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OBSERVABLE EFFECTS OF DUST ON THE IONIZATION OF NEBULAE*

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ABSTRACT

The effects of dust on the ionization and thermal structure of a photoionized nebula are explored by numerical model calculations. Particular emphasis is placed on the calculation of observable quantities such as the radio and infrared continua and the optical emission-line spectrum and the effects that dust might have on these quantities. The role of the dust in the nebular structure is summarized. Model calculations of planetary nebulae are shown to agree very well with much of the radio, infrared, and optical data. The importance of dust in the case of H II regions is called into question, especially as an explanation for the low abundance of ionized helium observed in several nebulae.

Subject headings: nebulae — planetary nebulae — spectra, infrared — spectra, radio

I. INTRODUCTION

The possible effects of dust on the ionization structure and emission properties of photoionized nebulae has been studied recently with renewed interest. Observations of several H II regions, especially the giant H II regions toward the galactic center, have been found to be anomalously deficient in He I recombination lines (e.g., Churchwell *et al.* 1974). Optical studies of planetary nebulae indicate a strong enhancement of the forbidden lines of low excitation such as those of [N II], [O II], and [S II] relative to theoretical predictions (e.g., Miller 1974). A possible explanation consisting of the preferential absorption of energetic photons by dust embedded within these nebulae has been proposed and studied in recent papers by Mathis (1971), Panagia (1974), Mezger *et al.* (1974), and Petrosian and Dana (1975).

In these theoretical papers the relationship between the dust and the hydrogen and helium ionization was explored. Aside from the infrared luminosity and the abundance of ionized helium relative to that of ionized hydrogen, the effects of dust on the various observational parameters at radio, infrared, and optical wavelengths were not discussed in detail. Consequently it has been difficult to effect quantitative measurements of dusty and "normal" ionized nebulae for the purpose of studying the possible role of the dust or investigating the ultraviolet absorption properties generally attributed to it.

In this paper the effects of dust on the experimentally measurable parameters are emphasized. A numerical approach is used because of the complexity of the equations which describe the ionization and thermal structure as well as the emission processes; also, fewer assumptions for the sake of analytic simplicity are required. In addition, studies of observable parameters under a wider range of physical conditions can be explored in this manner. For example, the single greatest uncertainty in this problem is the unknown absorption cross section of the dust, σ_d, above the Lyman limit; the numerical approach makes it possible to explore several different types of frequency dependence, and to study the effects of these different cross sections on the observable quantities.[1]

We begin by a brief discussion of the numerical method and review the important qualitative effects of the dust on the nebular structure. Next, the effects of dust on the observables are presented—mostly graphically—and discussed. Finally, we explore the viability of the dust as an explanation for the often mentioned observational problems. Specifically, we show that the observations of two planetary nebulae NGC 7027 and IC 418 can be well fitted by the model calculations of dusty nebulae. We also explore the possibility that dust can explain the anomalously low abundances of ionized helium observed in some H II regions.

II. THE MODEL

The model calculations were made using a computer program similar to that described by Hjellming (1966) and modified to include the effects of dust. Briefly described, the program integrates the optical depths for dust and for hydrogen and helium (at their respective ionization thresholds) radially from the stellar surface. A modified Runge-Kutta method is used to integrate in a stepwise fashion over the radial interval $r - \delta r$ to r until the hydrogen

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[1] It must be emphasized that we are investigating primarily the properties and consequences of some agent (assumed to be dust) required to explain the nebular observations. The creation, destruction, and composition of the "dust" lie beyond the scope of this paper.

becomes neutral or the available ionizing flux drops below a value where ionization equilibrium can safely be assumed. From the optical depths and the externally specified density distribution $N(r)$ and abundances, the flux of stellar ultraviolet radiation at r and the degrees of ionization x of H, He, and the trace elements can be computed. Then the thermal balance equation is solved iteratively; in the process the permitted and forbidden lines emissivities are calculated (using the full five-level calculations for the latter). Integrals over frequency are done using the 10-point Gaussian integration scheme within each of the ranges with endpoints $h\nu_0$ (ionization potential of H^0), 1.8 $h\nu_0$ (I.P. of He^0), 4 $h\nu_0$ (I.P. of He^+), and 9 $h\nu_0$. The program was run on a 7600 computer, and each model requires $\sim$1 second for execution.

The absorption cross section σ_d for the dust in the ultraviolet was taken to be completely unknown and thus arbitrarily specified. Five different models for the frequency dependence of σ_d were considered, and the dust content was scaled in magnitude by four powers of $\sqrt{10}$. Including the dust-free case, 21 models were run for each density distribution specified.

The various frequency dependences of the absorption cross sections are shown in Table 1. For convenience these are all scaled to the cross section $\sigma_d(\nu_0)$ predicted by the Mie theory at the Lyman edge ($\nu = \nu_0$) in the "linear" regime $2\pi a/\lambda \sim 1$, where a is the particle size. Since the effective dust particle sizes in H II regions are thought to be on the order of 1000 Å or less (Panagia 1974; Mezger *et al.* 1974), the Mie theory can be expected to suffice for estimating the magnitude of $\sigma_d(\nu_0)$. However, the exact functional form for σ_d is not known theoretically nor has it been measured experimentally, primarily because the composition of the dust is uncertain and because the requisite theoretical calculations must include complex quantum-mechanical effects. Often the dust in H II regions is taken to have properties similar to the dust in the interstellar medium (ISM) (Mezger *et al.* 1974; Pottasch 1974); however, arguments against this approach include the anomalously low strength of the 2200 Å resonance (Bless and Savage 1972) and the high value of selective to total extinction (Johnson 1968) toward the only visible stars which excite a dusty nebula (the Trapezium, or θ^1, stars in Orion). Other reasons have been discussed by Panagia (1974). Moreover, little has been published on the measured properties of any dust in the far-ultraviolet ($\lambda < 1000$ Å).

The functional forms of the absorption cross section include a "flat" model independent of ultraviolet frequency, two "linear" models which are proportional to frequency, and two "step" models which change discontinuously at $1.8\nu_0$ but are otherwise flat. One of the linear models is that predicted by the Mie theory, and the other has a steeper dependence with frequency. The "step" models must be considered totally ad hoc since no known or predicted resonances or other absorption features occur near λ500 Å. These models illustrate an extremum in which differences in ionization structure between hydrogen and helium are artificially enhanced. Mezger *et al.* have studied one such model and find that a ratio for σ_d of 1:7 at frequencies below and above $1.8\nu_0$ is required to explain the putative absence of ionized helium in the galactic center.[2]

Quantitatively, the absorption effects of dust are expressed by the optical depth of the dust,

$$\tau_d(r, \nu) = \int_{r_*}^{r} N_d(r')\sigma_d(\nu)dr' , \tag{1}$$

where N_d is the particle density of dust and r_* is the radius of the exciting star. A more convenient parametrization of the dust is obtained by replacing N_d by the more commonly used ratio of the mass of dust to the mass of gas, M_d/M_g, which is generally thought to be of the order of 0.01 in the ISM and in H II regions as well (Panagia 1974). It is easy to show that

$$N_d = 2.4 \times 10^{-25} \frac{M_d}{M_g}\frac{N_{\rm H}}{a^3}\ \mathrm{cm}^{-3} \quad (\mathrm{cgs})\,, \tag{2}$$

where $N_{\rm H}$ is the total density of hydrogen (neutral and ionized), a is the dust size, and a mass density of 2.3 g cm^{-3} is assumed for the dust. The cross section is scaled to the value predicted by Mie theory at $\nu = \nu_0$ for dielectric particles,[3]

$$\sigma_d^{\rm Mie}(\nu) = \pi a^2 Q_{\nu_0} \approx \pi a^2 \frac{16\,\pi a m}{3\lambda_0}\ \mathrm{cm}^{-2} \quad (\mathrm{cgs}) \tag{3}$$

$$= 2.63\frac{a^3}{\lambda_0} = 2.89 \times 10^{11} a^3 \frac{\nu}{\nu_0} \quad (m = 0.05) \tag{4}$$

[2] Brown and Lockman (1975) have detected ionized helium in Sgr B2 at 15 GHz since this paper was written. The ionized helium abundance is apparently "normal."

