

Fe I FLUORESCENCE IN T TAURI STARS. II. CLUES TO THE VELOCITY FIELD IN THE CIRCUMSTELLAR ENVELOPE

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ABSTRACT

Radial velocity measurements for the fluorescent iron lines $\lambda\lambda 4063, 4132$ for RW Aur compared with the primary emission lines of hydrogen and calcium show that Fe I $\lambda 3969.26$ has been shifted onto H ϵ , as was predicted. Profiles of Ca II H-H ϵ and Ca II K clearly demonstrate the effects of absorption by Fe I $\lambda 3969.26$. The relative line strengths indicate that H ϵ provides about 50 percent of the excitation for the fluorescent lines, and Ca II H the rest. Profiles of the fluorescent iron lines indicate that (1) the lines are formed at 3-10 stellar radii from the star, and (2) rotational velocities are equal to or exceed expansion velocities at this distance. The Ca II K profile gives evidence that within 1.5 stellar radii, rotational velocities also dominate. The conclusion is drawn that mass-loss rates calculated on the basis of a purely expanding envelope may be in error by factors of 2 to 10 for T Tauri stars. Implications of these results for the envelope structure are also discussed.

Subject headings: emission-line stars — mass loss — pre-main-sequence stars — rotation, stellar — stars, individual

I. INTRODUCTION

In a previous paper (Willson 1974; hereafter referred to as Paper I), an analysis was made of the process leading to enhanced emission of Fe I $\lambda\lambda 4063, 4132$ in T Tauri stars. The mechanism is based on the coincidence of Fe I $\lambda 3969.26$ with the Ca II H-H ϵ emission around $\lambda 3969$, which leads to an overpopulation of level $y^3F_3^o$ of Fe I. In Paper I, limits were obtained on the electron and hydrogen number densities of the form

$$10^{13} \leq N_H/W \leq 10^{16}; N_e/W \leq 10^{16}, \quad (1)$$

where W is the "dilution factor" and $T_e \simeq 3000$ K has been assumed.

In order to use the fluorescent lines to give information on the physical conditions in the gas surrounding a T Tauri star, we must first obtain information about where in the envelope the lines are formed. One way in which this may be done is by examining the mean radial velocity of these lines, and comparing this with a model for the mass flow around the star. A better approach is to examine the fluorescent-line profiles, and again compare this with some form of dynamic model.

The schematic model used here assumes that the primary emission lines, Ca II H and K and the hydrogen Balmer lines, are formed in a hot chromosphere near the star. The fluorescent, or pumped, lines are formed in a cooler region of the envelope at some distance from the star. The matter around the star is flowing outward and decelerating; thus the fluorescent Fe I lines are formed in a less rapidly moving region than the region in which the primary lines are formed.

In this paper we first confirm that the sign of the velocity difference between the fluorescent lines and the primary emission lines is consistent with the iron lines being formed outside the chromosphere. Second,

we confirm selective excitation as the mechanism producing $\lambda\lambda 4063, 4132$, and estimate the relative contributions made by H ϵ and Ca II H. Third, the evidence provided by the line profiles about the velocity fields are discussed, and arguments presented that large rotational velocities are present in T Tauri envelopes. This in turn implies that calculations done ignoring rotation give mass-loss rates that are too large by at least a factor of 2.

II. THE VELOCITY DIFFERENCES AND EXCITATION BY H ϵ

In Paper I it was suggested that H ϵ , not Ca II H, provides the excitation for the Fe I lines. This conclusion came from several arguments about the expected sign of the velocity difference between the primary emission lines and the region producing the fluorescent iron lines, giving the schematic model described above. The Doppler shift required to shift Fe I $\lambda 3969.26$ onto H ϵ at 3970.07 is 60 km s^{-1} ; thus we expect the region producing the pumped lines to be located farther from the star where the expansion velocity is about 60 km s^{-1} less than at the chromosphere.

