THE ASTROPHYSICAL JOURNAL, 199:436–442, 1975 July 15

THE $^{12}C/^{13}C$ RATIO IN STELLAR ATMOSPHERES. IV. ELEVEN G AND K TYPE GIANTS

J. TOMKIN, D. L. LAMBERT, AND R. E. LUCK
McDonald Observatory and Department of Astronomy, The University of Texas, Austin
Received 1974 December 2

ABSTRACT

High-resolution photoelectric scans of portions of the 4–0, 5–1, 2–0, 3–1, and 4–2 bands of the CN red system in the spectra of 11 late-type giants have been obtained. Analysis of the ^{12}CN and ^{13}CN line intensities gives the following $^{12}C/^{13}C$ abundance ratios: 20 ± 3 (ε Vir), 22 ± 3 (ζ Cyg), 19 ± 2 (β Cet), 11.5 ± 1.5 (ε Cyg), 22 ± 4 (46 LMi), 17 ± 4 (ζ Cep), 19 ± 3 (α Ari), 6.5 ± 1 (γ Leo A), 18 ± 3 (α Hya), 13 ± 2 (γ Dra), and 13 ± 3 (γ Sge). Revised $^{12}C/^{13}C$ ratios of 14 and 9 are derived for α Ser and α Tau, respectively. The low $^{12}C/^{13}C$ ratios of ε Cyg and α Ser are interpreted as evidence that they have been extensively mixed during the core helium flash.

Subject headings: abundances, stellar — late-type stars — molecules

I. INTRODUCTION

Recent analyses of lines due to the CN molecule in the spectra of K and M giants have shown that these stars have a high ^{13}C abundance. The observed $^{12}C/^{13}C$ isotope ratios cover a range from 5.1 for the supergiant ε Peg (Lambert and Tomkin 1974) to 18 for the K giant μ Leo (Tomkin and Lambert 1974). The terrestrial and solar $^{12}C/^{13}C$ ratio is 89.

In this paper we present $^{12}C/^{13}C$ ratios for a further 11 G and K type giants. The stars were selected primarily because they are relatively bright, and hence additional information about the range of $^{12}C/^{13}C$ ratios in G and K stars could be obtained with very modest investments of observing time.

Epsilon Virginis is a G8 IIIab star with a composition very similar to that of the Sun (Cayrel and Cayrel 1963). The star ζ Cyg has strong CN lines. It was classified as a barium star by Chromey *et al.* (1969), but Keenan (Morgan and Keenan 1973) remarks that the Ba II 4554 Å line is not noticeably strengthened, and he does not list it as a barium star. The star ε Cyg is metal-deficient by a factor of 2.5 relative to the Sun (Hansen and Kjaergaard 1971). Gamma Leonis A is a K0 III star separated by 4″.4 from a G7 III companion. Both stars are metal-deficient by a factor of 5 relative to the Hyades (Helfer and Wallerstein 1968). We had intended to obtain the $^{12}C/^{13}C$ ratios of γ Leo A and B, but it is more difficult to do this for the secondary, both because it is fainter and because its CN lines are weaker. Its $^{12}C/^{13}C$ ratio, estimated from one scan of ^{12}CN and ^{13}CN lines, is greater than that of γ Leo A. As a K1.5 Ib star, ζ Cep is in the same luminosity class as ε Peg. Analysis of narrow-band photoelectric photometry (Hansen and Kjaergaard 1971; Williams 1971) shows that the remaining six stars investigated all have approximately solar metal abundances.

