THE ASTROPHYSICAL JOURNAL, 205:580–591, 1976 April 15

ARRIVAL-TIME ANALYSIS FOR A PULSAR IN A BINARY SYSTEM*

ROGER BLANDFORD
Institute for Advanced Study
AND
SAUL A. TEUKOLSKY†
Cornell University
Received 1975 August 14; revised 1975 September 25

ABSTRACT

A method is described for analyzing the arrival times of pulses from the binary pulsar PSR 1913+16, in terms of the orbital elements and their possible secular variations. Estimates are given for the times necessary to measure such secular changes and to detect various relativistic effects. If measurement errors ~ 1 ms are the dominant source of error and ~ 1000 independent observations are made per year, then ~ 5 years of observations are necessary for a dynamical determination of the component masses accurate to 10 percent and ~ 15 years for the possible detection of gravitational radiation. Other sources of error are briefly discussed.

Subject headings: pulsars — stars: binaries

I. INTRODUCTION

The binary pulsar PSR 1913+16, discovered by Hulse and Taylor (1975), possesses a companion object in a bound gravitational orbit. In several recent papers it has been pointed out that because pulsars are very reliable clocks, this system presents a unique opportunity for observing a wide variety of Newtonian and relativistic effects. In particular, as discussed for example by Wagoner (1975) and Blandford and Teukolsky (1975, hereinafter Paper I), there is the possibility of both a dynamical determination of the component masses and a test of general relativity based on the detection of changes in the apparent orbital period.

In this paper, we present a discussion of the inferences that might be drawn directly from observations of pulse arrival times and estimate the quality and quantity of observations that will be necessary. In § II a timing formula, equation (2.46), to which the observations can be fitted is derived. A brief discussion is given of the relationship between this analysis and conventional orbit perturbation calculations of celestial mechanics. Our timing formula is in a sense merely a parametrization of the arrival times. We show how to measure secular variations in the parameters (i.e., in the orbital elements), and which variations are measurable. While we give estimates of some effects that should exist, the choice of which theoretical interpretation best explains the measurement of a particular secular variation is beyond the scope of this paper.

In § III approximate values for the variances of interesting physical quantities appropriate for PSR 1913+16 are calculated. These variances are used to estimate the time necessary to observe particular effects. Some physical factors that might complicate the preceding analysis are mentioned in § IV.

II. DERIVATION OF THE TIMING FORMULA

Let T_p be proper time as measured by a hypothetical clock on the pulsar. The time of emission of the Nth pulse is given in terms of the frequency ν (rotation frequency of the pulsar) by

$$N = N_0 + \nu T_p + \dot{\nu} T_p{}^2/2 + \ddot{\nu} T_p{}^3/6 \,. \tag{2.1}$$

Here N_0 is an arbitrary constant while $\dot{\nu}$ and $\ddot{\nu}$ are the first and second time derivatives of the frequency. From past experience, we expect to need no higher derivatives in the expression (2.1), although discontinuous frequency jumps (glitches) and random noise may occur (see § IV). Our aim is to find the relation between N and the time of arrival of the Nth pulse at the Earth as measured by an observer on the Earth.

The metric in a coordinate system with origin at the center of mass of the binary system is

$$ds^2 = -[1 + 2\Phi + O(v^4)]dt^2 + O(v^3)dx^i dt + [1 - 2\Phi + O(v^4)](dx^2 + dy + dz^2) \,. \tag{2.2}$$

* Supported in part by the National Science Foundation [GP-40768X] at the Institute for Advanced Study, and [MPS 74-24273] at Cornell University.
† Alfred P. Sloan Foundation Fellow.

Here the Newtonian potential is given by

$$\Phi(\boldsymbol{r}, t) = \Phi_1 + \Phi_2 = \frac{-M_1}{|\boldsymbol{r} - \boldsymbol{r}_1(t)|} - \frac{M_2}{|\boldsymbol{r} - \boldsymbol{r}_2(t)|}, \tag{2.3}$$

where the subscripts 1 and 2 refer to the pulsar and its companion, respectively. (We are using units with $c = G = 1$, so that $M_\odot = 1.477$ km $= 4.925 \times 10^{-6}$ s). Errors in the line element (2.2) are shown as powers of a typical orbital velocity v in the system, where $v^2 \approx M/r \approx 10^{-6}$.

The orbital plane of the binary system can be chosen as the equatorial plane of a polar coordinate system related to x, y, and z in the usual way, with the x-axis intersecting the orbit at pericenter. The (r, θ, ϕ) coordinates of the pulsar and companion are

$$\boldsymbol{r}_1 = (r_1, \pi/2, \phi)\,, \qquad \boldsymbol{r}_2 = (r_2, \pi/2, \phi + \pi)\,, \tag{2.4}$$

where ϕ is the true anomaly and r_1 and r_2 are, in the first approximation, ellipses about the center of mass:

$$r_1 = \frac{M_2}{M_1 + M_2} r\,, \qquad r_2 = \frac{M_1}{M_1 + M_2} r\,, \qquad r = \frac{a(1 - e^2)}{1 + e\cos\phi}\,. \tag{2.5, 2.6}$$

Here a is the semimajor axis of the relative orbit and e the eccentricity.

The proper time T_p is related to the coordinate time t by the metric (2.2):

$$(dT_p)^2 = -ds^2 = dt^2[1 + 2\Phi - v_1^2 + O(v^4)]\,, \tag{2.7}$$

or

$$\frac{dT_p}{dt} = 1 + \Phi(\boldsymbol{r}_1) - \tfrac{1}{2}v_1^2 + O(v^4)\,. \tag{2.8}$$

The terms Φ and $-\frac{1}{2}v_1^2$ can be interpreted as the gravitational redshift and the transverse Doppler shift, respectively. In integrating equation (2.8), we can ignore any overall multiplicative constants, which can be absorbed in the definition of ν, and any additive constants which can be absorbed in N_0. Since

$$v_1^2 = \frac{M_2^2}{M_1 + M_2}\left(\frac{2}{r} - \frac{1}{a}\right), \tag{2.9}$$

we find, on dropping constant terms,

$$\frac{dT_p}{dt} = 1 - \frac{M_2}{r} - \frac{M_2^2}{M_1 + M_2}\frac{1}{r}\,. \tag{2.10}$$

Note that $\Phi_1(\boldsymbol{r}_1)$ is not really infinite; the clock is at the emitting radius of the pulsar and Φ_1 merely contributes a constant gravitational redshift. Equation (2.10) is most easily integrated in terms of the eccentric anomaly E, which is related to t by

$$E - e\sin E = t/\mathscr{P} + \sigma\,, \tag{2.11}$$

where $\mathscr{P} = P/2\pi$, P is the orbital period, and the constant σ is related to the time of periastron passage. Using

$$r = a(1 - e\cos E)\,, \tag{2.12}$$

we find from equation (2.10) that

$$T_p = t - \frac{M_2(M_1 + 2M_2)}{a(M_1 + M_2)}\mathscr{P}E = t - \frac{M_2(M_1 + 2M_2)}{a(M_1 + M_2)}\mathscr{P}e\sin E\,, \tag{2.13}$$

where we have dropped the constant in equation (2.11) and ignored an overall multiplicative constant as well.

