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AN ELEMENTARY THEORY OF ECLIPSING DEPTHS OF THE LIGHT CURVE AND ITS APPLICATION TO BETA LYRAE

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ABSTRACT

An elementary theory of the ratio of depths of secondary and primary eclipses of a light curve has been proposed for studying the nature of component stars. It has been applied to light curves of Beta Lyrae in the visual, blue, and far-ultraviolet regions with the purpose of investigating the energy sources for the luminosity of the disk surrounding the secondary component and determining the dominant radiative process in the disk. We have found no trace of the spectrum of primary radiation in the disk and have therefore suggested that LTE is the main radiative process in the disk, which radiates at a temperature of approximately 12,000 K in the portion that undergoes eclipse. A small source corresponding to 14,500 K has also been tentatively detected and may represent a hot spot caused by hydrodynamic flow of matter from the primary component to the disk.

Subject headings: stars: eclipsing binaries — stars: individual

I. RELATIVE DEPTHS OF PRIMARY AND SECONDARY ECLIPSES

As is well known, if we assume that the stellar disk is uniformly bright, the ratio of depths of secondary and primary eclipses $(d_2/d_1)_{\rm bol}$ in the bolometric measure is related to the ratio of the effective temperatures T_j ($j = 1, 2$) of the eclipsed areas of the two components. In general, it can be shown easily that the ratio of depths of secondary and primary eclipses in any wavelength region from λ to $\lambda + \Delta\lambda$ denoted by $(d_2/d_1)_{\Delta\lambda}$ is given by

$$\left(\frac{d_2}{d_1}\right)_{\Delta\lambda} = \frac{\int_{\Delta\lambda} d\lambda \iint_{\Delta A_2} F_2(\lambda;\, \xi, \eta) d\xi d\eta}{\int_{\Delta\lambda} d\lambda \iint_{\Delta A_1} F_1(\lambda;\, \xi, \eta) d\xi d\eta}\,, \tag{1}$$

where $F_j(\lambda;\, \xi, \eta)$ represents the radiative flux of the primary ($j = 1$) or the secondary ($j = 2$) coming from the projected areas of stellar objects on the tangential (ξ, η) plane to the celestial sphere at the point where the binary is located, and where ΔA_j denotes, respectively, the obscured area of the primary ($j = 1$) and secondary ($j = 2$) during the mid-eclipses. In most cases, including the present one, $\Delta A_1 = \Delta A_2 = \Delta A$. If one assumes the emerging fluxes of both component stars are independent of (ξ, η) and are, furthermore, given by the Planck function $B(\lambda, T_j)$, ($j = 1, 2$), equation (1) reduces to the well-known bolometric relation mentioned in the beginning after being integrated over all wavelengths.

In practice the wavelength regions used in photometry, such as U, B, V, etc., are not too broad. Thus, for the sake of simplicity, we may use as an approximation an effective wavelength of each region as the representative of the entire region. Thus if λ_i is the effective wavelength of the ith region and if $\Delta A_1 = \Delta A_2 = \Delta A$, equation (1) can be simplified to

$$\left(\frac{d_2}{d_1}\right)_{\lambda_i} = \frac{\iint_{\Delta A} F_2(\lambda_i;\, \xi, \eta) d\xi d\eta}{\iint_{\Delta A} F_1(\lambda_i;\, \xi, \eta) d\xi d\eta}\,. \tag{2}$$

Equations (1) and (2) represent, of course, the general form for the ratio of the depths of secondary and primary eclipses. While the derivation of these equations is simple, they have so far failed to catch the attention of astronomers, because groundbased observations cover a region of wavelengths too narrow to make them very useful. However, with the recent advances in the study of the far-ultraviolet region as well as in the infrared region, equation (1) or (2) will provide a powerful means for understanding the nature of component stars in an eclipsing system. It will be especially useful for those systems where one component is peculiar and cannot be ascertained by the usual means of study, such as those systems where an opaque or a semitransparent disk is associated with the secondary component. In such cases equation (1) or (2) can give us, as we will see later in the paper, some clues as regards the nature of the disk. This is indeed the purpose of the present paper, since we will apply this idea to the disk in the β Lyrae system where light curves in different colors from the optical region (Larsson-Leander 1969; Lovell and Hall 1970, 1971; Landis *et al.* 1973) to the far-ultraviolet region (Kondo *et al.* 1972) are now available. Needless to say, the same procedure may be used for a study of any eclipsing system with or without a disk.

II. SOURCE OF THE DISK LUMINOSITY

According to the disk model, the light variation in the β Lyrae system is caused by the eclipse of the primary component by the disk (the primary eclipse) and vice versa (the secondary eclipse). That the secondary eclipse of β Lyrae is clearly seen leads to the conclusion that the disk surface is luminous, as can be seen from equation (1). That being so, we immediately encounter the problem of finding the energy source of the luminous disk. In order to investigate this problem we propose a new procedure of study which has been briefly stated in a paper presented before the AAS meeting at Bloomington, Indiana (Huang and Brown 1975) and which is being developed here in full. Basically, we apply equation (2) to the observed light curves in different wavelength regions.

