# DISTANCES AND LUMINOSITIES OF IRREGULAR VARIABLES OF TYPE N* 

Benjamin F. Peery, Jr.<br>Department of Astronomy, Indiana University<br>Received 1974 July 26; revised 1975 January 13


#### Abstract

Near-infrared photometric data of Baumert are shown to suggest that the variety in the observed $(0.78-1.08) \mu$ color index of N irregular variables ( $\mathrm{L} b$ and $\mathrm{SR} b$ ) is primarily a consequence of interstellar reddening. On this hypothesis, color excesses are used to estimate distances of 60 N irregulars. Luminosities follow from photometry relative to Vega and the observed flux distribution of 19 Psc out to $14 \mu$; the latter star is taken as the prototype N irregular. Derived distances are supported by the Oort double-sine distribution of radial velocities, by some apparent association with galactic spiral features, and by agreement with bolometric corrections of Mendoza and Johnson. Within a Keenan-Morgan C subclass there is a dispersion in $M_{\text {bol }}$ of $\sim 1 \mathrm{mag}$ or more. $M_{\text {bol }}$ increases from $\left\langle M_{\text {bol }}\right\rangle=-4.4$ in C4 to $\left\langle M_{\text {bol }}\right\rangle=-5.6$ in C7. Near-infrared CN bands do not correlate with luminosity, but the Merrill-Sanford bands emerge as negative luminosity indicators.


Subject headings: carbon stars - luminosities - semiregular variables

## I. INTRODUCTION

The determination of distances and luminosities of stars of type N has been a persistent challenge for decades. Early efforts were usually statistical; the most recent report based upon this approach (Baumert 1974) contains the relevant bibliography. Gordon (1968) exploited observations of N stars in clusters; and she, and later Richer (1972), estimated absolute visual magnitudes from assumed absolute magnitudes of the companions of a few N stars. Eggen (1972a, b) has based his estimates for individual N stars on UBVRI photometry and kinematics. Peery (1970) attempted to find the mean absolute visual magnitude that best associates early-type N stars, as a group, with the spiral arms.

Advances in knowledge of the gross parameters that characterize N stars have been slow because of the paucity of observations in the infrared. Baumert (1972) has now provided a large and useful set of 8 -band (Wing 1970), near-infrared observations of 360 carbon stars. The results reported here are based upon Baumert's observations of the subgroup of 107 irregular (L $b$ ) and semiregular (SR $b$ ) variables (Table 1). We emphasize our exclusion here of Mira-type long-period variables, SR $a$ variables, and stars of type R. Among the N stars to be considered, the dis-" tinction between "irregular" and "semiregular" variables is of doubtful utility; available light curves of representatives of both variability types show only erratic behavior with no convincing differences between them (e.g., Eggen 1972a). We will refer to the entire subgroup as " N irregular variables" ( N Irr). The General Catalog of Variable Stars has been used to identify the N Irr. However, light curves of carbon

[^0]stars are still inadequately observed, and our subgroup may well contain some improperly designated variables.

## ii. characteristics of N Irr

Figure 1 is a composite display of measured fluxes from the bright N Irr 19 TX Psc (N0; C7,3), reduced to emergent flux at the stellar surface. I am indebted to Dr. Theodore D. Faÿ for his generosity in allowing me to use this diagram. Faÿ has constructed the curve from (i) spectrum scans out to $1.2 \mu$ at Indiana University, calibrated absolutely via Hayes (1970) standards; (ii) the Fourier spectroscopy of Johnson and Mendez (1970) between, 1.3 and $4 \mu$, which Faÿ calibrated via these authors' spectroscopy of Vega; and (iii) the calibrated infrared spectroscopy of Gillett, Merrill, and Stein (1971) from 2.8 to $14 \mu$. Calibration error may render the peak flux uncertain by 10 percent. Reduction to emergent flux density at the stellar surface is based upon the assumed angular diameter of $0 " 008$ (deVegt 1974). (Lasker et al. [1973] have deduced $0 " 009$. Angular diameters are derived from observations of lunar occultations and assumption of a uniformly bright stellar disk.) Baumert's passbands at $0.78,1.04$, and $1.08 \mu$ fall at flux peaks between CN absorption bands. The color index $[m(0.78 \mu)-$ $m(1.08 \mu)]$ will be assumed to be free of the influence of CN absorption.
The sample of N Irr display these characteristics:

1. Variability at $1.04 \mu$ is typically less than 0.2 mag among the stars for which Baumert presents multiple observations. This magnitude of variability stands in contrast to $\operatorname{SR} a$ and long-period variables of type N , for which the corresponding amplitude is typically 1-2 mag.
2. The $(0.78-1.08) \mu$ color index is sharply limited to values redder than 0.1 mag (Fig. 2).