[3] The choice of dielectric particles (for which $Q \propto \lambda^{-m}$ where $m = 1$) rather than crystalline particles ($m = 2$) or blackbodies ($m = 0$) is consistent with Mie theory but is otherwise arbitrary and was chosen for computational convenience. There is some evidence that $m \approx 1$ in M42 (Werner *et al.* 1975).

TABLE 1

MODEL ABSORPTION CROSS SECTIONS FOR DUST

Model No.	Designation	$\sigma_d(\nu > \nu_0)/\sigma_d(\nu_0)$	Description
1........	Mie slope	ν/ν_0	Mie theory in limit $2\pi a/\lambda < 1$
2........	Steep slope	$2\nu/\nu_0$	Twice as steep as 1
3........	Small step	1:3	$\sigma_d(\nu < 1.8\nu_0) = \sigma_d(\nu_0)$, $\sigma_d(\nu \geq 1.8\nu_0) = 3\sigma_d(\nu_0)$
4........	Large step	1:10	Same as 1 except $\sigma_d(\nu \geq 1.8\nu_0) = 10\sigma_d(\nu_0)$
5........	Flat	1	$\sigma_d(\nu > \nu_0) = \sigma_d(\nu_0)$

(Kaplan and Pikel'ner 1970), where Q_{ν_0} is the radiation efficiency at $\nu = \nu_0$, $\lambda_0 = 912$ Å, and m is the imaginary part of the index of refraction. With the normalization the expression for the dust optical depth is independent of particle size (for dielectric particles only)

$$\tau_d(r, \nu_0) \approx 7.00 \times 10^{-20} \frac{M_d}{M_g} \int_0^r N_{\rm H}(r')dr' , \tag{5a}$$

and

$$\tau_d(r, \nu) = \tau_d(r, \nu_0)[\sigma_d(\nu)/\sigma_d(\nu_0)] . \tag{5b}$$

The functional forms for the term in brackets in equation (5b) used in the present calculations are listed in Table 1.

We consider now the changes δy in various quantities y across the integration integral δr. The photon flux $F(r, \nu)$ is given by

$$F(r, \nu) = F(r_*, \nu) \left(\frac{r_*}{r}\right)^2 \exp\left[-\tau_d(r, \nu) - \tau_g(r, \nu)\right] , \tag{6}$$

where $F(r_*)$ is the emergent photon flux at the stellar radius r_*, and τ_g is the gas optical depth which is nonzero for $\nu \geq \nu_0$. The total number of photons absorbed per second at frequency ν between $r - \delta r$ and r is then

$$\delta\mathscr{F}(r, \nu) = \mathscr{F}(r_*, \nu)\{\exp\left[-\tau(r - \delta r, \nu)\right] - \exp\left[-\tau(r, \nu)\right]\} , \tag{7a}$$

where

$$\int_{\nu_0}^{\infty} \mathscr{F}(r_*, \nu)d\nu = 4\pi r_*^2 \int_{\nu_0}^{\infty} F(r_*, \nu)d\nu \equiv \Phi \tag{7b}$$

at frequency ν. Here $\tau = \tau_d + \tau_g$, and Φ is the total stellar photon flux above the Lyman limit ($\nu > \nu_0$); $\Phi \approx 10^{49}$ photons s^{-1} for early Population I O stars which excite H II regions.

Since

$$\frac{d\mathscr{F}(r, \nu)}{dr} \equiv \frac{d\mathscr{F}_g(r, \nu)}{dr} + \frac{d\mathscr{F}_d(r, \nu)}{dr} = -\mathscr{F}(r, \nu)\left(\frac{d\tau_g}{dr} + \frac{d\tau_d}{dr}\right) , \tag{8}$$

we find the ratio of photons absorbed by gas and dust, respectively, to be

$$\delta\mathscr{F}_g(r, \nu)/\delta\mathscr{F}_d(r, \nu) \approx \delta\tau_g(r, \nu)/\delta\tau_d(r, \nu) , \tag{9}$$

where $\delta\mathscr{F} = \delta\mathscr{F}_g + \delta\mathscr{F}_d$. Equations (7) and (8), when combined, multiplied by the energy of the photons absorbed, and integrated over all relevant frequencies, give the luminosity absorbed by dust L_d (which is reemitted mostly in the infrared and optical regions), and by gas L_g (reemitted mostly in the optical and near-ultraviolet). Thus

$$\delta L_d = \int_0^{\infty} \left[h\nu\delta\mathscr{F}_d + \frac{1}{1.1}\left(\frac{3}{4}h\nu_0\right)\zeta\delta\mathscr{F}_g\right]d\nu \text{ ergs s}^{-1} , \tag{10a}$$

$$\delta L_g = \int_{\nu_0}^{\infty} \left[h\nu\delta\mathscr{F}_g - \frac{1}{1.1}\left(\frac{3}{4}h\nu_0\right)\zeta\delta\mathscr{F}_g\right]d\nu \text{ ergs s}^{-1} , \tag{10b}$$

where ζ is the fraction of recombinations of hydrogen which result in the emission of a Lα photon, given approximately by Panagia (1973) as

$$\zeta = \frac{1.0 + 1.35 \times 10^{-4}N(\mathrm{H}^+, r)}{1.5 + 1.35 \times 10^{-4}N(\mathrm{H}^+, r)} . \tag{11}$$

It is implicitly assumed that Lα is efficiently scattered, i.e., as many Lα photons are absorbed in any volume element by the dust as are emitted there by the gas. This assumption is not quite correct because Lα emission is weighted by $N(\mathrm{H}^+)N_e$ whereas Lα absorption is weighted by $N(\mathrm{H}^0)$. Moreover, we ignore the fact that a (relatively small) fraction of the Lα photons escape through the surface of the nebula; but the error is not large, and the Lα is generally only a small portion of the energy absorbed by the dust in any case for the types of models considered here.

For comparison to the observations we calculate the dust temperature at each point (by equating δL_d per dust particle to the energy reemitted in the infrared),

$$T_d(r) = \left[\frac{(\delta L_d/N_d\delta V)}{0.00318\, ma^3}\right]^{1/5} \quad (\text{cgs}) \tag{12a}$$

$$= \left[\frac{\delta L_d}{3.86 \times 10^{-28}(M_d/M_g)N_{\mathrm{H}}\delta V}\right]^{1/5} \quad (m = 0.05) \tag{12b}$$

(Kaplan and Pikel'ner 1968 as modified by Mezger *et al.* 1974), where the radiation efficiency factor Q_ν implicitly assumed is appropriate for dielectrics and is given by equation (3). The quantity δV is the volume element given by $4\pi r^2 \delta r$. We also calculate the infrared flux by summing the flux δS_ν from each volume element, where

$$\delta S_\nu = N_d \frac{4\pi a^2}{4\pi D^2} Q_\nu \frac{2h\nu^3}{c^2} \frac{\delta V}{\exp(-h\nu/kT_d) - 1} \tag{13a}$$

$$\approx 0.081 \frac{M_d}{M_g} \frac{N_{\mathrm{H}}}{D^2} \frac{\lambda_\mu^{-4}\delta V}{\exp(14388/\lambda_\mu T_d) - 1}\,, \tag{13b}$$

where D is the distance to the nebula, λ_μ is the infrared wavelength in microns, the flux is in flux units (1 flux unit = 10^{-23} ergs s^{-1} cm^{-2} Hz^{-1} = 10^{-26} W m^{-2} Hz^{-1}), and other quantities are expressed in cgs units. Typically, $(M_d/M_g) \approx 10^{-2}$, $N_{\mathrm{H}} \approx 10^3$, $D \approx 10^{21}$, $\delta V \approx 10^{55}$, $\lambda_\mu \approx 10^2$, and $\delta L_d \approx 10^4 L_\odot$ so that $T_d \approx 100$ K and $\delta S_\nu \approx 10^4$ f.u. The infrared fluxes for $\lambda > 20\,\mu$, especially the slope of the far-infrared spectrum, are affected if the radiating particles are not dielectric.

