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A SEARCH FOR STRONG MAGNETIC FIELDS IN RAPIDLY ROTATING Ap STARS

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ABSTRACT

A search for strong magnetic fields ($\gtrsim 10^3$ gauss) in rapidly rotating peculiar A stars has been made using the solar magnetograph technique on Balmer lines. Observations of 16 stars have been made with standard errors typically in the range of 300 to 700 gauss. No definite fields are found, and the results indicate that fields in excess of 10^3 gauss are significantly less common among rapidly rotating Ap stars than among slowly rotating ones.

Subject headings: magnetic stars — peculiar A stars — rotation, stellar

I. INTRODUCTION

Virtually all the well-established magnetic fields known in nondegenerate stars are found in peculiar A stars. These fields have been detected using the photographic technique developed by Hale (1908) for the study of sunspot fields, and applied by Babcock (1947, 1958, 1960, 1962) to the study of stellar magnetism. The accuracy of this technique is strongly dependent on line width, being much better for sharp-lined stars, and for this reason fields are known with certainty to exist only in stars having $v \sin i \lesssim 30$ km s^{-1}. However, about half the Ap stars have $v \sin i \gtrsim 30$ km s^{-1}, and some have projected rotational velocities in excess of 100 km s^{-1} (Abt *et al.* 1972). Almost nothing is known about the presence of magnetic fields in such stars.

It has been shown by Angel and Landstreet (1970) that fields can be detected in Ap stars using a technique similar to that employed in the solar magnetograph (Babcock 1953), but isolating a Balmer-line wing, for instance with an interference filter of a width of a few angstroms, rather than isolating a metal-line wing with a high-dispersion spectrograph. Because of the large intrinsic width of the Balmer lines and of the filter used, the field measurement errors are almost independent of rotational velocity, and the technique is well suited to the study of magnetic fields in rapidly rotating stars. In this paper, we report a search for magnetic fields based on this method in 16 Ap stars having $v \sin i \gtrsim 30$ km s^{-1}, made with the aim of determining if the frequency with which large fields ($H_e \gtrsim 10^3$ gauss) occur in such stars is similar to the frequency in the slowly rotating Ap's.

II. OBSERVATIONS

The observations reported here were made with four different polarimeters. All have Pockels cell modulators similar to the one described by Angel and Landstreet (1970), but two are essentially photometers employing interference filters of 5 Å passband to isolate line wings, while the other two are scanning photoelectric grating spectrometers. The instruments and telescopes used for various observations are listed in Table 1, each with an identifying number which is used in the table of observations (Table 2) to identify the source of individual measurements.

In all observations, the circular polarization was measured at two points symmetrically placed on either side of the line center, located far enough into the wings to be only slightly affected by radial-velocity differences from star to star. In the presence of a net longitudinal field H_e, measurements V_b and V_r of the polarization in the blue and red line wings will be equal and opposite, so a mean polarization $\langle V\rangle = (V_b - V_r)/2$ is defined which is related to the mean longitudinal field by

$$\langle V\rangle = \frac{ez}{4mc^2}\lambda^2 H_e \frac{dI/d\lambda}{I(\lambda)} \qquad (1)$$

(e.g., Bray and Loughhead 1964), where $I(\lambda)$ is the observed line profile and z is the Landé splitting of the transition. All measured polarizations and their standard errors are converted to field measurements using equation (1) with $z = 1.0$, which is approximately correct for the Balmer lines. Typical sensitivities

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TABLE 1

TELESCOPES AND INSTRUMENTS USED FOR OBSERVATIONS

Identifying Number	Telescope	Instrument	Line Bandwidth (Å)	Typical Sensitivity ($G/1\%$)	Observer
1..........	Kitt Peak National Observatory, No. 2, 1-m	*	Hγ 5	1.1×10^4	JRPA, JDL
2..........	University of Western Ontario, 1.2-m	*	Hγ 5	1.1×10^4	JDL
3..........	Mount Wilson Observatory, 1.5-m	†	Hα 20	2×10^4	EFB
4..........	Columbia University, 60-cm	‡	Hγ 5	$0.8\text{–}2.7 \times 10^4$	RMEI
5..........	McDonald Observatory, 2.1-m	†	Hα 1	0.5×10^4	JRPA, RMEI
6..........	University of Western Ontario, 1.2-m	*	Hα 5	1.5×10^4	JDL