In order to use equation (2) we must know $F_j(\lambda_i; \xi, \eta)$ ($j = 1, 2$). As a first approximation we may equate $F_1(\lambda_i; \xi, \eta)$ to the Planck function $B(\lambda_i, T_1)$ corresponding to the effective temperature T_1 of the primary component. But it is much more difficult to write down $F_2(\lambda_i; \xi, \eta)$, because we must first know the nature of the energy source of the disk as well as the radiative process inside it. It is obvious that the disk does not generate thermonuclear energy, nor could its own gravitational contraction be significant. So what it emits must come directly or indirectly from external sources, namely the primary and secondary component stars. The relation between the nature of energy sources and the characteristics of the disk luminosity has been extensively discussed in the paper on BM Orionis (Huang 1975), so we will not repeat it here. Suffice it to say that the luminosity of the disk surface can be broken down into two terms, one term with cylindrical symmetry and the other term varying with the azimuthal angle around the disk. The first term could be due to energy coming, in whatever way, from either or both component stars, while the second term is necessarily associated with energy fed by the primary component, because the secondary component located in the center of the disk can only give rise to a brightness distribution over the disk surface having cylindrical symmetry.

Let us now examine the relative importance of these two terms in connection with the disk in β Lyrae. The asymmetric term with energy source from the primary component must have a maximum at the point closest to the primary and a minimum at the point on the opposite side away from the primary. Such a brightness distribution will create a slope in the light curve between primary and secondary minima. The light would increase from primary to secondary eclipse and decrease from secondary to primary eclipse (Huang 1975). Although the time interval between the two eclipses of β Lyrae is too short to measure the exact slope, a close examination of the light curve in both B and V colors does not indicate any such slope at all. On the other hand, since the primary component is a highly luminous star, it must create an asymmetric brightness distribution over the disk surface. The reason that we have not observed the asymmetry could be caused by (1) a relatively large inclination of the disk, (2) the relatively high luminosity of the secondary component, and/or (3) a high degree of dilution of primary radiation when it reaches the disk surface.

We can detect by observation the asymmetric component of the brightness distribution over the disk surface most clearly when the disk inclination is close to $\pi/2$ because then we see the edge surface of the disk. Since we observe different parts of the edge surface at different phases, the asymmetric brightness distribution over the edge surface will produce the slope of the light curve we have mentioned. However, if the disk inclination deviates increasingly from $\pi/2$, we will see more and more one of the two base surfaces and less and less the edge surface of the disk. The actual brightness distribution on the base surface may be asymmetric, but we cannot detect it, because we observe the same base surface at all phases. If this is the case, the inclination of the disk in β Lyrae could differ greatly from $\pi/2$. The second possibility is more obvious. If the secondary component should be highly luminous, then the brightness distribution over the disk surface will be dominated by energy from the secondary and, consequently, will have cylindrical symmetry. The third possibility arises when the relative size of the primary star is not large, a requirement that is contrary to the accepted interpretation of the light curve of β Lyrae. However, at this point we should not completely rule out this possibility.

In general, $F_2(\lambda_i; \xi, \eta)$ comes from the energy fed to the disk by several sources. Three sources can be mentioned, although not all of them are necessarily present, let alone significant, in equal degrees. These three sources are electromagnetic radiation from the primary, the radiation and other forms of energy flow from the secondary component, and the gaseous flow (including resulting shocks, hot spots, etc.) and corpuscular radiation from the primary component. When the disk has received energy from these sources, it reemits radiation and becomes luminous itself. The secondary eclipse is a result of this luminosity of the disk.

What kind of spectrum will the disk surface radiate? The answer, of course, depends upon the physical nature of the disk. If the density of the disk is high, so that local thermodynamic equilibrium (LTE) holds, the disk will radiate in the first approximation as a blackbody, and one can define an effective temperature T_d for the disk. In such a case, equation (2) may be approximated by

$$\left(\frac{d_2}{d_1}\right)_{\lambda_i} = \frac{w_d B(\lambda_i, T_d)}{B(\lambda_i, T_1)}, \tag{3}$$

where T_1 is the effective temperature of the primary component, and w_d is a numerical factor which takes care of two facts. In the first place, it absorbs the departure of the radiative flux from that given by the Planck function as a result of the presence of emission lines, etc. In this respect w_d serves as a correction factor. Second, it takes into account the possibility that only a fraction of ΔA covers the effective radiative surface of the disk. This means that LTE may be, and likely is, established below the geometrical surface of the disk. In either case we should expect w_d to be of the order of magnitude of unity.

