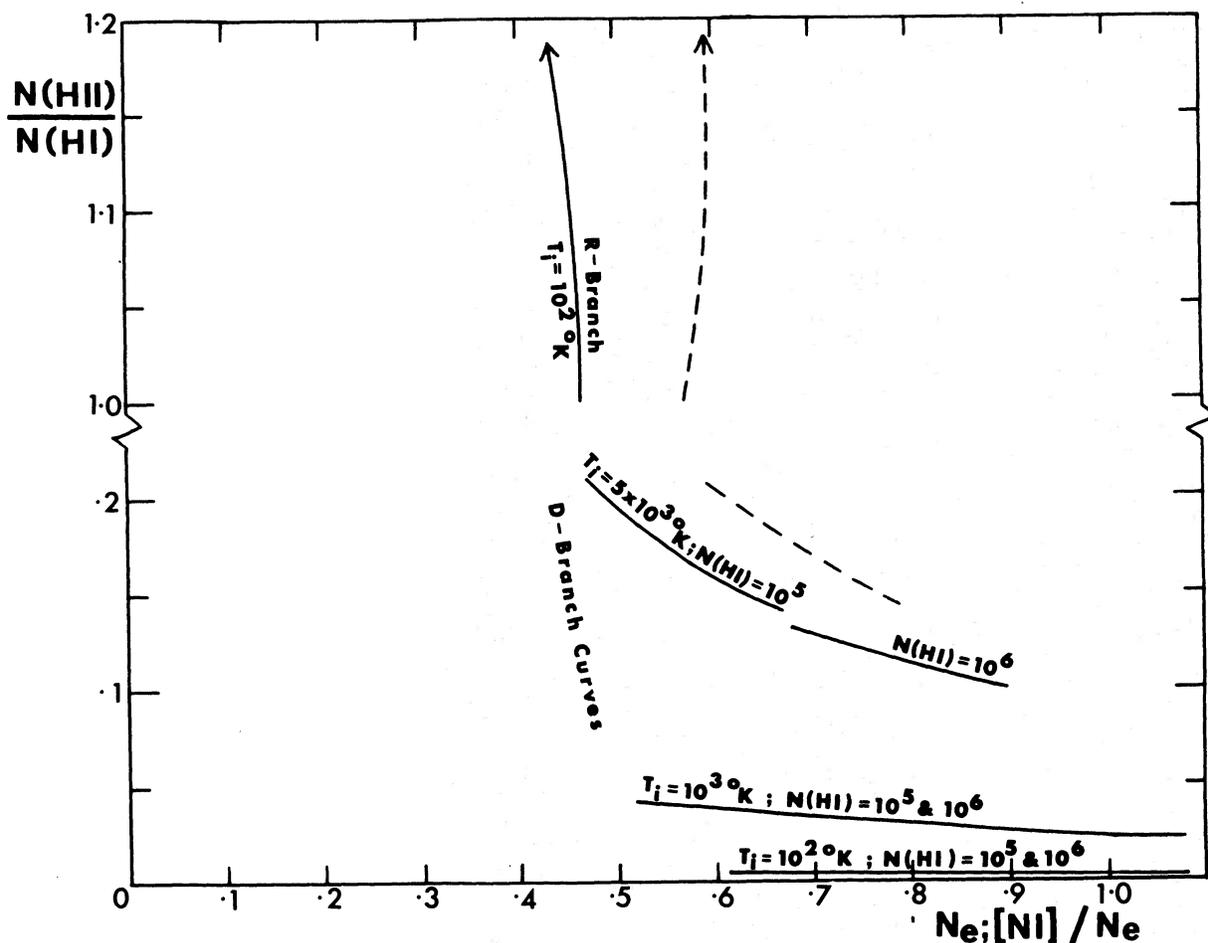


FIG. 2.—The ratio of undisturbed H I hydrogen density to final H II hydrogen density plotted against the ratio of electron density given by the [N I] ratio to final electron density in the H II region. The dotted lines correspond to cases of observation along the direction of motion of the photoionization front and the solid lines to observation at right angles to this. The arrows indicate the sense of evolution of the fronts.  $T_i$  is the initial temperature.

[N I] emitting region is detached from the nebula proper would give strong support to the shock-heating hypothesis.