* Two-channel polarimeter with interference filters.
† Photoelectric Cassegrain scanner.
‡ Single-channel polarimeter with interference filter.

for the various systems, expressed in terms of the field strength required for a polarization of 1 percent, are given in the penultimate column of Table 1. Use of a mean polarization $\langle V \rangle$ determined from both wings insures that instrumental polarization (such as is encountered in the Mount Wilson observations due to linear polarization produced by the diagonal flat which brings the light to the broken Cassegrain focus, combined with instrumental crosstalk between Stokes components) cancels out of the measurement, and only polarization due to the Zeeman effect is detected.

The stars observed and field measurements obtained are given in Table 2. In the first row for each star are given the star name, HD number, V magnitude, Balmer-line spectral type and main peculiarities as determined by Osawa (1965), projected rotation velocity (in km s^{-1}) from Abt *et al.* (1972), the period of variation (in days) if one is well established, the type of variations which are definitely present (l = light, sp = spectrum, m = magnetic field), and a reference to previous magnetic observations reported by Babcock (1958; T1, T2, and T4 refer to his Tables 1, 2, and 4). Below this line our measurements are reported in three series of four columns, where for each observation we give the Julian date (less 2,400,000), the observed field strength and standard error of measurement (in gauss), and the observer (as numbered in Table 1).

It may be seen from the data in Table 2 that the technique used here is reasonably sensitive. A standard error of $\sigma_{\rm H} \sim 300$ gauss may be obtained in 1 hour with an interference filter polarimeter on a 1.2-m telescope for a fifth magnitude star, which compares reasonably well with $\sigma_{\rm H} \sim 150\text{–}200$ gauss obtained from high-dispersion Zeeman-analyzed photographic spectra of sharp-line stars obtained with much larger telescopes (Preston 1969*a*).

A number of observations of stars already known to be magnetic were obtained to determine the sign of magnetic fields and to verify that the instruments were working properly. These observations are given in Table 3, using a format similar to that in Table 2 except that the column ϕ gives the phase of observation and H_{pg} gives the magnetic field determined photographically for that phase according to the references given in the notes. Within the quoted errors, the fields determined by our method agree reasonably well with photographically measured ones, except for α^2 CVn, where our fields are systematically lower than photographic measurements. A plausible explanation of the discrepancy found for α^2 CVn is that it may be an example of an effect discussed by Borra (1974), who has shown that an appropriate combination of a nonuniform longitudinal field and appreciable rotational velocity can result in photographic measurement overestimating the true mean longitudinal field by a factor of order 2. Further observations of this star are planned.

III. DISCUSSION

Examining the 110 observations of 16 stars in Table 2, we find no obvious evidence for the presence of fields much larger than the errors in any of the stars except possibly for ϵ UMa, in which two of the 10 measurements differ from zero by about 3 standard errors and two more by more than 2 standard errors, but even in this star the evidence for a field is not compelling. This absence of obvious fields is perhaps surprising in view of rather common occurrence of fields of $\sim 10^3$ gauss in sharp-line Ap stars, but because of the fairly large errors (~ 300 to 700 gauss typically) of our observations, it is worthwhile to make a more precise comparison between sharp- and

