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PRECESSION OF THE NODES IN SOME TRIPLE STELLAR SYSTEMS

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ABSTRACT

Long period variability of apparent binary stellar systems may be caused by the presence of a third companion, via the precession of the nodes phenomenon. We show that this phenomenon may indeed have already been observed in the confirmed triplet λ Tauri, as well as in three systems for which a binary structure was confirmed and a possible long period has been observed—Her X-1, HD 217061, and CV Serpentis. Suggestions for some immediate observations of these systems are made.

Subject headings: stars: binaries — stars: variable — X-rays: sources

I. INTRODUCTION

Multiple stellar systems are quite common in our Galaxy; there are estimates as high as 0.2 for the fraction of stars that are members of multiple systems (Batten 1973; Worley 1967). Recently, suggestions for the existence of a third component in some of the binary X-ray sources have been put forward (Bahcall *et al.* 1974; Brecher and Wasserman 1974). With no detailed evolutionary model implying otherwise, this seems to be a plausible assumption on statistical grounds.

The stellar three body problem is a very old one, and has been discussed in numerous works (see, for example, Szebehely 1974). The standard procedure is to separate the motion of the closest pair from the motion of the third around the center of mass of the pair. The problem is then equivalent to that of having two interacting bodies orbiting a center of $1/r^2$ force, which gives two perturbed elliptical motions. The interaction between the two bodies depends on their respective individual distances from the center, as well as on their relative distance.

The observational signature of a stable triple system containing a close pair includes, in addition to direct viewing of the three members (when possible), variations of the dynamical parameters of the motion of the close pair. These are (i) periodic motion of the center of mass of the close pair (to be normally detected by the Doppler effect); (ii) periodic variation in the eccentricities of the two motions; (iii) periodic variation in the periastron passage; and (iv) periodic variation of the planes of the two motions ("precession of the nodes"). Of these, only the first has been commonly utilized. The reason is traced mainly to the difficulty in observing (ii), (iii), and (iv), due to the smallness of the effect and its normally very long period.

One major theoretical obstacle of the stellar three body problem is the determination of system stability. Instabilities show up mostly by too large a growth of the eccentricity of one motion. When this is accompanied by a growth of the corresponding axis, the system loses one of its members; when that axis remains bound, the finite size of the stars makes a collision imminent. Whereas it is quite clear that one should be able to describe instabilities in terms of inability to have matched energy and angular-momentum transfers between the two motions, an analytical treatment is yet to be formulated. Estimates do exist for some limiting cases (Harrington 1972; Bahcall *et al.* 1974), but in the general case one must resort to numerical simulations. They show, for example, that coplanar motion is not necessarily more stable than the noncoplanar one (in the latter case, one may be able to match the angular momentum transfer by the additional option of adjusting the relative directions of the angular momenta vectors); they also show that with initially large eccentricities, noncoplanar motion is likely to be less stable than at low eccentricities. But, again, no systematic description exists.

In this *Letter* we wish to point out that whenever a long period is being observed in a system which is a confirmed binary, one may want to check whether a third companion might not be present in the system. We point out such possible periods in Her X-1, HD 217061, and CV Ser. In the case of accreting systems such as Her X-1, one must carefully discuss the influence of a possible third member on the gas and disk dynamics; we leave that discussion for a later publication. Here we concentrate on the λ Tauri system, which is a confirmed triplet, and show, by a reanalysis, based on the precession of the nodes, of existing spectroscopic data, that the two planes are noncoplanar. This is in contrast with a photometric analysis based on the same effect, recently carried out by Söderhjelm (1975). Further observations are suggested.

