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A CYCLIC THERMAL INSTABILITY IN CONTACT BINARY STARS

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ABSTRACT

We show analytically that contact binary stars coupled by a common convective envelope, the Lucy model, are almost invariably unstable when subjected to mass transfer: if either component begins to exchange mass, it will continue to do so. A detailed sequence of models is calculated which follows the thermal evolution of a 2 $M_\odot$ contact binary of normal Population I abundances ($X = 0.70$, $Z = 0.02$), starting at nearly equal mass. The initial instability develops into a cyclic exchange of mass with the mass fraction oscillating between $0.56 \leq M_1/(M_1 + M_2) \leq 0.62$ with a period of $\sim 10^7$ years. The system achieves thermal equilibrium on the average, but individually both stars are perpetually out of equilibrium. The instability is of a general nature and such oscillating systems can satisfactorily populate the short-period, red region of the period-color relation for W UMa stars.

Subject headings: stars: binaries — stars: mass loss — stars: W Ursae Majoris

I. INTRODUCTION

Theoretical models of contact binary stars (CBS) in both hydrostatic and thermal equilibrium have been developed over the last several years, based on the key idea of Lucy (1968*a*) that the components share a common convective envelope in which the entropy should be essentially constant. In hydrostatic equilibrium the surface of each star should coincide with the same equipotential surface which, in the Roche approximation, requires

$$\frac{R_2}{R_1} \propto \left(\frac{M_2}{M_1}\right)^{0.46} . \quad (1)$$

Thus the average surface gravity of the two components, $\sim GM/R^2$, is nearly equal, and the ratio of surface areas is proportional to the mass. If both the entropy in the surface convection zone and the mean surface gravity are equal, then the envelopes of the two stars are similar, and their effective temperatures will be equal. Note that this requires a large flux of energy from the primary to the secondary. Lucy (1968*b*, 1973) has shown that this model satisfactorily reproduces the characteristic features of the light curves of W Ursae Majoris stars. Primary and secondary eclipse are of nearly equal depth because the brightness primarily depends on the visible projected surface area; and the anomalous mass-luminosity relation, $L \propto M$, reflects the ratio of surface areas for equal-temperature stars. Thus the Lucy model reproduces the observational features characterizing the surface of CBS.

However, thermal-equilibrium, zero-age models do not satisfactorily populate the short-period domain of the period-color relation observationally established by Eggen (1967) for W UMa stars. Lucy's original work demonstrated that, even with the surface energy exchange, a fundamental interior structural difference between the two stars is also necessary if they are to fit into the equipotential mass-radius relation of equation (1). His models indicated that agreement could be obtained if the more massive component produced nuclear energy predominantly by CN reactions while the secondary burned by *p-p* reactions. For short-period systems this requires large mass ratios to maintain a massive enough primary, and the resultant color of the object is too blue. Furthermore, only for carefully selected combinations of mass and mass ratio will the model be in thermal equilibrium where the luminosity radiated from the surface equals the combined interior energy generation. Systems where both components are less massive than the Sun, and therefore both burn by *p-p* reactions, cannot be explained theoretically, but are relatively common among W UMa stars (Mauder 1972; Lucy 1973). In fact detailed models, based on the Lucy idea (Moss and Whelan 1970; Hazlehurst and Meyer-Hofmeister 1973), indicate that zero-age systems cannot exist in thermal equilibrium (except at equal mass) unless the metallicity is extreme Population I, say $Z \gtrsim 0.04$. Nonetheless, several lines of evidence strongly suggest that W UMa stars represent a long-term "main sequence" evolutionary phase which exists from age zero (e.g., see Lucy 1968*a*; Kraft 1967). So theoretical models of CBS in hydrostatic and thermal equilibrium, employing the Lucy equal-entropy common convective envelope, do not account for the short-period W UMa stars.

In this paper we investigate the behavior of a short-period CBS which need not be in thermal equilibrium. Nonetheless, in contact we expect the Lucy surface conditions to be fulfilled. Hydrostatic equilibrium requires that the surface radii satisfy equation (1), and equal entropy in the outer envelopes is maintained by the interchange of convecting elements which occurs

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with far shorter time scales than are required for thermal readjustment. But as a result of nonthermal equilibrium the components may secularly expand or contract at a thermal time scale, and mass exchange can occur. Calculations for a 2 $M_\odot$ CBS starting with equal mass components show that (1) equilibrium at equal mass is unstable, and (2) a cyclic mass exchange follows in which one component persists as the primary but its relative mass fraction oscillates on a thermal time scale. An analysis of the initial instability producing the transfer of mass indicates that it can occur for a large range of initial configurations, at least if the components are not appreciably evolved. We are led to the conclusion that a contact binary never achieves true equilibrium during its main-sequence phase; instead, oscillating departures from thermal equilibrium cyclically pump mass between the two stars. With this interior structure the distribution of CBS in the period-color diagrams is understandable in terms of a distribution of systems with various values of total mass and angular momentum. The difficulties of theoretically producing short-period systems in exact thermal equilibrium is overcome by the existence of an average equilibrium state.