#### IV. IONIZATION STATE OF THE HERBIG-HARO OBJECTS

Herbig-Haro objects along the T Tauri nebulae, and so-called cometary nebulae, are characterized by very strong forbidden lines of ions with low ionization potential such as O I, N I, and S II. There have been several attempts to explain the state of ionization of these objects. Magnan and Schatzman (1965) suggested that the emission lines are excited by inelastic collisions of high-energy ( $\sim 100$  keV) electrons with atoms. Gurzadyan (1974) believes that 1.5 MeV electrons produced at the central star give rise to transition radiation (by collisions with dust grains) of sufficient energy to give the observed degree of ionization. Strom, Grasdalen, and Strom (1974) give evidence to support the idea that the Herbig-Haro objects are

reflection nebulae and that the line spectrum is excited in the vicinity of the central object. However, Schwartz (1975) prefers a picture of radiating shocks produced by collisions between the stellar wind gas and circumstellar clouds. He points out that Coulomb scattering in the high-energy electron-proton collisions would thermalize the electrons in a distance of only a hundredth of what is required.

In order to give more information on the ionization balance of these objects we have calculated two useful ratios. The first is the ratio of the [N II]  $\lambda 6584$  to the sum of [N I]  $\lambda 5198 + \lambda 5200$ . We have used atomic data given by Nussbaumer (1971) for the transition probabilities and by Seaton (1975) for the collision strengths. A temperature/density plot of the ratio is shown in Figure 3 with the assumption that the number of singly ionized atoms equals the number of neutral atoms.

This ratio is a strong function of both temperatures and density, and so to derive the degree of ionization of nitrogen, both these parameters must be known

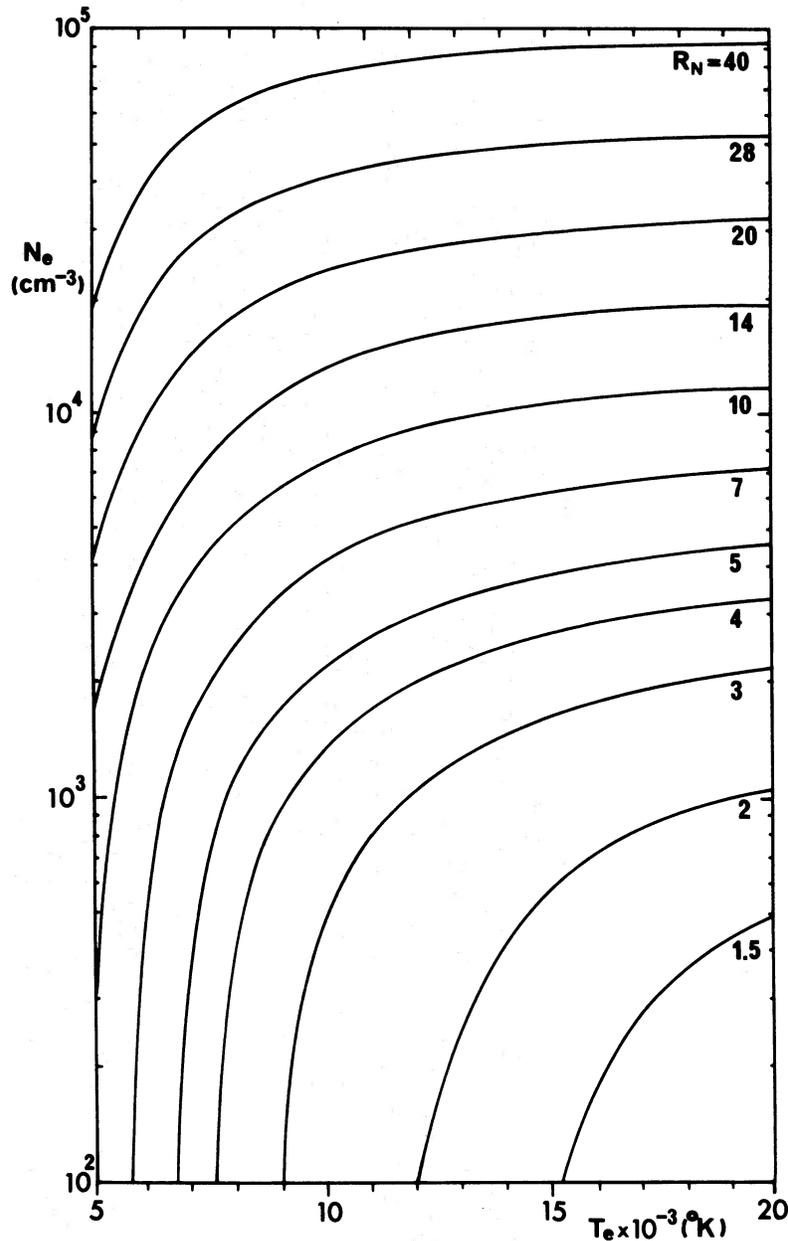


FIG. 3.—A temperature/density plot of the ratio  $R_N = I(6584 \text{ \AA}) / I(5198 \text{ \AA} + 5200 \text{ \AA})$  assuming equal abundances of N I and N II.