II. OBSERVING THE PRECESSION OF THE NODES THROUGH THE CLOSE PAIR

The precession of the nodes is a precession of the angular momentum vector of the close binary $\boldsymbol{G}_1$ (which is perpendicular to the instantaneous plane orbit), around the (constant) total angular momentum $\boldsymbol{G}$, accompanied by a corresponding motion of the second angular momentum vector, $\boldsymbol{G}_2$. The angular frequency ω of the precession is given to lowest order by

$$\omega = \frac{m_3}{m_1 + m_2}\left(\frac{a_1}{a_2}\right)^3 \Omega\eta\,, \qquad (1)$$

where Ω is the binary angular frequency; a_1 and a_2 are the semimajor axes of the first (binary) and second ellipses; m_1, m_2, m_3 are, respectively, the masses of the components of the binary and the third (more distant) star; and η is a geometrical factor which is normally of order unity (except for extreme cases):

$$\eta = \frac{3}{4}\frac{G}{G_2}\cos\theta\,\frac{1}{(1-e_2{}^2)^{3/2}}\,.$$

Here, e_2 is the eccentricity of the orbit of m_3 and θ is the angle between $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$, which is constant, barring changes in eccentricity. Equation (1) can be derived, for example, by using the mathematical machinery of the Delaunay variables (Harrington 1968, 1969; see also Söderhjelm 1975).

As a result, the angle between $\boldsymbol{G}_1$ and any direction which is relatively constant in space (but not coinciding with $\boldsymbol{G}$), undergoes periodic changes. These can lead to genuine, as well as apparent, changes in the binary system.

When the fixed direction is our line of sight, we have periodic changes in the inclination of the binary plane relative to us. These bring about periodic changes in amplitude of apparent velocity modulations, in surface brightnesses (due to changes in average gravity over the projected stellar disk or in anisotropic illumination), and in eclipse curves (the binary could even be an eclipsing binary for part of the precession period, and a noneclipsing one for the rest).

The fixed direction can also be the spin axis of one of the stars. Physical processes depending on the orientation of that spin axis relative to the binary plane, e.g., accretion in compact X-ray sources which are apparent binaries (Pines, Pethick, and Lamb 1973), or tidal distortions, will undergo genuine periodic changes.

If α is the angle between the fixed direction and $\boldsymbol{G}$, and ψ_1 is the angle between that direction and $\boldsymbol{G}_1$, we have

$$\cos\psi_1 = \cos\alpha\cos(\boldsymbol{G}_1, \boldsymbol{G}) + \sin\alpha\sin(\boldsymbol{G}_1, \boldsymbol{G})\cos(\omega t + \phi_1)\,, \qquad (2)$$

where ϕ_1 is a constant angle. Thus the maximum variation in the angle ψ_1 is the smallest of $(\boldsymbol{G}_1, \boldsymbol{G})$ and α.

Note that the corresponding expression for ψ_2 involves $\phi_2 = \phi_1 + \pi$; thus $\cos\psi_1$ and $\cos\psi_2$ are in phase or out of phase depending on $(\boldsymbol{G}_1, \boldsymbol{G})$ and $(\boldsymbol{G}_2, \boldsymbol{G})$.

One may expect to observe the precession of the nodes when a_2/a_1 and $1/\Omega$ are not too large and when the motion is sufficiently noncoplanar. The λ Tauri system has the smallest a_2/a_1 known (Ebbighausen and Struve 1956) and is hence a natural candidate for observation. Also, X-ray binaries have rather short orbital periods, and this too will give rise to a sufficiently large ω if a_2/a_1 is not enormous.

III. λ TAURI

The λ Tauri system is an eclipsing system which undergoes a partial eclipse every ~4 days. Ebbighausen and Struve (1956) carefully examined spectroscopic data extending over 50 years and confirmed Schlesinger's conjecture (1916*a*, *b*) that λ Tau is a triple system. Four different sets of radial velocities have been independently analyzed, and the Keplerian parameters for each group of data were derived. A sine function was fitted to the residuals of three sets of data, assuming these residuals to be due to a perturbation by a third star on the binary velocity curve, whose amplitude is K_1. The period of the sine curve was $33^{\rm d}025$, and its amplitude, K_2, was determined by the least-squares method. The relevant results of Ebbighausen and Struve are summarized in Table 1.

