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$^{12}C/^{13}C$ RATIOS IN STARS ASCENDING THE GIANT BRANCH THE FIRST TIME

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ABSTRACT

Stellar evolution calculations were carried out for 1 $M_\odot$ and 2 $M_\odot$ red giants in order to understand the observed $^{12}C/^{13}C$ ratios. Although very low values of 5 to 7 in some red supergiants can be understood in terms of thermally unstable helium shell burning models, there remains a problem of moderately low values (10 to 20) in stars not luminous enough for this process. In addition to the standard mixing at the base of the giant branch, which can give $^{12}C/^{13}C$ ratios in the region of 25 to 30, we consider: variation in initial composition; meridional mixing; mass loss; zero-age composition gradient; mixing during the helium flash; and instability of the hydrogen-burning shell. Of these, only the mass loss mechanism and the hydrogen shell instability produce values below ~20.

Subject headings: stars: abundances — stars: late-type — stars: mass loss — stars: evolution

I. INTRODUCTION

The purpose of this paper is to examine possible explanations for observed $^{12}C/^{13}C$ ratios in red giants. Recent observations (Thompson and Johnson 1974; Geballe *et al.* 1972; Tomkin and Lambert 1974; Tomkin *et al.* 1975; Dearborn *et al.* 1975) of $^{12}C/^{13}C$ ratios in K and M subgiants should be a quantitative measure of the amount of mixing of surface material with interior material which has been nuclearly processed via CNO tri-cycle reactions. We will primarily consider the stars observed by Tomkin *et al.* (1975) and Dearborn *et al.* (1975) because they have applied the same technique to a large number of red giants and subgiants. They divide the observed stars (Table 1) into three groups according to luminosity.

The three subgiants of the first observational group ($\log L < 1.1$) have $^{12}C/^{13}C$ ratios of 50, 30, and 51 $\pm$ 5. Their low luminosities suggest they have not evolved very much, so these ratios may represent zero age values, although they are lower than the solar and terrestrial values of 89. There is the possibility that ^{13}C in the interstellar medium has been enriched since the Sun's formation (Audouze and Lequeux 1974; Bertojo *et al.* 1974; Wannier *et al.* 1975). However, these stars are probably older than the Sun. Alternatively their $^{12}C/^{13}C$ ratios may reflect inhomogeneities in the interstellar medium: we shall assume that a range of 40–90 is reasonable for initial stellar $^{12}C/^{13}C$ ratios. As a third alternative they may have already experienced some mixing of zero-age material with nuclearly processed material.

The seven members of the second group of red giants ($1.1 < \log L < 1.6$) are presumably more evolved than the first group, but are not luminous enough to be burning helium. They should be ascending the giant branch for the first time, in a shell hydrogen-burning configuration. The average $^{12}C/^{13}C$ ratio in this group is 22, with a range from 12 to 34. The two stars significantly below 20, ι Cep and θ Cet, have log luminosities above 1.5; with possible errors in distance they may actually belong to the third group.

The third group of red giants ($1.6 \le \log L < 3$), with 21 members, presumably contains stars ascending the giant branch for both the first and the second (post–helium flash) time. A typical $^{12}C/^{13}C$ ratio for this group is ~14, with a range from 6.5 to 25.

It is difficult to determine the stage of evolution of a field red giant with $\log L > 1.6$. It may be burning hydrogen only (in a shell), hydrogen in a shell and helium in the core, or both hydrogen and helium in shells. In stars with two burning shells and with $\log L > 3$ it is possible to understand very low $^{12}C/^{13}C$ ratios (Iben 1975; Scalo *et al.* 1975). For example, in the models of Scalo *et al.* (1975) the surface convection zone in a high-luminosity star ($\log L \approx 3.5$ for $M = 1.5\,M_\odot$) can have a base temperature high enough to allow the CNO tri-cycle to occur. This sort of mixing would eventually change the CNO material in the envelope from its initial value to an equilibrium value with $^{12}C/^{13}C = 3.5$ and $^{12}C/^{14}C = 40$ to 200, depending on the exact base temperature reached. Iben (1975) has followed a 7 $M_\odot$ star through 10 double shell flashes and found that during a flash the intershell region became convective. After the flash