II. PHYSICAL ASSUMPTIONS AND APPROXIMATIONS

Following conventional procedure, we calculate the structure of each component of the binary as though it were a single, spherical, nonrotating star, but two forms of coupling are introduced. First, if the stars are in contact, we artificially introduce an exchange of energy such that the entropy in the surface convection zone of each star will be the same. Physically, the exchange must result from a circulation of convective elements along common equipotential surfaces, where the circulation is driven by small pressure gradients which will exist if the entropy is not constant. But if the exchange occurs in the adiabatic part of each convection zone, only its magnitude must be specified, since the luminosity effectively does not enter the stellar structure equations.

The second coupling explicitly affects the surface boundary conditions. We assume that equipotential surfaces in the binary are given by the Roche approximation (this requires stars with a pointlike mass distribution, synchronously rotating in circular orbit), and that a spherical star fills its Roche lobe if its radius exceeds the radius, H, of a sphere of equal volume. Three states exist for the binary. In contact, both stars fill their Roche lobe, and their radii must terminate on a common equipotential for hydrostatic equilibrium to obtain. In a detached state, neither star fills its critical lobe, so any radius less than the critical value is permissible. In a semidetached state, only one component fills its lobe, and the radius of this star must be very nearly H, since material outside H will flow onto the companion, driven by unbalanced pressure gradients. Over the range of interest to us, the critical radii can be accurately approximated by

$$H_{1,2} = D\left[0.38 \pm 0.2 \log\left(\frac{\mu}{1-\mu}\right)\right] \qquad (2)$$

(Paczynski 1971), where D is the binary separation, μ is the mass fraction in star "1," i.e., $\mu = M_1/(M_1 + M_2)$ and the minus sign applies to the critical radius of star "2." With mass-exchange the binary separation can vary. We assume conservation of mass and orbital angular momentum, J, so that

$$D = \frac{J^2}{G(M_1 + M_2)^3}\frac{1}{[\mu(1-\mu)]^2}\,. \qquad (3)$$

Note that minimum separation, but not minimum critical radius, occurs at $\mu = 0.5$.

In the Appendix we give the explicit form of the surface boundary conditions and the method used to calculate the transfer of energy, which is zero if the stars are not in contact. Because mass is exchanged in the calculated models, it is conceivable that any of the three states described above will occur during the evolution. The formulae in the Appendix have been expressed so that no discontinuities exist in the coupling conditions.

III. ANALYSIS OF THE MASS-EXCHANGE INSTABILITY

In this section we investigate the stability of a contact binary subjected to an infinitesimal transfer of mass. We assume that no mass exchange will occur if both stellar surfaces are at the same potential, a condition approximated as $R_A/H_A = R_B/H_B$, where R and H are the stellar and critical radii of the components. However, if $R_A/H_A > R_B/H_B$, the surface of A is at a higher potential than B, so that an unbalanced pressure gradient will force gas from A onto B. The analysis considers the variation of the surface potential of the components as mass is transferred.

The variation of the critical radii follows directly from the binary model, equations (2) and (3), but the variation of the stellar radii is a more complicated problem. The detailed evolutionary models of the next section automatically follow the time-dependent, nonthermal equilibrium adjustment of the stars. Here we consider the mass-radius relation for two sequences: first, the zero-age main sequence (ZAMS) relation for which both hydrostatic and thermal equilibrium hold; second, a contact relation for stars with equal entropy outer envelopes for which hydrostatic (but not necessarily thermal) equilibrium holds.

To investigate the response of a system in which the mass of component B, of mass fraction $\mu = M_B/(M_A + M_B)$ is increased, we define a stability parameter

$$\mathscr{S} = \ln(R_A/H_A) - \ln(R_B/H_B)\,, \qquad (4)$$

and consider its derivative with respect to $\ln\mu$.

Rearranging terms in equation (4), we find

$$\frac{d\mathscr{S}}{d\ln\mu} = S = S_H - S_R \qquad (5)$$

with

$$S_H = \frac{d \ln (H_B/H_A)}{d \ln \mu}$$

$$= \frac{0.457}{(1-\mu)[\![1 - 0.277\{\log [\mu/(1-\mu)]\}^2]\!]}, \quad (6)$$

$$S_R = \frac{d \ln (R_B/R_A)}{d \ln \mu}$$

$$= \frac{d \ln R_B}{d \ln M_B} + \frac{\mu}{1-\mu}\frac{d \ln R_A}{d \ln M_A}. \quad (7)$$

The expression for S_H follows from equations (2) and (3). For a system initially in equilibrium, $\mathscr{S} = 0$. So, if $S_H > S_R$, a positive increment to the mass of star B leads to additional mass exchange from A to B, since the surface potential of A will exceed that of B after the exchange.