In this formulation scattering has been entirely neglected. This has been done for computational simplicity, and the expected effects scattering might have on the macroscopic nebular properties investigated in this paper are not

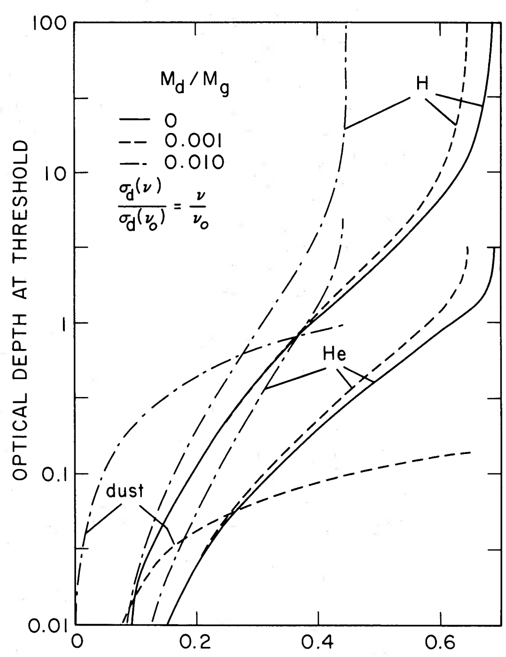

FIG. 1.—The optical depths of dust, hydrogen, and helium at $h\nu_0$, $h\nu_0$, and 1.8 $h\nu_0$, respectively, for different dust contents. The nebular density distribution is $N_{\mathrm{H}}(r) = 1000$ cm^{-3}, and the nebular radii corresponding to M_d/M_g = 0, 0.001, and 0.01 are 0.69, 0.65, and 0.45 pc. The exciting star is a main-sequence star of $T_* = 40{,}000$ K and $\log g = 4.0$.

large (these will be discussed below). Furthermore, the discovery that radio recombination lines of He do not peak at the same place in NGC 2024 where the H lines are strongest (MacLeod *et al.* 1975) indicates that scattering does not render the ultraviolet flux uniform throughout the nebula; consequently scattering may not play the important role in the far-ultraviolet that an extrapolation of the scattering efficiency curves for dust in the ISM (Witt and Lillie 1973) might otherwise imply.

III. EFFECTS OF DUST

In this section we first consider the physical structure of dusty photoionized nebulae, and then discuss the effects of dust on their observable properties. Some of the results have already been discussed in previous papers on the subject, so we shall emphasize only the new results or reemphasize certain conclusions whose importance may have been overlooked.

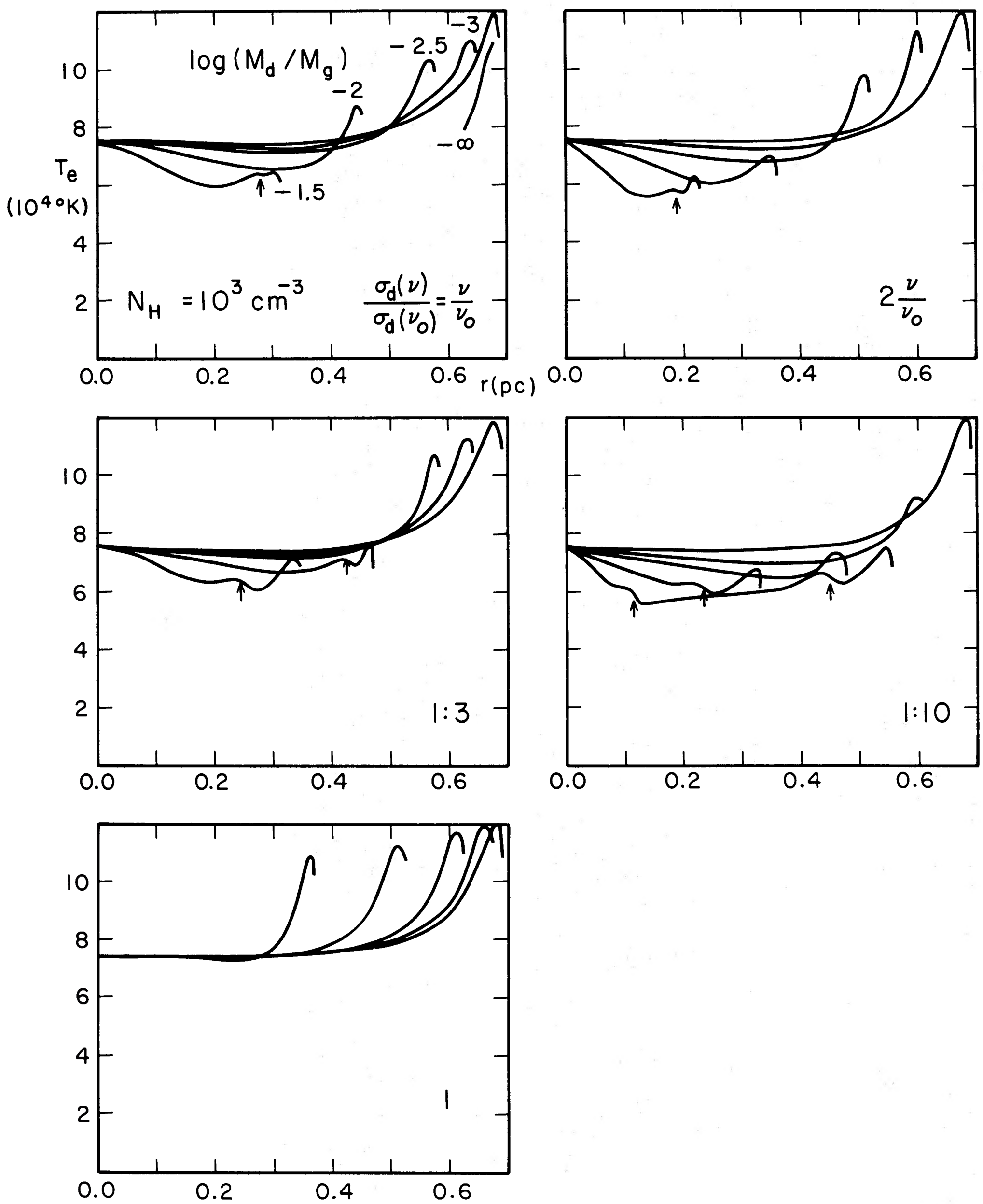

FIG. 2.—The thermal structure of nebulae excited by a star of temperature 40,000 K for various types of dust and different dust contents. Arrows indicate radii where helium becomes predominantly neutral within the H II region.

In this section we consider only constant density nebulae at a distance of 500 pc excited by a single star of surface temperature $T_* = 40{,}000$ K and $\log g = 4.0$. The model atmosphere calculations of Auer and Mihalas (1972) were used for the emergent stellar flux for $\nu > \nu_0$, and the stellar radius and total luminosity were taken from Panagia (1973). A 40,000 K star is representative of stars which excite many normal H II regions (e.g., M42) and giant H II regions (Mezger *et al.* 1974, Appendix A). Abundances were adopted from Peimbert and Costero (1969) as derived for M42. The density of hydrogen was varied from 10^2 cm^{-3} to 10^4 cm^{-3}, and no clumping was assumed for these calculations whose purpose is entirely illustrative.

The general role of the dust in nebulae can be stated quite simply; to wit., the dust serves as a "presoftener" to the radiation absorbed by the gas. Consider a dust-free nebula. The ultraviolet radiation is actually absorbed efficiently only in the outer parts of the nebula where $N(\mathrm{H}^0)$ rises slightly (because geometric dilution and re-ionization of foreground gas weaken the radiation field). Since the gas preferentially absorbs the less energetic photons, the radiation field in the outer parts of the nebula is "hardened," and the temperature rises as the average