The mass of the closed pair is about 10 $M_\odot$, and the mass function for the third is 0.002 $M_\odot$. Ebbighausen and Struve estimated $m_3 \approx 1.0\ M_\odot$ and $\sin i_2 \approx 1$. From the ratio $T_2/T_1 \approx 8$ one obtains $a_2/a_1 \approx 4$. By equation (1), the period for the precession of the nodes comes out to be of the order of 10 years. We may therefore expect variations in K_1, K_2, and the light curve on time scales of that order. Table 1 does indeed suggest such variations in K_1 and K_2; moreover, as K_2 gets its maximum value when K_1 is minimal, it is suggestive to think of the precession of the nodes effect, in spite of the poor accuracy of those values. As Allegheny and Michigan–Victoria radial velocities extend over time intervals greater than the precession period, we analyzed four concentrated subsets of residuals, taken from Ebbighausen and Struve's paper. For each set, a sine function with the $33^{\rm d}025$ period was fitted, and the amplitude was determined by least squares. The subsets and the results are given in Table 2. One notes a change of K_2 in the different data subsets which appears to be real, as it is larger than the statistical errors involved. One must keep in mind that the precession of the nodes also changes K_1, so that the Ebbighausen and Struve figures for K_1, quoted in Table 1, should represent only averaged quantities. Thus the resulting residuals are not well known, and neither are the K_2 values. Therefore, it is too early to speculate on the geometry of the triple system from these data. Nevertheless, a relative angle (between the planes of motion) as large as 30° will not be surprising. In such a case, the precession period would be around 7 years, depending on m_3. Therefore, there might be a variation of i_1 and i_2 between ~90° and ~60°, causing variations of K_1, K_2, and the light curve of the eclipsing binary. This effect might be seen by a series of spectroscopic or photometric observations, one per season. If the precession does exist in λ Tau, it should show a sharp minimum of i_1 each 7–8 years, possibly one of them in 1975. Söderhjelm (1975) has also considered the relative angle in λ Tau based on photometric data. From somewhat uncertain photometric elements, the work of Nijland (1932), and his own model, he concludes that the relative angle is, in fact, no larger than 7°. However, that angle seems to be incompatible with our analysis of the spectroscopic data, and observations for the above-mentioned sharp minimum should cast some light on that point.

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TABLE 1

Results of Ebbighausen and Struve 1956

Source	Time of Observation	K_1(km s^{-1})	K_2(km s^{-1})	Number of Points
Potsdam	1901–1902	59.1 ± 2.0	...	18
Allegheny	1906–1916	54.4 ± 1.0	11.6	108
Michigan-Victoria	1923–1939	55.1 ± 1.0	8.6	83
Mount Wilson-Lick	1952–1955	57.3 ± 1.0	8.7	70

TABLE 2

A Finer Analysis of Part of Table 1

Source	First Point (JD 2,400,000+)	Last Point (JD 2,400,000+)	Number of Points	K_2 (km s^{-1})	K_2 from Table 1 (km s^{-1})
Allegheny I	17,831	18,022	66	12.1	11.6
Allegheny II	20,072	20,212	26	9.7	
Michigan-Victoria I	23,743	24,157	47	7.2	8.6
Michigan-Victoria II	28,048	28,868	34	9.9	

The stability of a triple star system depends mainly on the ratio $a_2(1 - e_2)/a_1$ (Harrington 1972). In λ Tau we can derive a_2/a_1 independent of i_2, just by dividing one Kepler equation by another. Therefore our noncoplanar model is just as stable as the coplanar usual one. We have carried out numerical simulations to check this point over $\sim 10^3$ orbit periods of the close binary. Treating the three stars as point masses we start with a circular binary orbit, with $m_1/m_2 = 3.8$, $m_3/(m_1 + m_2) = 0.1$, and $a_2/a_1 = 4$. Various eccentricities and relative inclinations for the distant orbit were chosen for these initial conditions. The three components of the total angular momentum of the system served as indicators for the accuracy of our computer simulations; they were numerically conserved to a few parts in 10^4 throughout all of our calculations. As long as the relative angle was less than 40° and $e_2 < 0.3$, only very small modulations appeared in a_1, e_1, e_2, and θ throughout the motion (that fact by itself already guarantees the accuracy of our numerical calculations for these angles). At higher inclinations, and for $e_2 = 0$ as a starting point, only e_1 and θ were strongly modulated, as predicted by Harrington's first order perturbation theory (Harrington 1968).