First consider the case for components of equal mass. Such a system is in thermal equilibrium and can satisfy all the conditions described in §§ I and II, yet no equal mass W UMa stars are found in nature. For $\mu = 0.5$, $S_H = 0.914$, and the ZAMS relation near a solar mass is $R \propto M^{0.8}$. We find

$$S_{\rm ZAMS} = 0.91 - 1.60 = -0.69 \quad \text{(stable)}.$$

In other words, if mass were transferred from A to B, the surface of B would shift to a higher potential relative to A so that the mass would flow back to A. However, the additional consideration of energy exchange from primary to secondary results in a much shallower mass-radius relation for stars in contact than for detached ZAMS stars. Lucy (1968*a*, Fig. 2) presents the mass-radius relation for stars with equal-entropy surface convection zones. Note that while two stars of equal mass can be in thermal equilibrium, in general the surface luminosity of two unequal components will not match their combined energy generation rate. Also, after mass transfer, the stars in contact should have a common entropy in their outer envelope, but the entropy will not be what it was at equal mass. Nonetheless, the contact relation near a solar mass is $R \propto M^{0.13}$, so that

$$S_{\rm CONTACT} = +0.91 - 0.26 = +0.65 \quad \text{(unstable)}.$$

Although the radius of component A shrinks after losing mass, the new ratio of stellar to critical radius for A exceeds the similar ratio for B. Consequently the surface potential of A is above that of B, and the mass transfer is self-sustaining.

The problem is slightly more complicated if the contact system is initially in equilibrium at some non-equal mass ratio, as must be the case if W UMa stars are not in a perpetual state of mass exchange. Equation (6) can, of course, be evaluated for any μ (but note that the approximate relation, eq. [2], is accurate to 2% or better only for $0.23 < \mu < 0.95$, which includes the observed mass ratios of most W UMa stars). As previously mentioned, detailed models do not succeed in producing contact binary stars of zero age and normal metallicity which are in thermal equilibrium. However, we assume, following Lucy, that such systems at least require that one component should burn via CN reactions with $R \propto M^{0.5}$ while the secondary burns by *p-p* reactions with $R \propto M^{0.13}$, where the mass-radius relations are again from Lucy (1968*a*, Fig. 2). We evaluate S for two initial mass fractions $\mu = 0.24, 0.75$) and find: $S_H = (0.65, 1.95)$, $S_R = (0.30, 0.89)$. In either case the system is unstable for mass transfer onto component B, regardless of whether B is the primary or secondary!

One is invited to substitute values for some other initial configuration, but the conclusion seems to be that for stars near a solar mass the contact mass-radius relation is too shallow to inhibit mass-exchange. It is primarily the shallow relation, $R \propto M^{0.1}$, of the secondary which generates the instability. If the mass of each component exceeded 1.3 $M_\odot$, and if a sufficiently deep convection zone remained, then the system would be stable in this approximation. But for both stars less than 1 $M_\odot$, the system is unstable at all mass ratios of interest, and for transfer to either the primary or the secondary. This result might not be valid once both stars have evolved so that their mass-radius relations have been appreciably altered.

IV. THERMAL EVOLUTION OF A 2 $M_\odot$ CONTACT BINARY

Since earlier detailed models of zero-age CBS suggest that it is impossible to form systems of unequal mass in both hydrostatic and thermal equilibrium without a highly enriched metallicity, this study was undertaken to investigate what response would occur if a pair of stars with normal abundances were forced into contact. In particular, would the binary contrive to separate, or would some equilibrium state be achieved? The calculations begin with a contact system of 1.97 $M_\odot$ with a mass ratio of 1.01, so that the system is nearly in thermal equilibrium at the start. The orbital angular momentum is 6.14×10^{51} cgs, resulting in a contact configuration with a period of 6.6 hours. Solution of the models follows the normal Henyey relaxation scheme, but with a simultaneous solution for both stars coupled by the surface boundary conditions and an energy exchange as described above. The numerical code is that of Eggleton, as described in Eggleton (1971) and Eggleton, Faulkner, and Flannery (1973). Evolution of the composition was not evaluated since the total time span covered by the models is $\sim 3 \times 10^7$ years. The composition is $X = 0.70$, $Z = 0.02$, and the convective mixing length is 1.5 times the pressure scale height.

a) *The Cyclic Mass Exchange*

The cyclic instability resulting from the combined binary plus stellar evolution is most easily described in terms of the variation of radii versus mass-fraction of the primary—star 1—as shown in Figure 1. Two pairs of fiducial curves are plotted in Figure 1: the ZAMS mass-radius relation, and the critical radii for

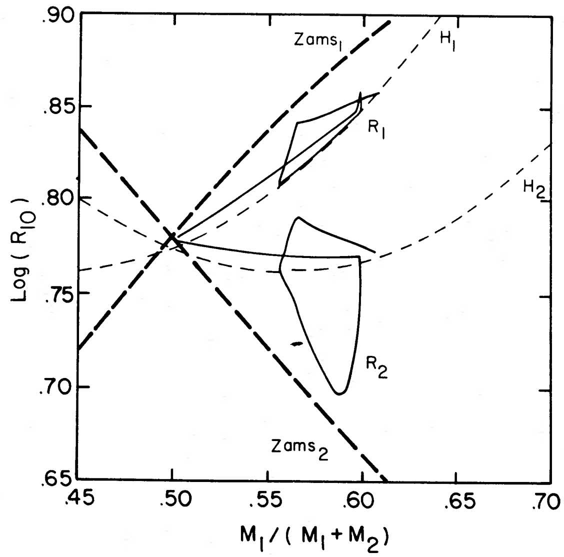

FIG. 1.—Evolution of the radii of the components of a 2 $M_\odot$ contact binary as a function of mass fraction in the primary. Four fiducial curves are drawn: $ZAM_{1,2}$ are the thermal equilibrium radii and $H_{1,2}$ the critical Roche radii of the components. The evolution starts near $\mu = 0.5$ with the stellar radii near equilibrium, but both are greater than their respective critical radii, so the system is in contact. The system evolves to the right, i.e., mass transfer onto the primary until contact breaks near $\mu = 0.6$ because the binary is separating. (See the text for further discussion.)