TABLE 1
N IRREGULAR VARIABLES: OBSERVATIONAL DATA, DISTANCES AND LUMINOSITIES

| Star | Spectrum | $\mathrm{m}(1.04 \mu \mathrm{~m})$ | $\begin{aligned} & (0.78- \\ & 1.08)_{\mu^{m}} \end{aligned}$ | $\delta(B-\mathrm{V}) *$ | r (kpc) | z (pc) | L/L® | $\mathrm{M}_{\mathrm{Bol}}$ | M-S** | $\begin{aligned} & V_{R}(\text { corr }) \dagger \\ & \left(\mathrm{km} \mathrm{~s}^{-1}\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SU And | C6, 4 | 4.42 | 0.23 | 0.14 | - | - | - | - | - | - |
| UY And | C5,4 | 6.56 | 0.15 | 0.04 | - | - | - | - | - | - |
| AQ And | C5,4 | 3.84 | 0.20 | 0.10 | - | - | - | - | - | - |
| EW And | C7,3 | 5.13 | 0.33 | 0.27 | 2.0 | 370 | 12000 | -5.6 | - | -11 |
| U Ant | - | 1.64 | 0.15 | 0.04 | - | - | - | - | - | - |
| $V$ Aql | C5,4 | 2.41 | 0.33 | 0.27 | 0.3 | 29 | 3400 | -4.2 | 10 | +49 |
| V391 Aq1 | - | 5.89 | 0.39 | 0.35 | 1.2 | 150 | 2300 | -3.8 | - | +14 |
| T Ara | - | 5.10 | 0.44 | 0.42 | 1.3 | 160 | 6000 | -4.8 | - | - |
| RV Aur | - | 5.20 | 0.24 | 0.16 | - | - | - | - | - | - |
| TX Aur | C5,4 | 4.64 | 0.44 | 0.42 | 1.1 | 13 | 6600 | -4.9 | 1 | -12 |
| UU Aur | C5,4 | 1. 59 | 0.31 | 0.25 | 0.3 | 71 | 7000 | -5.0 | 2.5 | +6 |
| XZ Aur | - | 6.58 | 0.37 | 0.33 | 2.4 | 510 | 4900 | -4.6 | - | +15 |
| AF Aur | - | 5.57 | 0.34 | 0.29 | 2.3 | 350 | 11000 | -5.4 | - | -15 |
| CM Aur | C5,4 | 5.93 | 0.50 | 0.49 | 2.7 | 23 | 13000 | -5.6 | - | -33 |
| EL Aur | C5,4 | 4.08 | 0.60 | 0.62 | 0.8 | 77 | 7000 | -5.0 | 0 | -14 |
| FU Aur | C7,2 | 4.19 | 0.39 | 0.35 | 1.3 | 24 | 13000 | -5.6 | 0 | $+8$ |
| T Cae | - | 4.32 | 0.08 | - | - | - | - | - | - | - |
| U Cam | C5,4 | 3.24 | 0.37 | 0.33 | 0.5 | 52 | 4600 | -4.5 | 9 | - 1 |
| ST Cam | C5,4 | 2.56 | 0.30 | 0.23 | 0.4 | 100 | 5100 | -4.6 | 5 | -10 |
| UV Cam | C5,4 | 3.89 | 0.42 | 0.39 | 0.8 | 99 | 6800 | -4.9 | 0.5 | - 7 |
| X Cnc | C6,4 | 2.69 | 0.12 | 0.00 | - | - | - | - | - | - |
| Y CVn | C5,4 | 1.47 | 0.27 | 0.19 | - | - | - | - | - | - |
| W CMa | C6, 3 | 2.93 | 0.25 | 0.17 | 1.0 | 31 | 21000 | -6.2 | 0 | $+7$ |
| BE CMa | C5,5 | 5.41 | 0.29 | 0.22 | 1.7 | 110 | 6500 | -4.9 | - | +29 |
| BO CMa | - | 5.60 | 0.48 | 0.47 | 3.0 | 220 | 21000 | -6.2 | - | +51 |
| RT Cap | - | 3.06 | 0.19 | 0.09 | - | - | - | - | - | - |
