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SCATTERING BY DUST AND THE PHOTOGRAPHIC APPEARANCE OF ETA CARINAE

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ABSTRACT

We have attempted to construct simple isotropic-scattering models which reproduce the apparent surface brightness of the "halo" of η Carinae, as a function of projected radius; unfortunately, the most applicable observational data are now about 30 years old, and improved observations are highly desirable.

Subject headings: circumstellar shells — luminous stars — nebulae, individual — stars, individual

I. INTRODUCTION

At visual wavelengths, η Carinae seems to consist of a small (but not quite stellar) "nucleus" or "core," surrounded by an expanding "halo" or "shell" whose present size is of the order of 10″—corresponding to about 3×10^{17} cm at a likely distance of 2 kpc. This appearance has been described by Thackeray (1949, 1950, 1953), Gaviola (1950), Ringuelet (1958), and Gehrz and Ney (1972). The spectrum and polarization of the halo suggest that it is essentially a scattering nebula, illuminated by the central object (Thackeray 1956, 1961; Visvanathan 1967; Craine 1974). The far-infrared luminosity discovered by Westphal and Neugebauer (1969) almost surely indicates that the halo is optically thick at visual wavelengths. Most of the visual and ultraviolet radiation from the central object is evidently absorbed by dust grains in the halo and reradiated at wavelengths around 20 μ; only a small fraction of the visual photons escape, either directly or after some scattering by the dust grains (see Pagel 1969*b*; Burbidge and Stein 1970; Gehrz *et al.* 1973).

Additional plausibility is lent to this general picture by the fact that the observed far-infrared luminosity could have been "predicted" fairly well from certain visual and near-infrared data, under the rather naïve assumption that the central object is just an early-type star surrounded by some photoionized gas (Davidson 1971). The most likely model involves a very massive star (mass $\gtrsim 50\ M_\odot$, to provide sufficient luminosity) which seems to have an effective surface temperature around 30,000 K. The star is noticeably above the main sequence; or alternatively, it may be hotter than 30,000 K but surrounded by an extended envelope which lowers the effective temperature of the emergent radiation. In either case, Lyman-continuum opacity supplements electron scattering to bring the stellar atmosphere—or the outer layers of the extended envelope—precariously close to the Eddington limit, where outward radiation pressure would counteract gravity. It is easy to suspect that this situation may not be very stable; so occasional outbursts, like that which ejected the present "halo" during the nineteenth century, are not particularly surprising at this rather naïve level of analysis. The dusty absorbing/scattering halo, which is expanding at several hundred km s^{-1}, may consist of numerous unresolved filaments or condensations; it is noteworthy that the dust grains must have condensed rather quickly, probably no later than the years 1858–1868 (about 20 years after the major outburst), when the visual brightness of the object was observed to fade by about 7 mag.

In this paper we consider the scattering of light in the "scattering nebula" or "halo" or "dust shell." Our intent is to construct a simple, spherically symmetric model of a scattering shell which, as seen from outside, approximately reproduces the observed distribution of visual-wavelength surface brightness as a function of projected radius. In § II we describe our simplifying assumptions and treatment of the scattering problem; in § III we describe some results.

II. THE SIMPLIFIED SCATTERING PROBLEM

The halo of η Carinae is somewhat elongated, with several large condensations; but at least within a diameter of 5″ or 6″ a spherically symmetric model should be adequate for our purposes. We shall assume that dust is distributed spherically between inner radius $r = R_1$ and outer radius $r = R_2$, with radially dependent opacity $\kappa(r)$ (scattering plus absorption, per unit distance, at some observed wavelength). If the dust shell consists of small filaments or condensations, the above assumption requires that these condensations must shadow each other so that the configuration scatters light in roughly the same manner as a continuous shell. We suppose the dust-grain albedo ϖ (= scattering/total opacity, at the wavelength of interest) to be constant throughout the configuration. Also, our simplified method requires that we pretend that the scattering is isotropic. Since real dust grains can cause preferentially forward scattering, perhaps our calculations should be regarded as referring to a limiting case.

Our calculative method is somewhat different from most treatments of related problems (e.g., Code 1973 and references therein; Jones 1973). Given $\kappa(r)$ and ϖ, we divide the region $R_1 \leq r \leq R_2$ into a number of shells, each shell being optically and geometrically thin. For simplicity, we assign to each shell i a uniform opacity κ_i, representing the average of $\kappa(r)$ for that shell. Now, suppose that a photon has been scattered at some radius r, into some particular direction. It is not difficult to calculate the probability that the photon will experience its next scattering or absorption event in shell number i. Isotropically averaging over all directions, and also averaging over values of r in shell number j, we find the probability P_{ij} that a scattering event in shell j will be directly followed by absorption or scattering in shell i. The probability of escape is of course