both components. These curves display a reflection symmetry about $\mu = 0.5$, since primary and secondary simply interchange at this point. Note that the critical radius of the secondary, H_2, decreases until $\mu \approx 0.56$, when the effects of increasing binary separation dominate over the decreasing logarithmic term in equation (2).

The evolution begins near $\mu = 0.5$ with $R/H > 1$ for both stars; the initial models are ZAMS stars of 0.98 and 0.97 $M_\odot$. When placed into contact, the mass-radius relation of the stars is perturbed by the energy-exchange such that mass is transferred to the primary and the system evolves to the right in Figure 1. A single star, at zero age, losing mass at a constant rate would rapidly decrease in radius, and then evolve along a mass-radius relation essentially parallel to, but below, the ZAMS relation. This behavior, in response to removal of a star's outer layers, occurs because thermal energy is absorbed as the star's interior attempts to expand, which results in an overall decrease of the surface radius. An analogous behavior occurs for a single star receiving mass at a uniform rate: following a rapid initial expansion, the star evolves along a sequence parallel to, but above, the ZAMS mass-radius relation. In a contact binary the star's response to mass loss is drastically altered. The injection of energy into the secondary slows the radius decrease, while the primary is relatively deflated by loss of energy. By adjusting the rate of mass transfer, the system can achieve surfaces in hydrostatic equilibrium, i.e., $R_1/H_1 = R_2/H_2$. During this initial phase both components increasingly depart from thermal equilibrium, as will be described below, and the system evolves smoothly with the primary's mass increasing.

But the situation cannot persist. The binary is increasingly separating, so that eventually, near $\mu \approx 0.6$, the critical surfaces migrate upward to coincide with the stellar surface. If the full energy-exchange required to maintain equal entropy in both envelopes could be transported even as the connecting throat shrinks to a point, then the next iota of mass transfer causes the system to detach, and the energy exchange terminates. A new phase ensues as the separated stars attempt to adjust to their ZAMS equilibrium radii: the secondary recedes inside its Roche lobe, while the invigorated primary swells. Immediately mass overflows onto the secondary, and the direction of mass transfer reverses. The system is now evolving back toward equal mass.

In the reversed mass-transfer phase the binary is semidetached, but the secondary's radius never becomes less than about 0.86 of its critical radius. With the important exception of the mass exchange, the secondary is essentially decoupled from the binary. As described for a single star, the radius of the secondary eventually evolves along a track parallel to, and above, its ZAMS value. The rate of mass loss from the primary adjusts so that the stellar radius of the primary equals the critical radius. If the rate were smaller, the star would swell outside its lobe, stimulating more rapid mass transfer. If the rate were larger, the star would shrink inside its critical lobe, completely shutting off the exchange.

Again, the phase of reversed mass transfer cannot persist since the separation of the binary is decreasing. The increasing radius of the secondary eventually re-establishes a contact configuration, and energy is once again transferred to the secondary. This forces the direction of mass transfer to reverse, and the cycle is ready to repeat.

b) *Nonthermal Equilibrium of the Components*

Once the mass transfer begins, the stars cannot adjust rapidly enough to maintain thermal equilibrium. This is seen in Figure 2, which plots the variation of luminosity with radius for each component just prior to the first breaking of contact. The asterisk plotted for each star represents the ZAMS surface radius and luminosity appropriate to the star's mass, and the *C* denotes the base of the surface convection zone. The primary and secondary are undersized and oversized, respectively, as a result of the energy exchange which directly affects the entire structure of each star's convection zone, even though it is physically applied at the critical radius. However, very little energy is absorbed or released in the outer convection zones; it is the relative compression (expansion) of the outer 75 percent, by radius, of the primary (secondary) where large departures from thermal equilibrium occur. The secondary swallows nearly its entire interior luminosity in expanding its outer regions,