TABLE 1

STELLAR OBSERVATIONS

Star	Temperature (K)	Log L	$^{12}C/^{13}C$	Source
δ Eri.........	4870	0.51	>50	1
η Cep.........	4850	0.96	>30	1
ν^2 CMa.......	4800	1.01	51 ± 5	1
γ Cep.........	4750	1.14	24 ± 3	1
53 Eri........	4550	1.46	34 ± 4	1
ι Cep.........	4680	1.54	16 ± 2	1
46 LMi.......	4700	1.55	22 ± 4	2
θ Cet.........	4700	1.56	12 ± 2	1
ε Vir.........	4950	1.59	20 ± 3	2
λ Sgr.........	4650	1.60	22 ± 2	1
ξ Dra.........	4470	1.61	20 ± 2	1
α Ari.........	4440	1.61	19 ± 3	2
β Cet.........	4800	1.65	19 ± 2	2
ε Cyg.........	4750	1.65	11.5 ± 1.5	2
β Gem........	4750	1.67	16 ± 2	3
α Ser.........	4420	1.73	14 ± 2	2
ρ Boo........	4250	1.93	15 ± 2	1
δ Sgr.........	4150	1.94	18 ± 2	1
μ Leo.........	4460	2.00	18 ± 2	3
δ And........	4250	2.11	25 ± 2	1
α Boo........	4165	2.26	7.2 ± 1.5	4
γ Leo A......	4300	2.39	6.5 ± 1	2
γ Dra.........	3780	2.42	13 ± 2	2
α Hya........	4100	2.50	18 ± 2	2
α Tau.........	3790	2.68	9 ± 1	2
γ Sge.........	3780	2.73	13 ± 3	2
ζ Cyg.........	4950	2.97	22 ± 3	2
ζ Cep.........	4700	3.45	17 ± 4	2
ε Peg.........	4100	3.88	5.1 ± 0.5	5
α Sco.........	3600	4.42	12 ± 2	6
α Ori.........	3500	4.78	7 ± 1.5	7

SOURCE.—(1) Searborn *et al.* 1975. (2) Tomkin *et al.* 1974. (3) Tomkin and Lambert 1974. (4) Day *et al.* 1973. (5) Lambert and Tomkin 1974. (6) Lambert 1974. (7) Lambert *et al.* 1974.

the intershell convection stops, and material from the outer regions of this zone is mixed out to the outer convection zone. This would bring carbon as well as *s*-process material from the intershell region to the surface. Again the high temperatures at the base of the surface convection zone will allow CNO processing eventually to reduce the $^{12}C/^{13}C$ down to near 3.5. The 7 $M_\odot$ star of Iben's calculations had a luminosity of $\log L \approx 4.0$. A calculation by Gingold (1974) for a very low mass star (0.6 $M_\odot$) finds that in this case the double shell flashes begin at $\log L \approx 3.0$. It is therefore possible to understand low $^{12}C/^{13}C$ ratios in double shell source models with $\log L > 3$, at least provided that such low values go along with enhanced carbon and *s*-process elements relative to iron. This paper will therefore consider the implications of low $^{12}C/^{13}C$ ratios in stars that have not yet reached the double shell flashing stage.

In these less evolved red giants the $^{12}C/^{13}C$ ratio can certainly be reduced from a zero-age value of ~90 (or perhaps ~50) to ~25 by the following process (Iben 1967). Nuclear processing of CNO material forms a local ^{13}C peak at the outer edge of the hydrogen-burning core during main-sequence evolution. When core hydrogen is exhausted, the star moves to the giant branch, developing a deep convective envelope which then mixes the ^{13}C peak to the surface. But as the star evolves further up the giant branch, the convective envelope retreats so that no further enrichment should take place, at least until this evolution is terminated at the helium flash (at $\log L \approx 3.3$ [Eggleton 1967, 1970]), and perhaps until the double-shell-source mixing process takes place, at $\log L \approx 3$. The problem we have to consider is therefore how red

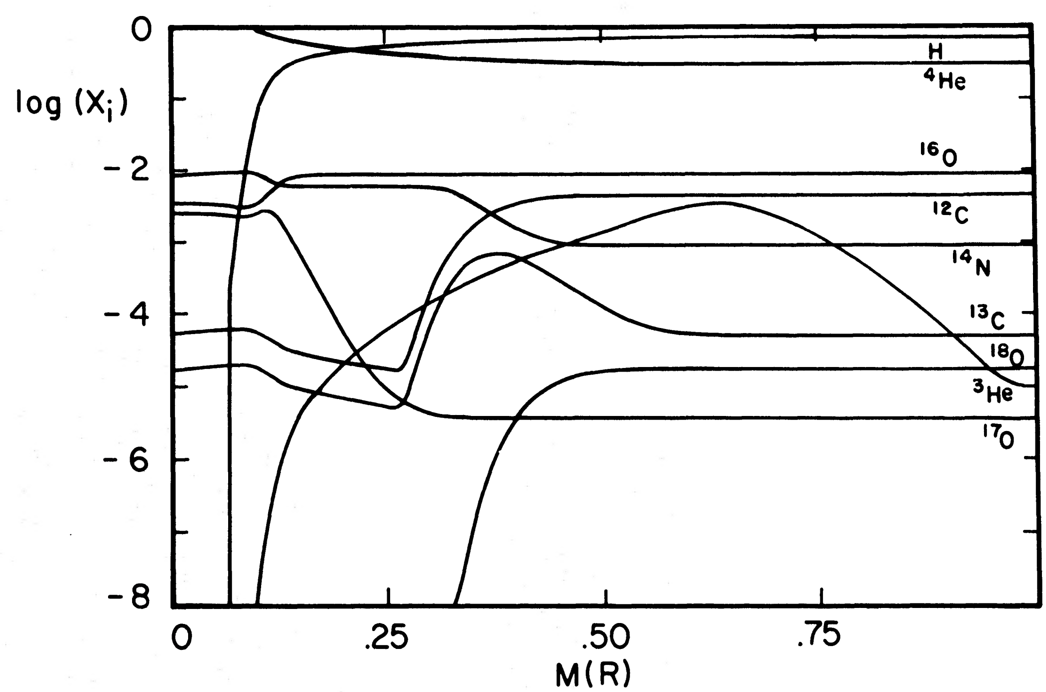

FIG. 1.—Composition of a 1 $M_\odot$ star near the end of its main-sequence evolution