Situations may also occur such that even when an effective temperature for the disk surface T_d can be defined, there are superposed on the general "photosphere" some hot and cold spots (e.g., Smak 1970) or shock fronts over each of which a different effective temperature T_j ($j = 3, 4, \ldots, N$), where $N - 2$ is the total number of spots covered by ΔA, prevails. In such a case, we would have

$$\left(\frac{d_2}{d_1}\right)_{\lambda_i} = \frac{w_d B(\lambda_i, T_d) + \sum_{j=3}^{N} w_j B(\lambda_i, T_j)}{B(\lambda_i, T_1)}, \quad (4)$$

where w_j ($j \geq 3$) is the fraction of area of the ($j - 2$)th spot of the effective temperature T_j within ΔA.

In the other extreme the radiation in the disk is scattered monochromatically. This will be true if the density is low, such as in the supergiant atmosphere, and/or if the temperature is high, so that electron scattering is the dominant source of opacity. Under these conditions radiation in the disk maintains the characteristic spectrum of its sources. In such cases the emerging flux is composed of as many components as there are sources, and we cannot assign an effective temperature to the disk. In general, we may express equation (2) approximately as follows

$$\left(\frac{d_2}{d_1}\right)_{\lambda_i} = \frac{\sum_{j=1}^{N} w_j B(\lambda_i, T_j)}{B(\lambda_i, T_1)}, \quad (5)$$

where T_2 now denotes the effective temperature of the secondary component star, w_1 and w_2 are dilution factors, while w_j and T_j ($j \geq 3$) have the same meanings as before.

From what has been said, the physical meaning of equation (5) differs greatly from that of equation (4), even though mathematically the two equations assume the same form. Thus in the mathematical analysis we do not have to separate these two physical cases. However, when we come to the interpretation, we must discuss them separately.

Our previous discussion of two radiative processes in the disk applies also to stellar atmospheres. This is especially true when we come to the study of what is generally called the reflection effect. Astronomers in the field of eclipsing binaries usually discuss the reflection effect in terms of the LTE condition. However, when we discuss the atmospheres of supergiant stars, especially of early spectral types, or other hot tenuous media where the opacity is dominantly provided by electron scattering, the approximation given by monochromatic scattering is far better than that given in LTE. In such cases the reflection effect must be considered in terms of monochromatic scattering. In the case of the disk in β Lyrae we have no a priori knowledge of which one of the two radiative processes provides a better approximation. We will try to find out which one does by the present study.

III. GEOMETRIC APPROXIMATION FOR THE DISK IN ANALYZING THE LIGHT CURVE

What counts in studying the light curve in a system where a semitransparent or a completely opaque disk is present is the projection of the disk on the celestial sphere. This projected area may be characterized by two parameters, a width and a length, which will not change greatly, even though the light curve may be fitted in an infinite variety of ways (e.g., Huang 1973). For example, consider the projection of a disk in the form of a circular cylinder, say a coin, at different perspective angles. The projected area is exactly an elongated rectangle when it is viewed edgewise. But as the viewer moves farther away from the edgewise direction, the projection is better represented by an ellipse, even though it is not an exact ellipse. Wilson (1974) takes the disk as an ellipsoid of revolution whose projections on the sky at different angles of inclination can be represented likewise by ellipses. In either case the projected area can be characterized by the two parameters mentioned above, which are the width and length in the case of a rectangle and the minor (polar) and major axes in the case of an ellipse. Hence empirically the two points of view (circular cylinder and ellipsoid of revolution) do not differ appreciably as far as the observations are concerned. Wilson has given five solutions by assuming five values (2, 3, 4, 5, 6) for the mass ratio $q = m_2/m_1$, where m_1 and m_2 denote, respectively, the masses of the primary and secondary components. The sizes of the disk and the primary component do not vary greatly with q.

We will denote the semimajor and the semiminor axes of the projected ellipse, respectively, by a and b in the unit of the separation of the two components. In the same unit system, we denote by r_1 and r_2, respectively, the radii of the primary and secondary components. Of course, in the disk model the secondary component is not directly seen by the observer.

The observed points in both B and V colors obtained during the 1959 international campaign (Larsson-Leander 1969) show large scatter, especially at primary mid-eclipse. Hence whether r_2 exceeds b is hard to say for sure. However, if r_2 did exceed b, and the secondary star protruded from the projected ellipse, we would expect to see a sudden dip or a change of slope in the light curve when the protruding portion of the secondary component begins to move across the face of the primary. In the observed light curves we can see some changes of slope in some cycles, but such a change has never been permanent. Consequently, we may assume in what follows that r_2 does not exceed b by any large amount.

IV. SINGLE-COMPONENT ANALYSIS AND ITS INTERPRETATION

Both LTE and monochromatic scattering processes give the same expression for $(d_2/d_1)_{\lambda_i}$ in the case that one energy source is assumed for the disk. Let us therefore write

$$\left(\frac{d_2}{d_1}\right)_{\lambda_i} = \frac{w_x B(\lambda_i, T_x)}{B(\lambda_i, T_1)} \quad (6)$$

and use T_x and w_x in order to note that we do not commit ourselves to any definite interpretation for the

TABLE 1

RATIO OF ECLIPSE DEPTHS

λ_i	$(d_2/d_1)_{\lambda_i}$
5470	0.58
4400	0.54
3330	0.60
2980	0.58
2460	0.65
1920	0.53
1500	0.75
1380	0.82

time being. Thus, if the LTE process turns out to be right, the index x becomes d. But if the scattering process dominates the radiation field, x becomes j, where j could be either 1 or 2.