Because of interstellar dispersion, the pulse travels with a group velocity less than unity as measured in the local orthonormal reference frame of an observer at rest in the (t, x, y, z) coordinate system:

$$\left|\frac{d\hat{\boldsymbol{x}}}{d\hat{t}}\right| = 1 - \epsilon\,. \tag{2.14}$$

From the line element (2.2),

$$d\hat{t} = (1 + \Phi)dt\,, \qquad |d\hat{\boldsymbol{x}}| = (1 - \Phi)|d\boldsymbol{x}|\,. \tag{2.15}$$

Therefore

$$\left|\frac{dx}{dt}\right| = 1 - \epsilon + 2\Phi ,$$

or

$$t_{\rm arr} - t_{\rm em} = \int_{r_1(t_{\rm em})}^{r_e(t_{\rm arr})} (1 + \epsilon - 2\Phi)|dx| . \tag{2.16}$$

Here $t_{\rm arr}$ is the coordinate time of arrival of the pulse, $t_{\rm em}$ the time of emission, and $\boldsymbol{r}_e$ the position vector of the Earth with respect to the orbit barycenter. To the required accuracy, the integral can be done along a straight line joining $\boldsymbol{r}_1$ and $\boldsymbol{r}_e$. The integral of ϵ gives a term D/f^2, where the dispersion constant is as usual

$$D = \frac{e^2}{2\pi m}\int_{r_1}^{r_e} n_e|dx| \tag{2.17}$$

and f is the frequency of the radiation.

The integral of Φ is responsible for the relativistic time delay across the orbit. In doing the integral, we can take the pulsar and its companion to be stationary to leading order, and we can ignore Φ_1 which simply adds a constant term. Then

$$-2\int_{r_1}^{r_e} \Phi|dx| \approx 2M_2\int_{t_{\rm em}}^{t_{\rm arr}} \frac{dt}{|\boldsymbol{x}(t) - \boldsymbol{r}_2(t_{\rm em})|} .$$

Since

$$\boldsymbol{x}(t) = \boldsymbol{r}_1(t_{\rm em}) + \frac{t - t_{\rm em}}{t_{\rm arr} - t_{\rm em}}[\boldsymbol{r}_e(t_{\rm arr}) - \boldsymbol{r}_1(t_{\rm em})] ,$$

the integral is equal to

$$2M_2\frac{t_{\rm arr} - t_{\rm em}}{|\boldsymbol{r}_e - \boldsymbol{r}_1|}\log\frac{|\boldsymbol{r}_e - \boldsymbol{r}_1||\boldsymbol{r} + \boldsymbol{r}_e - \boldsymbol{r}_1| + |\boldsymbol{r}_e - \boldsymbol{r}_1|^2 + \boldsymbol{r}\cdot(\boldsymbol{r}_e - \boldsymbol{r}_1)}{|\boldsymbol{r}_e - \boldsymbol{r}_1|r + \boldsymbol{r}\cdot(\boldsymbol{r}_e - \boldsymbol{r}_1)} . \tag{2.18}$$

To a first approximation $t_{\rm arr} - t_{\rm em} = |\boldsymbol{r}_e - \boldsymbol{r}_1|$. Since $r_e \gg r_1$, expression (2.18) reduces to

$$2M_2\log\left(\frac{2r_e}{r + \boldsymbol{r}\cdot\boldsymbol{n}}\right) , \tag{2.19}$$

where $\boldsymbol{n} = \boldsymbol{r}_e/r_e$ is a unit vector pointing to the Earth or, to high accuracy, to the solar system barycenter. The term $\log(2r_e)$ can be ignored since it is very nearly constant. The term $\boldsymbol{r}\cdot\boldsymbol{n}$ is equal to $-r\sin(\omega + \phi)\sin i$, where i is the inclination of the orbital plane and ω is the angular distance of the pericenter from the line of nodes measured in the direction of orbital motion. Thus the time-varying contribution of the integral (2.19) is

$$2M_2\log\left(\frac{1 + e\cos\phi}{1 - \sin i\sin(\omega + \phi)}\right) . \tag{2.20}$$

(The formula for this term derived by Wheeler 1975 is in error.) Therefore,

$$t_{\rm arr} - t_{\rm em} = |\boldsymbol{r}_e(t_{\rm arr}) - \boldsymbol{r}_1(t_{\rm em})| + \frac{D}{f^2} + 2M_2\log\left(\frac{1 + e\cos\phi}{1 - \sin i\sin(\omega + \phi)}\right) . \tag{2.21}$$

On using the current values of e and ω, we find that the relativistic time delay term has a maximum variation in a single orbit of 14 μs $(M_2/M_\odot)$ for $i = 0°$ and 98 μs $(M_2/M_\odot)$ for $i = 89°$. This means that it can probably not be detected at present. Also, it is comparable with the post-Newtonian corrections to the Keplerian orbit for $i \lesssim 89°$. We will therefore omit it from the formulae in the remainder of this section.

Now

$$|\boldsymbol{r}_e - \boldsymbol{r}_1| \approx r_b + (\boldsymbol{r}_{be} - \boldsymbol{r}_1)\cdot\boldsymbol{n} ,$$

where $\boldsymbol{r}_b$ is the position vector of the barycenter of the solar system and $\boldsymbol{r}_{be}$ the position vector of the Earth with respect to the barycenter. Thus equation (2.21) gives

$$t_{\rm arr} = t_{\rm em} + r_b + \boldsymbol{r}_{be}(t_{\rm arr})\cdot\boldsymbol{n} + \frac{a_1 \sin i(1 - e^2)\sin(\omega + \phi)}{1 + e\cos\phi} + \frac{D}{f^2}\,. \tag{2.22}$$

A coordinate system at rest with respect to the barycenter is related to the coordinate system of the center of mass of the binary system by an unknown Lorentz transformation. If the relative acceleration of the barycenter and the center of mass is sufficiently small and the separation sufficiently large, the transformation introduces only constant factors. We can then take $t_{\rm arr}$ to be the coordinate time of arrival at the Earth in the barycenter reference frame (ephemeris time), and set r_b equal to zero. Note, however, that these factors introduce intrinsic uncertainties $\sim\dot{r}_b$ in measurements of the orbital elements. If we expand r_b to first order in changes in the direction and relative velocity of the solar system barycenter with respect to the binary center of mass, then r_b contains a contribution $[\ddot{\boldsymbol{r}}_b\cdot\boldsymbol{n} + (\dot{\boldsymbol{r}}_b \times \boldsymbol{n})^2/r_b]t_{\rm em}^2/2$ ($t_{\rm em}$ being measured from a convenient origin), which might have observable consequences. For the remainder of this section we shall ignore this term, which is discussed further in § III.