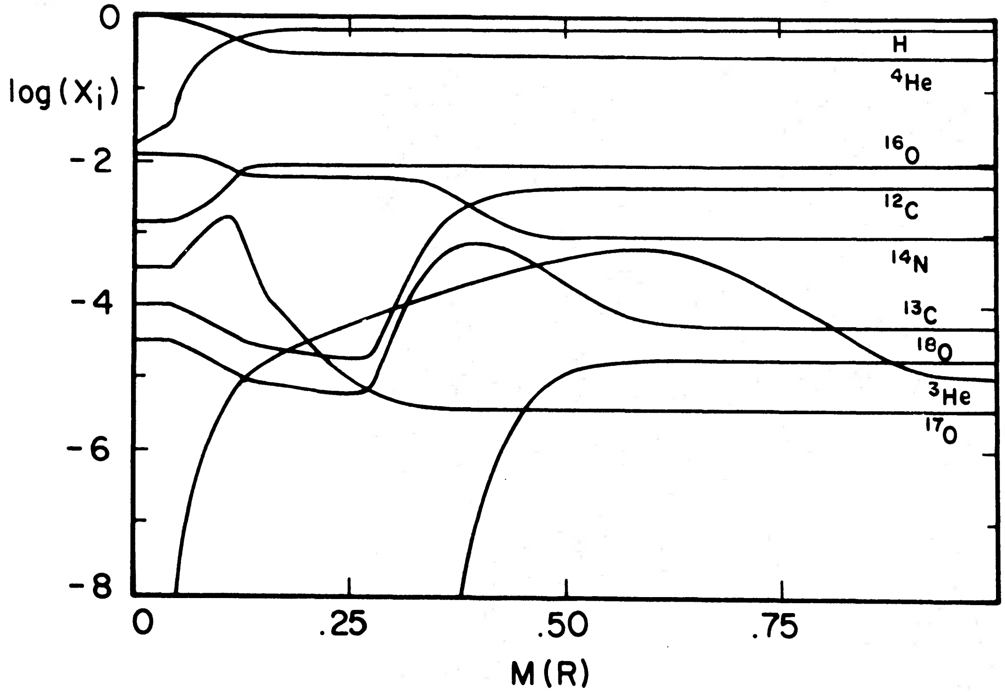

FIG. 2.—Composition of a 2 $M_\odot$ star near the end of its main-sequence evolution

giants in the luminosity range $1 < \log L < 3$, and more especially in the lower half of this range, can have a $^{12}C/^{13}C$ ratio significantly less than 25. We illustrate the problem with calculations of the evolution of 1 $M_\odot$ and 2 $M_\odot$ stars up to the helium flash, explicitly following the stable CNO isotopes. Our evolutionary tracks are essentially identical with other 1 and 2 $M_\odot$ models (cf. Iben 1967). Thus discussions regarding our standard model can be taken to be generalizable. In § II we discuss the standard enrichment of ^{13}C in the envelopes of red giants that burn hydrogen only. In §§ III–VIII we discuss methods by which the $^{12}C/^{13}C$ ratio may be additionally changed.

II. STANDARD CALCULATIONS

While a star is on the main sequence, there is a temperature region where, during the main-sequence life of the star, a substantial portion of the ^{12}C can capture a proton and increase the ^{13}C abundance via $^{12}C(p, \gamma)^{13}N(\beta^+\nu)^{13}C$. Slightly deeper in the star, the temperature is high enough [the $^{12}C(p, \gamma)^{13}N$ rate varies as T^{19} at the temperature where this occurs] that a substantial amount of ^{13}C is produced prior to being depleted to ^{14}N via $^{13}C(p, \gamma)^{14}N$. At higher temperatures, the rates are sufficiently rapid that the carbon is converted to nitrogen. A peak of ^{13}C is, therefore, formed while the star is on the main sequence (see Figs. 1 and 2). Such a peak is formed even in stars where the primary energy source is the *pp* chains. Outside this peak the material retains its original composition, while inside the peak the material rapidly attains CN equilibrium. In CN equilibrium for $\log T = 7.2$, $^{12}C/^{13}C \approx 3.4$ and $^{12}C/^{14}N \approx 1/250$; so while the $^{12}C/^{13}C$ ratio is low, almost all of the carbon has been converted to nitrogen. In the ^{13}C peak itself, the $^{12}C/^{13}C$ ratio reaches 2.5, and the carbon has not yet been fully depleted to nitrogen. The star holds this composition structure until deep convective mixing can bring the material to the surface. It is well known that as a star evolves to the giant branch, the atmosphere expands and the surface convection zone reaches deeper into the star. Point A on Figure 3 shows when the surface convection zones in a 1 $M_\odot$ and a 2 $M_\odot$ star reach the ^{13}C peak. By point B the ^{13}C peak is entirely mixed, and the $^{12}C/^{13}C$ ratio has reached its limiting value. The surface convection zone does go slightly deeper, but there is so little carbon that the surface $^{12}C/^{13}C$ ratio remains unaffected. However, mixing deeper than the peak does affect the $^{12}C/^{14}N$ ratio. By point C, the surface convection zone has reached its deepest penetration and begins to retreat as the hydrogen burning shell moves outward. At this point, the C-N-O isotope ratios have finished changing in the surface convection zone.