The primary component has often been classified as B8 or B9. If we should take it to be B8.5, we have T_1 = 11,350 K (Morton and Adams 1968). Eight $(d_2/d_1)_{\lambda_i}$ values in eight colors are available: two colors in the standard B and V (Larsson-Leander 1969) and six colors in the ultraviolet region obtained recently by the OAO (Kondo *et al.* 1972). We have measured $(d_2/d_1)_{\lambda_i}$ from light curves in these eight wavelength regions and give the results, together with the effective wavelengths, in Table 1. It appears that the light curve in the wavelength region of 1920 Å differs greatly from light curves in other wavelength regions. It is believed that there occurs in this region some abnormal behavior, which has also been found in another star (Underhill and Fahey 1973). Therefore, in each study we have performed two sets of analyses, one including the 1920 Å region (8 colors) and one excluding this region (7 colors).

For each color, equation (6) provides a relation between two unknowns, w_x and T_x. We can compute T_x by assigning a definite value for w_x. In this way we derived seven (eight) values for T_x from the seven (eight) $(d_2/d_1)_{\lambda_i}$ values in Table 1.

In principle the seven (eight) values of the temperature should all be the same if the analysis is exact and if the correct value of w_x is chosen. Actually several approximations have been made in the procedure, and we do not expect such a perfect result. But we can always look for the most consistent result by the following procedure. For each value of w_x we can obtain seven (eight) values of T_x and compute afterward the standard deviation or the root mean square (rms) of the deviations of the seven (eight) values of the temperature from their average value. Thus, for each assumed value of w_x, we can obtain a mean value of T_x and a rms value of the deviations for seven (eight) values of temperature from the mean value. In this way we have obtained the average temperatures and rms deviations for a series of values of w_x. The mean temperature as well as the standard deviation were found to change with w_x. The results are plotted in Figure 1. In order to save space we have combined in one figure the present results of our one-component analysis with those of our two-component analysis, which will be discussed later. The one-component result corresponds to the two curves marked "one-component." The

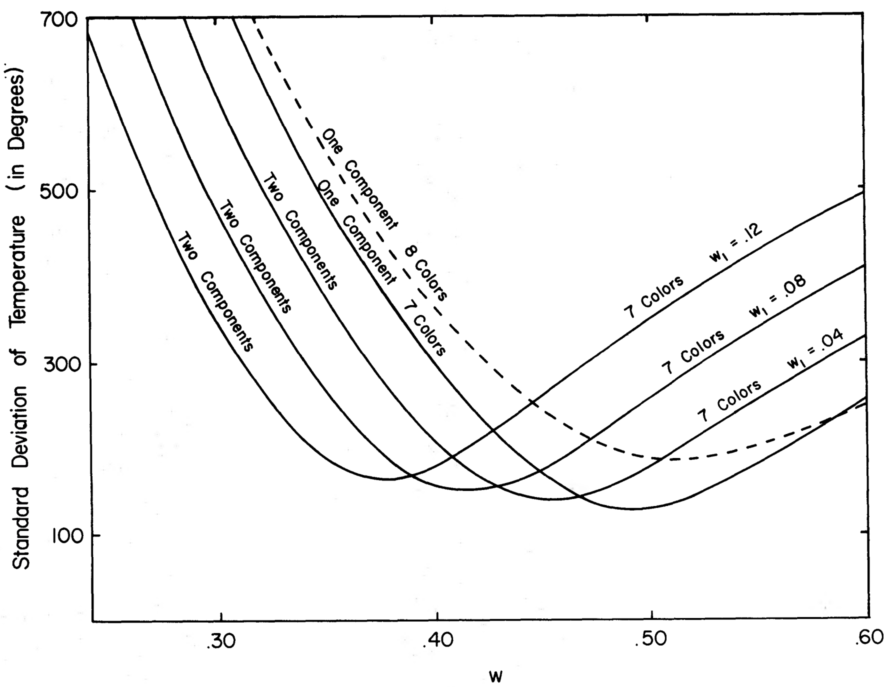

FIG. 1.—Standard Deviation Curves. The two curves labeled "one component" represent the standard deviation of T_x as a function of w_x calculated from equation (6) in seven colors (without the 1920 Å color, *solid line*) and in eight colors (with the 1920 Å color, *broken line*). The three curves marked "two-components" represent the standard deviation of T_y as a function of w_y calculated from equation (10) in seven colors and with T_1 = 11,350 K for the three cases of w_1 = 0.12, 0.08, 0.04.