TABLE 2

MAGNETIC FIELD MEASUREMENTS OF BROAD-LINE Ap STARS

Star	HD JD	V $H_e \pm \sigma_H$	Sp Obs	v sin i JD	P $H_e \pm \sigma_H$	var Obs	ref JD	$H_e \pm \sigma_H$	Obs
γ Ari S	11503	4.75	B9 Si	85	2.61	ℓ, sp	T4		
	1918.92	600±500	3	1922.87	-160±500	3	1947.89	0±500	3
	1919.89	-400±500	3	1923.88	0±500	3	1949.94	900±360	3*
	1920.94	260±700	3	1924.89	240±500	3	2250.85	-160±270	6*
	1921.90	-780±700	3	1925.89	-300±500	3			
α Psc A	12447	4.33	B9 Cr	92	-	-	T4		
	878.71	- 30±370	2	1922.95	0±500	3	1949.83	180±360	3*
	1918.98	0±500	3	1923.95	-940±700	3	1951.85	600±360	3*
	1919.96	-140±500	3	1947.82	-600±420	3			
	1921.94	490±500	3	1948.88	100±360	3			
63 And	14392	5.58	B5 Si	105:	-	-	T4		
	2265.83	- 60±360	6*						
ι Cas	15089	4.51	B9SrCr	54	1.74	ℓ, sp	T4		
	767.93	70±340	1	883.59	-770±500	2	1928.91	1030±630	4
	878.79	-590±340	2	952.66	- 20±280	2*	1948.78	- 40±500	3
	879.80	860±340	2	1179.84	-170±290	2*	1951.77	160±500	3
20 Eri	22470	5.22	B6 Si	80	-	-	T4		
	1947.96	500±1000	3*	1951.91	-800±1000	3*			
θ Aur	40312	2.69	B9 Si	45:	-	-	T2		
	1762.62	90±240	4	1923.99	0±280	3	1950.02	-200±260	3
	1919.99	-100±320	3	1924.98	580±260	3	1951.96	100±320	3
	1921.98	-360±420	3	1925.98	400±240	3*			
	1922.98	480±320	3	1948.00	360±220	3*			
ε U Ma	112185	1.76	B9 Cr	49	5.09	ℓ, sp	T2		
	1685.85	960±310	4	1765.80	-520±230	4	1804.79	120±220	3
	1693.90	200±360	4	1766.74	470±280	4	1805.78	-100±160	3
	1694.71	90±260	4	1769.77	100±260	4	1806.77	140±220	3
	1760.87	-640±200	3	1788.67	360±140	4*	1808.78	100±160	3
	1762.82	-110±190	4	1789.60	- 90±170	4	1809.69	120±220	3
	1762.9	-220±320	3	1793.65	- 60± 40	5*	1810.71	140±220	3
CU Vir	124224	5.00	B8	135:	0.52	ℓ, sp	T4		
	767.70	360±310	1*	1809.72	1260±700	3			
	1761.03	680±1000	3	1810.74	400±700	3*			
ι Lib	134759	4.53	B7 Si	56	-	-	T4		
	1793.90	120±360	5*	1809.89	-320±700	3	1810.92	-660±700	3*
χ Ser	140160	5.33	A0SrCr	62	1.59	ℓ, sp	T4		
	767.86	-220±550	1*	1810.92	760±700	3	1922.68	0±1000	3
	1793.84	50±480	5*	1919.68	-200±1000	3	1924.68	-1000±700	3
	1809.93	-1260±700	3	1921.65	-1840±700	3			
HR 5857	140728	5.39	B8 Cr	62	1.31?	ℓ?	T4		
	767.81	20±550	1*						
ω Her	148112	4.56	B9 Cr	54	-	-	T4		
	1808.97	1960±700	3	1919.74	-340±500	3	1922.73	500±700	3
	1809.97	-520±700	3	1920.72	-800±500	3	1924.75	-1360±700	3
	1810.97	300±700	3	1921.79	940±500	3*	1925.68	-700±500	3*
φ Dra	170000	4.18	B9 Si	93	-	-	T4		
	878.59	60±330	2*	1810.99	960±800	3	1925.78	- 60±360	3
	1793.96	90±470	5	1918.83	-360±500	3	1947.72	200±360	3
	1804.82	400±700	3	1919.80	140±500	3	1948.67	-500±320	3*
	1806.93	80±800	3	1920.81	260±500	3	1949.65	-160±400	3
	1808.92	200±660	3	1922.77	120±500	3	1950.68	-700±360	3
	1809.99	-860±800	3	1923.79	-360±400	3	1951.67	200±360	3
4 Cyg	183056	5.12	B9 Si	34	-	-	T2		
	2249.79	-460±270	6*	2250.73	- 20±270	6*			
HR 8407	209515	5.60	A0 Mg	100	-	-	T4		
	2264.71	330±580	6*	2265.71	560±330	6*			
κ Psc	220825	4.95	B9CrSr	38	0.59	ℓ, sp	T1		
	2250.80	80±270	6*	2265.78	140±330	6*			