It is also interesting to follow the oxygen isotope ratios. The main-sequence temperature of the mass zone finally reached by the surface convection zone is not high enough for a great deal of ^{17}O to be processed through the NO parts of the CNO tri-cycle. The original $^{16}O/^{17}O$ ratios, however, are so large that it does not require much ^{16}O processed to ^{17}O in order to change the $^{16}O/^{17}O$ ratio significantly. The measurement of the $^{17}O(p, \alpha)^{14}N$ rate by Rolfs and Rodney (1974) showed that it is much slower than previously thought and thus significantly affects CNO processing (Dearborn and Schramm 1974). The amount of ^{16}O processed to ^{17}O and not processed further during the

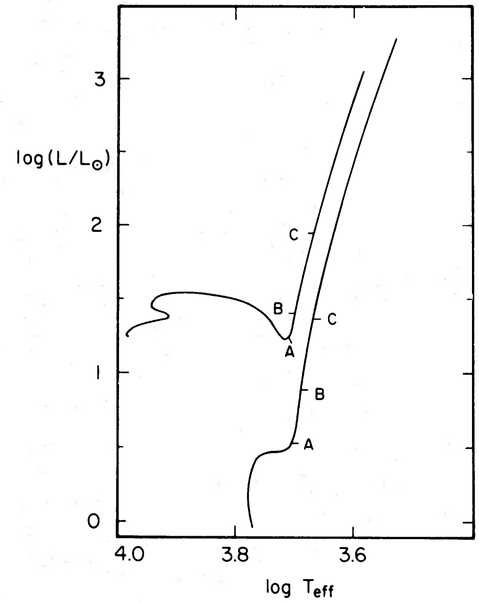

FIG. 3.—Evolutionary tracks for a 1 $M_\odot$ and a 2 $M_\odot$ star. Point A is where the $^{12}C/^{13}C$ ratio begins to change; point B is where the $^{12}C/^{13}C$ ratio achieves its final value; and point C is where the surface convection zone reaches its maximum depth.

main-sequence life appears to be sensitive to mass. If the surface convection zone mixes deeply enough, it will be able to mix sufficient amounts of ^{17}O to the surface and affect the observed $^{16}O/^{17}O$ ratio. The 2 $M_\odot$ star decreased its $^{16}O/^{17}O$ ratio from 3160 to 817, while in the 1 $M_\odot$ only enough ^{17}O was mixed to change the ratio to 2840. The ^{18}O is rapidly burned to ^{15}N through a (p, α) reaction. The $^{16}O/^{18}O$ ratios increased from 560 to 930 for the 2 $M_\odot$ star and went to 750 for the 1 $M_\odot$ star. So in the 2 $M_\odot$, the ^{17}O has become more abundant than ^{18}O. This suggests that in stars where only this simple mixing appears to have taken place, the $^{16}O/^{17}O$ ratio may be an interesting mass indicator.

The resultant $^{12}C/^{13}C$ ratio from this deep mixing gives 25 for 2 $M_\odot$ and 28 for 1 $M_\odot$ for an initial $^{12}C/^{13}C$ ratio of 90. Calculations of Dearborn *et al.* (1974) indicate that the $^{12}C/^{13}C$ for a 0.75 $M_\odot$ star will be 31, and the figure for the composition of a 5 $M_\odot$ star by Iben (1967) indicates a final ratio of 25. The range 25 to 30 is consistent with some of the observed stars such as δ And and γ Cep; but as can be seen from Table 1, there are several stars with $\log L/L_\odot \leq 2.5$ with significantly lower $^{12}C/^{13}C$. In the next section we will discuss some possible mechanisms for changing the resultant $^{12}C/^{13}C$ ratio.

III. VARIATION IN INITIAL COMPOSITION

As was mentioned above, there is likely to be a range of initial $^{12}C/^{13}C$ values from 40 to 90 due to evolution of the interstellar medium since the formation of the Sun. If this is correct, then red giants with masses greater than 1.5 $M_\odot$ would have started with a lower

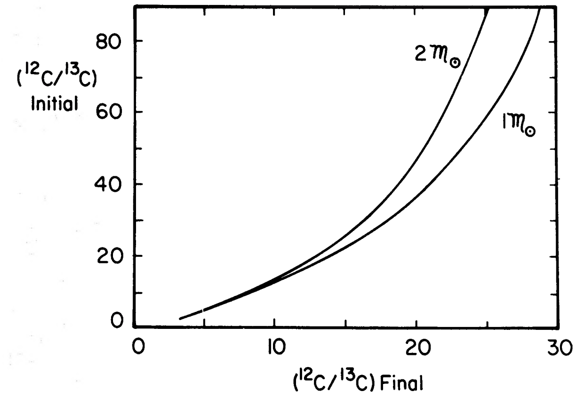

FIG. 4.—The resulting $^{12}C/^{13}C$ ratio for different initial values.