separately. Using observations by Schwartz (1975) for Burnham's nebula (around T Tauri), the [O II] lines give  $T_e \approx 15,000 \text{ K}$ ,  $N_e \approx 5 \times 10^3$ , the [N II] lines give  $T_e \approx 14,600 \text{ K}$ , and the [S II] doublet  $N_e \approx 1.2 \times 10^4$ . These figures suffice to give the degree of ionization of nitrogen in the range  $0.55 < N(N^+) / N(N) < 1.00$ . On the other hand, Herbig-Haro object No. 1 (HH1) yields  $N_e \approx 1.4 \times 10^4$  for the [S II] ratio,  $N_e \approx 1.5 \times 10^3$  for the [O II] ratio, and  $T_e = 11,300 \text{ K}$  from the [N II] lines. Owing to the large uncertainty in the density, the degree of ionization can only be deter-

mined as  $0.5 < N(N^+) / N(N) < 2.2$ . This may be compared with Osterbrock's (1958) value for oxygen,  $N(O^+) / N(O) = 1.1$ .

In fact, owing to the very close correspondence of the ionization potentials of nitrogen and oxygen, we expect their ionization to be similar regardless of the particular mechanism causing ionization (cosmic-ray ionization excepted). A more useful indicator of ionization would seem to be the ratio of the [N II] to [N I] lines, given above, multiplied by the ratio of [O II] lines ( $\lambda\lambda 3726, 3729$ ) to the [O I] lines ( $\lambda\lambda 6300,$

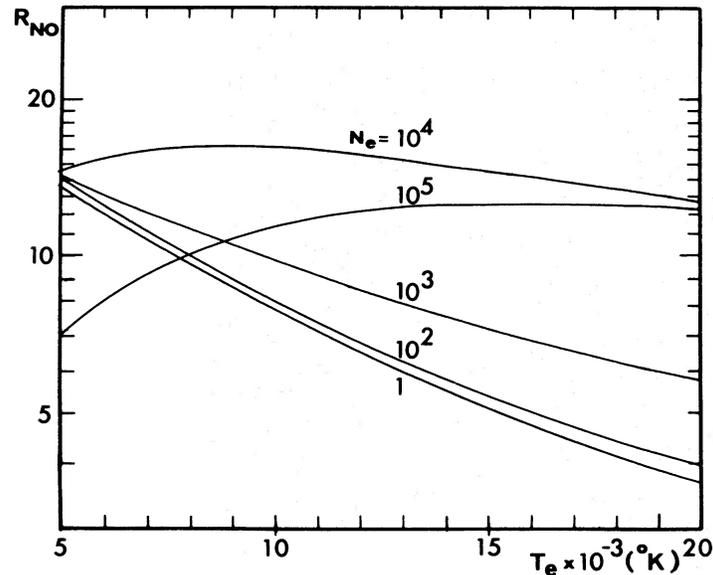


FIG. 4.—The mixed ratio  $R_{N,O} = [I(6584 \text{ \AA})I(3726 + 3729 \text{ \AA})]/[I(5198 + 5200 \text{ \AA})I(6300 + 6363 \text{ \AA})]$  plotted as a function of temperature for various densities assuming 50 percent ionization of both oxygen and nitrogen. Note that the ratio depends little on temperature and density, but is very sensitive to the state of ionization of gas.

6363). This dual ratio has the advantages that (1) the density dependence of the [N I] lines is more or less canceled by the [O II] lines; (2) the temperature dependence is reduced; (3) the dependence on reddening corrections is also reduced; (4) the ratio is much more sensitive to ionization conditions. The computed values of this ratio are shown in Figure 4. We have used  $\gamma(i, j)$ 's for [O I] calculated from collision strengths published by Saraph (1973), those of Czyzak *et al.* (1970) for O II, and transition probabilities from Wiese, Smith, and Glennon (1966) for both ions.