TABLE 3

OBSERVATIONS OF KNOWN MAGNETIC STARS

Star	HD JD	V $H_e \pm \sigma_H$	Sp Obs	v sin i ϕ	P H_{pg}	var	ref
53 Cam	65339	6.03	A2SrCrEu	14	8.03	ℓ,sp,m	(1)
	632.75	3600± 700	(2)	.10	3500		
	878.89	-3200± 800	2	.76	-5000		
α^2CVn	112413	2.90	B7SiHg CrEu	18	5.47	ℓ,sp,m	(3)
	1793.72	410± 100	5	.49	1800		
	2249.66	-1350± 300	6	.85	-1800		
	2265.58	- 650± 180	6	.76	-1800		
CS Vir	125248	5.89	B9CrEu	11	9.30	ℓ,sp,m	(4)
	767.76	+2400± 800	1	.01	+2200		
HR 5597	133029	6.36	B9SiCr	20	2.89	ℓ	(5)
	1760.96	300±1400	3		2300		
	1763.01	5300±2000	3		2300		
	1804.90	1500±1000	3		2300		
	1805.88	2200± 700	3		2300		
	1806.85	3600± 820	3		2300		
	1808.86	2000± 700	3		2300		
	1809.88	2600± 820	3		2300		
	1810.83	2400±1000	3		2300		
β Cr B	137909	3.72	F0SrCrEu	≤3	18.49	ℓ,m	(6)
	2265.64	1020± 180	6	.34	1000		

(1): Preston and Stępień (1968).
(2): The value of H_e given here is revised from that reported by Angel and Landstreet (1970).
(3): Pyper (1969), group 3 elements.
(4): Babcock (1960).
(5): Babcock (1958) reports field always positive, 1000 to 3000 G. Bonsack (unpublished) gives the field as 2300±200(p.e.)G with occasional non-periodic excursions to 1500 G.
(6): Wolff and Bonsack (1972).

broad-line stars to determine the reality and extent of the difference.

Ideally, we would like to compare a sample of short-period Ap's with a sample of long-period ones, but even in the best studied sample of Ap's, less than half the stars have well-established periodic variation of light, spectral line strength, or magnetic field, so we must separate long- and short-period stars using projected rotational velocities, which have been determined homogeneously for nearly all Ap's north of $\delta = -20°$ and brighter than $V = 6.0$ by Abt *et al.* (1972). We will use this list as our basic sample, deleting all stars having the Hg-Mn peculiarity, as they seem to be quite different from other Ap's (Preston 1971), and adding four stars not included in the observations of Abt *et al.* although they have appropriate V and δ, namely α Psc A (HD 12447, Sp = B9 Cr, $v \sin i = 92$ km s^{-1}), HR 1217 (HD 24712, F0 SrCrEu, $\leqslant 6$ km s^{-1}), μ Lib A (HD 130559, A0 SrCr, ~ 20 km s^{-1}), and 10 Aql (HD 176232, A5 SrCr, $\leqslant 5$ km s^{-1}). Spectral types for these additions are taken from Osawa (1965), and projected rotational velocities from Bernacca and Perinotto (1971), Preston (1970), and Babcock's (1958) index of line width w assuming $v \sin i \sim 50\, w$ (km s^{-1}) (Preston 1968). This gives us a sample of 52 stars to work with; all the broad-line stars observed by us are included in this sample.