| SZ Car | - | 3.70 | 0.22 | 0.13 | 0.6 | 43 | 4000 | -4.4 | - | - |
| TZ Car | - | 4.66 | 0.48 | 0.47 | - | - | - | - | - | - |
| ST Cas | C4,4 | 5.52 | 0.13 | 0.01 | - | - | - | - | - |  |
| UX Cas | - | 5.51 | 0.62 | 0.65 | 2.0 | 31 | 13000 | -5.6 | - | - 4 |
| WW Cas | C5, 5 | 4.99 | 0.45 | 0.43 | 2.3 | 190 | 21000 | -6.2 | - | -54 |
| FR Cas | - | 5.93 | 0.50 | 0.49 | 2.7 | 160 | 13000 | -5.6 | - | -40 |
| NQ Cas | C4, 5 | 5.82 | 0.32 | 0.26 | - | - | - | - | - | - |
| RV Cyg | C6,4 | 2.86 | 0.33 | 0.27 | 0.5 | 99 | 6200 | -4.8 | 10 | +13 |
| RY Cyg | C6,4 | 5.09 | 0.30 | 0.23 | 1.0 | 24 | 3100 | -4.1 | - | +27 |
| SV Cyg | C7,4 | 4.36 | 0.41 | 0.38 | 1.2 | 170 | 9800 | -5. 3 | 2 | $+5$ |
| AW Cyg | C4, 5 | 4.32 | 0.19 | 0.09 | - | - | - | - | - | - |
| Ax Cyg | C5,5 | 3.74 | 0.27 | 0.19 | 0.5 | 69 | 2600 | -3.9 | 10 | +8 |
| AY Cyg | C4, 8 | 5.50 | 0.53 | 0.53 | 1.0 | 79 | 2700 | -3.9 | - | +35 |
| HV Cyg | - | 6.96 | 0.45 | 0.43 | - | - | - | - | - | - |
| V460 Cyg | C6, 3 | 2.32 | 0.28 | 0.21 | 0.4 | 90 | 6200 | -4.8 | 1 | +21 |
| V744 Cyg | - | 6.61 | 0.24 | 0.16 | - | - | - | - | - |  |
| v778 Cyg | - | 5.90 | 0.29 | 0.22 | 1.4 | 280 | 2800 | -4.0 | - | -30 |
| HH Del | , | 6.93 | 0.18 | 0.08 | - | - | - | . | - | - |
| RY Dra | C4,4 | 2.59 | 0.24 | 0.16 | 0.4 | 310 | 4600 | -4.5 | 10 | - |
| SY Eri | C6, 3 | 4.54 | 0.23 | 0.14 | - | - | - | - | - | - |
| TU Gem | C6,4 | 3.30 | 0.36 | 0.31 | 0.8 | 47 | 11000 | -5.4 | 3 | +39 |
| VW Gem |  | 4.96 | 0.23 | 0.14 | - | - | - | - | - | - |
| BM Gem | C5,4 | 4.90 | 0.17 | 0.06 | - | - | - | - | - | - |
| CY Gem | - | 6.70 | 0.37 | 0.33 | - | - | - | - | - | - |
| DH Gem | C4,5 | 5.46 | 0.30 | 0.23 | 1.9 | 39 | 7900 | -5.1 | (2) | -19 |
| HX Gem | - | 6.58 | 0.38 | 0.34 | 2.3 | 120 | 4500 | -4.5 | (2) | $+6$ |
| U Hya | C6.5,3 | 1.29 | 0.13 | 0.01 | - | - | , | . 5 | - | +6 |
| Y Hya |  | 2.64 | 0.17 | 0.07 | - | - | - | - | - | - |
| T Ind | C7,3 | 2.65 | 0.18 | 0.08 | - | - | - | - | - | - |
| TX Lac | - | 6.41 | 0.33 | 0.27 | - | - | - | - | - | - |
| 2 Lup | - | 4.33 | 0.17 | 0.19 | - | - | - | - | - | - |
| T Lyr | C6, 5 | 3.33 | 0.26 | 0.18 | - | - | - | - | - | - |
| HK Lyr | C6,4 | 4.00 | 0.17 | 0.07 | - | - | - | - | - | - |
| W Mon | C4, 5 | 5.68 | 0.33 | 0.27 | 1.3 | 69 | 2900 | -4.0 | - | -14 |