The third and fifth terms on the right-hand side of equation (2.22) are familiar from ordinary pulsar timing. The quantity $\boldsymbol{r}_{be}$ is assumed to be known from a good ephemeris of the Earth's motion, and so $\boldsymbol{r}_{be}\cdot\boldsymbol{n}$ depends only on the pulsar's right ascension and declination and possibly proper motion, which are parameters that will be determined from the timing data. The dispersion constant D must also be determined, since f is related to the known fixed frequency of observation on the Earth f_e by the Doppler formula $f = f_e(1 - \boldsymbol{v}_e\cdot\boldsymbol{n})$, where $\boldsymbol{v}_e$ is the known velocity of the Earth with respect to the barycenter.

We define the infinite-frequency barycenter arrival time as

$$t = t_{\rm arr} - \boldsymbol{r}_{be}(t_{\rm arr})\cdot\boldsymbol{n} - D(1 - 2\boldsymbol{v}_e\cdot\boldsymbol{n})/f_e^2\,. \tag{2.23}$$

The pulsar position parameters are found from equation (2.23) in the standard manner. (See, e.g., Manchester, Taylor, and Van 1974; Manchester and Peters 1972, for explicit expressions.) Note that since $t_{\rm arr}$ is measured by an atomic clock on the Earth, it must be corrected to ephemeris time by an equation analogous to equation (2.13) with $M_2 \to M_\odot$, $M_1 \to M_\oplus \approx 0$. Thus $t_{\rm arr}$ in equation (2.23) should be

$$t_{\rm arr} = t_{\rm arr}^{\rm (atomic\ clock)} + 2M_\odot\mathscr{P}_\oplus e_\oplus a_\oplus^{-1}\sin E_\oplus\,. \tag{2.24}$$

Expanding $\sin E$ for small eccentricity gives

$$t_{\rm arr} = t_{\rm arr}^{\rm (atomic\ clock)} + 1.66145[(1 - \tfrac{1}{8}e_\oplus^2)\sin l + \tfrac{1}{2}e\sin 2l_\oplus + \tfrac{3}{8}e_\oplus^2\sin 3l]\quad{\rm ms}\,, \tag{2.25}$$

where l is the mean anomaly of the Earth. Equation (2.25) corrects a slight error in the formula given by Clemence and Szebehely (1967).

Substituting equation (2.23) in equation (2.22) gives

$$t = t_{\rm em} + a_1\sin i[\sin\omega(\cos E - e) + (1 - e^2)^{1/2}\cos\omega\sin E]\,, \tag{2.26}$$

where we have substituted the eccentric anomaly $E(t_{\rm em})$ for the true anomaly. The quantity $t_{\rm em}$ is related implicitly to T_p by equation (2.13). We can get an explicit expression by defining a new eccentric anomaly $\tilde{E}$:

$$\tilde{E} - e\sin\tilde{E} = T_p/\mathscr{P} + \sigma\,. \tag{2.27}$$

Since by equation (2.13), $t_{\rm em} = T_p[1 + O(v^2)]$, we have $\tilde{E} = E[1 + O(v^2)]$, and so equation (2.13) gives

$$t_{\rm em} = T_p + \gamma\sin\tilde{E}\,, \tag{2.28}$$

where

$$\gamma = \frac{M_2^2(M_1 + 2M_2)\mathscr{P}e}{a_1(M_1 + M_2)^2}\,. \tag{2.29}$$

Thus equation (2.26) becomes, to the same accuracy,

$$t = T_p + \alpha(\cos\tilde{E} - e) + (\beta + \gamma)\sin\tilde{E}\,, \tag{2.30}$$

where

$$\alpha = x\sin\omega\,,\qquad \beta = (1 - e^2)^{1/2}x\cos\omega\,,\qquad x = a_1\sin i\,. \tag{2.31}$$

We can turn equation (2.30) into an explicit expression for T_p by defining another eccentric anomaly E':

$$E' - e \sin E' = t/\mathscr{P} + \sigma \,. \tag{2.32}$$

To zeroth order $T_p = t$ and $\tilde{E} = E'$. Thus to $O(v)$,

$$T_p = t - \alpha(\cos E' - e) - (\beta + \gamma) \sin E' \,.$$

But

$$T_p - t \simeq \mathscr{P}(1 - e \cos E')(\tilde{E} - E') \,,$$

and so

$$\tilde{E} - E' = -\frac{\alpha(\cos E' - e) + (\beta + \gamma) \sin E'}{\mathscr{P}(1 - e \cos E')} + O(v^2) \,.$$

Thus equation (2.30) gives

$$T_p = t - \alpha(\cos E' - e) - (\beta + \gamma) \sin E' - \frac{(\alpha \sin E' - \beta \cos E')[\alpha(\cos E' - e) + (\beta + \gamma) \sin E']}{\mathscr{P}(1 - e \cos E')} \,. \tag{2.33}$$

Substitute equation (2.33) in equation (2.1) and find (on dropping the primes)

$$N = N_0 + \nu t - \nu\alpha(\cos E - e) - \nu(\beta + \gamma) \sin E - \frac{\nu(\alpha \sin E - \beta \cos E)[\alpha(\cos E - e) + \beta \sin E]}{\mathscr{P}(1 - e \cos E)} + \dot{\nu}t^2/2 - \dot{\nu}t[\alpha(\cos E - e) + \beta \sin E] + \ddot{\nu}t^3/6 \,, \tag{2.34}$$

where we have omitted some negligible terms proportional to $\dot{\nu}$ and $\ddot{\nu}$.

At this point it is worthwhile relating the above procedure to the conventional concepts of celestial mechanics. Equation (2.34), together with equations (2.31) and (2.32), defines

$$N = N(t; N_0, \nu, \dot{\nu}, \ddot{\nu}, x, \omega, e, \mathscr{P}, \sigma, \gamma) \,. \tag{2.35}$$

The actual orbit of the pulsar will not be an ellipse because of relativistic corrections and various other perturbations. It can, however, be described by an osculating ellipse: the six parameters of the elliptic orbit are allowed to vary slightly in such a way that the elliptic relations and their first time derivatives are still valid. Equations for the variation of the parameters produced by arbitrary perturbations can be found in standard treatments of celestial mechanics. For example, Brouwer and Clemence (1961) use the set of parameters (a, e, i, σ, ω, Ω), where Ω is the longitude of the ascending node. Note that our timing formula does not depend on Ω, which is therefore not measured by an analysis of pulse arrival times. The relation between our parameters x and $\mathscr{P}$ and the conventional a and i is

$$x = \frac{a \sin i}{1 + M_1/M_2} \,, \tag{2.36}$$

$$\mathscr{P} = \frac{a^{3/2}}{(M_1 + M_2)^{1/2}} \,. \tag{2.37}$$

Secular variations $\dot{a}$, $\dot{e}$, ..., can in principle be determined from the timing formula (2.35) by the replacements

$$x \to x + \dot{x}t \,, \quad \omega \to \omega + \dot{\omega}t \,, \quad e \to e + \dot{e}t \,, \quad \mathscr{P} \to \mathscr{P} + \tfrac{1}{2}\dot{\mathscr{P}}t \,. \tag{2.38}$$