Virtually all well-established magnetic fields in peculiar A stars are found in stars having $v \sin i \leqslant 30$ km s^{-1}, so we divide the above sample at this value of the projected rotational velocity, which corresponds to a period of $\sim 5^d.4$ for a star viewed at $i = 90°$ (Preston 1971). There are 25 stars having $v \sin i \leqslant 30$ km s^{-1} in the sample, which we will refer to as sharp-line stars, and 27 broad-line stars of $v \sin i >$

30 km s^{-1}. Using the smoothed distribution of true rotation velocities found by Abt *et al.* (1972) for the Ap stars, it is easily shown that about seven or eight (~30%) of the 25 sharp-line stars actually have $v >$ 30 km s^{-1}, so that our sample of sharp-line stars is significantly contaminated by stars having $v >$ 30 km s^{-1} and $P < 5^d.4$, such as 21 Per ($P = 2^d.88$), 78 Vir ($P = 3^d.72$), and 108 Aqr ($P = 3^d.73$). In spite of this contamination, however, we take the properties of the sharp-line stars to be approximately representative of slow rotators of $P \geqslant 5^d$.

We next investigate the distribution of magnetic fields observed in the sharp-line stars for comparison with our observations of broad-line stars. We may conveniently compare the distribution of longitudinal field strengths which would be measured by an observer who observes a large number of stars at random phases, making the same number of observations of each star. Almost all the 25 sharp-line stars in the basic sample used here have been observed for magnetic fields. We have determined approximately the probability of measuring a field in the ranges 0–500 gauss, 500–1000 gauss, etc., in a series of measurements at random phases for each sharp-line star separately, using the mean photographic magnetic curves when they are available (see particularly Babcock 1960; Preston 1967*b*, 1969*a*, 1969*b*, 1972; Preston *et al.* 1969; Bonsack and Pilachowski 1974; Pyper 1969; Stępień 1968; Wolff 1973; Wolff and Bonsack 1972), or the distribution of the field measurements given by Babcock (1958) for stars for which no period is known. If fewer than four published measurements are available, the star is assumed to have zero field, so that we underestimate rather than overestimate the occurrence of large fields among the sharp-line stars. The resulting individual field distributions were then averaged together, giving the result shown in Table 4. This distribution can be represented approximately by a Gaussian distribution centered on 0 gauss with a standard deviation of $\sigma = 700$ gauss, as shown in the last row of Table 4, a fact which will be useful in later discussion. From Table 4, we observe that in field measurements of a sample of sharp-line stars, we expect a field in excess of 1 kgauss about 15 percent of the time; essentially this is due to the fact that the observed fields of about 30 percent of the sharp-line stars exceed 1 kgauss for a substantial part of their magnetic cycle.

TABLE 4

DISTRIBUTION OF OBSERVED FIELDS IN UNIFORM SAMPLE OF SHARP-LINE STARS

	FIELD RANGE (gauss)				
	0–500	500–1000	1000–1500	1500–2000	2000–
Observed Relative Frequency...	0.59	0.25	0.13	0.016	0.006
Gaussian Fit, $\sigma = 700$ gauss	0.53	0.32	0.12	0.028	0.004

We now examine the data of Table 2 to determine whether there is evidence in our observations for the presence of a distribution of observed fields like that given in Table 1. Almost all the stars in Table 2 were observed at least two times. In order to obtain a distribution of observed fields which gives essentially the same weight to each star, we consider only two measurements per star, choosing (for greatest statistical leverage) the two of smallest standard error. We shall arbitrarily use the most recent measurements when several of the same precision are available. This gives us a list of 30 measurements (two of the 16 stars in our list only have one observation), with errors ranging from 40 to 1000 gauss. The observations contained in this list are marked with an asterisk in Table 2. In this sample we do not have any measurements which exceed 1 kgauss, although if the distribution of longitudinal field measurements is the same in broad-line stars as given in Table 4 for sharp-line stars, we would expect four or five measurements in excess of 1 kgauss, even disregarding the large errors of some of the observations in the list.