table 1 - Continued

| Star | Spectrum | $\mathrm{m}(1.04 \mu \mathrm{~m})$ | $\begin{aligned} & (0.78- \\ & 1.08)_{\mu^{m}} \end{aligned}$ | $\delta(B-v) *$ | $\mathbf{r}$ (kpe) | 2 (pc) | L/LC | $M_{\text {Bol }}$ | M-S** | $\begin{aligned} & \mathrm{V}_{\mathrm{R}}(\text { corr })+ \\ & \left(\mathrm{km} \mathbf{s}^{-1}\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RV Mon | C4,4 | 3.50 | 0.11 | - | - | - | - | - | - | - |
| bG Mon | - | 6.04 | 0.22 | 0.13 | 1.6 | 98 | 3000 | -4.0 | - | +59 |
| BN Mon | - | 5.05 | 0.50 | 0.49 | 0.8 | 44 | 2600 | -3.9 | - | +16 |
| CO Mon | C5, 5 | 6.07 | 0.22 | 0.13 | - | - | - | - | - | - |
| Cz Mon | C4, 5 | 5.04 | 0.43 | 0.40 | 1.2 | 0 | 5300 | -4.7 | (4) | +14 |
| DF Mon | - | 5.73 | 0.40 | 0.36 | - | - | - | - | - | - |
| GY Mon | C6,3 | 4.28 | 0.34 | 0.29 | 1.3 | 260 | 11000 | -5.5 | 0 | +24 |
| T Mus | - | 4.06 | 0.24 | 0.16 | - | - | - | - | - | - |
| TW Oph | C5,5 | 4.64 | 0.63 | 0.66 | 1.0 | 110 | 6700 | -4.8 | - | - 5 |
| W Ori | C5,4 | 2.02 | 0.16 | 0.05 | 0.45 | 170 | 7500 | -4.9 | 7 | - |
| RT Ori | C5, 6 | 4.14 | 0.34 | 0.29 | 1.1 | 430 | 9300 | -5.3 | 0 | - 7 |
| BL Ori | C6,3 | 2.66 | 0.26 | 0.18 | 0.6 | 11 | 10000 | -5.3 | 0 | $+4$ |
| GK Ori | - | 5.02 | 0.39 | 0.35 | 1.5 | 93 | 8100 | -5.1 | - | +34 |
| GS Ori | C5,4 | 6.48 | 0.19 | 0.09 | - | - | - | - | - |  |
| V431 Ori | C5,5 | 4.46 | 0.55 | 0.56 | 1.1 | 280 | 8800 | -5.2 | 5 | -22 |
| Y Pav | C7,3 | 2.33 | 0.32 | 0.26 | - | - | - | - | - | - |
| RX Peg | C4,4 | 4.39 | 0.23 | 0.30 | - | - | - | - | - | - |
| VZ Per | C4 | 6.55 | 0.53 | 0.53 | 1.75 | 120 | 3200 | -4.1 | - | -18 |
| AC Per | C7,3 | 4.95 | 0.53 | 0.53 | 2.2 | 300 | 22000 | -6.2 | 0 | -34 |
| BS Per | C5,5 | 5.23 | 0.29 | 0.22 | - | - | - | - | - | - |
| W Pic | - | 3.60 | 0.31 | 0.25 | - | - | - | - | - | - |
| 2 Psc | C7,2 | 2.79 | 0.31 | 0.25 | 0.6 | 360 | 9300 | -5.3 | 0.5 | - |
| TX Psc | C7,3 | 1.30 | 0.24 | 0.16 | - | - | - | - | - | - |
| RT Pup | C6,2 | 4.62 | 0.50 | 0.38 | 1.7 | 110 | 17000 | -5.9 | - | +12 |
| RU Pup | - | 4.24 | 0.24 | 0.16 | - | - | - | - | - | - |
| AC Pup | - | 4.95 | 0.20 | 0.10 | - | - | - | - | - | - |
| GO Pup | - | 5.52 | 0.29 | 0.22 | 1.5 | 180 | 4600 | -4.5 | - | +19 |
| BF Sge | - | 5.49 | 0.36 | 0.31 | 1.3 | 120 | 3900 | -4.3 | - | +11 |
| SS Sgr | C5,0 | 4.88 | 0.52 | 0.52 | 0.9 | 49 | 3900 | -4.3 | - | +11 |
| SZ Sgr | c7,3 | 4.37 | 0.51 | 0.51 | 1.25 | 120 | 12000 | -5.5 | 0 | +29 |
| UW Sgr | C6,5 | 5.48 | 0.31 | 0.25 | - | - | - | - | - | - |
| AQ Sgr | C7,4 | 2.98 | 0.30 | 0.23 | - | - | - | - | - | - |
| V1942 Sgr | C6,4 | 3.10 | 0.27 | 0.19 | - | - | - | - | - | - |
| SX Sco | C5,4 | 4.02 | 0.26 | 0.18 | - | - | - | - | - | - |
| TT Sco | - | 3.94 | 0.32 | 0.26 | 0.9 | 90 | 7400 | -5.0 | - | +20 |
| V 450 Sco | - | 5.88 | 0.68 | 0.73 | 1.8 | 85 | 7300 | -5.0 | - | -45 |
| S Sct | C5,4 | 2.89 | 0.24 | 0.16 | 0.5 | 29 | 5400 | -4.7 | 2 | +12 |
| T Sct | C5,4 | 4.99 | 0.41 | 0.38 | 1.5 | 120 | 8600 | -5.2 | - | +27 |
| RX Sct | C4,8 | 4.05 | 0.67 | 0.72 | 0.6 | 47 | 4300 | -4.4 | - | $+5$ |
| DR Ser | - | 4.83 | 0.61 | 0.64 | 1.0 | 59 | 5500 | -4.7 | - | - 3 |
| TT Tau | C4, 2 | 3.41 | 0.62 | 0.65 | - | - | - | - | - | - |
| TU Tau | C5,4 | 4.10 | 0.39 | 0.35 | 1.2 | 51 | 12000 | -5.6 | 0 | -33 |
| CP Tau | C5,4 | 5.41 | 0.38 | 0.34 | 1.2 | 150 | 3600 | -4.2 | - | +9 |
| V Tra | C5, 5 | 4.72 | 0.18 | 0.08 | - | - | - | - | - | - |
| X TrA | C5,5 | 1.70 | 0.21 | 0.11 | 0.25 | 46 | 4000 | -4.3 | - | - 7 |
| vY UMa | C6,3 | 2.46 | 0.16 | 0.05 | - | - | - | - | - | - |
| x Vel |  | 2.90 | 0.19 | 0.09 | - | - | - | - | - | - |

*Color excess of a fictitious $O B$ star with the same $(0.78-1.08) \mu^{m}$ color excess as the $N$ Irr.
${ }^{* *}$ Merrill-Sanford band strength estimates on a scale of 10 (Yamashita, 1967, 1972). Values in parentheses are inferred from the correlation of Shane $N$ subclass with band strength.

+ Sanford (1944) radial velocities after correction for basic solar motion (lo $=51^{\circ}, b \odot=+23^{\circ}$,
$\mathrm{V} \odot=15.4 \mathrm{~km} \mathrm{~s} \mathrm{~s}^{-1}$ ).

3. The upper limit of the $11-\mu$ excess (Gillett et al. 1971) is about 1 mag, in contrast to the typical $1.5-2.5 \mathrm{mag}$ excesses of N -type long-period variables.
4. The Keenan-Morgan temperature classes are exclusively C4-C7.
5. Balmer lines, both emission and absorption, are absent in the spectra of nearly all N Irr (Yamashita 1967, 1972). We have omitted from our analysis the few emission-line stars that have been designated $\mathrm{L} b$ or SR $b$.