$^{12}C/^{13}C$ ratio. It is also possible that the observations from which Audouze and Lequeux claim that the interstellar medium is now enriched in ^{13}C by a factor of 2.3 are actually observations of the inhomogeneity of the interstellar medium. If that were the case, then even red giants slightly less massive than the Sun could have initial $^{12}C/^{13}C$ ratio less than 90.

The predicted range of $^{12}C/^{13}C$ for deep mixing on the first ascent of the giant branch was 25–30 for an initial value of 90. Allowing the initial value to vary down to 40 will obviously increase the range of the resulting $^{12}C/^{13}C$ ratios.

The change in the $^{12}C/^{13}C$ ratio is caused by enrichment of ^{13}C from mixing out the ^{13}C peak. Using a stellar model of the temperature and density, it is possible to calculate the position and magnitude of the ^{13}C peak without calculating a complete evolutionary sequence of models. The depth to which the mixing goes is known from the standard calculation with $^{12}C/^{13}C = 90$. By mixing out the ^{13}C peak, the $^{12}C/^{13}C$ ratio can be determined for any initial values given in a 1 $M_\odot$ and 2 $M_\odot$ star (see Fig. 4). For an initial ratio of 40, resultant $^{12}C/^{13}C$ ratios of 21 for the 1 $M_\odot$ and 19 for the 2 $M_\odot$ model are obtained. It would thus seem reasonable to expect from the variation in the initial value of the $^{12}C/^{13}C$ ratio a range in the resultant post-mixing ratio of 19–30. In order to have a resultant ratio of 10–13, initial ratios of 12–20 would be required. Observations of molecular lines (especially CO) in interstellar clouds would appear to exclude these low values (Wannier *et al.* 1975; Bertojo *et al.* 1974).

IV. MERIDIONAL MIXING

Meridional mixing or Eddington-Vogt circulation is usually not considered in a stellar evolution calculation because in order to have a significant effect on the composition, the star must be rapidly rotating. According to Sweet (1950) the time scale for meridional mixing is

$$\tau_{mm} = R/V_{mm} \approx \tau_{KH}/\bar{X}$$

where R is the stellar radius; V_{mm}, the velocity of meridional mixing; τ_{KH} is the Kelvin-Helmholtz time scale; and $\bar{X}$ is the mean ratio of the centrifugal to gravitational force. In early A or B stars, the rotation velocity is often large enough that τ_{mm} from the above equation is less than the main-sequence lifetime, τ_{ms}. Mestel (1953) showed, however, that inhomogeneities resulting from meridional mixing reduce the circulation velocity such that even for most early main-sequence stars $\tau_{mm} > \tau_{ms}$, so the effect of meridional mixing is insignificant.

The temperature outside the convective core of a 2 $M_{\odot}$ star is 16×10^6 K. Since meridional mixing is inhibited by even a small change in the mean molecular weight (Fricke and Kippenhahn 1972) it will not, operate across the composition discontinuity at the edge of the convective core. If $\tau_{mm} > \tau_{ms}$, meridional mixing does not have enough effect on the composition structure to affect the resulting CN isotope ratios.

If meridional mixing is operating with $\tau_{mm} < \tau_{ms}$, the ^{12}C in the envelope will be processed to ^{14}N, and the envelope will be converted to CN equilibrium while the star is on the main sequence ($^{12}C/^{13}C \approx 3.5$; $^{12}C/^{14}N \approx 1/200$). Paczynski (1973) has studied the effects of meridional mixing on the $^{12}C/^{14}N$ ratio in greater detail. He found that in stars less massive than 1.5 $M_{\odot}$ the temperature outside the convective core was too low to process a significant fraction of the ^{12}C; while in stars of more than 5 $M_{\odot}$ the convective cores were so large that the temperatures outside were again too low. He found that stars from 3 to 5 $M_{\odot}$ had temperatures outside the cores sufficient to nuclearly process carbon. In such stars, however, the rotation must be very fast because at the temperatures outside the core a composition gradient sufficient to stop meridional mixing can build up rapidly. Paczynski found that if meridional mixing were operating, a $^{12}C/^{14}N$ ratio of 1/200 would be expected.

If the rotational velocity is such that $\tau_{mm} \approx \tau_{ms}$, it is possible to obtain values of the $^{12}C/^{13}C$ ratio between 25 and 3.5 (for stars of 2–5 $M_{\odot}$). In this case, only part of the envelope is processed through the deep-burning region to CN equilibrium. The ^{13}C peak will be deformed (moved farther out near the pole and farther in near the equator), but the total material in the ^{13}C peak should remain the same. Using this, the effect of partial mixing while on the main sequence can be estimated by converting a given fraction of the envelope to CN equilibrium and then mixing the ^{13}C peak. The results of such a calculation are given in Table 2.

Initially this appears to be an attractive method for changing the $^{12}C/^{13}C$ ratio. There is, however, a strong velocity dependence. The fraction of the envelope mixed is proportional to $1/\tau_{mm} \propto V^2$ (rotation). Therefore, while a star that is rotating fast enough to process 20 percent of the envelope changes the $^{12}C/^{13}C$ ratio only from 25 to 22, a star rotating 2.25 times faster processes all of the envelope and results in $^{12}C/^{13}C = 3.5$, $^{12}C/^{14}N = 1/200$.