We next calculate the *expected* distribution of measurements in the sample of 30 observations from the distribution of standard errors of measurement on two hypotheses: (1) that the broad-line stars have no magnetic fields, and (2) that the broad-line stars have a Gaussian distribution of longitudinal fields of standard deviation 700 gauss, like that in the last row of Table 4 (which is assumed to be hidden in the statistical noise of measurement). The expected distribution of field measurements on these two hypotheses is compared with the actual distribution of measurements found in the list in Table 5. It is clear that hypothesis (1), that no fields are present, is much more successful in accounting for the observed distribution than is hypothesis (2), and the difference between the expected number of observations above 1 kgauss on hypothesis (2) (almost 7) and the actual number (0) is great enough to be reasonably significant. We are therefore led to the tentative conclusion that if the *broad-line* Ap stars possess longitudinal fields, they are statistically considerably smaller than those of the sharp-line stars.

This conclusion is strengthened if we consider the well-observed broad-line stars for which four or more observations with $\sigma_H \leqslant 500$ gauss are available. There are seven such stars: γ Ari S, α Psc A, ι Cas, θ Aur, ϵ UMa, ω Her, and ϕ Dra. None of the field measurements of any of these stars with $\sigma_H \leqslant 500$ gauss exceed 1 kgauss, although from the distribution of fields found for the sharp-line stars we expect about 30 percent (i.e., two stars) of a well-measured sample to have fields which exceed 1 kgauss for a substantial fraction of their cycle.

This apparent lack of large magnetic fields in broad-line stars is not likely to be an aspect effect. The sharp-line stars include some (about 30%) rapid rotators seen from near the rotation poles, while the broad-line star group is somewhat deficient in pole-on stars. However, from discussions by Preston (1967*a*, 1971) and Landstreet (1970), it appears that the magnetic poles, which are the regions of largest longitudinal field in a magnetic star, are more likely to be located near the rotational equator than near the rotational poles in Ap stars, so that the deficiency of pole-on stars in the broad-line sample should not bias its field distribution against strong fields if slow and rapid rotators possess the same distribution of true magnetic fields. We are thus led to the further tentative conclusion that *rapidly rotating* Ap stars tend to have systematically smaller magnetic fields than slowly rotating ones, and indeed may usually be without fields. It is not clear where exactly the dividing line is, or how sharp it is, but the most rapidly rotating Ap stars with well-established fields are 78 Vir ($P = 3^{d}.72$) and HD 133029 ($P = 2^{d}.89$) (Winzer 1974) so that the dividing line may lie somewhere in the range $P = 3-5^{d}$.

A possible alternative explanation of the apparent systematic difference between the fields of rapidly and slowly rotating stars should be considered. As mentioned earlier, Borra (1974) has shown that photographic field measurements may in some particular cases systematically overestimate the true averaged longitudinal field by a factor of order 2, and we have pointed out that this may be the case for α^2 CVn. If this effect is common, the distribution of field strengths for sharp-line stars obtained from photographic measurements would need revision toward smaller field strengths, and the difference between sharp- and broad-line stars might be undetectable in the measurements presented here. However, from the limited observations given in Table 3, it appears that substantial systematic differences between photographic and Balmer-line field measurements are the exception rather than the rule, so that we consider it more likely that a real difference exists between the field distributions of slowly and rapidly rotating Ap's. Further Balmer-line magnetic measurements of several known magnetic stars, for the purpose of clarifying the relationship between photographic and photoelectric measurements, are planned.

Two further conclusions are suggested. First, stars of large period may have become slow rotators by virtue of having large magnetic fields to slow the rotation, and the rapid rotators were left rotating relatively rapidly (more like normal B and A stars) because of their relatively weak fields. If this is correct, one would expect that at least a few (young) rapidly rotating stars should be found to be magnetic.

Secondly, if the magnetic field is in some way related to the other peculiarities of Ap stars, as has been frequently suggested, rapid rotators should be "less peculiar" than other Ap's.

TABLE 5

Distribution of Field Values in List of Best Measurements

	Field Interval				
	0–500	500–1000	1000–1500	1500–2000	2000–
Observed number of measurements in interval...	21	9	0	0	0
Expected number, hypothesis (1) (no fields)...	23.9	4.8	1.0	0.3	0.1
Expected number, hypothesis (2) (Gaussian distribution, $\sigma = 700$ gauss)	13.8	9.4	4.6	1.6	0.6

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