Fig. 1.-The spectrum of 19 Psc. $F_{v}$ is the emergent monochromatic flux density at the stellar surface (ergs $\mathrm{cm}^{-2} \mathrm{~s}^{-1} \mathrm{~Hz}^{-1}$ ).
6. The galactic distribution shows a distinct concentration to the galactic plane, in contrast to the diffuse distribution of SR $a$ and long-period variables. Figure 3 shows the distribution upon the sky of moderately bright N Irr. The plot also reveals a concentration of N Irr where one looks "down" the local spiral arm toward Gemini and Orion. We therefore anticipate that our distances for N Irr will reveal some degree of association with the spiral features of the Galaxy.

## III. Distances of N Irr

## a) Interstellar Reddening

Figure 2 is a display of $(0.78-1.08) \mu$ color indices observed among N Irr at various galactic latitudes. It is evident that redder N Irr lie at lower galactic latitudes. The high-latitude N Irr are blue; they are also bright and presumably near. We infer that the variety in observed color is primarily a consequence of interstellar reddening, rather than attributable to dispersion in the intrinsic color temperature of N Irr. Baumert has fitted blackbody curves through observed "continuum" points, including the flux peaks at 0.78 and $1.08 \mu$. A temperature change of 100 K alters the $(0.78-1.08) \mu$ color index by 0.08 mag . Therefore, in the present coarse treatment we proceed on the basic assumption that differences in intrinsic color temperature among N Irr are negligible, and that interstellar absorption alone accounts for the observed dispersion in the $(0.78-1.08) ~ \mu$ color index.

Each observational quantity listed in Table 1 is the
mean value of the extremes observed during variation of the star. Each derived luminosity and bolometric magnitude is consequently the mean value during variation. The $(0.78-1.08) \mu$ color index changes by typically $0.05-0.1 \mathrm{mag}$ during light variation. However, not all stars are equally well observed; some values are those of a single epoch only. We take the mean color index of the well-observed, high-latitude N Irr X Cnc as the intrinsic mean color index that characterizes N Irr: $(0.78-1.08) \mu=0.12$ mag.

Variation of color excess with distance is usually found from studies of OB stars and cataloged as color excess in $(B-V)$. The Wing bandwidths are narrow $(\sim 50-75 \AA)$, and so the $(0.78-1.08) \mu$ color excess of an N Irr is the same as the one that an OB star would suffer if it were at the same location. Given the color excess of an N Irr, $\delta(0.78-1.08)$, we use van de Hulst's (1968) theoretical curve 15 to find the implied color excess, $\delta(B-V)$, of a similarly reddened, fictitious OB star. The result is that $\delta(B-V)=$ $1.3 \delta(0.78-1.08)$. Also, van de Hulst's curve relates reddening to absorption at $\lambda 1.04 \mu: \delta(1.04)=$ $1.2 \delta(0.78-1.08)$.

## b) Distances

Color excesses in $(B-V)$ thus derived for fictitious OB stars at the locations of the N Irr have been translated into distances by use of FitzGerald's (1968) study of the dependence of reddening on distance in various directions near the galactic plane. The most uncertain distances are those of N Irr that are little


Fig. 2.-Observed ( $0.78-1.08$ ) $\mu$ color index of N Irr plotted against galactic latitude. Each entry is the mean of the extremes observed for that star.
reddened or that lie in directions of marked inhomogeneities in the interstellar medium. We have succeeded in estimating distances of only 60 N Irr -about half the sample listed in Table 1.

## c) Validation of Derived Distances

Sanford (1944) has determined the radial velocities that appear, corrected for the basic solar motion, in

Table 1. The Oort double-sine distribution of radial velocities should dominate over peculiar velocities among the more remote N Irr, and its presence may be taken as a coarse validation of the derived distances. Because our sample is small, we have divided the stars into a group that is more remote than 1.7 kpc , for which the double-sine variation should be conspicuous, and a second group nearer than 1 kpc , for which the


Fig. 3.-Apparent distribution of all 40 N Irr in the infrared magnitude interval $4 \leq m(1.08 \mu) \leq 5$ (filled circles), supplemented with all the additional 19 N Irr from Sanford's (1944) catalog in the magnitude interval $8 \leq m_{v} \leq 9$ (open circles), to which the former group roughly corresponds. The concentration in the direction of Orion and Gemini, i.e., in the direction roughly parallel to the local arm, is indicated.