Radial velocities for selected hydrogen lines, Ca II H and K, and the fluorescent iron lines are shown in table 1. These were taken from two of the plates used by Gahm for his (1970) catalog of emission lines in RW Aur, and labeled by him S1 and S2; they are Lick plate numbers EC1478 and EC7150, taken by G. Herbig in 1962 and 1968. The sign of the velocity differences is as anticipated: the fluorescent lines are shifted longward with respect to the other emission lines. The magnitude of the velocity shift is harder to interpret, due to the complex structure of the primary emission lines; however, if it has the same shift as the fluorescent emission lines, the Fe I $\lambda 3969$ absorption line should lie between the blue peak of H ϵ and its central absorption. The observations are thus at least

TABLE 1
RADIAL VELOCITIES FOR THE BLUE EMISSION PEAK (e) AND THE
ABSORPTION COMPONENT (a) FOR REPRESENTATIVE EMISSION
LINES ON TWO PLATES OF RW AURIGAE

LINE	S1		S2	
	e	a	e	a
Hydrogen lines:				
H γ	-175	-45	-200	-50
H δ	-170	-45	-200	-40
H ϵ	-150	-55	-200	-30
Ca II K.....	-160	-55	-200	-90
Ca II H.....	-180	-25	-190	-80
Fe I lines:				
λ 4063.....	+ 20	...	+ 20	...
λ 4132.....	+ 12	...	+ 18	...

qualitatively in agreement with the predictions of Paper I.

It is also significant that the fluorescent emission lines for S1 and S2 are centered on $\sim +20$ km s $^{-1}$. If we assume that RW Aur has a radial velocity of $+25$ km s $^{-1}$ (in agreement with the velocities of other stars in the Taurus-Auriga complex; see Herbig 1962), then the fluorescent lines must arise (a) in a region at rest with respect to the star, or (b) far enough out that occultation effects are minimal, and we can see both the near and far sides of the shell. These possibilities are discussed in more detail in § IV.

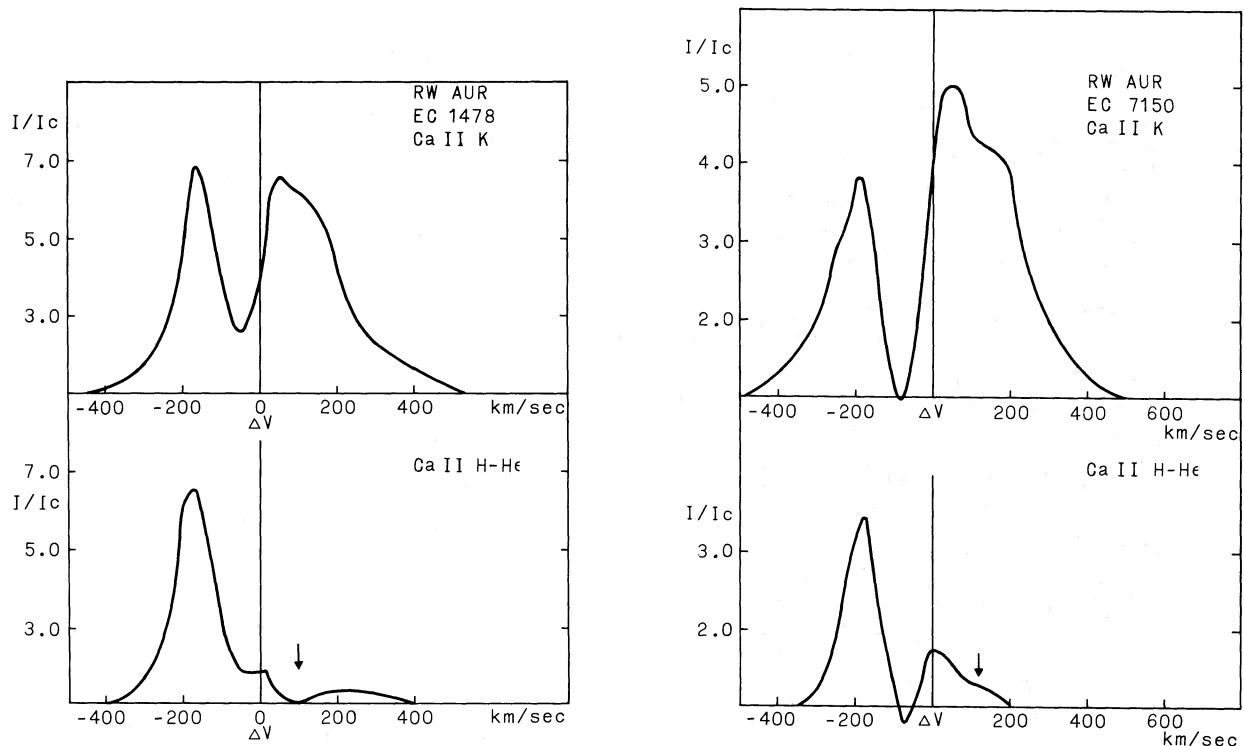
As a further confirmation that the Fe I lines are formed by absorption of H ϵ , consider the profiles of