TABLE 2

MERIDIONAL MIXING

Percent of Envelope Processed	$^{12}C/^{13}C$	$^{12}C/^{14}N$
0	25	1.6
20	22	1.0
50	16	0.5
80	8	0.2
100	3.5	0.005

Another problem with meridional mixing is the effect on the $^{12}C/^{14}N$ ratio. Greene (1969) has observed $^{12}C/^{14}N$ ratios in three of the stars in Table 1. They are α Boo (7.2, 4.7), α Ser (14, 3.1), and β Gem (16, 3.1), where the first number in parentheses is the $^{12}C/^{13}C$ ratio and the second number is the $^{12}C/^{14}C$ ratio. From Table 2 it can be seen that α Ser and β Gem should show $^{12}C/^{14}N$ ratios more like 0.5 and α Boo more like 0.2. Meridional mixing appears to require more ^{14}N enhancement than is observed.

If the Am star phenomena is a diffusion effect as is believed, it is evident that meridional mixing is not operating in ~20 percent of the stars between A0 and F1 (Smith 1973). It should not, however, be discounted in all A and B type stars. Some Ae and Be stars are rotating near breakup velocity, and meridional mixing may well be converting their envelopes to CN equilibrium with $^{12}C/^{13}C \approx 3.5$ and $^{12}C/^{14}N \approx 1/200$.

Admittedly our estimates of meridional mixing effects are crude, and more work on circulation current is certainly merited. However, we do feel that the points mentioned are at least qualitatively correct.

V. MASS LOSS

According to Iben (1967) it is necessary to assume mass loss during the hydrogen-burning phase in order to account for horizontal-branch stars. Rood (1973) found that a mass loss of 0.2 $M_{\odot}$ with a dispersion of several hundredths of a solar mass was necessary to fit the observed variation of the horizontal branch on a color-magnitude diagram. Observations of horizontal-branch stars are, of course, pertinent mainly to Population II stars, but there is no reason to believe that Population I stars do not also lose mass. In fact, observations (cf. Woolf 1973) clearly show that large amounts of mass loss are occurring in some red giants.

Using typical properties for the solar wind given by Axford (1968), it can be calculated that the Sun itself is losing 10^{-13} $M_{\odot}$ yr^{-1}. Arcturus and β Gem have been observed to have coronae by Gerola *et al.* (1974) and Moos *et al.* (1974). Using a corona temperature of 260,000 K, they calculate a mass loss rate of 2×10^{-10} $M_{\odot}$ yr^{-1} for β Gem, but the mass loss could be greater if the temperature they used were in error.

Stars develop their ^{13}C peak during their early life on the main sequence. In a 2 $M_{\odot}$ star, the peak has already formed after $\sim 3 \times 10^7$ years. If mass is lost during the later stages of main-sequence development or while moving to the giant branch, the size of the ^{13}C peak is unaffected. There will, however, be less ^{12}C-rich material to mix with the ^{13}C when the star reaches point A on the giant branch in Figure 3. This

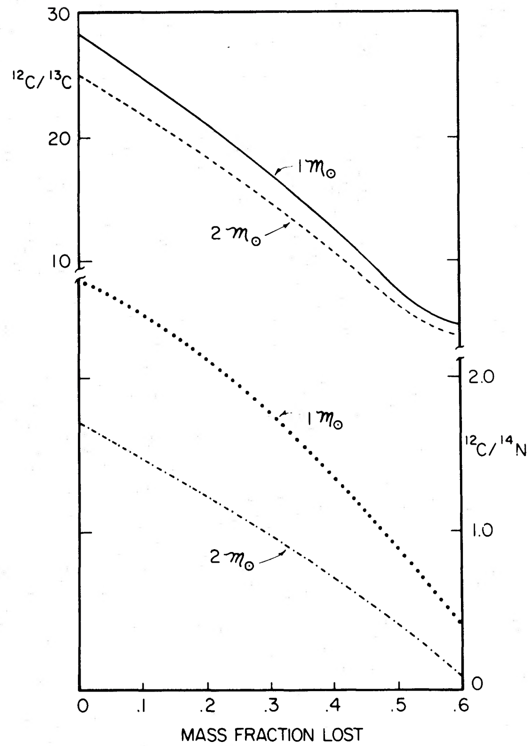

FIG. 5.—$^{12}C/^{13}C$ and $^{12}C/^{14}N$ ratio versus the fraction of the mass of the star lost before deep mixing. The ratios are shown for both 1 and 2 $M_\odot$ stars.

results in a lower $^{12}C/^{13}C$ ratio. Figure 5 shows the resultant $^{12}C/^{13}C$ ratio (assuming an initial value of 90) for a given fraction of the star having been lost.