A secular term $\dot{\sigma}$ merely modifies the apparent unperturbed value of $\mathscr{P}$ and is therefore not measurable by the observation of pulse arrival times. The reason for the factor 1/2 in front of $\dot{\mathscr{P}}$ is that we defined $\mathscr{P}$ by

$$E - e \sin E = t/\mathscr{P} + \sigma \,, \tag{2.39}$$

whereas the actual definition is equation (2.37) and (cf. Brouwer and Clemence 1961)

$$E - e \sin E = (M_1 + M_2)^{1/2} \int a^{-3/2} dt + \sigma \,. \tag{2.40}$$

If we let $a \rightarrow a + \dot{a}t$ in equations (2.37) and (2.40), we find

$$E - e \sin E = t/(\mathscr{P} + \tfrac{1}{2}\dot{\mathscr{P}}t) + \sigma . \tag{2.41}$$

Thus the factor $\frac{1}{2}$ is necessary for $\dot{P} = 2\pi\dot{\mathscr{P}}$ to be interpreted in the conventional manner as an anomalistic period change.

Return now to equation (2.35), and suppose we have a first guess $N_0^{(1)}$, $\nu^{(1)}$, $\dot{\nu}^{(1)}$, ... for the parameters. This defines the "computed" values

$$N^{(1)} = N(t; N_0^{(1)}, \nu^{(1)}, \cdots) . \tag{2.42}$$

The "observed" values

$$N = N(t; N_0, \nu, \ldots) \tag{2.43}$$

depend on the true values of the parameters; the residuals $R(t)$ (in seconds) are defined by

$$-\nu^{(1)}R(t) \equiv N - N^{(1)} = \frac{\partial N}{\partial N_0}\bigg|_1 \delta N_0 + \frac{\partial N}{\partial \nu}\bigg|_1 \delta\nu + \cdots + \frac{\partial N}{\partial \gamma}\bigg|_1 \delta\gamma , \tag{2.44, 2.45}$$

where $\delta N_0, \delta\nu, \ldots, \delta\gamma$ are estimates for the corrections to the parameters. These corrections can be found by a least-squares fit to the residuals (with appropriate weights), and then used to provide a new first guess for the parameters. The residuals are computed in equation (2.44) using the full expression (2.34) for N. To solve equation (2.45) for the corrections $\delta N_0, \delta\nu, \ldots, \delta\gamma$, however, it is only necessary to keep the largest term in each partial derivative since the procedure is iterative. Thus equation (2.45) becomes

$$\begin{aligned} R(t) = {} & -\delta N_0/\nu - t\delta\nu/\nu - t^2\delta\dot{\nu}/(2\nu) - t^3\delta\ddot{\nu}/(6\nu) \\ & + [\sin\omega(\cos E - e) + (1 - e^2)^{1/2}\cos\omega\sin E]\delta x + x[\cos\omega(\cos E - e) - (1 - e^2)^{1/2}\sin\omega\sin E]\delta\omega \\ & - [W\sin E + x\sin\omega + (1 - e^2)^{-1/2}xe\cos\omega\sin E]\delta e + W(E - e\sin E - \sigma)\delta\mathscr{P}/\mathscr{P} - W\delta\sigma + \sin E\delta\gamma , \end{aligned} \tag{2.46}$$

where

$$W \equiv x[\sin\omega\sin E - (1 - e^2)^{1/2}\cos\omega\cos E]/(1 - e\cos E) . \tag{2.47}$$

Additional terms resulting from errors in the pulsar position and proper motion can be added to equation (2.46) (Manchester, Taylor, and Van 1974).

Note that of the terms on the right-hand side of equation (2.46), only $\sin E\delta\gamma$ is of relativistic origin. The remainder are purely Newtonian. As discussed below, the term $\gamma\sin E$ cannot be distinguished from $\beta\sin E$ in equation (2.34) except over a time scale long enough for ω to change significantly. At present only the combination $\beta + \gamma$ can be measured; and in fitting equation (2.46), $\delta\gamma$ must be set equal to zero.

The secular variations (2.38) can be incorporated by making the replacements

$$\begin{aligned} &\delta x \rightarrow \delta x + t\delta\dot{x} , \qquad \delta\omega \rightarrow \delta\omega + t\delta\dot{\omega} , \\ &\delta e \rightarrow \delta e + t\delta\dot{e} , \qquad \delta\mathscr{P} \rightarrow \delta\mathscr{P} + \tfrac{1}{2}t\delta\dot{\mathscr{P}} , \end{aligned} \tag{2.48}$$

in equation (2.46).

One already has good first guesses for the parameters N_0, ν, x, ω, $\dot{\omega}$, e, $\mathscr{P}$, and σ from analysis of the velocity curve (Hulse and Taylor 1975). An initial guess of zero should be adequate for the smaller quantities $\dot{\nu}$, $\ddot{\nu}$, $\dot{x}$, $\dot{e}$, and $\dot{\mathscr{P}}$ at present. Should nonzero values of these smaller quantities be "measured," it is important that any such values be tested for statistical significance and that the resulting fit be successfully predictive.

III. VARIANCE ESTIMATES

We now derive some estimates of the accuracy with which the parameters of § II can in principle be determined in a long series of measurements of the infinite-frequency barycentric arrival time defined in equation (2.23). One method of doing this would be numerical simulation using equation (2.46); but as the future frequency and accuracy of the observations are at present unknown, a simpler analytic treatment is sufficient. It is convenient for analytic work to fit the residuals to a slightly different set of independent parameters:

$$R(t) = \delta K + (\cos E - e)\delta\alpha + \sin E\delta\eta + \left[\frac{\alpha\sin E - \beta\cos E}{1 - e\cos E}\right]\frac{\delta\Sigma}{\mathscr{P}} - \left[\frac{(\alpha\sin E - \beta\cos E)\sin E}{1 - e\cos E} + \alpha\right]\delta e , \tag{3.1}$$

where

$$
\begin{aligned}
\delta K &= -\delta N_0/\nu - t\delta\nu/\nu - t^2\delta\dot{\nu}/(2\nu) - t^3\delta\ddot{\nu}/(6\nu)\,,\\
\delta\eta &= \delta\beta + \delta\gamma\,,\\
\delta\Sigma/\mathscr{P} &= -\delta\sigma + (t/\mathscr{P})\delta\mathscr{P}/\mathscr{P}\,.
\end{aligned}
\tag{3.2}
$$