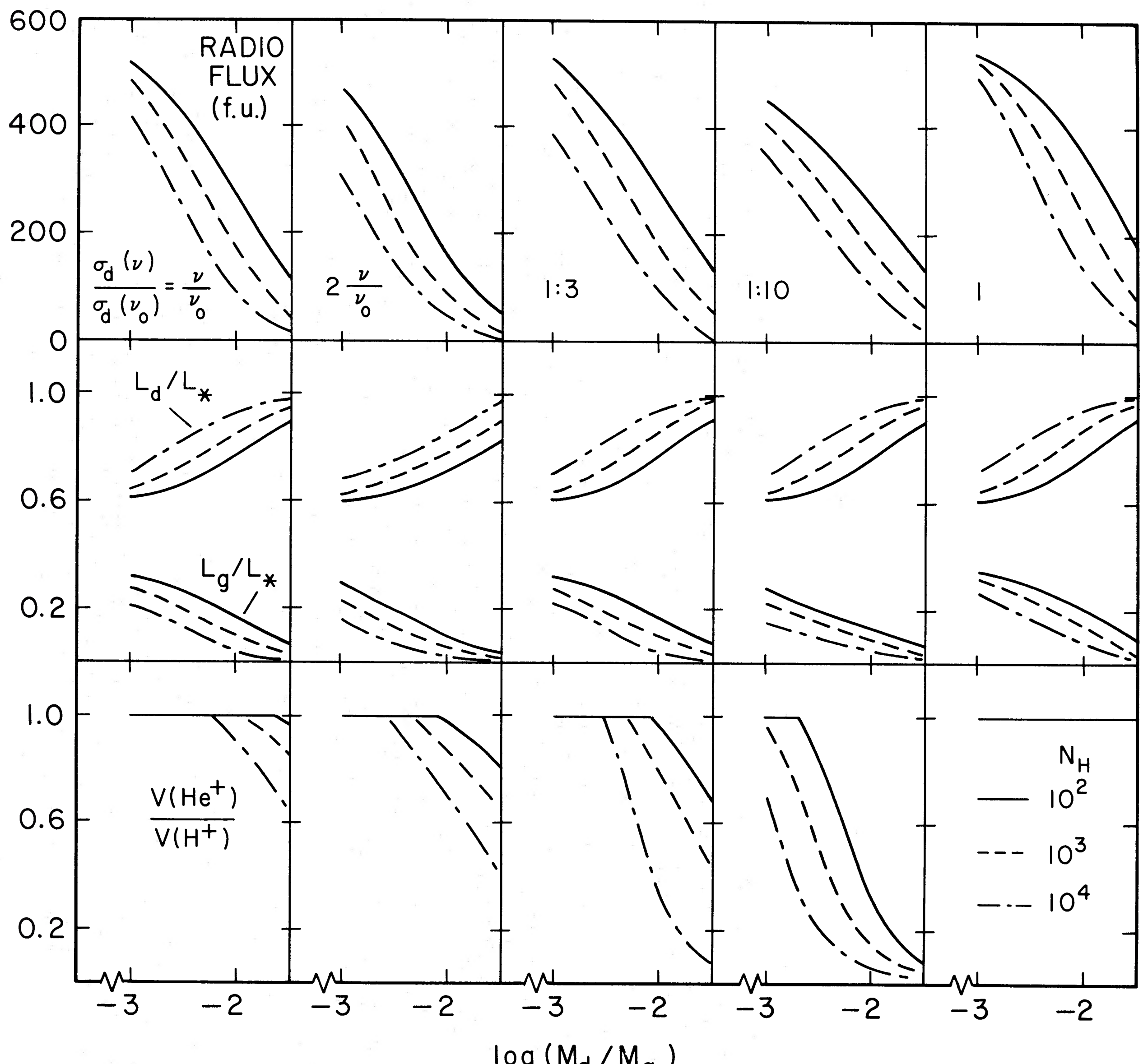

FIG. 3.—The radio flux, fractions of the stellar luminosity L_* absorbed by dust (L_d) and gas (L_g), and the ratio of volumes of ionized helium and hydrogen calculated for nebulae excited by a star of temperature 40,000 K and various densities, dust contents, and types of dust.

free energy released per photoionization increases. The absorption cross section of dust to photons of $\nu > \nu_0$ is unknown, but for this study it rises or remains constant with energy. Since the optical depth of the dust is proportional to $N(\mathrm{H})$ and not $N(\mathrm{H}^0)$, τ_d rises linearly with distance (for these constant-density nebulae). Hence the dust serves to presoften (and attenuate) the ultraviolet radiation before the absorption by gas can become efficient, especially for dense nebulae.

The relative roles of the dust and gas are illustrated in Figure 1. The optical depths of H and He at ionization threshold, and the optical depth of dust at $\nu = \nu_0$, are plotted as a function of distance from the star. The dust absorption cross section is assumed to be given by Mie theory in the linear regime (§ II), but the results are actually quite general.

The case of a dust-free nebula is shown with solid lines. When only small amounts of dust ($M_d/M_g \approx 10^{-3}$) are considered, the optical depth of He begins to rise from the solid line at smaller radii than $\tau(\mathrm{H})$ (also, the nebular radius shrinks slightly). At higher values of M_d/M_g the softening effect of dust is yet more pronounced; for $M_d/M_g \approx 0.03$, $\tau(\mathrm{He})$ exceeds 10 at radii where $\tau(\mathrm{H}) \ll 1$. Note that the nebular boundary is always determined by the absorption from gas. Within their respective Strömgren spheres both the hydrogen and helium are always highly ionized, even in the dustiest nebulae.

The thermal structure of dusty nebulae is illustrated in Figure 2 for a nebula with $N(\mathrm{H}) = 10^3\ \mathrm{cm}^{-3}$. The radiation softening by the dust is manifested by a decrease in electron temperature, T_e, in the inner parts of the nebula where $\tau_g < \tau_d \sim 1$, and then an increase where absorption by gas becomes efficient. Wiggles in the temperature profile occur where helium becomes neutral within the H II region as $\tau_g = \tau(\mathrm{H}) + \tau(\mathrm{He})$ exceeds τ_d for $\nu > 1.8\nu_0$. Eventually all helium-ionizing photons are absorbed, and dust may again dominate the transfer of the remaining ionizing radiation.

A sensitive indicator of the total number of ionizing photons absorbed by gas, $\Phi_g \equiv \Phi - \Phi_d$, is the radio flux, where Φ_g is defined by equation (7b) and

$$\Phi_g \approx 4.76 \times 10^{48}\left(\frac{S_\nu}{\mathrm{f.u.}}\right)\left(\frac{D}{\mathrm{kpc}}\right)^2\left(\frac{\nu_t}{\mathrm{GHz}}\right)^{0.1}\left(\frac{T_e}{\mathrm{K}}\right)^{-0.45} \tag{14}$$

at a radio frequency ν_t where the nebula is optically thin (Rubin 1968). Here it is assumed that all ionizing radiation is absorbed within the nebula. The predicted values for the radio flux as a function of density, dust content, and absorption characteristics are shown in Figure 3. If Φ and D could be accurately determined (say, by direct observation of *all* exciting stars) and $N(\mathrm{H})$ found from optical or radio studies, then the ratio M_d/M_g could be reliably estimated without a detailed knowledge of the frequency dependence of $\sigma_d(\nu)$. It is important to emphasize that for nebulae with "considerable" amounts of dust ($M_d/M_g \geqslant 0.01$), no more than about half of the ionizing photons are absorbed by gas. In most studies, Φ_g is replaced by Φ in equation (14), and thus the flux of ionizing photons is seriously underestimated, especially for nebulae excited by very hot stars (see below), or those with high electron density.

Also shown in Figure 3 is the fraction of the total stellar luminosity L_* absorbed by dust and gas. For a 40,000 K star, only about half of the stellar flux can be absorbed by the gas [since $\tau_g(\nu < \nu_0) = 0$], and many of the photons absorbed by the gas are converted into hydrogen Lα which are then assumed to be absorbed by dust. The results for L_d and L_g are similar to results published elsewhere. It should be noted that L_d/L_* is much less sensitive to the dust content M_d/M_g than is L_g/L_*; moreover, L_g and L_* are difficult to estimate observationally, especially since L_* is often estimated from radio flux measurements and substitution of Φ for Φ_g in equation (14).

A fairly sensitive discriminator of the dust content and absorption characteristics is afforded by the average degree of ionization x of various ionized species R^{+p},

$$x(R^{+p}) = \frac{V(R^{+p})}{V(\mathrm{H}^+)} = \frac{N(\mathrm{H})}{N(R)}\frac{\int_{\mathrm{neb}} N(R^{+p})N_e dV}{\int_{\mathrm{neb}} N(\mathrm{H}^+)N_e dV}, \tag{15}$$

where the integral extends over the nebular volume, V is the volume of the nebula containing the species, and $N(R)/N(\mathrm{H})$ is the fractional abundance of element R. The most commonly observed species is He^+, especially at radio frequencies where extinction and reddening corrections to the recombination line intensity are negligible, even to distant nebulae (see Churchwell *et al.* 1974 for a discussion of radio observations). $V(\mathrm{He}^+)/V(\mathrm{H}^+)$ is plotted in Figure 3 for different densities, types, and contents of dust; clearly this ratio is sensitive to the absorption characteristics of dust above and below the ionization potential of helium.