FIG. 4.-Galactic distribution of radial velocities of N Irr with $|b|<15^{\circ}$. (a) N Irr with $r<1 \mathrm{kpc}$. Correct distances and pure galactic rotation would place these radial velocities between the horizontal axis and the double-sine curve. (b) N Irr with $r \geq 1.7 \mathrm{kpc}$. Correct distances and pure galactic rotation would place these radial velocities outside the double-sine curve. The curves are based on Oort constant $A=15 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-1}$.
amplitude of variation should be small. Figure 4 shows the double-sine variation to be present in both samples, and with appropriately larger amplitude in the remoter group.
A few aberrant velocities in Figure 4 cannot be attributed to erroneous estimates of distance; e.g., the radial velocity of DH Gem has the wrong sign for its quadrant of galactic longitude and is about $40 \mathrm{~km} \mathrm{~s}^{-1}$ too negative for its estimated distance. TT Sco shows the same peculiarity. The radial velocity of V Aql, if due to differential galactic rotation, would place the star 12 times as distant as we find it and brighter in absolute magnitude by 5.4 mag -leading to the unlikely bolometric magnitude of -9.6 mag. Yamashita (1967) has pointed out that spectroscopic peculiarities
of TU Gem are suggestive of a CH star. TU Tau is a known spectroscopic binary; at galactic longitude $184^{\circ}$, its radial velocity should be positive, rather than $-33 \mathrm{~km} \mathrm{~s}^{-1}$ as observed (Table 1). Superposed orbital velocity may account for this discrepancy and prompts the suggestion that other aberrant N Irr may also be binaries. However, TT Sco and V Aql show no violet excesses that would indicate the presence of early-type companions (Richer 1971).
Perceptible association with galactic spiral features should also serve as a validation of the derived distances. N Irr are projected upon the galactic plane in Figure 5. Evidently there is a loose association with spiral features that are defined by various Population I objects (Becker 1964). Bias toward such association


Fig. 5.-Projection of N Irr upon the galactic plane
may well have been insinuated into the derivation of distances, however; FitzGerald's (1968) relations between interstellar reddening and distance are based largely upon observations of extreme Population I objects.
IV. LUMINOSItiEs of N Irr

## a) Procedure

We write the luminosity of an N Irr as
$\frac{L(\mathrm{~N})}{L_{\odot}}=\frac{L(\text { Vega })}{L_{\odot}} \frac{L(\mathrm{Vega})_{1.04}}{L(\text { Vega })} \frac{L(\mathrm{~N})}{L(\mathrm{~N})_{1.04}} \frac{L(\mathrm{~N})_{1.04}}{L(\text { Vega })_{1.04}}$.
We may evaluate each factor in this expression with the input parameters listed in Table 2.

$$
\text { 1. } \begin{align*}
\frac{L(\mathrm{Vega})}{L_{\odot}} & =\frac{L(\mathrm{Vega})}{4 \pi r^{2}(\mathrm{Vega})} \frac{4 \pi r^{2}(\mathrm{Vega})}{L_{\odot}} \\
& =\frac{\int f_{\lambda}(\mathrm{Vega}) d \lambda}{f(\mathrm{Vega})_{0.5556}} \frac{4 \pi r^{2}(\mathrm{Vega})}{L_{\odot}} f(\mathrm{Vega})_{0.5556} \tag{2}
\end{align*}
$$

where $r$ (Vega) is Vega's distance, $f_{\lambda}$ (Vega) is the monochromatic flux from Vega above the Earth's atmosphere, and $f(\mathrm{Vega})_{0.5556}$ is the flux from Vega at $0.5556 \mu$. We write $F_{\lambda}$ as the emergent monochromatic flux at the surface of Vega, and

$$
\begin{equation*}
\frac{L(\text { Vega })}{L_{\odot}}=\frac{\int F_{\lambda}(\mathrm{Vega}) d \lambda}{F(\mathrm{Vega})_{0.5556}} \frac{f(\mathrm{Vega})_{0.5556}}{L_{\odot}} 4 \pi r^{2}(\mathrm{Vega}) \tag{3}
\end{equation*}
$$

TABLE 2
Input Parameters for N Irr Luminosities

| $\frac{f(\text { Vega })_{0.5556}}{L_{\odot}}$ | $\frac{\int F_{\lambda}(\text { Vega }) d \lambda}{F(\text { Vega })_{0.5556}}$ | $\frac{F(\text { Vega })_{1.04}}{\int F_{\lambda}(\text { Vega }) d \lambda}$ | $\frac{\int F_{\lambda}(N) d \lambda}{F(N)_{1.04}}$ |
| :---: | :---: | :---: | :---: |
| $8.72 \times 10^{-39} \mathrm{~cm}^{-2} \mu^{-1}$ | $0.899 \mu$ | $0.181 \mu^{-1}$ | $1.61 \mu$ |

We use the atmospheric model of Oke, Schild, and Peterson (1971) to evaluate the total-to-monochromatic flux ratio, and the absolute calibration of the monochromatic flux at $0.5556 \mu$ reported by Oke and Schild (1970).

$$
\begin{equation*}
\text { 2. } \quad \frac{L(\text { Vega })_{1.04}}{L(\text { Vega })}=\frac{F(\text { Vega })_{1.04}}{\int F_{\lambda}(\text { Vega }) d \lambda}, \tag{4}
\end{equation*}
$$

which we evaluate as above.
3. We take 19 Psc as the prototype N Irr and evaluate

$$
\begin{equation*}
\frac{L(\mathrm{~N})}{L(\mathrm{~N})_{1.04}}=\frac{\int F_{\lambda}(\mathrm{N}) d \lambda}{F(\mathrm{~N})_{1.04}} \tag{5}
\end{equation*}
$$

with the data employed in the construction of Figure 1. 4. The flux from Vega in the $1.04-\mu$ band at the Earth defines the zero point of Baumert's infrared magnitude scale. Therefore,

$$
\begin{align*}
\frac{L(\mathrm{~N})_{1.04}}{L(\mathrm{Vega})_{1.04}} & =\frac{L(\mathrm{~N})_{1.04}}{4 \pi r^{2}(\mathrm{~N})} \frac{4 \pi r^{2}(\mathrm{Vega})}{L(\mathrm{Vega})_{1.04}} \frac{r^{2}(\mathrm{~N})}{r^{2}(\mathrm{Vega})} \\
& =\operatorname{dex}\{-0.4[m(1.04)-\delta(1.04)]\} \frac{r^{2}(\mathrm{~N})}{r^{2}(\mathrm{Vega})} \tag{6}
\end{align*}
$$

where $\delta(1.04)$ corrects Baumert's observed $m(1.04)$ for interstellar absorption.