We also rediscuss the $^{12}C/^{13}C$ ratios of α Ser (Day,

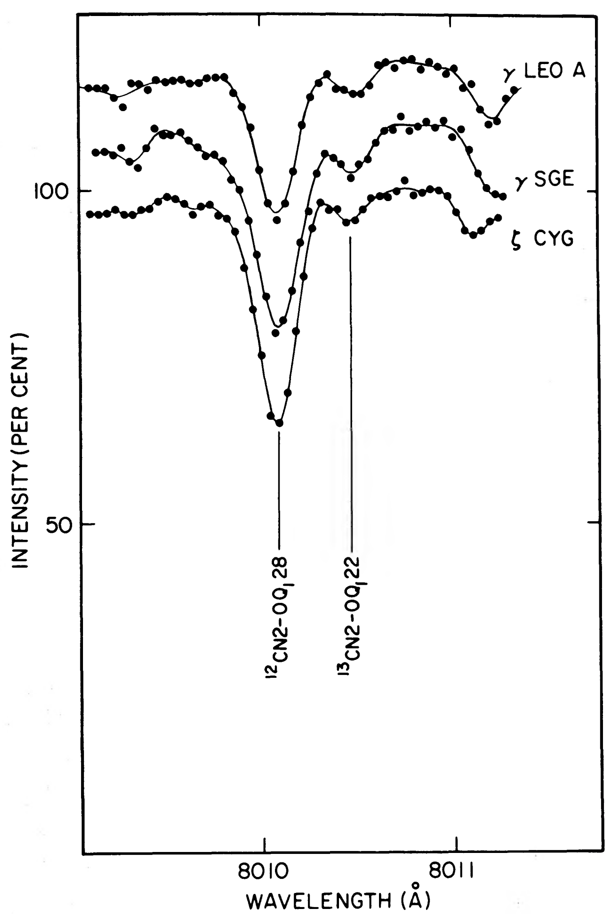

FIG. 1.—Scans at 8010 Å showing a ^{12}CN 2–0 *Q*-branch line and an adjacent ^{13}CN 2–0 *Q*-branch line in the spectra of γ Leo A, γ Sge, and ζ Cyg. The intensity scale is for the scan of ζ Cyg.

Lambert, and Sneden 1973) and α Tau (Tomkin and Lambert 1974).

II. OBSERVATIONS

Photoelectric spectral scans of lines belonging to the red CN system in the program stars were obtained with the McDonald 2.7-m telescope and the Tull coudé scanner (Tull 1972).

Observations were made of ^{12}CN 4–0 and 5–1 lines in the red (between 6200 and 6400 Å) and of ^{12}CN and ^{13}CN 2–0 lines in the near-infrared (between 7900 and 8100 Å). Additional scans of ^{12}CN 3–1 and 4–2 lines near 8400 Å were made for the K5 spectral type stars γ Dra, γ Sge, and α Tau. Scans of ^{12}CN 4–0 and 5–1 lines in α Ser were also obtained.

The scans typically covered 2–4 Å at a signal-to-noise ratio of 100:1 or better. The resolution was 0.07 Å in the red and 0.09 Å in the infrared. The excellent photometric quality of the data can be seen in Figure 1, which shows ^{12}CN and ^{13}CN 2–0 lines in the spectra of three stars at 8010 Å.

The wavelength intervals scanned were chosen for lines that were known to be relatively well defined and unblended from previous observations of CN lines in red giants (Day *et al.* 1973; Tomkin and Lambert 1974). The lines were also chosen to be free of contamination by telluric lines at the time of observation.

The equivalent widths of the ^{12}CN and ^{13}CN lines are given in Tables 1, 2, and 3. We also give the equivalent widths for the ϵ Peg observations used by Lambert and Tomkin (1974). In Table 3, the corrected equivalent widths are given for those lines where corrections have been made to allow for the effect of a blending line.

III. ANALYSIS

The effective temperatures, gravities, and luminosities of the 11 stars are listed in Table 4. The temperatures and gravities of ϵ Vir and γ Leo A are taken from Cayrel and Cayrel (1963) and Fawell (1970), respectively. The temperatures of the remaining stars were obtained from their infrared colors (Johnson *et al.* 1966). For ζ Cep, the unreddened colors for a star of the same spectral type were used instead of the observed colors, which are affected by interstellar reddening. Their gravities were estimated from their