In order to affect the CN isotope ratios, the mass must be removed before the ^{13}C peak is mixed out. Removing 15 percent of the mass of a star before this point (point A on Fig. 3) reduces the $^{12}C/^{13}C$ ratios of the 1 $M_\odot$ star from 28 to 23, while it reduces those of the 2 $M_\odot$ star from 25 to 20. Removing 15 percent of the mass of a star before point A requires mass loss rates for 1 $M_\odot$ star of 1.6×10^{-11} $M_\odot$ yr^{-1} from the zero-age main sequence, or 3.3×10^{-10} $M_\odot$ yr^{-1} from the time that the surface convection zone begins to descend into the star. Similarly, a rate of 1.3×10^{-8} $M_\odot$ yr^{-1} beginning when the surface convection zone starts to descend is required to remove 15 percent of the mass of a 2 $M_\odot$ star. It is a simple matter to determine the maximum amount of mass loss that can be supported by the kinetic energy flux of the surface convection zone from

$$\text{KE flux} = \tfrac{1}{2}\,\text{mV}^2 V/l = \tfrac{1}{2}V^3\rho \times \text{Area}\,,$$

or

$$\dot{M} \leq 2\pi R^3 \rho \bar{V}^3/GM\,,$$

where V is the convective velocity, M is the total mass, R is the radius, and ρ the mean density of the convective region. The kinetic energy flux was found sufficient to support a mass loss rate an order of magnitude higher than those stated above.

That the surface convection zone contains sufficient kinetic energy flux for a sizable amount of mass loss does not prove that the energy is being used in this way; but the energy is there. It is encouraging that two of the stars listed with $^{12}C/^{13}C \approx 19$ (β Gem and α Boo have been observed to have a corona, so they are probably losing mass). A 2 $M_\odot$ star with a low initial $^{12}C/^{13}C$ ratio ($\sim$40) losing 15 percent of its mass would reduce its $^{12}C/^{13}C$ ratio down to 17. In order to reduce the $^{12}C/^{13}C$ ratio to 9, a 2 $M_\odot$ star must lose $\sim$50 percent of its mass regardless of the initial $^{12}C/^{13}C$ ratio. While this much mass loss may be a bit excessive in a pre–helium-flash star, it demonstrates that for very low $^{12}C/^{13}C$ ratios, the initial $^{12}C/^{13}C$ ratio is of little significance. Ultraviolet observations of dwarfs and subgiants are needed to place limits on their coronae and thus get a limit on their mass loss rates. Mass loss appears to be a good mechanism for lowering the $^{12}C/^{13}C$ ratio, but it requires the mass loss prior to mixing the ^{13}C peak out (during the subgiant phase). Most mass loss mechanisms proposed have mass loss increasing with luminosity. If this is correct, these stars would lose more mass after the ^{13}C peak is mixed than before. This would seem to require an extremely large (possibly excessive) amount of mass loss for the first ascent of the giant branch.

VI. COMPOSITION GRADIENT

If the carbon abundance decreased with increasing radius, the ^{13}C peak formed at the higher abundance would have more effect when mixed to the surface. Such a composition gradient is, however, in the opposite direction to that required to lower the solar neutrino rate; and if chemically fractionated material were accreted after the Hayashi convection phase, the heavy-element abundance would be larger on the surface, and not smaller.

VII. THERMAL INSTABILITIES IN HYDROGEN-BURNING SHELLS

If thick hydrogen-burning shells surrounding degenerate cores are unstable as suggested by Bolton and Eggleton (1973), the $^{12}C/^{13}C$ ratio could be substantially reduced. As the shell moves slowly out, it forms a ^{13}C peak in front of it. After the pulse, the material outside the burning shell would expand and cool. This could allow the surface convection zone to extend down and sweep out the ^{13}C peak. When the material recollapsed and heated, the surface convection zone would retreat back out, and a new ^{13}C peak could form. Since the $^{12}C/^{13}C$ ratio is lower than on the main sequence, and the temperature structure is steeper, a ^{13}C peak produced on the giant branch would be much smaller than that formed on the main sequence. The results of such mixing were studied by Dearborn *et al.* (1974), and it was found that 50 mixings were required to reduce the $^{12}C/^{13}C$ ratio from 30 to 10 for a 0.75 $M_\odot$ star. This instability has only been studied for Population II compositions, but it is believed that it could operate in Population I stars. This mechanism would, however, apply to any instability which could induce mixing on the giant branch.

It should be pointed out that the actual mechanism of Dearborn *et al.* (1974) has not occurred as a natural consequence of stellar evolution but was somewhat artificially induced. Similarly the ratio of the flash to quiescent times was arbitrary. Thus, although such H flashes produce interesting results it is not at all clear that they actually occur to the extent needed, if at all.