We evaluate and combine all factors to produce our working equation

$$
\begin{equation*}
\frac{L(\mathrm{~N})}{L_{\odot}}=0.274 \operatorname{dex}\{-0.4[m(1.04)-\delta(1.04)]\} r^{2}(\mathrm{~N}) \tag{7}
\end{equation*}
$$

where $r(\mathrm{~N})$ is in parsecs.
The resulting luminosities (Table 1) are not dramatically different from previous estimates. However, Gordon's (1968) list contains two stars in common with Table 1; we find BL Ori 0.5 mag fainter and V460 Cyg 1.5 mag fainter than her estimates. Richer's (1972) distance of 1.5 kpc for TU Tau accords well with that of Table 1, but his 0.7 kpc for SZ Sgr is just over half our value. Eggen's (1972a,b) estimates of $M_{\text {bol }}=$ -5 mag for BL Ori and W Ori are in good agreement with Table 1, but his estimates for all five other stars in common with Table 1 are fainter by $1-2$ mag. RY Cyg may be a member of NGC 6883 (Gordon
1968); Becker's (1963) distance modulus would increase $M_{\text {bol }}$ to -4.9.
Mendoza and Johnson (1965) have integrated their calibrated UBVRIJKLN observations of carbon stars to determine bolometric corrections to $V$ magnitudes. Table 1 contains distances for five of their program stars. If valid, our procedure should recover these bolometric corrections at the epoch of each observation. From the definition of bolometric correction and equation (7),

$$
\begin{align*}
\mathrm{BC} & =M_{\mathrm{bol}}-V-5+5 \log r+A_{v} \\
& =\text { const. }+m(1.04 \mu)-\delta(1.04)-V-5+A_{v} \tag{8}
\end{align*}
$$

Equation (8) is therefore independent of derived distance, but our assumption of a single intrinsic color index for all N Irr remains implicit in the term $\delta(1.04)$.

Mendoza and Johnson could not correct their observations for interstellar reddening. The stellar flux peaks at $\sim 1.6 \mu$ (Fig. 1), where van de Hulst's (1968) theoretical curve 15 indicates $A_{1.6 \mu} \approx 0.1 A_{v}$. We therefore take interstellar absorption into account approximately by writing

$$
\begin{align*}
\mathrm{BC}(\text { corrected }) & =\mathrm{BC}(\text { Mendoza-Johnson })-A_{\mathrm{bol}}+A_{v} \\
& =\mathrm{BC}(\text { Mendoza-Johnson })+0.9 A_{v} \tag{9}
\end{align*}
$$

Equations (8) and (9) produce the bolometric correction from independent approaches. Observational error in $V, m(1.04 \mu)$, and color index, as well as variability in color index, may combine to account for $\sim 0.1 \mathrm{mag}$ of discrepancy between bolometric corrections from equations (8) and (9). Dispersion in color temperature among N Irr of the order of 100 K may introduce another 0.1 mag of discrepancy through spurious color excess and consequently spurious $\delta(1.04)$. The extinction $A_{v}=R \delta(B-V)$, where $\delta(B-V)$ is the color excess of the N Irr. The latter quantity is uncertain because the intrinsic $(B-V)$ color index depends upon the strengths of the Swan bands and the violet deficiency, and these are peculiar to each N Irr. However, Honeycutt (1972) has shown that although the total-to-selective absorption ratio for N Irr approaches the value of 4 , one may use

TABLE 3
Bolometric Corrections

| Star | Spectrum | $\begin{gathered} V^{*} \\ (M-J) \end{gathered}$ | $\begin{gathered} \mathrm{BC}^{*} \\ (M-J) \end{gathered}$ | $A_{v}(\mathrm{mag})$ | $\underset{\text { (this report) }}{\mathrm{BC}}$ | $\underset{\text { corrected }}{\mathrm{BC}(M-J)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BL Ori. | C6 | 6.18 | -2.51 | 0.54 | -2.0 | 2.02 |
| RY Dra. | C4 | 6.35 | -3.24 | 0.48 | -2.4 | 2.81 |
| RX Sct. | C4 | 8.93 | -4.21 | 2.16 | -2.3 | 2.27 |
| SV Cyg | C7 | 8.80 | -3.83 | 1.14 | -2.6 | 2.80 |
| V460 Cyg. | C6 | 6.05 | -2.73 | 0.63 | -2.2 | 2.16 |

[^1]