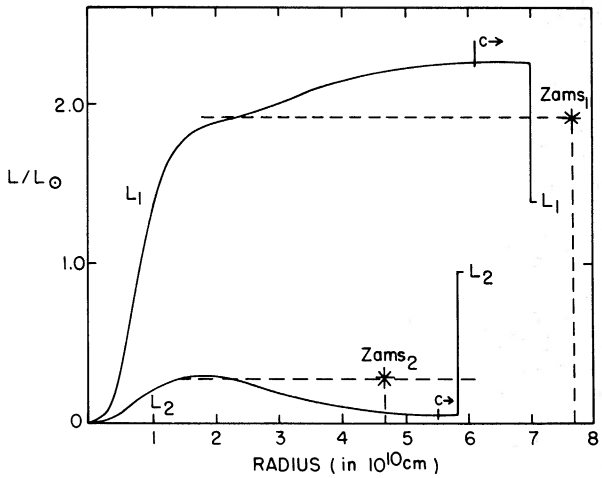

FIG. 2.—The luminosity distribution as a function of radius for both components just prior to the initial breaking of contact when the mass transfer rate is $dM_1/dt \approx +1.6 \times 10^{-8} M_\odot$ yr^{-1}. The equilibrium (ZAMS) surface luminosity and radius for stars of the same mass are indicated by the asterisks. C marks the start of the surface convection zone, and energy exchange occurs at the critical radii. Note that thermal energy is released (absorbed) in the outer 70% of the primary (secondary).

while the compression of the primary produces a 20 percent enhancement of luminosity—prior to the surface energy exchange—relative to its nuclear energy generation rate.

The nonthermal equilibrium features of the cycle have important effects. In Figure 3 we plot the variation of luminosity with time during the thermal evolution. It is the energy liberated by the primary, nuclear plus thermal, which controls the evolution of the binary; just prior to the initial breaking of contact the secondary supplies only ~2 percent of the surface luminosity. As the mass transfer reverses, the interior state of the primary switches from compression to expansion, and the absorption of energy produces a sudden drop of 30 percent in the surface luminosity.

The inability of the primary to achieve its equilibrium radius drives the oscillation. The primary attempts to expand throughout the cycle. In the semidetached phase the expansion is short-circuited by Roche-lobe overflow. In the contact phase the expansion is slowed by the energy transfer which, however, self-stimulates additional expansion by the return mass transfer. In turn the nonequilibrium of the secondary regulates the cycle. When the energy exchange ultimately is unable to pump the secondary sufficiently to maintain mass transfer to the primary, the contact breaks. During the semidetached phase, dumping of mass onto the secondary forces the system back into contact. Nuclear, as well as thermal, energy sources are utilized to drive the oscillation.

c) *Observational Characteristics of the Cycle*

We turn to a discussion of possible observational features of the models. Bear in mind that only one complete loop of the cycle has been calculated. While construction of models for one more loop would have been feasible, it seemed likely that nothing substantial would have been learned without following at least several more loops which would have been prohibitively expensive. Basically, the complete cycle lasts $\sim 10^7$ years and contains two phases of nearly equal length. Figure 4 plots the mass-transfer rate as a function of time. Except for the brief periods when the sign reverses $|dM_1/dt| \approx 1.5 \times 10^{-8} M_\odot$ yr^{-1}. The

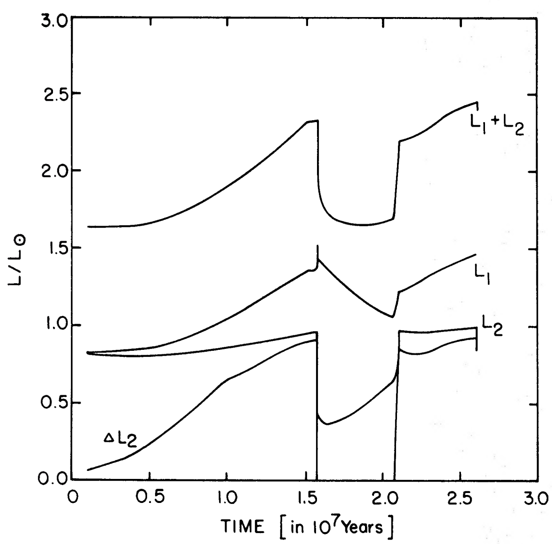

FIG. 3.—Luminosity as a function of time. $L_1 + L_2$, L_1, and L_2 are the total and individual surface luminosities, while ΔL_2 is the energy transferred into the secondary from the primary. Thus the interior luminosity of the primary, just before energy exchange, is $L_1 + \Delta L_2$.

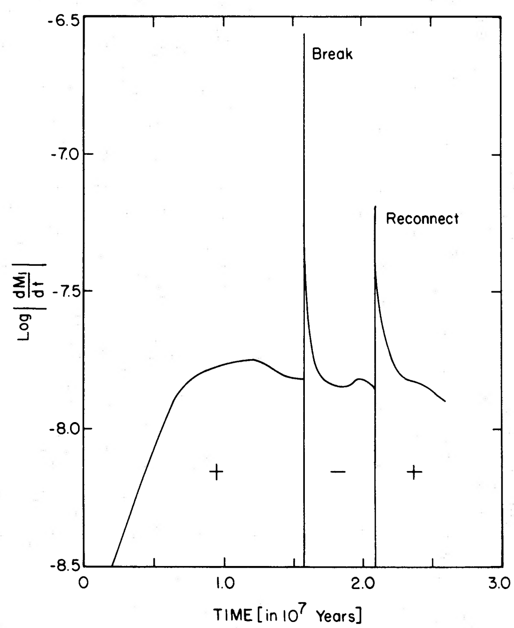

FIG. 4.—Absolute values of the mass transfer rate onto the primary as a function of time; the sign is indicated by + or −.

associated rate of change of period is about $|p^{-1}(dp/dt)| \approx 1.7 \times 10^{-8}\ \mathrm{yr}^{-1}$, i.e., $5 \times 10^{-4}\ \mathrm{s\ yr}^{-1}$ at a period of 7.5 hours. A relatively large literature exists regarding period changes in W UMa stars. Period changes are found of even larger magnitude than predicted by this study, but the changes are rarely monotonic. Thus there appears to be substantial "jitter" in the period which makes it difficult to sort out a secular trend.