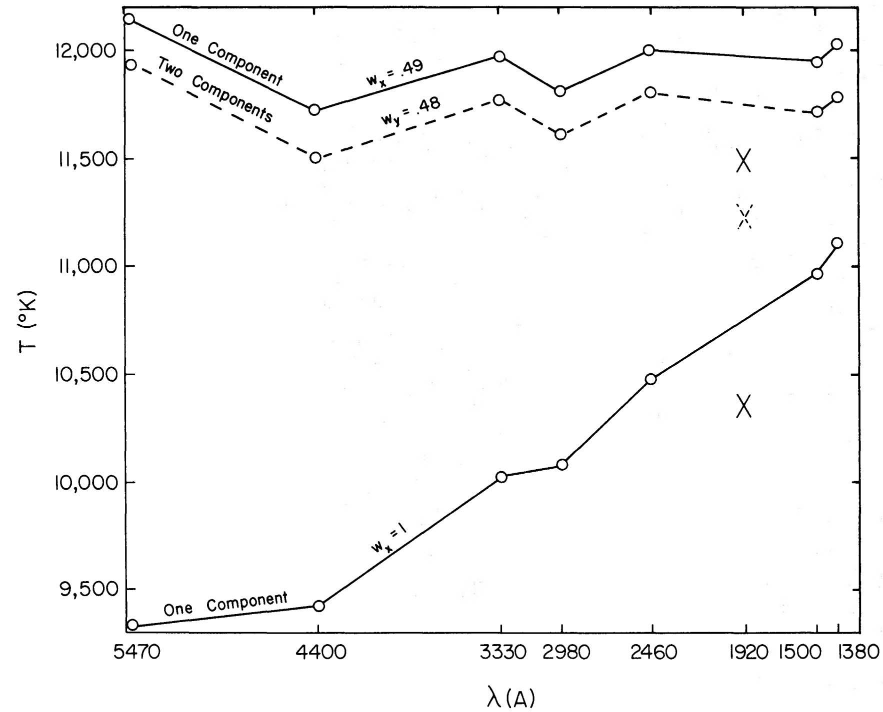

FIG. 2.—Individual Temperatures. The uppermost curve is a plot of the individual temperatures T_x calculated from equation (6) for seven colors and $w_x = 0.49$, the value of w_x corresponding to the highest internal consistency. The value of T_x computed for the 1920 Å region for this case is denoted by the uppermost ×. The lowest of the three curves represents the seven individual temperatures T_x computed from equation (6) for the case $w_x = 1$. The eighth value of T_x for this case in the 1920 Å region is denoted by the lowest × of the three. The dashed line represents the individual values of T_d computed from equation (10) in seven colors and for the case that $T_3 = 14{,}500$ K, $w_3 = 0.02$, $w_d = 0.48$. The dashed × is the value of T_d computed for the 1920 Å region from equation (10).

lower (upper) curve of the two represents the result derived from seven (eight) light curves excluding (including) the one obtained in the 1920 Å region. It is plotted as the solid (broken line). Obviously, the least value of the standard deviation, i.e., the minimum point in the curve, corresponds to the most consistent determination. Thus the seven-color determination gives w_x equal to about 0.49, which corresponds to a mean temperature $T_x = 11{,}950$ K, while the eight-color determination gives $w_x = 0.51$, which corresponds to a mean $T_x = 11{,}780$ K. Figure 2 illustrates the individual values of the temperature derived from the ratio $(d_2/d_1)_{\lambda_i}$ at different wavelengths, λ_i, for $w_x = 0.49$. For our present rough estimate the difference between the two sets of values (w_x, T_x) in seven and eight colors is too small to be significant. However, the minimum value of the standard deviation is considerably greater in the case of the eight-color determination indicating indeed that the 1920 Å region is distorted by some unusual phenomenon and therefore departs greatly from what would be expected from the Planck function.

The derived temperature of about 12,000 K for the eclipsed portion of the disk is much higher than what was originally derived for the secondary component by Struve (1958). His result was obtained by assuming that no opaque disk surrounds the secondary component. If secondary eclipse should be simply due to the obscuration of the secondary component itself by the primary component, we would have to set $w_x = 1$, and the light curve in the visual region would indicate that the secondary component is an A star. However, if we should set $w_x = 1$ and try to derive temperatures T_x from equation (6) from the light curves of seven colors as we have done before, we would find that the standard deviation is 646 K, which is way above the corresponding value of 129 K that is derived for the case of $w_x = 0.49$. This is a great increase in internal inconsistency. The individual temperatures T_x derived for the case $w_x = 1$ from various values of $(d_2/d_1)_{\lambda_i}$ are also given in Figure 2 by the line marked "one component," $w_x = 1$. Here we see that light curves observed in widely separated regions of wavelength confirm the assumption that the secondary object that is eclipsing and is being eclipsed during primary and secondary eclipses, respectively, is not an ordinary star (which requires w_x to be close to 1) but is a disk whose outer layer is tenuous. Since the OAO results for the light variation of β Lyrae were published, it has often been said that these light curves are abnormal. By the present analysis we have removed this idea of abnormality.