Fig. 6.-Bolometric magnitudes and luminosities of N Irr plotted against C subclass, and against Merrill-Sanford band strength estimated on a scale of $10 . M_{\text {bol }}$ and $L / L_{\odot}$ are mean values of the extremes during stellar variation. Filled circles, N Irr in subclass C5 only, identified to illustrate the variety of Merrill-Sanford band strengths to be found in a single C subclass; crosses, two supplementary stars with band strengths inferred from the correlation with violet deficiency.
instead the color excess of an OB star at the same location and $R \approx 3$ to derive the approximate value of $A_{v}$ for the N Irr. We have used this procedure with the $\delta(B-V)$ of fictitious OB stars listed in Table 1; however, the difference between equations (8) and (9) shows that error in $A_{v}$ can contribute only weakly to any discrepancy. We conclude that a discrepancy significantly larger than $\sim 0.2$ mag between the bolometric corrections produced by the two equations must call into question the assumption of a single flux distribution for all N Irr.
Table 3 displays bolometric corrections derived by the two procedures. Agreement between corrected bolometric corrections is excellent for four of the five stars; RY Dra alone presents the large discrepancy of 0.4 mag, for which we have no satisfying explanation. However, RY Dra is known to be spectroscopically peculiar; it is one of Gordon's (1967) " $\lambda 6168$ " stars. The peculiarities of RY Dra and the other four " $\lambda 6168$ " stars listed in Table 1 (UV Cam, Y CVn, TX Lac, RX Peg) may render the group inappropriate for this discussion.

## V. LUMINOSITY-CORRELATED FEATURES

Difficulties that attend the classifying of N stars on the Keenan-Morgan scheme lead to discrepancies among different workers. To improve the likelihood of consistency, we have taken the spectral classes of Table 1 primarily from Yamashita's (1967, 1972) large classification program. However, of the 60 N Irr with estimates of distance, only 42 have published classifications. These are plotted according to luminosity and spectral subclass in Figure 6. Evidently
there is a spread of 1-2 mag within a subclass-a part of which certainly expresses the error in derived distances. Mean luminosity increases with advancing subclass, as suggested by previous work (e.g. Gordon 1968).

Richer (1971) has conjectured that the near-infrared CN system may serve as a positive luminosity indicator,


Fig. 7.- $M_{\mathrm{bol}}$ plotted against Baumert (1972) CN index. This index expresses the depression of the CN bands near $0.81 \mu$ and $1.09 \mu$.
as it does for $G K M$ stars. Figure 7, however, reveals no correlation with bolometric magnitude; CN band strengths apparently reflect the CNO abundances peculiar to each star.
The molecular bands at visual wavelengths first noted by Merrill (1926) and Sanford (1926), and now usually attributed to $\mathrm{SiC}_{2}$ (Kleman 1956; Yamashita and Utsumi 1968), have defied correlation with other observable properties of N stars (except ultraviolet deficiency: e.g., Keenan and Morgan 1941). Figure 6 shows that the Merrill-Sanford bands are negative luminosity indicators. Yamashita's (1967, 1972) strength estimates of the Merrill-Sanford bands have been used, with greater weight given to his 1967 values because these were based on spectrograms of higher resolution. It is significant that this correlation emerges despite the coarseness of the distance-determination procedure, since the transition from maximum strength of the Merrill-Sanford bands to invisibility occurs over a range of only $\sim 1$ mag. The appearance of strong and weak Merrill-Sanford bands among stars within the same C subclass is no longer puzzling if there is a luminosity dispersion of $\geqslant 1$ mag within a subclass.

## IV. DISCUSSION

The Oort double-sine distribution of radial velocities, the galactic distribution, bolometric corrections, and the emergence of the Merrill-Sanford bands as luminosity indicators among N Irr appear to support the validity of our derived distances and the underlying assumption of small temperature dispersion among $N \operatorname{Irr}$ (we suggest $\pm 100 \mathrm{~K}$ ). Once again, then, arises the question of subclasses C4-C7 as a temperature sequence (Wyller 1957; Richer 1971). What might
be the physical significance of this sequence, if not a temperature sequence? It has been reported (e.g., Yamashita 1972) that excesses of rare earths frequently appear among subclasses C4-C5, while increasing abundances of such $s$-process nuclides as $\mathrm{Sr}, \mathrm{Y}, \mathrm{Zr}$, $\mathrm{Ba}, \mathrm{La}$ are observed with advancing subclass (e.g., all N Irr known to show spectral lines of technetium are $\mathrm{C} 6-\mathrm{C} 7$, and those known not to show them are C5: Peery 1971). This trend suggests that the subclasses C4-C7 may be a sequence of differing compositions.
Similarity of color temperature and dispersion in luminosity may imply that N Irr lie on pseudoHayashi tracks that precede carbon ignition (e.g., Paczynski, 1970). These tracks are steep in the $\left(\log L, \log T_{e}\right)$-plane, and a variety of masses and luminosities are funneled into a narrow temperature interval. On this view, we envisage a continuum of masses and ages among N Irr. Dean (1972) reaches a similar conclusion from a kinematic study. However, the galactic distribution of N Irr indicates that more of these stars are "young disk" (Eggen 1972b) objects (and therefore with masses greater than $\sim 2 \mathscr{N}_{\odot}$ ) than the 10 percent that Eggen finds.

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