Ca II K and Ca II H-H ϵ shown in figure 1. It is evident that the Ca II H-H ϵ profile has been "mutilated" by some overlying absorption. The absorption at $\Delta V \simeq -25$ km s $^{-1}$ undoubtedly corresponds to the self-absorption shown in Ca II K at $\Delta V \simeq -50$ km s $^{-1}$. The other absorption, displaced slightly longward of the expected wavelength of H ϵ , is produced by Fe I λ 3969.26.

From the intensities given by Gahm (1970) for the hydrogen lines and Ca II K, it is possible to estimate the expected intensity in the Ca II H-H ϵ blend. The difference between the expected and the observed intensity should then match the intensity observed at λ 4063, 4132. These numbers are presented in table 2 for the two plates for which Ca II K intensities were measured. The results are seen to be qualitatively correct. Approximately half the required energy is supplied by H ϵ ; the other half presumably comes from Ca II H. The evidence is thus overwhelming that the fluorescent iron lines are indeed produced by pumping of λ 3969 by H ϵ and Ca II H. Further, the velocity field plays an important role in determining the strength of the fluorescent lines by shifting the Fe I line relative to the peaks of the Ca II H-H ϵ emission.

III. VELOCITY FIELDS AND LINE PROFILES

Most of the evidence we have regarding the velocity fields in the chromospheric envelopes of T Tauri stars is contained in the broad, self-absorbed profiles of the hydrogen emission lines. Kuhi (1964, 1966), following Chandrasekhar (1934), fitted these



FIGS. 1a (left), 1b (right).—The Ca II H and K lines for two plates of RW Aur. The arrow indicates the wavelength of H ϵ with $\Delta V = 0$.

TABLE 2
COMPARISON OF OBSERVED AND PREDICTED INTENSITIES FOR
Ca II H-H ϵ

Line	S1	S3
Observed:		
H γ	70	37
H δ	74	26
H8.....	19	...
H11.....	31	...
Ca II K.....	386	241
Predicted:		
H ϵ	30	20
Ca II H.....	200	120
H ϵ + Ca II H.....	230	140
Observed:		
Fe I λ 4063.....	56	38
Fe I λ 4132.....	44	27
λ 4063 + λ 4132.....	100	65
H ϵ + Ca II H.....	149	47
Sum of iron and Ca II H-H ϵ ...	249	112

NOTE.—Observed intensities were taken from Gahm 1970. Predicted intensity for Ca II H is Ca II K intensity divided by 2; value for H ϵ is an interpolated value from all H lines in Gahm's catalog, only the principal lines being shown here.

using a spherically symmetric, ballistically expanding envelope; he obtained envelope sizes $\sim 3\text{--}10 R_*$ and mass-loss rates $\sim 10^{-7} M_\odot \text{ yr}^{-1}$. The best fit to the profiles had the ejection velocity approximately equal to the escape velocity for most of the stars examined. The envelope was assumed optically thin in the lines, and the absorption components were assumed to be produced in a thin shell at fixed V_{abs} surrounding the emission region. In this model, a peaked or rounded profile can only result from a decelerating envelope with $V_0 \leq V_{\text{escape}}$; if $V_0 > V_{\text{escape}}$, the profile is flat.