TABLE 1

^{12}CN LINES

Wavelength (Å)	Identity	Equivalent Width (mÅ)										
		ε Vir	ζ Cyg	β Cet	ε Cyg	46 LMi	ζ Cep	α Ari	γ Leo A	α Hya	α Ser	ε Peg
6263.992	4-0 $R_2$32	-	-	9	-	-	15	7	-	18	17	-
6264.361	4-0 $R_1$31	-	-	8	-	-	14	10	-	14	12	-
6324.446	4-0 $R_1$41	6	-	-	-	-	-	-	-	-	-	-
6331.661	4-0 $R_1$42	6	10	7	-	-	19	6	-	9	-	-
6332.683 .683	5-1 R_2 1 5-1 R_2 6	6	10	-	-	-	25	9	-	20	13	11
6333.186 .186	5-1 R_2 0 5-1 R_2 7	5	-	-	-	-	-	-	-	-	-	-
6333.877	5-1 R_2 8	7	-	7	-	-	18	7	-	17	12	14
6334.332	4-0 $Q_2$36	14	15	15	-	-	22	13	-	24	25	19
7061.301	3-0 $P_1$25	-	-	-	-	-	-	-	-	-	-	43
7062.488	3-0 $R_2$40	-	-	-	-	-	-	-	-	-	-	24
7105.584	3-0 $R_2$45	-	-	-	-	-	-	-	-	-	-	25
7106.144	4-1 $R_2$17	-	-	-	-	-	-	-	-	-	-	37
7873.961 .961	2-0 R_2 1 2-0 R_2 8	-	-	-	-	-	-	-	-	-	-	55
7874.844 .844	2-0 R_2 0 2-0 R_2 9	-	-	-	-	-	-	-	-	-	-	57
7875.964	2-0 $R_2$10	-	-	-	-	-	-	-	-	-	-	58
7934.836	2-0 P_1 9	-	-	-	28	37	94	39	-	-	-	52
7936.378	2-0 $P_2$13	-	-	-	19	25	52	30	-	-	-	40
7950.423	2-0 $Q_2$21	-	-	-	-	-	141	-	-	-	-	99
7958.692	2-0 $P_1$210	-	-	-	-	-	-	-	-	11	-	-
7960.693	2-0 $R_1$30	-	-	-	-	-	-	-	-	63	-	-
7962.606	2-0 $Q_1$21	-	-	-	-	-	-	-	-	109	-	-
7967.099	2-0 $R_1$31	44	-	-	-	-	-	-	-	-	-	-
7968.489	2-0 $Q_1$22	66	-	-	-	-	-	-	-	-	-	-
7973.782	2-0 $R_1$32	45	-	57	33	-	103	-	-	77	-	-
7974.664	2-0 $Q_1$23	71	-	81	51	-	-	-	-	101	-	-
7977.218	2-0 $Q_2$25	59	-	-	-	-	-	-	-	92	-	-
8000.261	2-0 $Q_2$28	-	-	-	-	-	-	-	-	83	-	-
8003.213	2-0 $P_2$22	-	-	-	-	-	94	-	-	-	-	-
8003.553	2-0 $R_1$36	-	-	-	-	36	-	-	-	-	-	-
8003.910	2-0 $R_2$37	-	-	-	-	38	-	-	-	-	-	-
8010.084	2-0 $Q_1$28	59	91	65	55	-	-	67	57	-	-	-
8012.546	2-0 $R_2$38	-	-	-	-	-	-	-	22	-	-	-
8015.668	2-0 $P_1$22	-	65	50	32	-	-	-	37	73	-	81
8017.012	2-0 $Q_2$30	-	-	72	-	-	-	62	-	82	-	99
8018.052	2-0 $Q_1$29	-	-	-	-	-	-	-	60	92	-	-
8020.222	2-0 $R_1$38	-	-	-	-	-	-	-	38	-	-	-
8034.964 .964	2-0 $Q_1$31 2-0 $Q_2$32	-	-	-	81	-	-	-	-	-	-	-
8051.070	2-0 $P_1$26	38	56	46	-	33	-	38	30	-	-	-
8053.064	2-0 $Q_1$33	69	-	-	-	52	169	79	51	-	-	-
8057.272	2-0 $R_1$42	38	-	-	-	35	94	-	30	58	-	62
8062.558	2-0 $Q_1$34	-	-	-	-	-	-	-	41	-	-	96
8064.110	2-0 $Q_2$35	-	77	67	50	-	142	61	55	105	-	-

TABLE 2

^{12}CN LINES MEASURED IN K5 GIANTS

WAVELENGTH (Å)	IDENTIFICATION	EQUIVALENT WIDTH (mÅ) γ Dra	γ Sge	α Tau
6334.332......	4–0 $Q_2 36$	26	34	...
7934.836......	2–0 $P_1 9$	...	61	...
7936.378......	2–0 $P_2 13$	...	48	...
8003.910......	2–0 $R_2 37$	48	...	...
8010.084......	2–0 $Q_1 28$	80	92	...
8015.668......	2–0 $P_1 22$	71	...	...
8017.012......	2–0 $Q_2 30$	75	73	...
8064.110......	2–0 $Q_2 35$	...	91	...
8397.924......	4–2 $Q_1 25$	39	...	33
8398.476......	3–1 $Q_1 45$	58	...	46
8399.101......	4–2 $R_2 35$	31	...	...
8399.756......	4–2 $R_1 34$	28	...	...
8429.957......	4–2 $Q_2 30$	39	42	35
8431.244......	4–2 $Q_1 29$	53	...	36
8432.374......	3–1 $P_2 41$	33	34	24
8433.241......	4–2 $P_2 24$	21	26	20
8476.355......	4–2 $P_2 28$	26	33	19
8476.671......	4–2 $R_1 42$	27	33	16
8477.057......	4–2 $P_1 27$	36	39	...
8477.520......	2–0 $Q_2 63$	35	42	22
8478.460......	2–0 $P_2 56$	41	42	28
8479.630......	3–1 $P_2 44$	36	32	22
8479.955...... 8480.033......	4–2 $R_2 43$ 5–3 $R_1 1$	22	...	13

effective temperatures, luminosities, and masses. The masses were inferred from the location of the stars in the H-R diagram.

The CN excitation temperatures for use in plotting the CN curves of growth for each star were derived from these effective temperatures and gravities. The method used to derive the excitation temperatures, and the molecular data used for the curves of growth, were described in Tomkin and Lambert (1974).

The fit of the theoretical curve of growth to the ^{12}CN curve of growth was used to determine the microturbulence. The results, which are given in Table 4, are in the range 0.5–3 km s^{-1}, and the uncertainty is typically ± 0.5 km s^{-1}. They are consistent with the determinations of microturbulence in G and K giants by Gustafsson, Kjaergaard, and Andersen (1974). The range of equivalent widths of ^{12}CN lines measured in 46 LMi was insufficient to set its microturbulence; a representative value of 1.0 km s^{-1} was assumed.