VIII. MIXING DURING HELIUM FLASH

Schwarzschild and Härm (1964, 1966) studied a 1 $M_{\odot}$ star at helium flash using a hydrostatic code. They found the helium flash to be a hydrostatic event in which the degeneracy was removed, and the core became convective to within two density scale heights. Later Edwards (1970), using one of Schwarzschild and Härm's models with a hydrodynamic code and a special description of time-dependent convection, found the event to be dynamic, resulting in a complete mixing of the star. Rood (1970) constructed models for stars which mixed the core with a substantial fraction of the envelope. He found that such models produced a carbon-rich subgiant branch. The stars then evolved to a second helium ignition (not a helium flash) and moved to the horizontal branch. The models of Rood are remarkably like the CH subgiants observed by Bond (1974). If it is possible that carbon-rich material from the core is mixed with the envelope during helium flash, the $^{12}C/^{13}C$ ratios for stars more luminous than $\log(L/L_{\odot}) = 1.6$ could change drastically. The results of such mixing would depend critically on how long the ^{12}C-rich triple-alpha material spent in a hot proton-rich region before being mixed out. If it spent too long in a burning region, the main result would be to enrich the nitrogen. Even in this case, the material may have enough carbon to affect the $^{12}C/^{13}C$ ratio. There would also be an enhancement of *s*-process material due to the neutrons produced from $^{17}O(\alpha, n)^{20}Ne$; $^{13}C(\alpha, n)^{16}O$; $^{14}N(\alpha, \gamma)^{18}O(\alpha, \gamma)^{22}Ne(\alpha, n)^{25}Mg$. This again agrees with the CH subgiants observed by Bond. It is not clear whether stars with low $^{12}C/^{13}C$ ratios and $1.6 \leq \log(L/L_{\odot}) < 3.0$ are pre– or post–helium flash. If they are pre–helium flash, one must consider the scenarios described in earlier sections. If they are post–helium flash, some degree of mixing may have occurred. This possibility has not been explored in detail in the present work due to the difficulties in following calculations through the helium flash. However, it should be noted that physical models of the helium flash constructed under the "standard" assumptions make mixing appear unlikely (Schwarzschild and Härm 1966; Demarque and Mengel 1971), though not impossible.

IX. CONCLUSION

The standard calculation with no mass loss and a solar composition does not fit the observed range of $^{12}C/^{13}C$ ratios in red giants (see Table 1). It predicts values ranging only from 25 to 30. Variation of the initial value of the $^{12}C/^{13}C$ ratio within reasonable limits extends the range of obtainable values down to 19. A final ratio of 13, however, demands an initial ratio of $\sim$20 which would seem excluded by observations of the interstellar medium. Meridional mixing may reduce the $^{12}C/^{13}C$ ratio in stars of 3 to 5 $M_{\odot}$ if they are rotating rapidly enough to overcome their composition gradient. The results of meridional mixing are strongly dependent on the rotational velocity. If a star is rotating only fast enough to process 20 percent of the envelope, there is only a slight change in the $^{12}C/^{13}C$ ratio; while if it is rotating at a little over twice that speed, the entire atmosphere is converted to CN equilibrium with $^{12}C/^{13}C \approx 3.5$ and $^{12}C/^{14}N \approx 1/200$. In partial meridional mixing, the ^{13}C peak near the poles is farther out than near the equator. In order to mix out the entire ^{13}C peak, substantial amounts of ^{14}N must be mixed out. Meridional mixing then appears to decrease the $^{12}C/^{14}N$ ratio excessively. Mass loss prior to mixing out the ^{13}C peak will allow the ^{13}C-rich material to mix with less ^{12}C-rich material, so a lower $^{12}C/^{13}C$ is produced. Since $^{12}C/^{14}N$ is a function of how far past the ^{13}C peak one mixes, the resultant value depends on the mixing length used. If a star mixes out its ^{13}C peak but does not mix into the ^{14}N below, the $^{12}C/^{13}C$ ratio is lowered and the $^{12}C/^{14}N$ ratio is relatively unaffected.

If stars achieve the low $^{12}C/^{13}C$ ratios on their first ascent of the giant branch, one of the above scenarios should be considered. If the low $^{12}C/^{13}C$ ratios can be attributed only to post–helium-flash stars, however, the possibility of core material mixing with the envelope should be considered. Because of the difficult problems associated with evolving a star through the helium flash, this possibility has not been followed through.

A combination of two of the effects mentioned above should also be considered. A 2 $M_{\odot}$ star with an initial $^{12}C/^{13}C$ ratio of 40 which loses 20 to 30 percent of its mass will have resulting $^{12}C/^{13}C$ ratios of 15 to 13 and $^{12}C/^{14}N$ of 2.0 to 1.7. Such low $^{12}C/^{13}C$ ratios account for all of the stars in Table 1 with the exceptions of α Tau, α Boo, and γ Leo A. Arcturus has a very low $^{12}C/^{13}C$ ratio (7.2 ± 1.5), but the high $^{12}C/^{14}N$ ratio of 4.7 measured by Greene would seem inconsistent with meridional mixing. Perhaps a more reasonable possibility is a combination of very substantial mass loss and a low-mass star in which the ^{13}C peak was just mixed out. The $^{12}C/^{14}N$ ratio could then remain high. Since α Boo and γ Leo A are metal-deficient with respect to the Sun, it is possible to subject them to hydrogen shell instabilities as discussed by Bolton and Eggleton (1973). If this instability does occur, it could reduce the $^{12}C/^{13}C$ ratio in a manner described by Dearborn *et al.* (1974). It would be of extreme interest to observe the $^{12}C/^{13}C$ ratios in a cluster or moving group. The original $^{12}C/^{13}C$ ratios could then be used to infer the amount of mass loss prior to mixing the ^{13}C peak. By choosing a cluster with a turnoff mass less than 1.5 $M_{\odot}$, any possible effects of meridional mixing could be ignored.

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