$$P_j^{\text{escape}} = 1 - \sum_i P_{ij} .$$

We arbitrarily set the relevant photon production rate of the central source equal to unity. Let S_i denote the rate at which photons, coming directly from the central source, are absorbed or scattered in shell i. Let X_i be the total rate at which absorption and scattering events occur in shell i; this is related to the photon density in shell i. (The rate of *scattering* events is ϖX_i.) The equilibrium equation of transfer is

$$X_i = S_i + \sum_j \varpi P_{ij} X_j .$$

In the form

$$\sum_j (\delta_{ij} - \varpi P_{ij}) X_j = S_i ,$$

this equation is readily solved for X_i by inverting the matrix $(\delta_{ij} - \varpi P_{ij})$.

The effective extinction factor or emergent fraction Φ is

$$\Phi = e^{-\tau} + \sum_i \varpi P_i^{\text{escape}} X_i ,$$

where $\tau = \int \kappa(r) dr$ is the total optical depth of the configuration. The first term represents light which escapes directly without being scattered, and which is seen from outside as the central core.

The radiation density in each shell can be found from X_i; then $I(r_{\text{proj}})$, the apparent surface brightness seen from outside as a function of projected radius, is straightforwardly calculable. In the calculations mentioned below, 50–60 thin shells were used for configurations with $R_2/R_1 \approx 6$ and $\tau \approx 4$.

III. THE HALO OF ETA CARINAE

Gaviola (1950) published an isophote map of η Carinae, derived from a set of photographs whose exposure times differed by successive factors of 2.

TABLE 1

"EQUIVALENT PROJECTED RADII" OF GAVIOLA'S ISOPHOTES

Isophote Number	Surface Brightness $I(r_{\text{proj}})$ (arbitrary units)	Projected Radius r_{proj} (arcsec)
1...........	9.4	0.53
2...........	5.4	1.02
3...........	3.06	1.50
4...........	1.75	1.98
5...........	1.00	2.27
6*..........	0.57	2.72
7...........	0.33	3.43

* Not including condensations c and d.

Gehrz and Ney (1972) have presented a more recent set of isophotes; but we shall consider only Gaviola's map, since its relative contour intervals are more easily estimated. (Note, however, that since the object has changed noticeably since the time of Gaviola's observations, our calculations should not be compared too closely with recent observations. A new isophote map, with a well-defined calibration at a well-chosen wavelength, should be made.)

Following Thackeray (1953), we assume that the inner nine isophote contours of Gaviola's map represent successive factors of about 1.75 in apparent surface brightness. Since the apparent halo is not exactly circular, we assigned equivalent projected radii according to the areas enclosed by the various isophotes: (enclosed area) = πr^2_{proj}. Adopted values are shown in Table 1 and Figure 1. The latter, showing the curve of $r^2_{\text{proj}} I(r_{\text{proj}})$ plotted against r_{proj}, indicates that much of the light comes from radii between 1″ and 2″. This curve has presumably been affected by finite angular resolution in two ways: (1) The "knee" at $r_{\text{proj}} = 2''$ may be slightly blurred, and (2) more

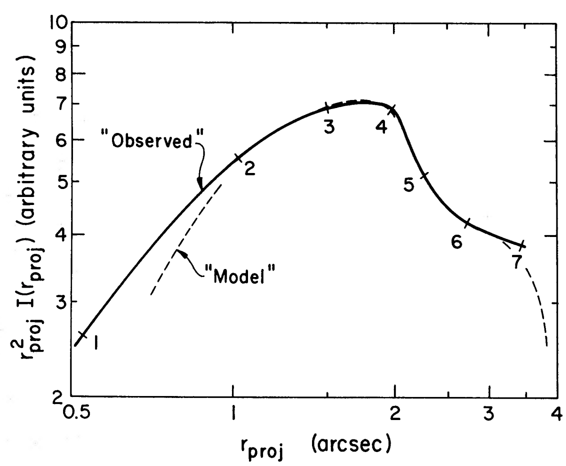

FIG. 1.—Surface brightness of the halo of η Carinae, as a function of projected radius, according to Gaviola (1950). Numbered marks on curve indicate Gaviola's isophotes. Results for the model described in the text are also shown.