The ^{12}C/^{13}C ratio for each star is obtained directly from the horizontal separation between its ^{12}CN and ^{13}CN curves of growth. Curves of growth for two of the stars, γ Leo A and α Hya, are shown in Figure 2.

The ^{12}C/^{13}C ratio of α Ser reported by Day *et al.* (1973) was based on equivalent widths of ^{12}CN and ^{13}CN 2–0 lines measured from photoelectric scans, and equivalent widths of ^{12}CN 4–0 lines that had been obtained from high-dispersion (2 Å mm^{-1}) photographic plates. We have redone the analysis with the equivalent widths for the ^{12}CN 4–0 and 5–1 lines, listed in Table 1, which were measured from scans.

The ^{12}C/^{13}C ratio of the K5 type star α Tau reported by Tomkin and Lambert (1974) was derived using weak ^{12}CN 4–0 lines. Generally these lines are not very well defined in the spectra of K5 and later type stars because of blending with numerous unidentified lines and the uncertain location of the continuum. The near-infrared spectra of K5 stars are relatively line-free and have many unblended ^{12}CN 3–1 and

TABLE 3

^{13}CN 2–0 LINES

Wavelength (Å)	Identification	Equivalent Width (mÅ) ϵ Vir	ζ Cyg	β Cet	ϵ Cyg	46 LMi	ζ Cep	α Ari	γ Leo A	α Hya	γ Dra	γ Sge	ϵ Peg	Comments*
7934.198	$R_2 17$	-	-	-	-	-	-	-	-	-	-	-	19	7-4 $R_2 52$
7935.627	Q_2 8	-	-	-	-	-	9	3	-	-	-	7	14	-
7952.555	$Q_2 13$	-	-	-	-	-	-	-	-	-	-	-	22	-
7964.022	$Q_1 11$	-	-	-	-	-	-	-	-	10	-	-	-	8-5 $Q_1 28$
7966.780 .863	$Q_1 12$ $P_2 11$	6	-	-	-	-	-	-	-	-	-	-	-	-
7973.181	$Q_1 14$	4	-	8	6	-	24	-	-	16	-	-	-	-
7976.821	$Q_1 15$	8	-	-	-	-	-	-	-	16	-	-	-	-
7998.205	$Q_2 22$	-	-	-	-	-	-	-	-	19	-	26	-	7-4 $Q_1 49$
7999.421 .421	$Q_1 20$ $R_2 31$	-	-	-	-	-	-	-	-	18	-	-	-	-
8004.554 .728 .781	$Q_2 23$ $Q_1 21$ $P_2 17$	-	-	-	-	12	48	-	-	27	30	28	91	-
8006.060	$R_2 32$	-	-	-	-	-	-	-	-	7	7	-	25	-
8010.458	$Q_1 22$	5	10	-	6	-	-	10	10	-	18	17	-	-
8015.166	$R_1 32$	-	7	5	-	-	-	-	7	9	9	11	24	-
8016.419	$Q_1 23$	-	12	11	9	-	-	-	11	15	20	20	48	-
8036.031	$Q_1 26$	-	-	-	7	-	-	-	-	-	-	-	-	-
8048.271 .399	$Q_2 29$ $P_1 21$	12	-	-	-	-	-	-	-	28	-	-	-	3-1 $R_{21} 19$
8050.551	$Q_1 28$	-	-	-	-	6	-	-	12	-	-	-	-	-
8051.731 .731 .894	$R_1 37$ $P_2 23$ $R_2 38$	10	11	14	-	6	-	11	18	-	-	-	-	3-1 $R_{21} 21$
8056.504	$Q_2 30$	-	-	-	-	8	-	-	-	13	-	-	-	-
8065.027	$Q_2 31$	-	10	7	7	-	15	6	11	13	16	18	41	8-5 $P_1 32$

* Corrections have been made for these blending ^{12}CN lines.

TABLE 4

DATA FOR THE PROGRAM STARS

Star	Spectral Type	T_e (K)	log g	log $L/L_\odot$*	θ_{exc}(CN)	Microturbulence (km s^{-1})	Source of Luminosity
ε Vir........	G8 IIIab	4950	2.7	1.59	1.19	1.0	Hansen and Kjaergaard 1971
ζ Cyg........	G8 II CN 1	4950	1.9	2.97	1.20	0.6	Keenan 1973
β Cet........	K1 III	4800	2.9	1.65	1.21	0.5	Parallax (Jenkins 1952)
ε Cyg........	K0$^-$ III	4750	2.9	1.65	1.21	1.4	Parallax (Jenkins 1952)
46 LMi......	K0 III–IV	4700	2.9	1.55	1.22	...	Hansen and Kjaergaard 1971
ζ Cep........	K1.5 Ib	4700	1.5	3.45	1.24	3.2	Keenan 1973
α Ari........	K2 IIIab	4440	2.5	1.61	1.26	1.1	Hansen and Kjaergaard 1971
γ Leo A......	K0 III	4300	1.7	2.39	1.27	1.7	Pagel and Tomkin 1969
α Hya........	K3 II–III	4100	1.9	2.50	1.28	2.0	Hansen and Kjaergaard 1971
γ Dra........	K5 III	3780	1.5	2.42	1.38	1.3	Hansen and Kjaergaard 1971
γ Sge........	K5–M0 III	3780	1.4	2.73	1.39	1.3	Keenan 1973

* Log $L/L_\odot$ was obtained using bolometric corrections taken from Johnson 1966.