A possibly important problem with these models is that during the semidetached phase the light curve is not in agreement with those found for W UMa stars. However, during the contact phase of the cycle with the period and luminosity increasing, the surface conditions are those of the Lucy model, for which theoretical and observed light curves are in satisfactory agreement.

During the semidetached phase the period decreases, and the luminosity is essentially constant following a rapid decrease. The envelopes are no longer coupled by energy exchange. The secondary's radius is never less than ~ 0.86 of the critical radius, so the degree of detachment is quite small. However, the effective temperature of the secondary is 1000 K less than that of the primary for a brief time and typically is 500 K cooler. If we simply assume that the observed brightness of each component is given by its surface area multiplied by the blackbody flux, B, appropriate to its temperature, then the ratio of observed flux at secondary and primary eclipse, for an orbital inclination of 90°, is

$$\frac{L_s}{L_p} = \frac{1}{1 + (R_2/R_1)^2[B_2/B_1 - 1]} \, .$$

At 5500 Å the difference ranges between ~ 0.25 and 0.35 V-magnitudes during the semidetached phase. The difference rarely exceeds 0.05 mag for W UMa stars; and β Lyrae binaries, for which substantial differences do occur, are rare at such short periods. Thus the eclipse light curves for one-half the cycle's duration do not tally with observations, a point to which we will return in the discussion.

Finally, consider the behavior of the models in the period-color diagram, Figure 5. As has been traditional in work on CBS, we use the transformation

$$B - V = \frac{(3.970 - \log T_e)}{0.31}$$

(Eggen 1961). The diagonal solid lines in the figure define the region populated by W UMa stars (Eggen 1967) and the dashed-line trapezoidal region corresponds to zero-age systems as selected by Lucy (1968*a*). The curve labeled L corresponds to thermal equilibrium models found by Lucy—though, as mentioned, more detailed calculations find it impossible to produce such systems without $Z > 0.04$. The dotted curve, labeled ZAMS, is the locus of points for equally massive stars just in critical contact, as found by the author for $Z = 0.02$.

The curve labeled F is the trajectory of the models calculated here. For the semidetached phase a mean color is plotted, since the temperatures are not equal. The curve starts at the color appropriate to 1 $M_\odot$ on the ZAMS relation, but at a shorter period since the stars are initially closer than just marginal contact. Through the cycle the curve oscillates over a region of size 0.06 in log (Period) and 0.5 in $B - V$.

Above F there is a vector symbol which represents a correction displacement which should be applied to account for effects of uniform synchronous rotation.

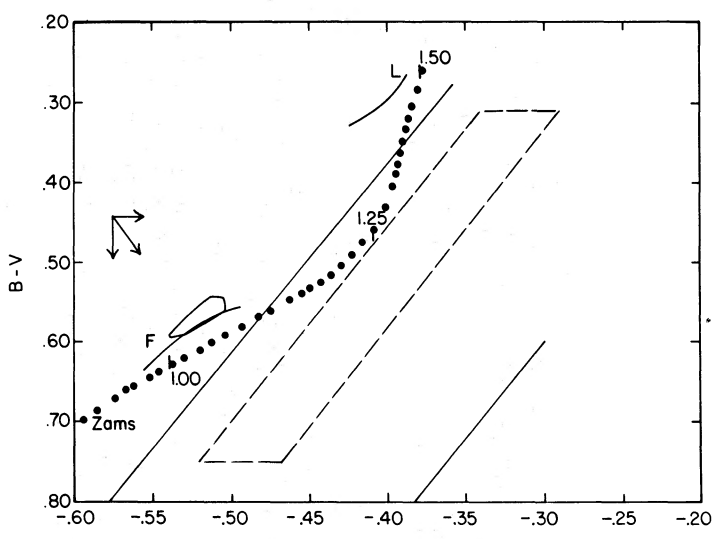

FIG. 5.—Evolution in the period-color diagram. The region observed to be populated is enclosed by the solid diagonal lines, while normal-abundance, zero-age systems are found in the trapezoidal region. ZAMS represents theoretical systems with components of equal mass just at marginal contact with the component's mass indicated. L is the sequence of thermal equilibrium systems of unequal mass originally calculated by Lucy. F is the trajectory followed by the 2 $M_\odot$ contact binary calculated here. The symbol above F represents corrections which should be applied to account for the rotation of the stars.