Mezger *et al.* (1974) considered the anomalously low values of $V(\mathrm{He}^+)/V(\mathrm{H}^+) \leqslant 0.1$ measured toward H II regions in the galactic center. They concluded that for normal dust contents, a "step" model of 1:7 for $\sigma_d(\nu)$ was required to explain the observations. Their results are corroborated in this study; furthermore, it appears that "linear" models for $\sigma_d(\nu)$ will not suffice in order to explain the observations, given the reasonable assumption that the effective temperature of the ionizing stellar radiation exceeds 40,000 K. However, the ad hoc "step" form of σ_d required to explain the observations lacks any laboratory or theoretical confirmation, and should be considered speculative at the present time.

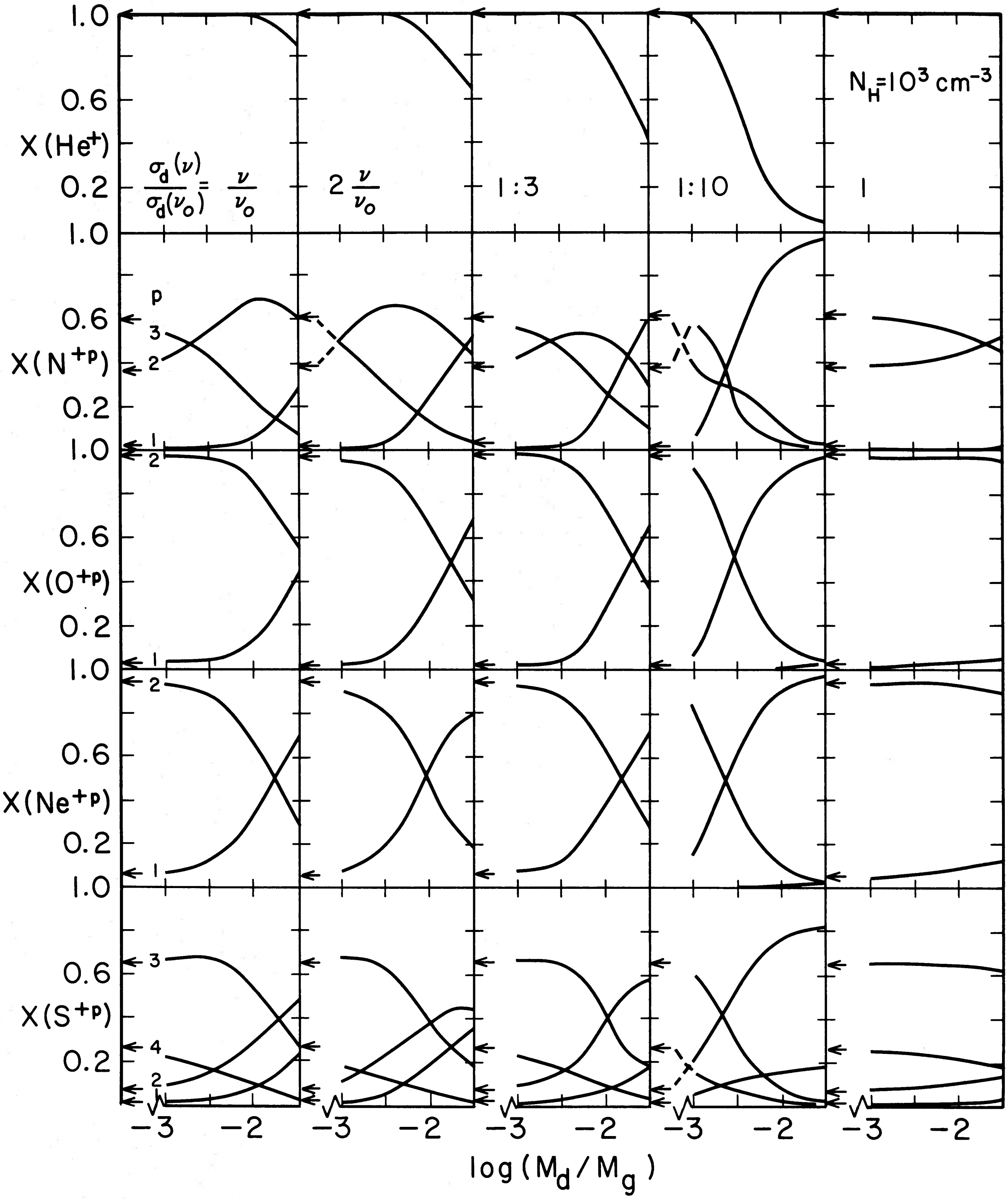

FIG. 4.—The average degrees of ionization of a nebula excited by a star of temperature 40,000 K as a function of dust type and content. The nebula has a constant density $N(\mathrm{H}) = 1000\ \mathrm{cm}^{-3}$. Values for $M_d/M_g = 0$ are indicated by arrows on the left side of each graph, and the ionization state p refers to these arrows.

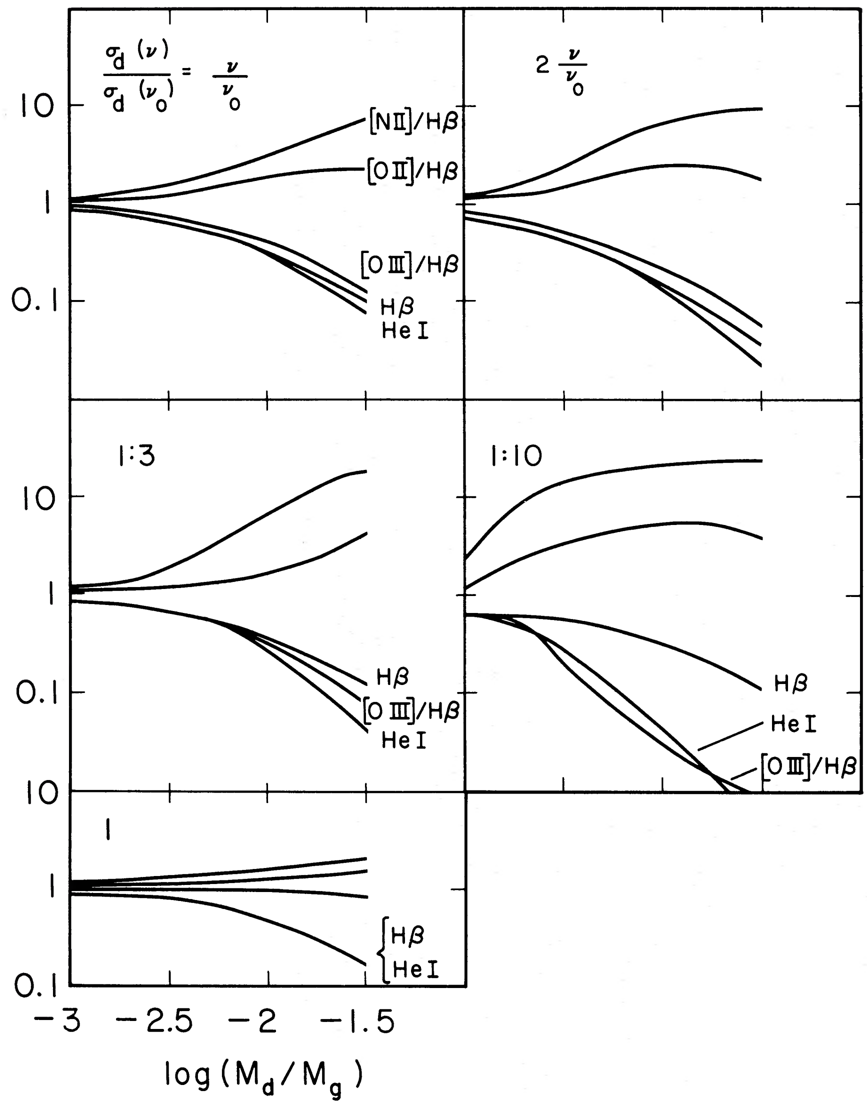

FIG. 5.—The "enhancement" (see text) of selected optical line intensities resulting from different types and contents of dust for uniform nebulae of density 1000 cm^{-3} and excited by a star of temperature 40,000 K. Labels for the various curves are the same as for the upper left figure unless otherwise indicated. The lines shown are Hβ (λ4861), He I (λ5876), [N II] (λ6548 + λ6583)/Hβ, [O II] (λ3726 + λ3729)/Hβ, and [O III] (λ4959 + λ5007)/Hβ. Results for [S II] (λ6717 + λ6731)/Hβ are intermediate between those of [O II]/Hβ and [N II]/Hβ.