More recently, models have been constructed for mass loss from Wolf-Rayet stars which omit the assumption that the envelope is optically thin, and which have the absorption component produced within the emission region (see, e.g., Castor 1970; Sobolev 1960). There, self-absorbed and peaked or rounded profiles correspond to optically thick envelopes. The profiles produced by Castor for an accelerating envelope reproduce some of the features of the T Tauri profiles: they are rounded, with absorption components displaced shortward. When self-absorption is included, the necessity for $V_0 \leq V_{\text{escape}}$ and a decelerating outward flow thus disappear.

The ejection velocity V_0 and the density at the inner edge of the envelope $N_{\text{H}}(r_0)$ which emerge from fitting the profile are used to estimate the mass-loss rates for T Tauri stars. Conservation of mass gives $dm/dt = 4\pi r_0^2 N_{\text{H}}(r_0) m_{\text{H}} V_0$. Kuhi found mass-loss rates that were very high—on the order of $10^{-7} M_\odot \text{ yr}^{-1}$ —indicating that a star might lose as much as 40 percent of its mass during the T Tauri stage. This is large enough to have a significant effect on the pre-main-sequence evolution of normal stars. Thus it is important to try to improve the estimates for $N_{\text{H}}(r_0)$ and V_0 in order to better determine the mass-loss rates.

All models which assume only radial expansion fail to fit the observed profiles for T Tauri stars in at least one significant detail—the portion of the profile which is occulted. For a thin shell which is expanding only, the red side of the profile drops off more rapidly to zero than the blue, due to occultation by the star of the most rapidly receding material. This leads to profiles with a very steep drop on the red side, and hence wings of equal central intensity but unequal width. If, however, the shell is only rotating instead, then the occulted portion lies at $V = 0$, giving a flattened profile. If the velocity is a combination of expansion and rotation, then the occulted velocities will be centered on V_r , while the width of the profile reflects $V_{\text{max}} = (V_r^2 + V_\theta^2)^{1/2}$. This leads to a profile in which the wings are of unequal width but where also the central peak is distorted, with the red side lower than the blue side and the entire peak flattened to some extent. Thus the shape of the central and red portions of the line profile allows some conclusions to be drawn regarding the amount of rotational velocity versus expansion velocity present. Since only expansion velocity enters into calculating the mass-loss rate, it is important to determine the amount of rotation. To interpret this quantitatively requires a model for the way in which the radial and tangential velocities vary with distance from the star—a problem to be discussed in a future paper. However, some conclusions may be drawn immediately which should hold for any model.

First, of course, any theoretical profile that fits the observed profile must have the same integrated intensity. If we further assume that the emission is proportional to the density squared, we find that, within a factor of 2 or so, all theories must yield the same “emission measure” $\int N_{\text{H}}^2 dV$. Kuhi's fit gave $\int N_{\text{H}}^2 dV \simeq 10^{56}$ for RW Aur. Then, since all fits yield roughly the same volume (given by r/r_0 of eq. [2], below), this implies that $N_{\text{H}}(r_0)$ will be essentially independent of the model used.