We can now discuss the physical nature of the disk luminosity. First, we observe that if it should be the result of illumination by the secondary component, our result would have led to a disk that is observable

TABLE 2

DILUTION FACTORS AND RATIO OF LUMINOSITIES

$q = m_2/m_1$	WILSON (1974) i	a	b_1	r_1	DERIVED HERE b	l_d/l_1	w_1
2.	90°	0.52	0.21	0.35	0.21	0.54	0.13
3.	89°	0.54	0.16	0.31	0.16	0.54	0.11
4.	85°	0.56	0.17	0.29	0.18	0.72	0.11
5.	85°	0.57	0.15	0.27	0.16	0.75	0.10
6.	85°	0.57	0.15	0.26	0.16	0.81	0.09

directly, whether the radiative process in the disk is LTE or monochromatic scattering. This can be easily seen by comparing the relative observable luminosity of the disk with respect to that of the primary component. The ratio of the projected area of the disk to that of the primary component being ab/r_1^2, the relative (bolometric) luminosity of the disk, l_d, with respect to that of the primary, l_1, is approximately equal to

$$\frac{l_d}{l_1} = \frac{w_x ab}{r_1^2} \frac{T_x^4}{T_1^4} . \tag{7}$$

In Table 2 we have listed l_d/l_1 computed according to equation (7) for $w_x = 0.49$, $T_x = 11{,}950$ K and for five values of q. Wilson has assumed as a model for the disk an ellipsoid of revolution and has given the semimajor axis, a, and the semiminor axis (denoted here by b_1) as well as the inclination, i, for each value of q. While the ellipse projected on the sky has the same semimajor axis as the ellipsoid, its semiminor axis, which has already been denoted by b, is related to a, b_1, and i by the following relation

$$b^2 = b_1^2 \sin^2 i + a^2 \cos^2 i . \tag{8}$$

It may be noted that although b has to be derived from b_1 and i, it is more accurately determined than b_1 and i. In other words, one can propose models with different combinations of b_1 and i. If such models should fit the light curve well, the final value of b will always come out approximately the same (for the same value of a). This is because the light curve is basically determined by the projected ellipse. Hence a and b are much better determined than b_1 and i in Wilson's or any other similar models.

We have computed l_d/l_1 according to equation (7). In every case l_d/l_1 is greater than 0.5. It follows that we should see the radiation coming from the disk. That we do not discern any absorption spectrum from the disk even during primary eclipse is a point that must be answered if we should take the secondary star as the sole source of illumination of the disk. The problem is currently being studied by us.

Let us consider electromagnetic radiation from the primary component as the sole source of the disk luminosity (the index $x = 1$). If such is the case, w_1 cannot have a value of 0.49. This can be easily seen by calculating the dilution factor of primary radiation at the disk from the geometrical configuration of the binary system. Considering the area on the disk surface that is nearest to the primary component, we have the conventional dilution factor given approximately by

$$w_1 = \frac{\pi r_1^2}{4\pi(1 - a)^2} \tag{9}$$

which may slightly underestimate the actual value in the present case. In Table 2 we give the values of w_1 for different solutions given by Wilson (1974). It shows that in no case does w_1 reach 0.49. Since w_1 as given by equation (9) has been computed for the point on the disk closest to the primary component, the dilution factor at other points should be less than this value. Hence the disk luminosity derived from our analysis cannot be explained by the illumination of electromagnetic radiation from the primary star.

V. TWO-COMPONENT ANALYSIS AND ITS INTERPRETATION

Let us next examine the two-component analysis. In this case we have

$$\left(\frac{d_2}{d_1}\right)_{\lambda_i} = \frac{w_y B(\lambda_i, T_y) + w_z B(\lambda_i, T_z)}{B(\lambda_i, T_1)} . \tag{10}$$

By using the subscripts y and z, we do not commit ourselves as regards the radiative process in the disk. For example, in the scattering case, z may be set to be 1. If LTE prevails in the disk, y is set to be d, and z may denote a hot spot, say $z = 3$. In any case, equation (10) means that in the area of the disk on which primary radiation impinges there are two components of radiation given by the two terms in the numerator.

If scattering dominates the radiative process, we know for sure that there exists a component of primary radiation; hence $z = 1$. We have seen from Table 2 that w_1 is of the order of magnitude of 0.1 but nevertheless have computed three cases with $w_1 =$ 0.04, 0.08, and 0.12. T_1 is set at the same value (11,350 K) as before. The same procedure that applies to w_x and T_x in the previous section can now be applied to w_y and T_y for each value of $(d_2/d_1)_{\lambda_i}$ in Table 1 and for a given value w_y. Thus, in the same way, we have computed the average T_y and respective standard deviations of individual temperatures from this average for various assumed values of w_y (for both