We now make several simplifications to reduce the calculation of the covariance matrix to manageable form. First, we regard the observations over the total time interval T (~ 5 years) as being split up into segments of duration d such that

$$\mathscr{P} \ll d \ll T\,. \tag{3.3}$$

Estimates of δK, $\delta\alpha$, $\delta\eta$, $\delta\Sigma/\mathscr{P}$, and δe obtained in every interval d are then to be used in a secondary fit to estimate the complete set of parameters. We assume that the observations of barycentric arrival times are made at regular intervals at a rate $\dot{n}$, each observation having constant variance ϵ^2 and zero covariance. If we now minimize the χ^2 for $\dot{n}d$ observations in a time interval d, the best fit for the parameters in equation (3.1) is obtained from the solution of the simultaneous linear equations

$$
\sum_i
\begin{bmatrix}
1 & c-e & s & \dfrac{\alpha s-\beta c}{1-ec} & \dfrac{-(\alpha s-\beta c)s}{1-ec}-\alpha \\
 & (c-e)^2 & (c-e)s & \dfrac{(c-e)(\alpha s-\beta c)}{1-ec} & \dfrac{-(c-e)(\alpha s-\beta c)s}{1-ec}-\alpha(c-e) \\
 & & s^2 & \dfrac{(\alpha s-\beta c)s}{1-ec} & \dfrac{-(\alpha s-\beta c)s^2}{1-ec}-\alpha s \\
 & \text{symm.} & & \left(\dfrac{\alpha s-\beta c}{1-ec}\right)^2 & \left[\dfrac{-(\alpha s-\beta c)s}{1-ec}-\alpha\right]\dfrac{\alpha s-\beta c}{1-ec} \\
 & & & & \left[\dfrac{(\alpha s-\beta c)s}{1-ec}+\alpha\right]^2
\end{bmatrix}
\begin{bmatrix}
\delta K \\ \delta\alpha \\ \delta\eta \\ \delta\Sigma/\mathscr{P} \\ \delta e
\end{bmatrix}
= \sum_i
\begin{bmatrix}
R_i \\ R_i(c-e) \\ R_i s \\ R_i\left(\dfrac{\alpha s-\beta c}{1-ec}\right) \\ R_i\left[\dfrac{-(\alpha s-\beta c)}{1-ec}-\alpha\right]
\end{bmatrix},
\tag{3.4}
$$

where $c = \cos E$, $s = \sin E$, and $R_i = R(t_i)$, each evaluated at the time of the ith measurement. The summations can be approximated by the product of the number of observations and the time average over an orbital period, i.e.,

$$\sum_i f(E) \simeq \frac{\dot{n}d}{2\pi}\int_0^{2\pi} f(E)(1 - e\cos E)dE + O(\mathscr{P}/d)\,. \tag{3.5}$$

In carrying out the averaging, α, β, e, and $\mathscr{P}$ are kept constant.

Averaging the matrix of coefficients of δK, $\delta\alpha$, $\delta\eta$, $\delta\Sigma/\mathscr{P}$, and δe in equation (3.4), we find for the covariance matrix of these five parameters

$$
V = \frac{\epsilon^2}{\dot{n}d}
\begin{bmatrix}
1 & -3e/2 & 0 & 0 & -3\alpha/2 \\
 & \frac{1}{2}+2e^2 & 0 & -\beta/2 & 2\alpha e \\
 & & \frac{1}{2} & \alpha/2 & 0 \\
 & \text{symm.} & & f_1(e) & f_2(e) \\
 & & & & f_3(e)
\end{bmatrix}^{-1},
\tag{3.6}
$$

where

$$f_1(e) = \alpha^2[1 - (1 - e^2)^{1/2}]/e^2 + \beta^2[(1 - e^2)^{-1/2} - 1]/e^2\,, \tag{3.7}$$
$$f_2(e) = 2\alpha\beta[1 - e^2/2 - (1 - e^2)^{1/2}]/e^3\,,$$
$$f_3(e) = \alpha^2[(1 - e^2)^{3/2} - 1 + 3e^2/2 + 2e^4]/e^4 + \beta^2[1 - e^2/2 - (1 - e^2)^{1/2}]/e^4\,.$$

The matrix inverse in equation (3.6) is cumbersome and not very illuminating in general. We therefore introduce a further approximation and ignore secular changes in the orbital elements. We substitute the present values of $e = 0.615$, $\alpha = 0$ (since $\omega = 180°$), in equation (3.6) and find

$$V = \frac{\epsilon^2}{\dot{n}d}\begin{bmatrix} 17 & 17 & 0 & 12/\beta & 0 \\ & 19 & 0 & 13/\beta & 0 \\ & & 2 & 0 & 0 \\ & \text{symm.} & & 11/\beta^2 & 0 \\ & & & & 6.4/\beta^2 \end{bmatrix}. \tag{3.8}$$

From equation (3.8) we can read off the variances

$$\text{var}\,(\delta K) = 17\epsilon^2/\dot{n}d\,, \tag{3.9a}$$
$$\text{var}\,(\delta\alpha) = 19\epsilon^2/\dot{n}d\,, \tag{3.9b}$$
$$\text{var}\,(\delta\eta) = 2\epsilon^2/\dot{n}d\,, \tag{3.9c}$$
$$\text{var}\,(\delta\Sigma/\mathscr{P}) = 11(\epsilon^2/\beta^2)(\dot{n}d)^{-1}\,, \tag{3.9d}$$
$$\text{var}\,(\delta e) = 6.4(\epsilon^2/\beta^2)(\dot{n}d)^{-1}\,, \tag{3.9e}$$

where the current value of β is -1.85 s. Note that $\delta\eta$ and δe each have zero covariance with any of the other parameters in this approximation. Neglecting the secular term $\dot{\omega}$ has produced fractional errors $\sim(\dot{\omega}T)^2 \sim (T/15\text{ years})^2$ in the estimates (3.9).