4–2 lines that are sufficiently weak to define the linear part of the curve of growth. Measures of ^{12}CN 3–1 and 4–2 lines in α Tau from three scans between 8400 and 8500 Å were used to derive a revised $^{12}C/^{13}C$ ratio. The analysis of the K5 stars γ Dra and γ Sge also used these lines.

IV. RESULTS

The $^{12}C/^{13}C$ ratios derived for the 11 program stars and the revised ratios for α Ser and α Tau are included in Table 5. The errors have been estimated from the scatter of the ^{12}CN and ^{13}CN lines about their respective curves of growth.

The new $^{12}C/^{13}C$ ratio for α Ser is not greatly different from the value of 12 derived by Day *et al.* (1973). The new value is preferred because it is based exclusively on the photoelectric scan data.

TABLE 5

RED GIANT $^{12}C/^{13}C$ RATIOS

Star	T_e (K)	log $L/L_\odot$	$^{12}C/^{13}C$	Source of $^{12}C/^{13}C$*
ε Vir........	4950	1.59	20 ± 3	...
ζ Cyg........	4950	2.97	22 ± 3	...
β Cet........	4800	1.65	19 ± 2	...
β Gem......	4755	1.67	16 ± 2	1
ε Cyg........	4750	1.65	11.5 ± 1.5	...
46 LMi......	4700	1.55	22 ± 4	...
ζ Cep........	4700	3.45	17 ± 4	...
μ Leo........	4460	2.00	18 ± 2	1
α Ari........	4440	1.61	19 ± 3	...
α Ser........	4420	1.73	14 ± 2	...
γ Leo A......	4300	2.39	6.5 ± 1	...
α Boo........	4165	2.26	7.2 ± 1.5	2
ε Peg........	4100	3.88	5.1 ± 0.5	3
α Hya........	4100	2.50	18 ± 3	...
α Tau........	3790	2.68	9 ± 1	...
γ Dra........	3780	2.42	13 ± 2	...
γ Sge........	3780	2.73	13 ± 3	...
α Sco........	3600	4.42	12 ± 2	4
α Ori........	3500	4.78	7 ± 1.5	5

* SOURCE.—(1) Tomkin and Lambert 1974. (2) Day *et al.* 1973. (3) Lambert and Tomkin 1974. (4) Lambert 1975. (5) Lambert, Dearborn, and Sneden 1974.

The revised result of 9 for α Tau compares with a value of 12 derived by Tomkin and Lambert (1974). It should be more accurate because the equivalent widths of the ^{12}CN 3–1 and 4–2 lines are more reliable than those of the ^{12}CN 4–0 lines. This is evident in the improved definition of the ^{12}CN curve of growth. The average energy difference between the lower levels of the ^{12}CN 3–1 and 4–2 lines and the ^{13}CN 2–0 lines is about 0.5 eV. Consequently, because the CN excitation temperature is not known precisely, there is an extra source of error in the $^{12}C/^{13}C$ ratio that is not present when ^{12}CN 4–0 lines are used in the analysis. The quoted errors in the $^{12}C/^{13}C$ ratios of α Tau, γ Dra, and γ Sge which are subject to this effect include a possible excitation error calculated assuming an uncertainty of ± 0.1 in θ_{exc}(CN).

Recently, Ridgway (1974) noted that the $^{13}C^{16}O$ band heads in the first overtone sequence of vibration–rotation bands near 2.3 μ were not detectable on spectra of α Ser at a resolution of 4 cm^{-1}. Similar spectra of other K giants yielded, by a spectrum-synthesis technique, $^{12}C/^{13}C$ ratios which are consistent with the results of the present program. For example, Ridgway finds $^{12}C/^{13}C = 10$ for β Gem, α Tau, and α Ari, with an uncertainty of less than a factor of 2. Our results (see Table 5) are $^{12}C/^{13}C = 16$ (β Gem), 9 (α Tau), and 19 (α Ari). From the absence of the $^{13}C^{16}O$ band heads in α Ser, he concludes that $^{12}C/^{13}C > 30$, which is inconsistent with our result. Ridgway remarks that in α Ser the total blocking due to $^{12}C^{16}O$ is greater than in α Ari, and at the same time the $^{12}C^{16}O$ band heads are 40 percent shallower. It may be that because of the unusual distribution of the CO band strength, the lower limit to the $^{12}C/^{13}C$ ratio implied by the absence of the $^{13}C^{16}O$ band heads is lower, and is compatible with the value 14 ± 2 that we have determined.