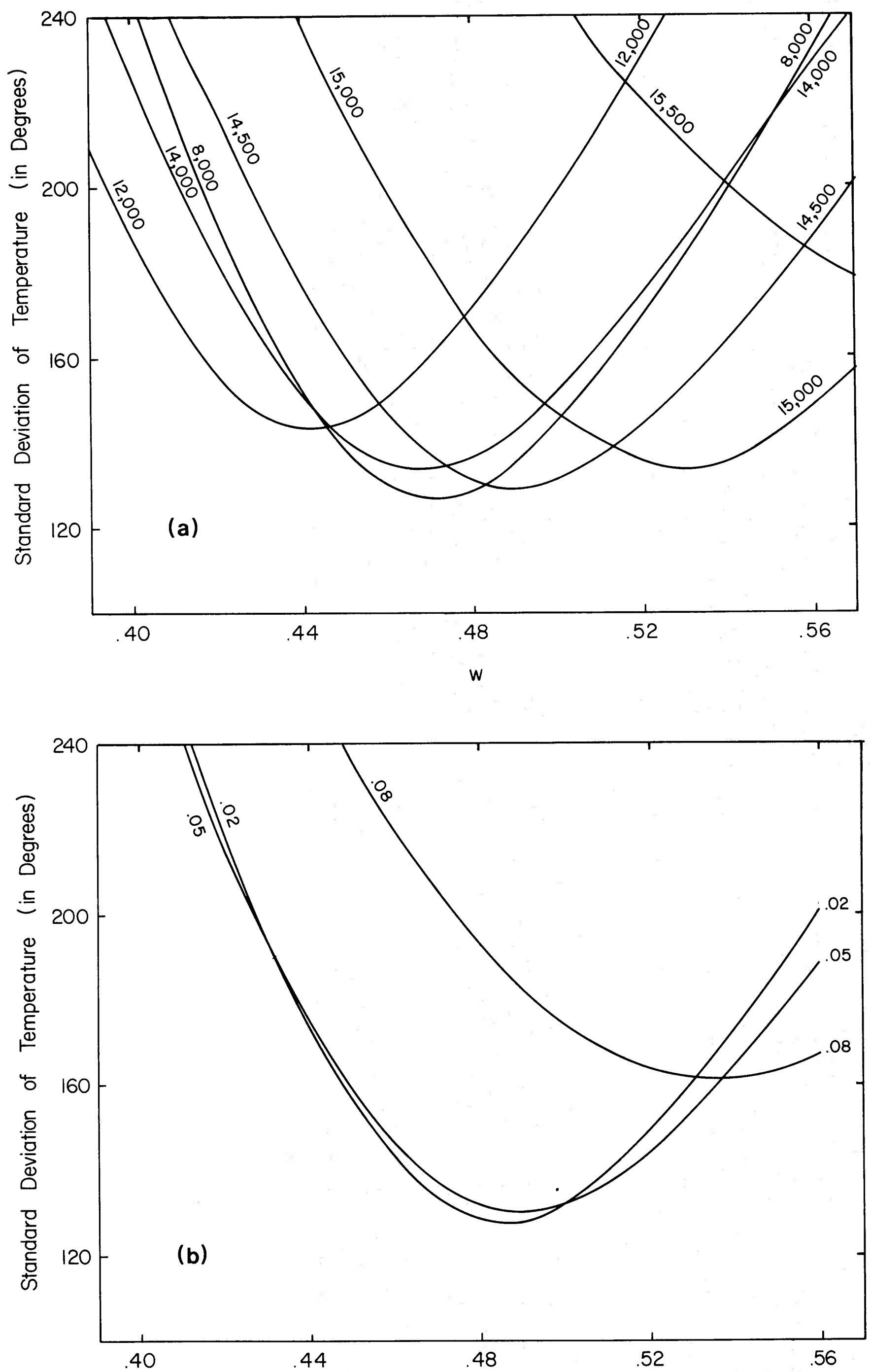

FIG. 3.—High Temperature Source. (*a*) Plotted is the standard deviation of T_d as a function of w_d for T_d calculated in seven colors from equation (10). The six curves correspond to six choices of T_3 (8000 K, 12,000 K, 14,000 K, 14,500 K, 15,000 K, 15,500 K) and are labeled as such. For all cases $w_3 = 0.05$. (*b*) The standard deviation of T_d, where T_d is computed from equation (10) for seven colors, is plotted against w_d for $T_3 = 14,500$ K and the three cases $w_3 = 0.02, 0.05, 0.08$.

7 and 8 colors). In Figure 1 we have illustrated the variation of rms with w_y, the curves (marked "two-components") being labeled by three values of w_1 (0.04, 0.08, and 0.12). Only the results derived from seven colors are shown here. Since the eight-color curves show consistently higher inconsistency (i.e., larger standard deviations), we consider the seven-color curves to be a better representation. Our results show that the average temperatures of the best internal consistency (i.e., the lowest standard deviation) for different values of w_1 (0.04, 0.08, 0.12) are all close to the value of 12,000 K derived from the one-component theory. As can be seen from Figure 1, the standard deviation becomes greater and greater (internal consistency worse and worse) as w_1 increases from zero (which corresponds to the one-component analysis) upward. Thus, judged simply by the standard deviation curves, the one-component theory is the best. On the other hand, we know that the primary component must contribute some radiation to the disk. If scattering should be important, we would observe radiation from the primary and, consequently, expect a considerable improvement in the internal consistency (i.e., lowering of the standard deviation) by introducing the primary radiation into the analysis. That this does not happen leads us to suggest that LTE instead of scattering dominates the radiative process in the disk.