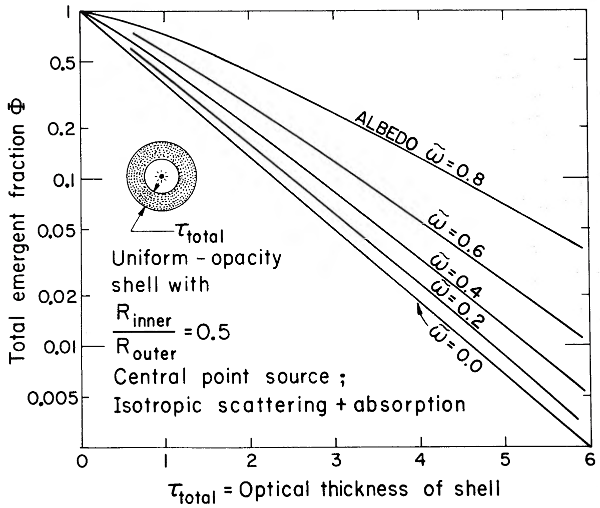

FIG. 2.—Emergent fraction Φ for a uniform dust shell illuminated by a central point source.

importantly, the region $r_{\text{proj}} < 1''$ includes the blurred image of the central core. In the following, we shall attempt to find a scattering model whose brightness distribution matches Figure 1 for $1'' \lesssim r_{\text{proj}} \lesssim 4''$, and whose central core brightness is in reasonable proportion to the total halo brightness. (No serious error should result from our placement of an imaginary, distinct outer boundary at a radius of 4″.)

We consider three aspects of the observations:

1. The total extinction in the halo—or equivalently, the emergent fraction Φ at bluish wavelengths—is roughly known. Partly from estimates based on various hydrogen and [Fe II] emission line intensities,[1] and partly from the consistency of such estimates with the observed far-infrared luminosity (see Pagel 1969*a*; Davidson 1971, and references therein), it appears that the total extinction around $\lambda \approx 5000$ Å is about 4 mag. Comparisons with other stars near η Carinae indicate that half (or slightly more) of this extinction is due to intervening dust clouds; the remainder occurs in the dusty halo. Unfortunately, we do not know how much the halo has changed since the time of Gaviola's observations; but we shall assume that at that time the effective extinction in the halo was at least 1.5 mag but not more than 3.5 mag. In other words, we adopt limits

$$0.04 < \Phi = \frac{\text{(total light emergent from halo)}}{\text{(light produced by central core)}} < 0.25\,.$$

2. As Thackeray (1953) emphasized, the halo appears brighter than the central core. If Φ is the total emergent fraction mentioned above, and $\Phi_{\text{core}} = e^{-\tau}$ is that portion of Φ which escapes directly without scattering (and therefore is seen as the core), then Gaviola's isophotes show that $\Phi_{\text{core}}/\Phi < 0.25$; allowing for a reasonable angular resolution, the actual value of this ratio is likely to be about 0.2.

3. Finally, Gaviola's isophotes should provide a fair representation of the shape of $I(r_{\text{proj}})$ for $1'' \lesssim r_{\text{proj}} \lesssim 3''$ or 4″.

Considerations (1) and (2) above provide some immediate information about possible albedos and optical depths. Figure 2 shows values of the emergent fraction Φ as a function of τ and ϖ, for the case of a uniform shell with $R_1/R_2 = 0.5$. A uniform filled sphere would give somewhat larger values of $\Phi(\tau, \varpi)$ while a very thin shell would give smaller values. However, these differences are not very great, and Figure 2 demonstrates the general situation fairly well. For "reasonable" albedos, $\varpi \lesssim 0.8$, the condition $0.04 < \Phi < 0.25$ indicates that $1.4 < \tau < 6$, and suggests that τ is likely to be close to 3. Another result follows from the condition $\Phi_{\text{core}}/\Phi < 0.25$: The curve for $\varpi = 0$ is just $\Phi = \Phi_{\text{core}} = e^{-\tau}$, so the required solution must be above this curve by a factor of at least 4; and for $\tau < 6$, this implies $\varpi > 0.6$.

Our calculated models confirm the above indications. We derived possible opacity functions $\kappa(r)$ by calculating series of models which fit the surface brightness curve in Figure 1 successively better, starting from the outside and working inward, readjusting $\kappa(r)$ model by model. Since all final models which fit the stated conditions are qualitatively similar, we shall describe only one. This model has $\varpi = 0.7$ and $\tau = 3.76$, with a relative opacity distribution as shown in Figure 3. The emergent fraction is $\Phi = 0.116$, and $\Phi_{\text{core}} = 0.0233$ so that $\Phi_{\text{core}}/\Phi = 0.20$. The form of $\kappa(r)$ for $r \gtrsim 3''$ is merely *assumed*, being too near our fictitious outer boundary. However, the steep gradient between 2″ and 3″ is presumably genuine if Gaviola's isophotes