V. DISCUSSION

The results bring the current total of red giants with accurately determined $^{12}C/^{13}C$ ratios to 19. These ratios are much lower than the solar-system value of 89, and are evidence that ^{13}C is markedly enhanced in

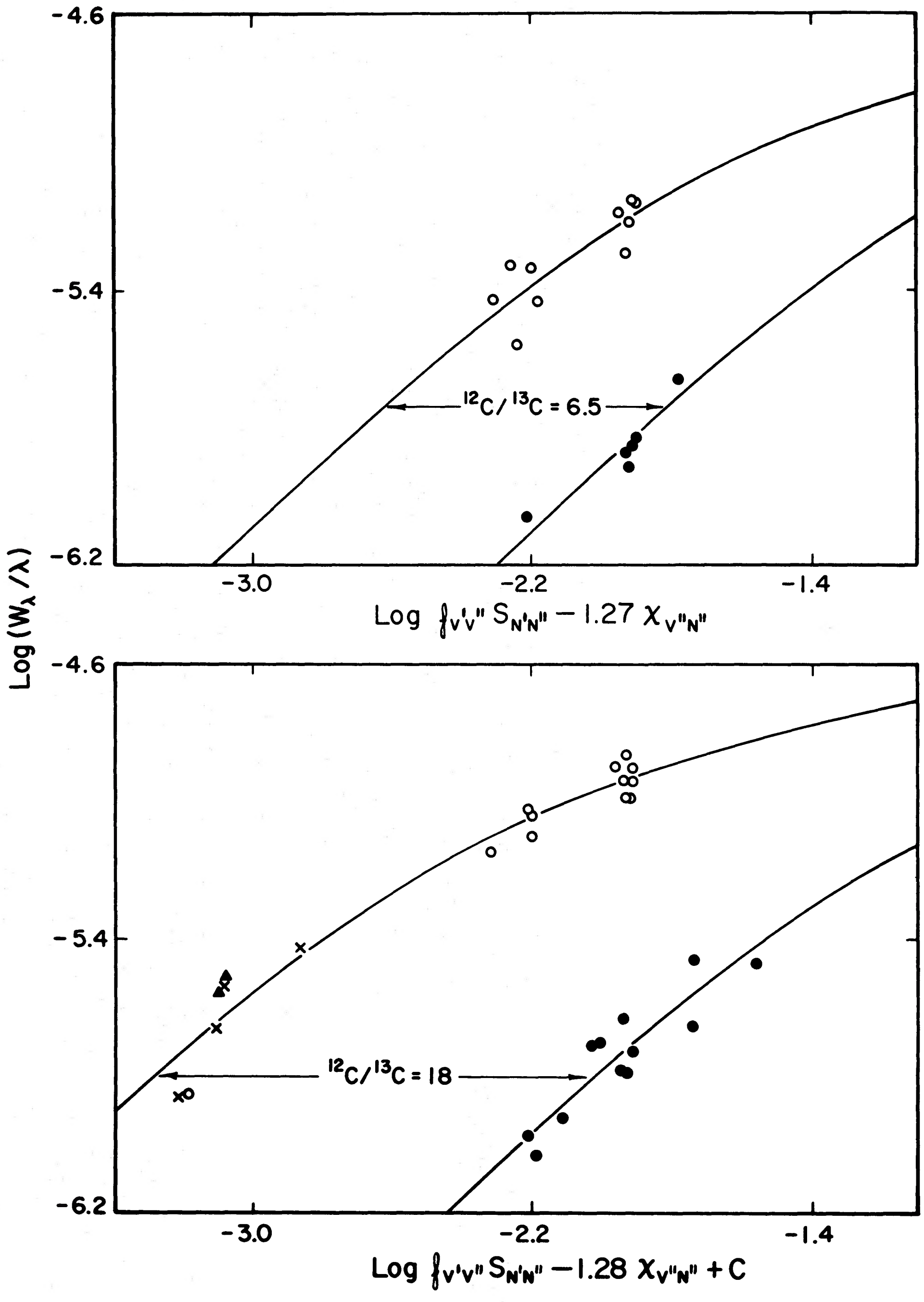

FIG. 2.—Curves of growth for ^{12}CN and ^{13}CN lines in γ Leo A (*upper diagram*) and α Hya (*lower diagram*). *Open circles, crosses,* and *triangles*: ^{12}CN 2–0, 4–0, and 5–1 lines, respectively; *filled circles*, ^{13}CN 2–0 lines.

red giants. The stars and their ^{12}C/^{13}C ratios with related data are listed in Table 5. The sources of temperatures and luminosities of the eight stars in Table 5 that are not in Table 4 have been given in Tomkin and Lambert (1974) and Lambert and Tomkin (1974). The locations of the stars in the H-R diagram are plotted in Figure 3. The evolutionary tracks are for stellar models of Population I stars calculated by Paczynski (1970).