This ratio gives, ignoring any possible error caused by inaccuracies in the atomic constants,

$$0.69 < \left[ \frac{N(N^+)}{N(N)} \frac{N(O^+)}{N(O)} \right]^{1/2} < 0.75 \text{ for Burnham's nebula}$$

and

$$0.47 < \left[ \frac{N(N^+)}{N(N)} \frac{N(O^+)}{N(O)} \right]^{1/2} < 0.58 \text{ for HH1.}$$

We have investigated the question of whether these figures are compatible with Schwartz's hypothesis. Here we are dealing with a collisionally ionized gas, and, provided the cooling rate is slower than the recombination rate, we can assume collisional ionization equilibrium for hydrogen, which at these densities is a function of temperature only. We have used coefficients given by Canto and Daltabuit (1974) to calculate this. For both oxygen and nitrogen, charge exchange (Field and Steigman 1971; Steigman, Werner, and Geldon 1971) dominates over recombinations and collisional ionizations in determining the ionization

balance, which may be written

$$\frac{N(N^+)}{N(N)} = (\epsilon + \beta) \left[ \alpha + \left( \frac{1-x}{x} \right) \beta' \right], \quad (7)$$

where  $x = N(H^+)/N(H)$ ,  $\epsilon$  is the collisional ionization rate,  $\alpha$  is the recombination rate obtained from Tarter (1971),  $\beta$  is the rate for reaction  $N + H^+ \rightarrow N^+ + H$ , and  $\beta'$  is the rate for the inverse charge exchange reaction. A similar expression applies for the oxygen balance.

The cooling processes which we consider are collisional ionizations, recombinations, free-free emission, collisional excitation of  $^2S$  and  $^2P$  levels of hydrogen, collisional excitation of metastable levels of the O I, O II, N I, N II, and S II ions, and spin change scattering of hydrogen atoms from the O I and O II ions (Wofsy, Reid, and Dalgarno 1971) which dominates at low temperature.

The computed ionization equilibria and cooling times for a hydrogen atom density of  $10^4$  are shown in Table 3. From this it is clear that collisional ionization equilibrium implies a theoretical temperature of about 13,900 K for Burnham's nebula and 13,400 K for HH1. This certainly appears to be consistent with the observations, and we believe that the Schwartz hypothesis is vindicated for these two objects. However, we want to make one further point regarding the nature of the radiating shocks.

If these were produced by a steady driving pressure (low density upstream, high density downstream), there would be a thick lower temperature (9000–13,000 K) zone containing predominantly neutral oxygen and nitrogen, and relatively few electrons. However, as a consequence of the large cooling time in this region,

TABLE 3  
COLLISIONAL IONIZATION EQUILIBRIA FOR GAS OF  $N_H = 10^{24}$ \*

| $T_e$  | 8000     | 9000     | 10,000   | 11,000   | 12,000   | 13,000   | 14,000   | 15,000  | 16,000  | 20,000   |
|--|----------|----------|----------|----------|----------|----------|----------|---------|---------|----------|
| Cooling Time (s).....                        | 2.7 + 10 | 2.8 + 10 | 2.4 + 10 | 9.2 + 9  | 1.9 + 9  | 5.2 + 8  | 1.9 + 8  | 1.2 + 8 | 9.7 + 7 | 8.6 + 7  |
| (Cooling time)/<br>(Recombination time)..... | 4.8 - 3  | 5.2 + 1  | 1.7 + 1  | 1.4 + 2  | 3.4 + 2  | 5.6 + 2  | 8.7 + 2  | 1.1 + 2 | 1.3 + 3 | 1.5 + 1  |
| $M(N^+)/N(H)$ .....                          | 6.78 - 5 | 7.26 - 4 | 4.81 - 3 | 2.33 - 2 | 8.70 - 2 | 2.53 - 1 | 7.13 - 1 | 1.50    | 3.55    | 3.57 + 3 |
| $N(N^+)/N(N)$ .....                          | 6.84 - 5 | 8.87 - 4 | 6.89 - 3 | 3.73 - 2 | 1.52 - 1 | 4.83 - 1 | 1.46     | 3.33    | 8.23    | 1.05 + 2 |
| $N(O^+)/N(O)$ .....                          | 5.84 - 5 | 6.32 - 4 | 4.21 - 3 | 2.05 - 2 | 7.68 - 2 | 2.24 - 1 | 6.34 - 1 | 1.33    | 2.50    | 3.19 + 1 |

\* Note that approximation breaks down for  $T_e \lesssim 9500$  K.

the net emission of [O I] and [N I] lines would be greatly enhanced over what is observed. A plausible alternative hypothesis is that the stellar mass loss phenomenon driving the shocks is highly irregular and unstable. Each shock produces a thin, hot, compressed, radiating shell of material which then relaxes to a much lower density. Such shocks (which are basically only driven for a short time) would soon decay, so that the process must be repeated with new outbursts of stellar mass loss. This would explain both the observed spectrum and the relatively small filling factor (about 0.05, Schwartz 1975).

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