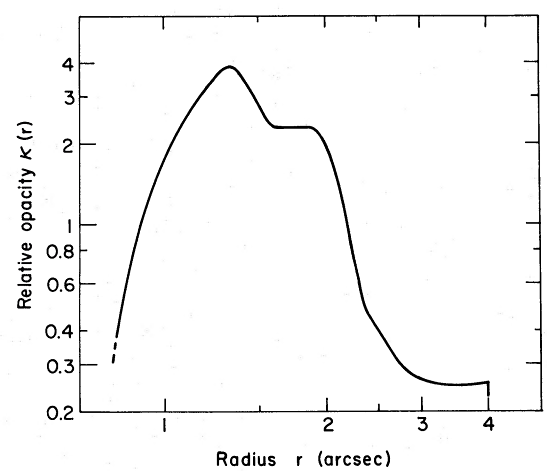

FIG. 3.—Relative dust or "opacity" distribution for one model which largely reproduces the surface brightness distribution of the halo of η Carinae. Required optical thickness and grain albedo are $\tau_{\text{total}} = 3.76$, $\varpi = 0.7$. As explained in the text, the "shelf" near the peak is not very significant.

[1] These lines probably originate in the stellar envelope (i.e., in the core) and/or near the inner edge of the halo. If the [Fe II] lines are excited indirectly through absorption of continuum photons by Fe II lines near 2400 Å (rather than by straightforward collisional processes), then Pagel's method of estimating the total reddening may be invalid. Rodgers (1971) found only a rather small amount of reddening from the [S II] lines; but the other methods agree on a larger value.

are correct. Between 1″ and 2″ the structure is not very well determined, since optical depths are rather large. The form of $\kappa(r)$ in Figure 3, with a narrow "plateau" just outside the maximum peak, results in a good match to the observed $r^2_{\text{proj}}I(r_{\text{proj}})$, as shown in Figure 1. However, a 20–40 percent modification of $\kappa(r)$, lowering and broadening the peak so that there is no separate "plateau," disturbs the shape of $r^2_{\text{proj}}I(r_{\text{proj}})$ by only 2 or 3 percent; thus, the double-shell structure suggested by Figure 3 is probably not significant. In the inner region, $r \lesssim 1''$, the slope of $\kappa(r)$ in Figure 3 is rather arbitrary; but there cannot be much additional opacity in this region, since the optical depth outside $r = 1''$ is sufficient to account for the likely amount of extinction.

To summarize: A scattering model for Gaviola's map of η Carinae, as discussed above, is likely to involve a dusty shell with optical depth $\tau \approx 3$ or 4, with inner and outer radii near 1″ and 2″.2 (roughly 3×10^{16} and 7×10^{16} cm). The only unexpected feature is the high dust-grain albedo, $\varpi \gtrsim 0.7$. (We were unable to construct a satisfactory model with $\varpi = 0.6$; for reasonable optical depths, the resulting core/halo brightness ratio was larger than observed.) *Some* of the surface brightness of the halo is actually due to forbidden lines emitted in the halo, rather than scattering (Thackeray 1956, 1961). But crude estimates of the total brightness of all forbidden lines (Visvanathan 1967) suggest that such emission is unlikely to account for more than perhaps 10 percent of the halo brightness—not enough to seriously affect the core/halo ratio. However, this conclusion is not observationally rigorous; and it is also conceivable that some of the observed halo brightness may be reflected light from sources outside the halo. If ϖ is indeed larger than 0.7, the dust grains may be of an unusual type.

Isotropic-scattering results of the type discussed above may be used to make some guesses about comparable models in which grains cause preferentially forward scattering. Following the example of Werner and Salpeter (1969), we can suppose that for some purposes, a forward-scattering event is practically equivalent to no photon deflection at all. In other words, if g is the scattering "phase factor" (= average cosine of scattering angle), then we pretend that we have isotropic scattering with a cross section equal to $(1 - g)$ times the actual scattering cross section. The actual optical depth and albedo are related to the "isotropic approximation" optical depth and albedo by

$$(1 - g\varpi_{\text{actual}})\tau_{\text{actual}} = \tau$$

and

$$\varpi_{\text{actual}} = \frac{\varpi}{1 - g + g\varpi}\,.$$

Comparing with calculations by Mathis (1972) for forward-scattering grains, one can verify that Figure 2 gives approximately correct results for such a case, provided that the isotropic parts τ, ϖ are used rather than τ_{actual}, ϖ_{actual}. Results for the intensity distribution $I(r_{\text{proj}})$ are more doubtful, although this too might be represented fairly well by an isotropic-scattering approximation, provided that the optical depth is sufficiently large.

Finally, we remark that before constructing improved models, for example with forward-scattering grains, better observations should be made. Remembering that Gaviola's observations were made in 1944 and 1945, we suspect that with modern equipment it would be possible to make calibrated isophote maps of η Carinae at well-defined wavelengths dominated by pure continuum light, by Balmer line emission, and by [Fe II] lines. Angular resolution superior to Gaviola's, while desirable, is not absolutely necessary. Furthermore, as infrared measurements improve, it may become possible to relate scattering models to infrared dust-emission models resembling those described by Apruzese (1975).

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