Theoretical interpretations of the results displayed in Figure 3 fall into two groups. Prior to a demonstration that ^{13}C was indeed overabundant, Iben (1967) pointed out that, as a star evolves up from the base of the red-giant branch, a deep convective envelope

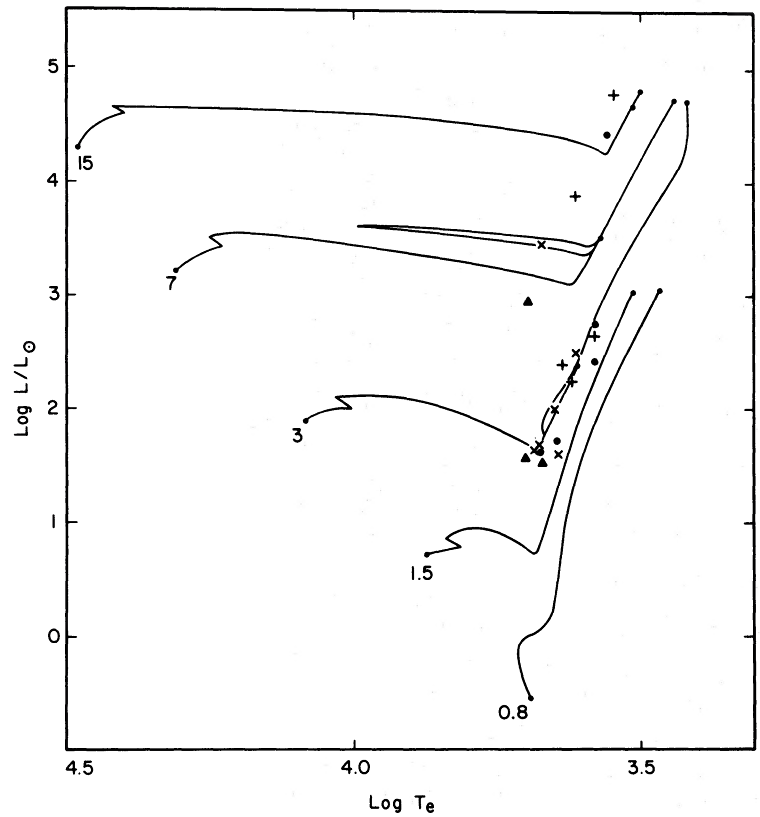

FIG. 3.—Location in the H-R diagram of G, K, and M giants with known $^{12}C/^{12}C$ ratios. The symbols denote different ranges of $^{12}C/^{13}C$ ratios; the key is: +, 5–10; ●, 10–15; ×, 15–20; and ▲, 20–25. The numbers at the start of each evolutionary track are the stellar masses in solar units. Dots on the tracks show the main sequence and the points of helium and carbon ignition in the cores of the models. The tracks for the 0.8 and 1.5 $M_{\odot}$ models stop at helium ignition.

develops, and material rich in ^{13}C is brought to the surface; this material was processed while the star was close to the main sequence. He predicted that the surface $^{12}C/^{13}C$ ratio would be lowered from an assumed terrestrial ratio to a value in the range 20–30. The least luminous stars of the sample, ϵ Vir and 46 LMi, are possible examples of this evolutionary stage.

At and after helium ignition, the complexity of the stellar structure calculations increases dramatically. Ulrich and Scalo (1972), Sackmann, Smith, and Despain (1974), and Iben (1975) have presented different treatments of the problem of mixing during helium burning. A general conclusion is that it is quite possible for the $^{12}C/^{13}C$ ratio to be lowered and to attain a value close to $^{12}C/^{13}C = 4$, which is the equilibrium value for the CNO cycle. The remarkably low ratio, $^{12}C/^{13}C = 5.1$, found for ϵ Peg is presumably attained during helium burning.

However, the current theoretical ideas about stellar evolution after helium ignition do not fully explain all the low $^{12}C/^{13}C$ ratios. The second phase of mixing and surface ^{13}C enrichment is supposed to take place after the model has a double-shell source structure, which is established during the second ascent of the giant branch when the luminosity becomes greater than it had been at helium ignition ($\log L/L_{\odot} \geq 2.4$ for the mass range 0.8–3 $M_{\odot}$). How is the ^{13}C enrichment of less luminous stars with low $^{12}C/^{13}C$ ratios like ϵ Cyg ($^{12}C/^{13}C = 11.5$) and α Ser ($^{12}C/^{13}C = 14$) accounted for?

The $^{12}C/^{13}C$ ratios of some K type subgiants we are presently investigating are all 20 or greater. The stars are of sufficiently low luminosity that it is certain they are on the first ascent of the red-giant branch. Their high ratios are evidence that there is only moderate surface enrichment of ^{13}C during the initial mixing phase, as Iben predicted. This suggests that an additional source of ^{13}C is responsible for the low $^{12}C/^{13}C$ ratios of stars like ϵ Cyg and α Ser. What the source is, and the stage of evolution at which it sets in, are not clear at present. The most likely possibility is that, in these stars, the core helium flash was violent enough to cause extensive mixing.