IV. OTHER SYSTEMS

Long period changes in the light curve or in the amplitude of the radial velocity of close binary systems may be an indication for the existence of a third distant star in the system. Three such systems, which may deserve further investigation and observations, are suggested below.

a) *HZ Herculis–Hercules X-1*

Jones *et al.* (1973) have pointed out that the modulation in the optical magnitude of HZ Her (caused, presumably, in part, by Her X-1 illumination) has active and inactive states of a duration of years. Moreover, "between 1914 and 1957 there does appear to be a tendency for active states to occur every 10 to 12 years." We suggest that these might have been caused by the precession of the nodes associated with a third star, such that

$$(a_2/a_1)^3 \frac{m_1 + m_2}{m_3} \approx 2000 .$$

It is quite clear that the sharp maximum of the X-ray pulse should enable one to detect small changes in the radial velocity of the center of mass of the binary system. Some frequency variations have, indeed, been reported by Giacconi (1973) and were interpreted by Brecher and Wasserman (1974) as being indeed caused by a third distant star, and by Lamb, Pines, and Shaham (1974) as representing either torque shot noise or Tkachenko oscillations. However, estimates using parameters of the first model, i.e., $a_2/a_1 \approx 50$ and $m_3/(m_1 + m_2) \approx 10^{-2}$, will not explain a 10 or 20 year precession of the nodes.

It seems that a third star with a period of about 31 to 40 days can fit the X-ray data, which are averaged over 35 day periods, with the same accuracy as Brecher and Wasserman's interpretation. Such a star, with $a_2/a_1 \approx 10$ and $m_3/(m_1 + m_2) \approx 10^{-1}$, would cause a precession of tens of years. Another possibility is that the period of the third star is the clock underneath the 35 day period, and therefore its influence on the Doppler shift of the close pair is minimal. However, the recently observed X-ray pulsations during the off part of the 35 day cycle should have a Doppler frequency difference as large as 10^{-5} s^{-1}.

We are now examining how such a third distant star could influence the dynamics of flowing gas in the binary system, and whether this could be connected with the recent model by Gerend and Boynton (1975) of a "precessing disk" (see also Katz 1973).

A precession of the nodes in HZ Her can be tested by looking carefully for a change in the amplitude of the Doppler shift of the pulsed X-rays over the next few years. If that conjecture is right, a variation in the amplitude has to emerge.

b) HD 217061

This star was recently discovered by Garmany (1971) to be a spectroscopic binary, without deriving the exact elements of orbit, except for the period $T \approx 2.7$ days and $K_1 \approx 125$ km s^{-1}. In spite of the short period and the large orbital velocity, Lanning and James (1975) failed to observe any optical variations. This somewhat surprising result could be interpreted as having the binary plane move to a more perpendicular position with respect to us between 1971 and 1975, due to a precession of the nodes. At this stage, it seems that one should perform photometric and spectroscopic observations at the same time, to be able to decide whether the effect is not that of the precession of the nodes.

c) CV Serpentis

A deep eclipse lasting 10 days in this close binary system was observed by Hjellming and Hiltner (1963) in 1962. A different type eclipse was observed by Tcherepaschuk (1969) while other observers failed to see any eclipse at all (see Cowley, Hiltner, and Berry 1971). Again, in order to see whether this phenomenon is indicating a third star in the system, further investigations and observations should be performed.

Naturally, one might at least want to consider all systems which are already confirmed triples, as candidates for observing the precession of the nodes effects. However, most of the known ones have rather long precession periods, of the order of hundreds of years, and it is therefore rather impractical to consider them at this stage.

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