Optical lines of H, He, or other species might also provide useful insight into the absorption characteristics of dust, especially if various parts of the nebula can be studied separately and extinction corrections determined. The degree of ionization for various species, averaged over the entire nebula, are shown in Figure 4 for a nebula of density N(H) = 10^3 cm^{-3}. The effects of different types and amounts of dust are especially pronounced in the trace elements. Unfortunately, however, the usual problems inherent in determining the species abundances complicate the analysis; moreover, the optical forbidden lines are often very sensitive to other factors such as N_e and T_e. It is first necessary to use line ratios or independent means in order to determine N_e and T_e (the optical continuum should be avoided because of the potentially large contribution of dust scattered starlight, as in NGC 2024) before inferring properties of the dust from observations of these lines.

The effects of dust on the brighter optical lines (in some cases normalized to the flux of the Hβ line) for the model of N(H) = 10^3 cm^{-3} are shown in Figure 5. The forbidden-line strengths vary as the species fractional abundance (Fig. 4), and in some cases of high dust content the lines become weakened because of the smaller nebular temperatures resulting from radiation softening of the dust. The enhancement effect of dust on the [O III]/Hβ line

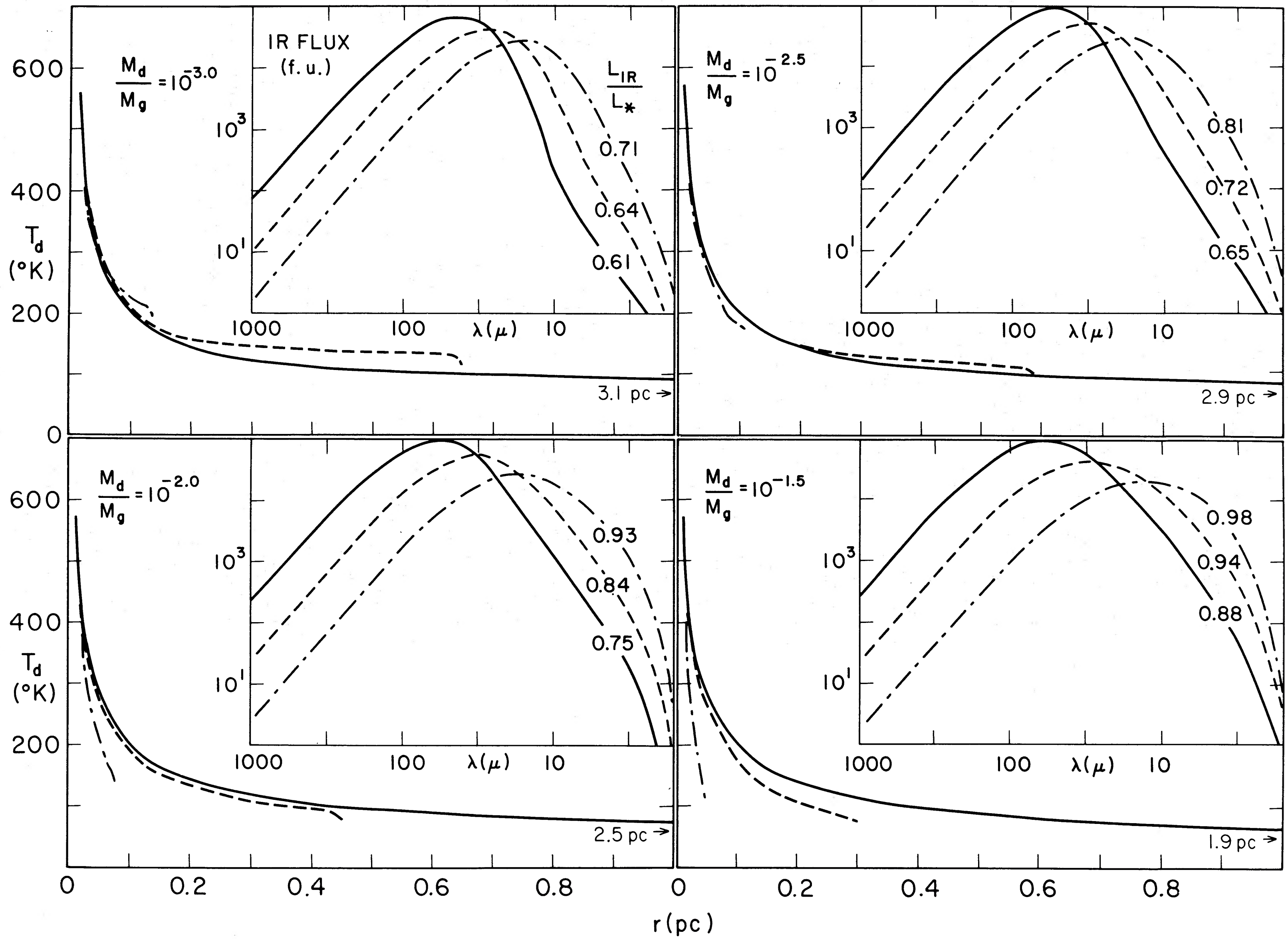

FIG. 6.—The temperature profiles of heated dust within the ionized gas and the resulting infrared spectra for uniform nebulae of different densities and dust contents. The exciting star has a temperature of 40,000 K and the absorption cross section of dust is assumed to be linear with frequency. Also shown are the fractions of the total stellar luminosity radiated in the infrared. The results shown here are weakly dependent on the functional form of absorption cross section of dust, but can vary substantially for exciting stars of different temperatures (not shown). Free-free radiation has not been added to the spectra, and can be important for $\lambda > 1$ mm and $\lambda < 5\ \mu$.

ratio, for example, first is positive with increasing dust content, and then decreases as the average nebular temperature declines (see Fig. 2). Nonetheless, it should be emphasized that dust can be an important factor in explaining the large [O II], [N II], and [S II] line intensities often observed in H II regions and planetary nebulae.

So far we have considered the effects of dust on the observable properties of the gas. The emission properties of the dust are also of interest. At optical wavelengths the scattering effects of dust have been studied, most recently by Schiffer and Mathis (1974; see also Osterbrock 1974, chapter 7). However, the dust particles which dominate these observations are necessarily different (i.e., larger) than those which affect the transfer of the ionizing radiation. Hence they probably absorb only a small fraction of the stellar energy and affect mainly the transfer of the nebular emission between 0.2 and 1 μ.

The dust heated by the stellar and Lα emission reradiates its energy at infrared wavelengths. The temperature profiles and infrared spectra for dust *within* the nebula are shown in Figure 6 for the case of the linear (Mie) absorption cross section of dust; the results are practically independent of the form σ_d and depend principally on M_d/M_g and N(H) (see Fig. 3 and eqs. [12] and [13]). The temperature profiles show that dust near the star is heated to $\sim$500 K, and then the temperature drops to about 100 K beyond $\sim$0.1 pc.

At smaller values of M_d/M_g the average dust temperatures increase weakly with $N(H)$ whereas they fall at $M_d/M_g \geqslant 0.01$. These differences can be ascribed to the varying importance of Lα relative to stellar photons in heating the dust. Note that at smaller values of M_d/M_g, $\tau_d < 1$ so that substantial amounts of sub-Lyman radiation ($\sim\frac{1}{3}L_*$) are available for heating dust outside the nebula. Note also that typical dust temperatures tend to be more nearly uniform and about 100 K throughout most of the volume of the larger, less dense nebulae.