Interpretation of Figure 3 is hampered by the crowding together of the giant branches. Uncertainties in the luminosity and effective temperature can lead to an intolerable mixing of stars of different masses in the H-R diagram. A way around the problem is to observe the giant branch of an open (or globular) cluster. It is of interest to note that ϵ Vir, γ Sge, and α Tau are assigned by Eggen (1974) to the Hyades moving group. Eggen notes that the Hyades giants correspond to a narrow mass range around 2.5 $M_{\odot}$. Our measurements show an increase of ^{13}C enrichment up the giant branch from ^{12}C/^{13}C = 20 (ϵ Vir) to 13 (γ Sge) and 9 (α Tau). A sample of about 10 Hyades giants is being observed to try to determine the dependence of the ^{12}C/^{13}C ratio on the stage of evolution.

A thorough check on theoretical calculations for evolution at the red-giant stage demands that the other predicted compositional changes be investigated. Changes in the C, N, and O abundances are predicted; for example, prior to helium ignition the N/C ratio should increase as processed material is mixed to the surface (Iben 1966). Observational evidence for such abundance changes was presented by Greene (1969). Further observational results on C, N, and O abundances in giants would be of great interest.

We wish to thank Mr. D. Dearborn for the use of his measures of the equivalent widths of four CN lines in 46 LMi, and Miss C. Webb for the loan of a grid of ATLAS 5 model atmospheres of late-type stars. This research has been supported in part by the National Science Foundation under grant GP-43959.

REFERENCES

Cayrel, G., and Cayrel, R. 1963, *Ap. J.*, **137**, 431.
Chromey, F. R., Faber, S. M., Wood, A., and Danziger, I. J. 1969, *Ap. J.*, **158**, 599.
Day, R. W., Lambert, D. L., and Sneden, C. 1973, *Ap. J.*, **185**, 213.
Eggen, O. J. 1974, *Pub. A.S.P.*, **86**, 129.
Fawell, D. 1970, Unpublished Ph.D. thesis, University of Sussex.
Greene, T. F. 1969, *Ap. J.*, **157**, 737.
Gustafsson, B., Kjaergaard, P., and Andersen, S. 1974, *Astr. and Ap.*, **34**, 99.
Hansen, L., and Kjaergaard, P. 1971, *Astr. and Ap.*, **15**, 123.
Helfer, H. L., and Wallerstein, G. 1968, *Ap. J. Suppl.*, **16**, 1.
Iben, I. 1966, *Ap. J.*, **143**, 516.
———. 1967, *Ann. Rev. Astr. and Ap.*, **5**, 571.
———. 1975, *Ap. J.*, **196**, 525.
Jenkins, L. F. 1952, *General Catalogue of Trigonometric Stellar Parallaxes* (New Haven: Yale University Press).
Johnson, H. L. 1966, *Ann. Rev. Astr. and Ap.*, **4**, 193.
Johnson, H. L., Mitchell, R. I., Iriarte, B., and Wisniewski, W. Z. 1966, *Pub. Lunar and Planet. Lab.*, **4**, No. 63, p. 99.
Keenan, P. C. 1973, *Problems of Calibration of Absolute Magnitudes and Temperatures of Stars*, ed. B. Hauck and B. E. Westerlund (Dordrecht: Reidel), p. 68.
Lambert, D. L. 1975, in preparation.
Lambert, D. L., Dearborn, D. S., and Sneden, C. 1974, *Ap. J.*, **193**, 621.
Lambert, D. L., and Tomkin, J. 1974, *Ap. J.* (*Letters*), **194**, L89.
Morgan, W. W., and Keenan, P. C. 1973, *Ann. Rev. Astr. and Ap.*, **11**, 29.
Paczynski, B. J. 1970, *Acta Astr.*, **20**, 47.
Pagel, B. E. J., and Tomkin, J. 1969, *Quart. J.R.A.S.*, **10**, 194.
Ridgway, S. T. 1974, *Ap. J.*, **190**, 591.
Sackmann, I. J., Smith, R. L., and Despain, K. H. 1974, *Ap. J.*, **187**, 555.
Tomkin, J., and Lambert, D. L. 1974, *Ap. J.*, **193**, 631.
Tull, R. G. 1972, in *Proceedings ESO/CERN Conference on Auxiliary Instrumentation for Large Telescopes*, p. 259.
Ulrich, R. K., and Scalo, J. M. 1972, *Ap. J.* (*Letters*), **176**, L37.
Williams, P. M. 1971, *M.N.R.A.S.*, **153**, 171.

J. TOMKIN, D. L. LAMBERT, and R. E. LUCK: Department of Astronomy, University of Texas, Austin, TX 78712