If a large number, $\mathscr{N} = T/d$, of measurements of one of the above parameters, δy say, are made regularly within the observing period T, then their individual variances given in equation (3.9), σ^2 say, can be used in variance estimates of secondary fits to determine the complete set of parameters. The variance estimates in linear and quadratic fits to δy are as follows:

i) If δy is fitted to

$$\delta y = \delta y_0 + t\delta\dot{y}\,, \tag{3.10}$$

then

$$\text{var}\,(\delta y_0) = 4\sigma^2/\mathscr{N}, \qquad \text{var}\,(\delta\dot{y}) = 12\sigma^2/\mathscr{N}T^2\,. \tag{3.11}$$

ii) If δy is fitted to

$$\delta y + \delta y_0 + \tfrac{1}{2}t^2\delta\ddot{y}\,, \tag{3.12}$$

then

$$\text{var}\,(\delta y_0) = 9\sigma^2/4\mathscr{N}, \qquad \text{var}\,(\delta\ddot{y}) = 45\sigma^2/\mathscr{N}T^4\,. \tag{3.13}$$

iii) If δy is fitted to

$$\delta y = \delta y_0 + t\delta\dot{y} + \tfrac{1}{2}t^2\delta\ddot{y}\,, \tag{3.14}$$

then

$$\text{var}\,(\delta y_0) = 9\sigma^2/\mathscr{N}, \qquad \text{var}\,(\delta\dot{y}) = 192\sigma^2/\mathscr{N}T^2\,, \qquad \text{var}\,(\delta\ddot{y}) = 720\sigma^2/\mathscr{N}T^4\,. \tag{3.15}$$

We now consider in turn some physical quantities that might be deducible from the observations.

i) $\dot{\nu}$

A value for the frequency derivative can be obtained from successive measurements of δK. If we set $\ddot{\nu} = 0$, then by using equations (3.2), (3.9a), and (3.15) we obtain

$$\text{var}\,(\delta\dot{\nu}/\nu) \approx 720 \times 17\epsilon^2/(\dot{n}T^5) \tag{3.16}$$

independent of d. Define an apparent age $\tau_\nu = |\nu/\dot{\nu}|$; then, since ν is known,

$$[\text{var}\,(\tau_\nu)]^{1/2} \approx 10^4\epsilon_{-3}\tau_{\nu 7}{}^2\dot{n}_3{}^{-1/2}T_y{}^{-5/2} \quad \text{years}\,, \tag{3.17}$$

where ϵ_{-3} is measured in milliseconds, $\tau_{\nu 7}$ in units of 10^7 years, $\dot{n}_3$ in units of 1000 observations per year, and T_y in years. The values $\dot{n}_3 = \epsilon_{-3} = 1$ are appropriate for numerical estimates based on current techniques.

Since τ_ν is already known to exceed 10^8 years (Taylor *et al.* 1976), it is unlikely that $\ddot{\nu}$ will be measurable.

ii) $\dot{\omega}$

A variance estimate for the apsidal motion can be determined by considering successive measurements of $\delta\alpha$ and $\delta\beta$. Since $\tan\omega = \alpha/\beta'$, where $\beta' = \beta(1 - e^2)^{-1/2}$, we find

$$\delta\omega = \cos^2\omega(\delta\alpha/\beta' - \alpha\delta\beta'/\beta'^2)\,. \tag{3.18}$$

On making replacements as in equation (2.48), we see that the same equation holds with $\delta\omega$, $\delta\alpha$, and $\delta\beta'$ replaced by $\delta\dot{\omega}$, $\delta\dot{\alpha}$, and $\delta\dot{\beta}'$. Taking the unperturbed value of ω to be π and noting that $\text{cov}\,(\delta\alpha, \delta\beta') = 0$ (ignoring the small term $\delta\gamma$), we obtain from equations (3.18), (3.9b), and (3.11)

$$\text{var}\,(\delta\dot{\omega}) = \text{var}\,(\delta\dot{\alpha})/\beta'^2 + O(\dot{\omega}T)^2 \approx 12 \times 19\epsilon^2/(\dot{n}T^3\beta'^2)\,. \tag{3.19}$$

Inserting numerical values gives

$$[\text{var}\,(\delta\dot{\omega})]^{1/2} \approx 10^{-2}\epsilon_{-3}\dot{n}_3{}^{-1/2}T_y{}^{-3/2} \quad \text{degrees per year}\,. \tag{3.20}$$

Changes in $\dot{\omega}$ with an associated time scale $\tau_{\dot{\omega}} = |\dot{\omega}/\ddot{\omega}|$ are detectable after a time T provided that

$$\tau_{\dot{\omega}} \ll 30T_y{}^{5/2}\dot{n}_3{}^{1/2}\epsilon_{-3}{}^{-1} \quad \text{years}\,. \tag{3.21}$$

Note that the general-relativistic prediction for the apsidal motion is

$$\dot{\omega} = 3M_2/[a_1(1 - e^2)\mathscr{P}] = 2.10[(M_1 + M_2)/M_\odot]^{2/3} \quad \text{degrees per year}\,, \tag{3.22, 3.23}$$

where we have substituted the known orbital parameters in equation (3.22). The observed value of $\dot{\omega} = 4.24$ degrees per year (Taylor, private communication) gives

$$M_1 + M_2 = 2.85M_\odot\,, \tag{3.24}$$

provided that the other possible sources of apsidal motion are negligible (Will 1975; Roberts, Masters, and Arnett 1975).

iii) γ

The measurement of γ is important because it provides an independent relation between M_1, M_2, and a_1. However, as pointed out in Paper I and by Brumberg *et al.* (1975) (cf. also Groth 1971; Hunt 1971), the directly measurable quantity is $\eta = \beta + \gamma$ and the value of γ can be isolated only in the presence of apsidal motion. Since

$$\delta\eta = (1 - e^2)^{1/2}\cos\omega\delta x - x(1 - e^2)^{1/2}\sin\omega\delta\omega + \delta\gamma\,, \tag{3.25}$$

we find on making the replacements (2.48)

$$\begin{aligned} \delta\eta &= -(1 - e^2)^{1/2}\delta x + \delta\gamma\,, \\ \delta\dot{\eta} &= -(1 - e^2)^{1/2}\delta\dot{x} + x(1 - e^2)^{1/2}\dot{\omega}\delta\omega\,, \\ \delta\ddot{\eta} &= (1 - e^2)^{1/2}\dot{\omega}^2\delta x + 2x(1 - e^2)^{1/2}\dot{\omega}\delta\dot{\omega}\,, \end{aligned} \tag{3.26}$$

where we have used $\omega = \pi + \dot{\omega}t$. Similarly,

$$\delta\alpha = -x\delta\omega\,, \qquad \delta\dot{\alpha} = -x\delta\dot{\omega} - \dot{\omega}\delta x\,. \tag{3.27}$$

Thus

$$\delta\gamma = \delta\eta - \delta\ddot{\eta}/\dot{\omega}^2 - 2(1 - e^2)^{1/2}\delta\dot{\alpha}/\dot{\omega}\,, \tag{3.28}$$

and so the dominant contribution to the variance is

$$\text{var}\,(\delta\gamma) \approx \text{var}\,(\delta\ddot{\eta})/\dot{\omega}^4 = 90\epsilon^2/(\dot{n}\dot{\omega}^4T^5)\,, \tag{3.29}$$

where we have used equations (3.13) and (3.9c) and set $\dot{x} = 0$. (Note that we need only fit to eq. [3.12] as the linear coefficient $\delta\dot{\eta}$ is separately determined from measurements of $\delta\alpha$.) Equation (3.29) differs by a factor 2 from equation (14) of Paper I because of a different definition of ϵ.