The infrared spectra show substantial differences in form, especially for different values of N(H). Note the behavior of $\lambda_{\max}$, the wavelength of maximum infrared flux, $S_{\max}$. In addition, the shapes of the spectra also vary; those with higher values of M_d/M_g have more gentle features for $\lambda \leqslant \lambda_{\max}$ because of the wider range of dust temperatures. The effects of the higher-temperature dust near the star are manifested at short wavelengths in some of the spectra. The total infrared luminosities are always a substantial fraction ($>60\%$) of L_* (see also Fig. 3), and much, but not all, of this energy emerges at wavelengths less than $\sim$30 μ. The actual infrared flux observed at $\lambda \geqslant 30$ μ can be expected to contain a contribution from extranebular dust for small values of M_d/M_g (Wright 1973).

Thus far all of the calculations of dusty nebulae have assumed an exciting star with $T_* = 40{,}000$ K. Obviously, the quality of nebular excitation can be affected by the temperature of the exciting star as well as the transfer of the radiation through the dust and gas. In order to explore the effects of dust on nebulae excited by other types of stars, we have used the model atmosphere calculations of Auer and Mihalas (1972), and the stellar luminosities, radii, and spectral type classifications reported by Panagia (1973) and Mezger *et al.* (1974) for stars with $T_* =$ 50,000, 45,000, 40,000, 37,500, 35,000, and 30,000 K and $\log g = 4.0$. Both dust-free and dusty ($M_d/M_g = 0.01$ and linear or "Mie" absorption cross section for $\nu \geq \nu_0$) models with densities of $N_{\rm H} = 10^2$, 10^3, and 10^4 cm^{-3} were calculated.

We limit the discussion to results of current observational interest; namely, we consider the radio flux of dusty nebulae $S_{\nu,d}$ divided by the radio flux in the dust free case $S_{\nu,0}$ and the degree of ionization of helium, averaged over the nebula $x(\mathrm{He}^+)$ (see eq. [15]). The results for $S_{\nu,d}/S_{\nu,0}$ are shown in Table 2. The increase in the relative fluxes for the cooler stars reflects the shift of the average photon energies to smaller values where the absorption cross section of gas increases sharply. Also shown in the table are the spectral types corresponding to the values of T_*, and the predicted radio fluxes for a dust-free nebula (the flux is essentially independent of density in this case).

TABLE 2

RADIO FLUXES OF DUSTY ($S_{\nu,d}$) AND DUST-FREE ($S_{\nu,0}$) NEBULAE

T_* (K)	$S_{\nu,d}/S_{\nu,0}$: $N_{\rm H} = 10^2$ cm^{-3}	$N_{\rm H} = 10^3$ cm^{-3}	$N_{\rm H} = 10^4$ cm^{-3}	$S_{\nu,0}$* (f.u.)	SPECTRAL TYPE
50,000......	0.29	0.15	0.06	3920	O4
	0.30	0.17	0.07	2550	O5
45,000......	0.40	0.22	0.10	1150	O5.5
	0.47	0.29	0.14	450	O7
40,000......	0.51	0.33	0.17	320	O6.5
	0.54	0.38	0.21	180	O8
37,500......	0.57	0.41	0.24	140	O7.5
	0.58	0.43	0.26	110	O9
35,000......	0.66	0.53	0.34	58	O8.5
	0.65	0.52	0.34	56	O9.5
30,000......	0.82	0.74	0.70	3	B0
	0.76	0.70	0.59	4	B0.5

* Distance = 500 pc.

NOTE.—Upper/lower entry based on calibrations of Panagia 1973/Mezger *et al.* 1974. All stellar spectra taken from Auer and Mihalas 1972 for $\log g = 4.0$.

Since the values of the stellar types and flux of ionizing photons taken from Panagia (1973) and Mezger *et al.* (1974) differ substantially, two entries are given for the fluxes and spectral types.

Interestingly, the calculations show that $x(\mathrm{He}^+)$ is not particularly affected by the presence of the dust in these models. For $T_* \geq 35{,}000$ K, the helium was ionized throughout at least 90 percent of the nebula (even for $N_\mathrm{H} = 10^4\ \mathrm{cm}^{-3}$), whereas for $T_* = 30{,}000$ K the amount of helium ionized by the star is negligible even for the dust-free case. At $T_* = 35{,}000$ K, $x(\mathrm{He}^+) = 1.0$, 0.90, 0.82, and 0.72 for the dust-free nebula and for dusty nebulae with $N_\mathrm{H} = 10^2$, 10^3, and $10^4\ \mathrm{cm}^{-3}$, respectively.

Finally, we discuss the problem of scattering. To first order, scattering will not affect the nebular emission averaged over the volume (although it is obvious that local irregularities might be washed out). To second order, however, scattering will have two effects which more or less cancel. The first effect is that the ionizing photons will be scattered back into the central regions of the nebula. The average stellar flux will thus be less diluted on the average, and the various constituents will tend toward a higher ionization state. On the other hand, the lesser abundances of H^0 and He^0 in these same regions mean that an increasing number of photons will be absorbed on dust; hence the radiation field will be softened, favoring a lower degree of ionization. Consequently, only minor changes are likely to occur in the average ionization structure of the nebula. Mathis (1971) and Petrosian *et al.* (1972) first studied this problem for the ionization of He and come to much the same conclusion. It is conceivable that somewhat smaller electron temperatures will result in models in which scattering is treated properly. In addition, the average dust temperature will increase (since the nebular volume decreases) with an attendant shift of the infrared spectrum to shorter wavelengths.

IV. DISCUSSION

We now illustrate the model calculations through practical application to well studied galactic nebulae. Models simulating high and low excitation planetary nebulae, NGC 7027 and IC 418, respectively, have been calculated. (The planetary nebulae were chosen for study because of their geometric simplicity. The more complex question of the importance of dust in H II regions will be deferred until further observations are made.) These results will not be discussed at length because the observations presently available can be fitted by a range of different models with various abundances, density distributions, clumping, dust content, etc. The question of the validity of model fits to observations of planetary nebulae, excluding dust, have recently been reviewed by Osterbrock (1974, chapter 5) and Miller (1974).

NGC 7027 is a bright, well studied planetary nebula at radio, infrared, and optical wavelengths. For the present calculations the cylindrical shell geometry and electron density ($N_e = 5 \times 10^4\ \mathrm{cm}^{-3}$) suggested by Scott (1973) were adopted, except that an electron density interior to the shell of $8000\ \mathrm{cm}^{-3}$ was included. Other geometries and densities have been suggested (see Bignell 1974; Miller and Mathews 1972), but the uncertainties of the macroscopic geometric properties are small compared to the uncertainties arising from the neglect of small-scale clumping. The stellar flux was taken from Hummer and Mihalas (1970) for a star with temperature 10^5 K and $\log g = 5.0$, and the total stellar luminosity was assumed to be $2.5 \times 10^4\ L_\odot$ (Becklin *et al.* 1973). The abundances for this nebula are not well known, so average values for many planetary nebulae (Peimbert and Torres-Peimbert 1971*a*) were used. The Mie, or linear, form for σ_d was adopted, and models with $\log (M_d/M_g)$ between -3 and -1.5 were considered. For all models, the volume was arbitrarily scaled by varying the cylindrical height (which is poorly known because of the small angle of inclination between the nebular axis of symmetry and the line of sight) in order that the calculated radio flux match the observations.

Briefly described, the model of NGC 7027 for which $M_d/M_g \approx 0.01$ gives an excellent fit to the radio continuum and infrared photometric measurements (Becklin *et al.* 1973), as well as the total infrared luminosity if the nebular volume is increased by 50 percent over that given by Scott. The infrared spectrum extends to shorter wavelengths than does a simple blackbody spectrum because of the hot dust core near the star. For $\lambda \gtrsim 5\ \mu$ the spectrum is approximately that of a 225 K blackbody. The high-excitation forbidden-line intensities relative to Hβ, such as [O III], are reasonably well fitted (errors are easily attributed to abundance uncertainties). However, as with all previous model calculations, the predicted low-excitation line intensities are far too small. It should be noted that the inclusion of dust strongly enhances the intensities of these lower excitation lines, and that a proper treatment with clumping and charge-exchange processes could conceivably give much better results. Strangely, the predicted value of $x(\mathrm{He}^{++})$ increases with increasing M_d/M_g, and a value of $x(\mathrm{He}^{++}) \approx 0.2$ is predicted for the best-fit model calculation, whereas a value of 0.4 is measured (Miller and Mathews 1972). Because of the extreme uncertainties in the value of $\sigma_d(\nu \geq 4\nu_0)$, it is not difficult to reconcile these differences.