The levitational effects of rotation lower the equilibrium central temperature of a star; consequently the luminosity is reduced. Also both the surface area and equatorial radius are increased. For a rotational parameter

$$\alpha = \frac{2}{3}\frac{\omega^2}{G}\frac{r^3}{M} \sim \frac{2}{3}\left(\frac{R}{D}\right)^3\frac{1}{\mu} = 0.07\,,$$

the tabulated results of Faulkner, Roxburgh, and Strittmatter (1968) indicate

$$\Delta \log T_e \approx -0.013 \quad (\Delta \log B - V \sim +0.05)$$

and

$$\Delta \log R \sim +0.01 \quad (\Delta \log P \sim +0.015)$$

at 1 $M_\odot$. Application of this correction shifts F into the populated region. It is interesting to note that the rotational corrections are larger for shorter period systems, not only because they rotate more rapidly but also because lower mass stars are less centrally condensed and have less concentrated nuclear burning. The increasing correction may offset the shallower slope of the ZAMS relation relative to the populated region.

V. DISCUSSION

The analysis of § III strongly suggests that the mass-exchange instability is ubiquitous among CBS. Although we have presented detailed calculations of the resultant cyclic behavior for only one model system, the thermal instability associated with rapid variations of mass seems likely to produce a similar cycle in most cases. The inability of a contact system to achieve true thermal equilibrium forces the system into an oscillatory state in which average equilibrium can be achieved over the cycle. No particularly constraining values for the chemical composition, mass, or mass ratio are necessary in this model. Given a system with a total mass, angular momentum, and mass ratio such that the system is in contact, evolution forces the system to remain very nearly in contact because the mass-transfer reverses direction before the system can separate sufficiently to detach. Viewed in this way the distribution of systems in the period-color diagram shows the current location of a distribution of systems with varying total mass and angular momentum in various phases of their mass-exchange cycle.

It would not be surprising if many details of these models were in error due to the many simplifying approximations, but the basic effects (energy exchange in contact producing an instability which allows mass transfer to occur, followed by binary separation which ultimately forces the mass transfer to reverse direction) seem likely to be present at all levels of refinement. Several authors have found that convective energy exchange is efficient enough to transport the necessary large-energy flux to the secondary (Hazlehurst and Meyer-Hofmeister 1973; Biermann and Thomas 1973), but no detailed physical model exists which would allow a better computation of how the flux is modulated as the degree of contact narrows. Also, use of the Roche approximation for the critical radii, and bypassing the problems associated with the interaction of rotational and orbital angular momentum, which actually control the variation in the binary separation, could substantially alter the overall time scale of the cycle and the degree of separation during the semidetached phase. Finally, we have neglected the hydrodynamic aspects of the mass exchange which are particularly important in the semidetached phase. The treatment used here is equivalent to assuming that the gas flows with negligible velocity and arrives with exactly the same thermodynamic properties as the gas at the surface of the star. However, the infall energy of the accreted gas during the semidetached phase would increase the luminosity of the secondary by ~10 percent. The possible development of a rotating equatorial rim which might even continue to allow circulation of convective elements during this phase is not an unreasonable possibility. For these reasons we are not particularly discouraged by the uncharacteristic eclipse light curves which are found in the semidetached phase. Rather it seems more appropriate to stress how very close to contact the stars do remain throughout the cycle.

These calculations suggest two alternative explanations for the existence of two equally numerous subclasses of W UMa stars. Ruciński (1973) has recently summarized some of the observationally distinguishing characteristics of the classes, referred to as types A and W. Type A systems are generally of earlier spectral type, higher luminosity, and have a deeper common envelope, i.e., the components are in contact to a greater degree. Type A systems are also less active photometrically and spectroscopically, and show more constant period change phenomenon. It is tempting to associate the more active, fainter, and erratic type W systems with the semidetached phase of the cycle. However, there is also evidence that the two classes are characterized by differing mass ratios, with type A systems having a larger fraction of mass in the primary. If so, that would rule out such a position since the cycle proceeds over the same mass ratios. Therefore a second interpretation might associate the entire cyclic phenomenon with systems of type W, with type A systems representing a later, perhaps calmer, phase of evolution.

In the realm of speculation it is extremely interesting to consider the ultimate evolution of a CBS. In this regard the most important aspect of these calculations is that one component remains the primary throughout the cycle—the mass ratio does not entirely reverse. This star should evolve much more rapidly than the secondary. As the luminosity of the system increases with time, the average mass fraction in the primary would probably grow because the mass-exchange from the secondary could be pumped for even larger excess radii of the secondary. This would allow the system a larger volume for expansion. If a rapid expansion of the primary occurs, as it commences ascent of the giant branch, the system might not be able to adjust rapidly enough for the binary to accommodate the swelling, so that the secondary would be swallowed. In this case large mass flow into a circum-binary disk might occur, or the star would contain two revolving cores inside the envelope. The possible merger of the two cores, one of them degenerate, would certainly present interesting possibilities. On the other hand, it is possible that the giant phase is bypassed. Mixing by meridional circulation currents is usually unimportant for solar mass stars which are slow rotators. The mixing time scale is $\tau_m \sim R/V_m$ with

$$V_m \approx \frac{2}{3}\frac{\omega^2 R^3}{GM} \times \frac{LR^2}{MGM} \approx 10^{-5}\ \mathrm{cm\ s^{-1}}\,,$$

which gives a mixing time of only 2×10^8 years since $\omega \sim 10^2\, \omega_\odot$. The problem of such mixing is not well understood, however (see Strittmatter 1969). Without additional comment we simply point out that homogeneous evolution without a giant phase would have implications for the possible formation of blue stragglers and, ultimately, cataclysmic variable stars from W UMa binaries.