We can rewrite equation (2.29), using equation (3.24), as

$$\gamma = 2.07 \times 10^{-3}\left(\frac{M_2}{M_\odot}\right)\left(1 + \frac{M_2}{2.85M_\odot}\right)\text{ s}\,. \tag{3.30}$$

Since

$$1.02M_\odot < M_2 < 2.85M_\odot\,, \tag{3.31}$$

(eq. [10] of Will 1975), we expect

$$2.88 \times 10^{-3}\text{ s} < \gamma < 1.18 \times 10^{-2}\text{ s}\,. \tag{3.32}$$

If $M_1 = M_2 = 1.41M_\odot$, then $\gamma = 4.42$ ms. A 10 percent measurement of the masses (assumed similar) requires

$$\frac{\delta M}{M} \sim \frac{1}{2}\frac{[\text{var}\,(\delta\gamma)]^{1/2}}{\gamma} \lesssim 0.1\,, \tag{3.33}$$

or

$$T \gtrsim 5\epsilon_{-3}{}^{2/5}\dot{n}_3{}^{-1/5}\quad\text{years}\,. \tag{3.34}$$

iv) $\dot{x}$

From equations (3.26) and (3.27), we find

$$\delta\dot{x} = -(1 - e^2)^{-1/2}\delta\dot{\eta} - \dot{\omega}\delta\alpha\,. \tag{3.35}$$

If $\dot{x}$ is to be determined simultaneously with γ, then

$$\text{var}\,(\delta\dot{x}) \approx (1 - e^2)^{-1}\,\text{var}\,(\delta\dot{\eta}) \approx 384\epsilon^2(1 - e^2)^{-1}/(\dot{n}T^3)\,, \tag{3.36}$$

where we have used equations (3.9c) and (3.15). If we define $\tau_x = |x/\dot{x}|$, a 10 percent measurement of τ_x requires

$$\tau_x < 0.1x[\text{var}\,(\delta\dot{x})]^{-1/2} \approx 200\dot{n}_3{}^{1/2}T_y{}^{3/2}\epsilon_{-3}{}^{-1}\quad\text{years}\,. \tag{3.37}$$

v) $\dot{\mathscr{P}}$

The measurement of $\dot{\mathscr{P}}$ is important as it may provide a test of general relativity. Using equations (3.2), (3.9d), and (3.15), we obtain for $\dot{\omega}T \ll 1$

$$\text{var}\,(\delta\dot{\mathscr{P}}) = \mathscr{P}^4\,\text{var}\,(\delta\ddot{\Sigma}/\mathscr{P}) \approx 7850\mathscr{P}^4\epsilon^2/(\beta^2\dot{n}T^5)\,. \tag{3.38}$$

If we define $\tau_{\mathscr{P}} = |\mathscr{P}/\dot{\mathscr{P}}|$, a 10 percent measurement of $\tau_{\mathscr{P}}$ requires that

$$\tau_{\mathscr{P}} \lesssim 0.1\mathscr{P}[\text{var}\,(\delta\dot{\mathscr{P}})]^{-1/2} \approx 5 \times 10^5\epsilon_{-3}{}^{-1}\dot{n}_3{}^{1/2}T_y{}^{5/2}\quad\text{years}\,. \tag{3.39}$$

If $\tau_{\mathscr{P}}$ arises solely from gravitational radiation, the general-relativistic prediction is (Wagoner 1975)

$$\tau_{\mathscr{P}} = 1.04 \times 10^9\left(\frac{M_1 + M_2}{M_\odot}\right)^{-4/3}\sin i\left[1 - \frac{0.51}{\sin i}\left(\frac{M_1 + M_2}{M_\odot}\right)^{-1/3}\right]^{-1}\quad\text{years}\,. \tag{3.40}$$

If $M_1 = M_2 = 1.42M_\odot$, $\tau_{\mathscr{P}} = 3.7 \times 10^8$ years. To measure this to 10 percent accuracy requires that

$$T \gtrsim 15\epsilon_{-3}{}^{2/5}\dot{n}_3{}^{-1/5}(\tau_{\mathscr{P}}/5 \times 10^8\text{ years})^{2/5}\quad\text{years}\,. \tag{3.41}$$

Eardley (1975) has pointed out that if the Dicke-Brans-Jordan theory of gravity is correct, then $\tau_{\mathscr{P}}$ can be less than 10^7 years as long as the companion is not a neutron star of the same mass as the pulsar. The time required to test this prediction is ~ 3 years.

Note that the observing time required to measure a period change has the same dependence on ϵ and $\dot{n}$ as the time to measure γ. Thus, with the assumptions we have made, the component masses will be measured before the effects of gravitational radiation are detected, independent of changes in the rate and accuracy of arrival time measurements.

In the discussion following equation (2.22) it was pointed out that $t_{\rm arr}$ contains a contribution proportional to $t_{\rm em}^2$. It can readily be seen that this contributes an artificial amount $[\ddot{\boldsymbol{r}}_b \cdot \boldsymbol{n} + (\dot{\boldsymbol{r}}_b \times \boldsymbol{n})^2/r_b]^{-1}$ to $\tau_{\mathscr{P}}$ (cf. Shklovsky 1969; Brumberg *et al.* 1975).

vi) $\dot{e}$

Using equations (3.9e) and (3.11), we obtain

$$\mathrm{var}\,(\delta\dot{e}) = 77\epsilon^2/(\beta^2\dot{n}T^3)\,. \tag{3.42}$$

A 10 percent measurement of $\tau_e = |e/\dot{e}|$ requires that

$$\tau_e < 0.1e[\mathrm{var}\,(\delta\dot{e})]^{-1/2} \approx 400\epsilon_{-3}{}^{-1}\dot{n}_3{}^{1/2}T^{3/2} \quad \text{years}\,. \tag{3.43}$$

Note that τ_e due to gravitational radiation emission ($\sim 10^9$ years) is completely unobservable.

vii) *Relativistic Time Delay and Post-Newtonian Terms*

In § II an expression was derived for the relativistic time delay. Unless $i \sim 90°$, the amplitude of this term is typically ~ 20 μs. Post-Newtonian corrections to the elliptic orbit are also typically of this magnitude and would have to be included. With existing timing accuracy ($\epsilon \sim 1$ ms), it is not appropriate to include the time delay term in the fitting equation (2.46). However, it is of interest to see what improvement would be necessary to make its detection feasible. We estimate the variance in a measurement of M_2 using this term by comparison with the variance estimates in equation (3.9), assuming that the variance after time d is the average of variances of $\delta\alpha$ and $\delta\eta$:

$$\frac{\mathrm{var}\,(M_2)}{M_2{}^2} \approx \frac{10\epsilon^2}{(20\,\mu\mathrm{s})^2\dot{n}T}\,. \tag{3.44}$$

Thus the time required for a 10 percent mass measurement using this term is

$$T \approx 2500\epsilon_{-3}{}^2\dot{n}_3{}^{-1} \quad \text{years}\,. \tag{3.45}$$

Provided that none of the additional sources of timing error discussed below interfere with this effect, we see that an improvement by a factor ~ 20 in the timing accuracy will be required to measure this term in 10 years of regular observations.