The radio, infrared, and optical observations of IC 418 (Milne and Aller 1975; Peimbert and Torres-Peimbert 1971*b*; see also Bussoletti *et al.* 1974) can be adequately modeled with a cylindrical shell model in which $N_\mathrm{H}(r < 0.045\ \mathrm{pc}) = 2 \times 10^3\ \mathrm{cm}^{-3}$, $N_\mathrm{H}(r > 0.045\ \mathrm{pc}) = 10^4\ \mathrm{cm}^{-3}$, $\log (M_d/M_g) \approx -2.5$, a linear form for $\sigma_d(\nu)$, and other parameters as given by Buerger (1973). The fit to the infrared spectrum is especially good for this model, including the observations for which $\lambda < 4\ \mu$. In addition, the agreement between the observed and predicted optical line intensities is extremely good, even for the lines of low excitation. (Buerger has also been able to match the line intensities with a dust-free model by clumping the gas in the outer cylindrical shell.) Of some observational interest are the predicted intensities of the [Ne II] 12.8-μ and the [S IV] 10.5-μ lines of 9 and 4×10^{-11} ergs cm^{-2} s^{-1}, respectively.

Compared to the planetary nebulae it is much more difficult to fit the macroscopic properties of H II regions with simple models for a variety of reasons. Foremost is the complex distribution of the spatial and kinematic features in these nebulae. Moreover, the sources of nebular excitation have not yet been identified in most H II regions, and so the geometric relationship between the star(s) and the gas and dust required for the calculation is not known. Also, the importance of clumping in H II regions is not well known. Thus detailed models of H II regions have not been attempted.

At the present time there is little except relatively weak evidence that dust does, in fact, affect the transfer of ionizing radiation in H II regions; in fact, there seems to be some evidence to the contrary. That some dust is mixed with the ionized gas has been well established by the well known optical scattering and polarization studies (see Schiffer and Mathis 1974 and papers cited therein). However, the component of the dust which absorbs most of the energy (and reradiates this energy in the infrared) is not always spatially distributed like the ionized gas. In some cases such as W43 the dust/gas correlation is high (Pipher *et al.* 1974), whereas in others such as NGC 2024 the strong infrared components at $\lambda < 20\ \mu$ are physically associated with but clearly displaced from the bright ionized gas (Grasdalen 1974; MacLeod *et al.* 1975), while still in others such as M17 the dust/gas correlation is known to change with wavelength (Harper 1974*a*, *b*). The observational picture is often made very complicated by strong infrared sources near but not directly related to the H II region, such as the KL nebula and the BN point source which dominate the infrared observations of Orion (see Wynn-Williams and Becklin 1974). *Until it can be established that dust* within *the nebula radiates most of the stellar luminosity, the importance of dust in* H II *regions is indeed most uncertain.*

Additional radio and infrared maps of high spatial resolution are necessary in order to locate the dust inside the H II region, especially at $\lambda \sim 50\ \mu$ and $\lambda \sim 1$ cm. Further studies of the emission properties of the gas in and near dusty features is also important; such observations will be reported in subsequent papers. As of this writing, observations have just been made of dusty regions in Orion and NGC 2024 using the 120-inch (3 m) telescope of the Lick Observatory. A preliminary analysis of the data shows no evidence of ionization variations resulting from dust. Specifically, variations in the He I (λ5876)/H I (λ6563) emission reported earlier by Liebowitz (1973) in the dusty regions were not seen.

In § II we discussed the frequency dependence of $\sigma_d(\nu > \nu_0)$ required to explain the extremely small values of $x(\mathrm{He}^+)$ seen toward H II regions in the galactic center region. The preliminary observational results reported above along with the unlikely frequency dependence of $\sigma_d(\nu)$ near $1.8\nu_0$ theoretically necessary to explain this lack of ionized helium argue strongly that dust does not yet provide an adequate explanation. Other possibilities should be explored in greater detail.

V. SUMMARY

We have considered the possible effects of dust on the ionization and thermal structure of photoionized nebulae. Particular emphasis has been placed on a presentation of the observable effects of the dust on the radio and infrared continua and the optical emission-line spectrum. The major results for a nebula excited by a star of temperature 40,000 K (typical of the ionizing radiation spectrum in many nebulae including the totally obscured "giant" H II regions) are listed below and illustrated in Figures 1–6.

1. The hydrogen and helium remain highly ionized within their respective Strömgren spheres for any reasonable type of dust $[\sigma_d(\nu)/\sigma_d(\nu_0)]$ or dust content (M_d/M_g).
2. Dust more successfully competes for the absorption of ionizing photons in the inner parts of the nebula, whereas absorption by gas dominates near the nebular boundary.
3. Dust absorption is strongly enhanced in regions of high nebular density.
4. Electron temperatures can decrease by up to 15 percent because of the radiation softening effects of dust.
5. A significant fraction of the ionizing photons are absorbed on dust (80% for nebulae of high density and dust content), and so the radio continuum flux density is a somewhat unreliable but commonly used measure of the stellar flux above the Lyman limit in dusty H II regions.
6. Likewise, 60–100 percent of the stellar luminosity is eventually absorbed on dust inside the nebula and reradiated in the infrared.
7. The fractional volume of ionized helium in the H II region is sensitive to dust type and content as well as density. Normally, however, the calculations indicate that this fraction will exceed 0.5 unless the dust cross section increases very rapidly at the helium ionization potential.
8. The average degrees of ionization of the trace elements such as nitrogen, oxygen, neon, and sulfur are usually more sensitive to the dust content and type than is the degree of ionization of helium.
9. The forbidden-line intensities (relative to Hβ) can be affected by even small amounts of dust and can be used effectively for ground-based studies of the dust cross section above the Lyman limit if the nebular density and temperature can be determined.
10. The temperature distribution of heated dust within the nebula is strongly dependent on density and dust content, but not on the type of dust. The infrared spectra of the heated dust clearly reflect this behavior.

Models in which the temperature of the exciting star is varied but the content and type of dust are fixed $[M_d/M_g = 0.01, \sigma_d(\nu)/\sigma_d(\nu_0) = \nu/\nu_0]$ show the following:

1. The fraction of ionizing photons absorbed on dust increases strongly with stellar temperature. For hot stars ($T_* = 50{,}000$ K), the radio flux is only 6–30 percent of its value in the dust-free case (depending on density), whereas for later stars ($T_* \leq 35{,}000$ K) this fraction ranges from 40 to 80 percent.

2. The average degree of ionization of helium is not strongly affected in dusty nebulae except for stars with $T_* \approx 35{,}000$ K. For $T_* = 35{,}000$ K (and with the dust type and content fixed), $x(\mathrm{He}^+)$ can be affected by $\lesssim 25$ percent, depending on the density.

3. The infrared spectra are very sensitive to the stellar temperature and total luminosity as well as the density.

Models were constructed of two planetary nebulae, NGC 7027 and IC 418, and the agreement between the measured and predicted radio and infrared continua and the high-excitation optical lines could be made extremely good. In order to match the low-excitation line intensities, dust or clumping (or both) are required. Detailed models of H II regions were not attempted because the spatial distribution of the exciting stars and the gas is complex and poorly understood. There is a possibility that dust does not strongly affect the transfer of the ionizing radiation in most nebulae since the luminosity of the infrared emission from dust within the nebula may be much weaker than the calculations predict, but further observations of high spatial resolution are a necessity in order to determine the dust/gas geometry. Based on the available evidence, there is no real evidence that dust can explain the anomalously small values of $x(\mathrm{He}^+)$ observed toward some H II regions.

Moreover, we wish to point out that at least in M42 where the source of excitation is directly observable, the radio flux seems to suggest that all of the ionizing radiation is absorbed by the gas. This means that M_d/M_g for dust particles which are efficient absorbers of ionizing radiation is considerably less than 10^{-2} in the central part of the nebula, or that the dust cross section is smaller than the value predicted by Mie theory. Much the same is probably the case in other nebulae.

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