VI. CONCLUSIONS

Two fundamental results are shown in this study of unevolved contact binary stars coupled by a common convective outer envelope at equal entropy—the Lucy model. First, the system is unstable against mass exchange, even if the system is initially in thermal and hydrostatic equilibrium. Second, the mass exchange sends the stars into a cyclic mass-transfer loop, during which the individual components never achieve thermal equilibrium instantaneously, but the system can achieve an average thermal equilibrium. Throughout the cycle the binary remains very close to a contact configuration, so that the system is trapped in contact. Compared with the constrained range of CBS which are allowed in thermal equilibrium, a broad range of configurations is possible for the oscillating systems; in particular, the short-period, red contact systems are allowed.

It is a pleasure to record that discussions with Martin Schwarzschild were of great help in overcoming difficulties associated with rapid thermal changes in the model calculations. I have also benefited from discussions with Schwarzschild and Leon Lucy on the general subject of contact binary stars.

APPENDIX

COUPLED SURFACE BOUNDARY CONDITIONS AND ENERGY EXCHANGE

In § II we outlined the physical conditions which couple the components of the binary. Here we describe the actual formulae used in the numerical solutions. It is useful to express the conditions so that no sharp discontinuities in the boundary occur between the three states. Such discontinuities slow, or altogether prevent, numerical convergence when the star is in transition between two phases.

We evaluate the energy exchange required to produce equal entropy in the two convective envelopes from the condition that the effective temperatures of the two stars will be equal. Let L_1^* and L_2^* be the luminosities of the stars just below the region of energy exchange; then

$$\Delta L_1 = f\left[L_2^* - R_2^2\left(\frac{L_1^* + L_2^*}{R_1^2 + R_2^2}\right)\right]\,, \tag{A1}$$

where f is a factor, described below, which varies from unity in contact to zero out of contact. The transition from 1 to 0 occurs smoothly as the radius of the star at lower surface potential becomes less than 1.005 of its critical radius, i.e., as the contact becomes exceedingly marginal. The exchange of energy, ΔL_1 in the primary, $-\Delta L_1$ in the secondary, is entirely applied at the first mesh point outside the star's critical radius. Both ΔL and the two mesh points are *explicitly* evaluated from the previous model; they are not varied during the iteration. Sufficiently small time steps are taken that the delayed adjustment of ΔL_1 does not influence the results.

The coupled surface boundary conditions are

$$M_1 = M_1^0 + \frac{dM_1}{dt}\Delta t\,, \tag{A2}$$

$$M_1 + M_2 = M\,, \tag{A3}$$

where M_1^0 is the mass of star 1 in the previous model, Δt is the time step, and M is the invariant total mass of the binary. As an approximate expression for the transfer of mass, which is solely meant to embody the boundary conditions described in § II and is not meant as a true physical model of mass exchange, we assume that if one star rises higher in the coupled potential, gas will flow at the sound speed through the area by which one component exceeds the other. This in turn is approximated as

$$\frac{dM_1}{dt} = R_k^2(\rho_k P_k)^{1/2}[\chi_2 - \chi_1]\,; \tag{A4}$$

and for $i = 1, 2$

$$\chi_i = R_i/H_i \quad \text{for } R_i/H_i \geq 1$$
$$= 1 \qquad R_i/H_i < 1\,, \tag{A5}$$

where R_k, ρ_k, P_k are the surface radius, density, and pressure of the component k with the larger χ parameter. Note that $\chi_2 - \chi_1 = 0$ for a detached configuration, and the sign produces transfer from 2 to 1 if star 2 has a higher surface potential. The factor $R^2(\rho P)^{1/2}$ is such that a departure from the two radii terminating exactly on an equipotential ($R_1/H_1 = R_2/H_2$) of $\Delta\chi = 10^{-3}$ produces a mass transfer rate of $10^{-8}\, M_\odot\, \mathrm{yr}^{-1}$. This sensitivity also demands a high degree of convergence in the models, which is necessary in any case to follow the nonthermal equilibrium evolution. Having defined χ_i, we complete the prescription for the coupling conditions by defining the factor f in equation (A1),

$$\alpha = \frac{\chi_m - 1}{0.005}\,,$$

$$f = 1\,, \qquad \alpha \geq 1\,,$$
$$= \tfrac{1}{2}[1 - \cos(\pi\alpha)]\,, \qquad 0 < \alpha < 1\,,$$
$$= 0\,, \qquad \alpha = 0\,,$$

where χ_m is smaller of the two χ-values. This factor produces a smooth cutoff in the transfer of energy when the region of common overlap between the two components is less than 0.005, as measured by the ratio of stellar to critical radius for the star at lower surface potential.

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