IV. DISCUSSION

The fitting procedure outlined in § II and the variance estimates given in § III have been derived on the assumption that the pulsar behaves as a clock whose phase is adequately described by a cubic polynomial. If this is correct and the apsidal motion and period change are predominantly relativistic, then equations (3.34) and (3.41) indicate that with existing timing accuracy and a feasible rate of measurement of arrival times (i.e., $\epsilon \sim 1$ ms, $\dot{n} \sim 1000\ \mathrm{yr}^{-1}$) the masses of both components and the effect of gravitational radiation can be detected within ~ 10 years. Should the system prove to be more complex (e.g., because of dynamical perturbations of a third body or spin-orbit interactions with a rapidly rotating companion), then the secular changes described by equations (3.21), (3.37), and (3.43) might in addition be detectable.

However, previous experience with the timing of pulsars and consideration of effects peculiar to the binary pulsar suggest that there may be noise components and secular drifts in the arrival times that are more important than measurement errors. In this case the variance estimates of § III will be significantly increased.

Probably the most important sources of noise are intrinsic to the pulsar. Several (single) pulsars display significant phase residuals after subtracting the best fitting polynomial to the barycentric arrival times (Groth 1975; Manchester and Taylor 1974). In Groth (1975) five years of arrival times from the Crab pulsar are analyzed, and it is shown that the majority of the noise component is well described by a random walk in frequency for which the diffusion coefficient is $\sim 0.5 \times 10^{-22}\ \mathrm{Hz^2\ s^{-1}}$. At present in observations of PSR 1913+16 using the Arecibo radio telescope, it takes approximately 10 days to sample a complete orbit, and this probably represents an estimate of the intermediate time scale, d, introduced in § III. Adopting the Crab pulsar diffusion coefficient, we find that the corresponding phase residual of PSR 1913+16 after 10 days is ~ 0.5 ms, which is comparable with the equivalent standard errors in estimates of the orbital elements indicated by equation (3.9). On this basis of this estimate we conclude that with careful analysis of the data, frequency noise need not seriously affect the determination of orbital parameters and their secular variation. It will, however, limit the accuracy with which the timing age can be measured in exactly the same fashion as for the Crab pulsar. Nevertheless, the only reliable estimate of the relative importance of intrinsic pulsar noise will be one that is empirically determined after several years' continuous observation. Similar remarks apply to possible glitches in PSR 1913+16, and timing errors introduced by Doppler shift of the emitted frequency, Galilean aberration, and geodetic precession (Smarr and Blandford 1976). Another noise component that may be detectable is that predicted by Eardley (1975) on the basis of the Dicke-Brans-Jordan theory provided the companion is not a black hole.

An additional source of spurious time delay might result from a variation of the dispersion measure. At 430 MHz, the total time delay arising from interstellar dispersion is ~ 4 s. If as much as 10^{-4} of the dispersion arises from within the binary orbit as a result of interaction with the companion star, this will also increase the variance estimates of § III. Such an effect (although unlikely) might be detectable by looking for a modulation of the dispersion measure with the orbital period. In addition, long-term changes in the dispersion measure may complicate the analysis.

By comparison with measurement errors, ephemeris and clock errors are probably negligible.

However, there is as yet no evidence that any of these possible noise components has an amplitude sufficiently large to affect the application of the timing formula derived in § II. In fact, the fitting procedure described in § II has been used successfully in analyzing a sequence of arrival times from PSR 1913+16 (Taylor *et al.* 1976). The measured variances of the quantities $a_1 \sin i$, e, $\mathscr{P}$, σ, ω, and $\dot{\omega}$ are all in agreement with the estimates presented in § III. This suggests that at least there is no serious additional time delay, with an orbital periodicity (e.g., arising from dispersion within the orbit). The principal result of § III is then that it is feasible to measure the mass of a neutron star and detect the effects of gravitational radiation. In view of the importance of these observations, we would urge that PSR 1913+16 be monitored as frequently and as accurately as is possible over the next 10 years.

We would like to thank E. J. Groth, W. H. Press, J. M. Rankin, L. Smarr, and J. H. Taylor for valuable discussions. R. B. thanks the Berkeley Astronomy Department for hospitality and support while part of this work was carried out.

REFERENCES

Blandford, R. D., and Teukolsky, S. A. 1975, *Ap. J.* (*Letters*), **198**, L27 (Paper I).
Brouwer, D., and Clemence, G. M. 1961, *Methods of Celestial Mechanics* (New York: Academic Press).
Brumberg, V. A., Zel'dovich, Ya. B., Novikov, I. D., and Shakura, N. I. 1975, *Astr. Letters*, **1**, 5.
Clemence, G. M., and Szebehely, V. 1967, *A.J.*, **72**, 1324.
Eardley, D. M. 1975, *Ap. J.* (*Letters*), **196**, L59.
Groth, E. J. 1971, unpublished Ph.D. thesis, Princeton University (available from University Microfilms, Inc., Ann Arbor, Michigan).
———. 1975, *Ap. J. Suppl.*, No. 293, **29**, 431.
Hulse, R. A., and Taylor, J. H. 1975, *Ap. J.* (*Letters*), **195**, L51.
Hunt, G. C. 1971, *M.N.R.A.S.*, **153**, 119.
Manchester, R. N., and Peters, W. L. 1972, *Ap. J.*, **173**, 221.
Manchester, R. N., and Taylor, J. H. 1974, *Ap. J.* (*Letters*), **191**, L63.
Manchester, R. N., Taylor, J. H. and Van, Y. Y. 1974, *Ap. J.* (*Letters*), **189**, L119.
Roberts, D. H., Masters, A. R., and Arnett, W. D. 1975, *Ap. J.* (in press).
Shklovsky, I. S. 1969, *Soviet Astr.—AJ*, **13**, 562.
Smarr, L., and Blandford, R. D. 1976 (to be published).
Taylor, J. H., Hulse, R. A., Fowler, L. A., Gullahorn, G. E., and Rankin, J. M. 1976, *Ap. J.* (*Letters*), in press.
Wagoner, R. V. 1975, *Ap. J.* (*Letters*), **196**, L63.
Wheeler, J. C. 1975, *Ap. J.* (*Letters*), **196**, L67.
Will, C. M. 1975, *Ap. J.* (*Letters*), **196**, L3.

ROGER BLANDFORD: Institute for Advanced Study, Princeton, NJ 08540

SAUL A. TEUKOLSKY: Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853