Next we consider the case of LTE, for which $y = d$. Let us label $z = 3$ for a hot or cold spot. Our problem is to investigate whether or not there is a component of radiation arising from a hot or cold spot. For this purpose we have analyzed T_d in the same way as before for different values of w_d according to equation (10) by assuming different values of T_3 from 6000 K up to 16,000 K and $w_3 = 0.05$ for all cases. We have found that for T_3 below 8000 K the standard deviation curves plotted against w_d are all close together and approach the result of the one-component analysis. This indicates that we are working mainly in the ultraviolet region and cannot discern the presence of any component corresponding to temperatures below 6000 K. In order to find out whether a low-temperature component (corresponding to a cold spot) exists, we will have to wait for observations in the infrared region.

In the high-temperature region we have found some interesting results which are illustrated in Figure 3*a*. When T_3 increases from 8000 K, the standard deviation curve in Figure 3*a* moves up and to the left indicating a worsening of the internal consistency. However, at about 12,000 K the minimum of the standard deviation curves reverses the trend. It moves down as T_3 increases until $T_3 = 14{,}500$ K, at which time it changes the trend and increases again. This would seem to show that there is a trace of radiation corresponding to a temperature of about 14,500 K in the disk.

Next we attempted to determine the value of w_3 corresponding to this component of radiation. In order to do so, we set $T_3 = 14{,}500$ K and analyzed the standard deviations of T_d as a function of w_d for different values of w_3. Our results for $w_3 = 0.02$, 0.05, 0.08 are shown in Figure 3*b*. It appears that the standard deviation curves have the lowest minimum somewhere near $w_3 = 0.02$. In Figure 2 we have plotted the values of T_d found from the various $(d_2/d_1)_{\lambda_i}$ values given in Table 1 for $T_3 = 14{,}500$ K, $w_3 = 0.02$, and $w_d = 0.48$, the approximate value of w_d for which there is the highest internal consistency. Hence, if this high-temperature component is not caused by observational errors, it indicates indeed a hot spot or a shock front. But whatever it is, it occupies only a small fraction in the eclipsed area ΔA.

VI. CONCLUSION

From the relative depths of primary and secondary eclipses in different colors from the visual to the far-ultraviolet region it has been found that the effective temperature of that part of the disk that it responsible for secondary eclipse is of the order of 12,000 K. It has also been found that there is no significant trace of radiation that belongs to the temperature of the primary star. It appears therefore that the radiative process in the disk is close to LTE, because from geometry we know a certain amount of primary radiation must be impinging on the disk. On the other hand, it appears from the available data, which could use improvement, that there is a trace of radiation corresponding to a temperature of 14,500 K in the eclipsed area of the disk. If this is not caused by observational errors, it may point to the presence of a small hot spot in the eclipsed area of the disk. What the brightness is outside the eclipsed area of the disk has not been examined in this study. Also, whether a component of radiation corresponding to a low temperature below 6000 K is present cannot be ascertained with the present data and requires observation in the infrared region.

Finally, it should be noted that the data in the far-ultraviolet region used in the calculations are provisional at best. Our analysis by using equation (4) or (5) instead of equation (1) is also crude, even though the result seems to be quite reasonable, as is indicated by the smallness of the minimum standard deviations shown in Figures 1 and 3 compared with the average temperature. Eventual improvement in observational data may require a more accurate analysis based on a theory of radiative transfer in the disk which at present is still lacking. However, the basic idea presented in this paper of utilizing equation (1) as a means for understanding eclipsing stars will become even more important than ever with the improvement in both observation and the theory of radiative transfer.

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Changing T_1 does not affect the general behavior shown in Figure 1. All curves in the figure will simply shift with minor variations up or down as the case may be when T_1 changes. For example, we obtain the minimum of curves at rms = 143°, $w_x = 0.49$ (corresponding to $T_x = 1.06T_1$) by setting $T_1 = 12{,}000$ K; and rms = 115°, $w_x = 0.49$ (corresponding to $T_x = 1.05T_1$) by setting $T_1 = 10{,}700$ K versus rms = 130° and $w_x = 0.49$ (corresponding to $T_x = 1.05T_1$) for the case $T_1 = 11{,}350$ K. The changes in both w_x and T_x/T_1 are very small when T_1 varies from 10,700 K to 12,000 K. (All cases are derived from the seven-colored data.)

Regretfully, we have only one set of observational data for d_2/d_1. However, if an error of 10 percent is arbitrarily introduced in any of the seven values of (d_2/d_1) it creates only some small changes in the behavior of curves in Figure 1.

Also a change of T_1 does not remove the small high-temperature component that is derived from Figure 3, even though we still consider this component as very tentative, because, as has been mentioned in the text, it could be due to observational errors in d_2/d_1.

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