Second, the percent of the profile which is occulted yields an “average” distance for the shell. Assume r_0 is the radius of the occulting stellar disk; if we assume a thin shell at r for the emitting region, and if f is the fraction of the light blocked out by occultation, then

$$r/r_0 = \frac{1}{2}(f - f^2)^{-1/2}. \quad (2)$$

For the fluorescent iron lines, the fraction occulted, found by integrating the two halves of each profile separately, is on the order of 0.01–0.05; this gives distances $\geq 3 R_*$. For the primary emission lines a direct estimate is more difficult, owing to the self-absorption; however, a conservative estimate yields 20–40 percent occulted for H λ , H δ , and H8, for these plates of RW Aur. This means that those lines are mostly formed within $\frac{1}{4} R_*$ of the star's surface. Kuhi's emission regions were larger than this, but that is at least partly due to his adopting an emission coefficient $\epsilon \propto N_e^2 V^\beta$ with $\beta = 2$ [or, roughly, $\epsilon \propto (r/r_0)^{-5}$ for most cases] so that very little of the emission comes from the outer part of the envelope. In fact, his outer

boundary was primarily determined by the requirement that the velocity there match the observed velocity of the absorption component.

Third, there are qualitative differences between line profiles produced by purely expanding envelopes and those produced by envelopes with rotational velocities equal to or exceeding the expansion velocity. If occultation occurs shortward of $\Delta V = 0$, then $V_r \leq V_\theta$ throughout a large part of the region producing a given line. If $V_r \geq V_\theta$, then the region with line-of-sight velocity near zero is mostly visible, and most of the occultation takes place at the red edge of the profile. If $V_r \ll V_\theta$, most of the occultation takes place near $\Delta V = 0$.

The actual profiles of both the Fe I lines and the primary emission lines show evidence of occultation around $\Delta V = 0$, indicating that rotational velocities are indeed high. Observed rotational velocities for the surfaces of T Tauri stars are $V \sin i \simeq 20\text{--}65 \text{ km s}^{-1}$ (see, e.g., Herbig 1962). For RW Aur, no stellar absorption spectrum is seen, so no estimate has been made. By inspecting the profiles to see at what velocity the major portion of the occultation takes place, we can estimate of V_r/V_{max} , where $V_{\text{max}} = (V_r^2 + V_\theta^2)^{1/2}$ is the maximum velocity present in the profile (roughly the same as V_θ in Kuhl's model). This gives $V_r/V_{\text{max}} \sim 0.1\text{--}0.5$ for the observed profiles shown by Kuhl (1964), and ~ 0.2 for RW Aur.

An independent measure of V_r/V_{max} can be found by comparing the Ca II K profiles with, for example, the profiles generated by Sobolev (1960, pp. 39–40) for a rotating, expanding disk viewed equatorially. His model includes self-absorption in the line, but does not consider occultation by a central star. With constant radial velocity and $1/r$ dependence of rotational velocity, the resulting profiles have wings of unequal width but similar height, and the absorption component falls roughly at V_r . For the Ca II K profiles in RW Aur, this gives $V_r = 50\text{--}80 \text{ km s}^{-1}$, or $V_r/V_{\text{max}} \sim 0.1\text{--}0.2$ again. This result is probably independent of the assumed law of rotation. If V_r is not constant, the velocity of the absorption component is a mean for the inner envelope.

Thus, both analysis of the position of the profile occulted and qualitative examination of the absorption feature in Ca II K lead to the same estimate: $V_r/V_\theta \leq 0.2$. These numbers are *very* approximate, and should serve only to encourage further investigation into the effects of a rotation on the line profiles. They do, however, indicate that mass-loss estimates which do not include rotation are likely to overestimate expansion velocity and hence mass-loss rates by a factor of 2 to perhaps 10.

IV. PHYSICAL CONDITIONS IN THE REGION PRODUCING THE FLUORESCENT LINES

In Paper I, both upper and lower limits were found for N_{H}/W for a given temperature. The upper limit merely tells us when collisions begin to dominate over radiative processes, forcing all line ratios to their LTE values. This limit is also quite high; for 5000 K, $N_{\text{H}}/W \leq 10^{14} \text{ cm}^{-3}$, and for lower temperatures this

limit is higher. Temperatures above 5000 K were shown in Paper I not to favor the Fe I lines. A distance of $5 R_*$ gives $W = 0.01$, and thus $N_{\text{H}} \leq 10^{12} \text{ cm}^{-3}$ (or greater, if $T < 5000 \text{ K}$). This is certainly trivially satisfied.

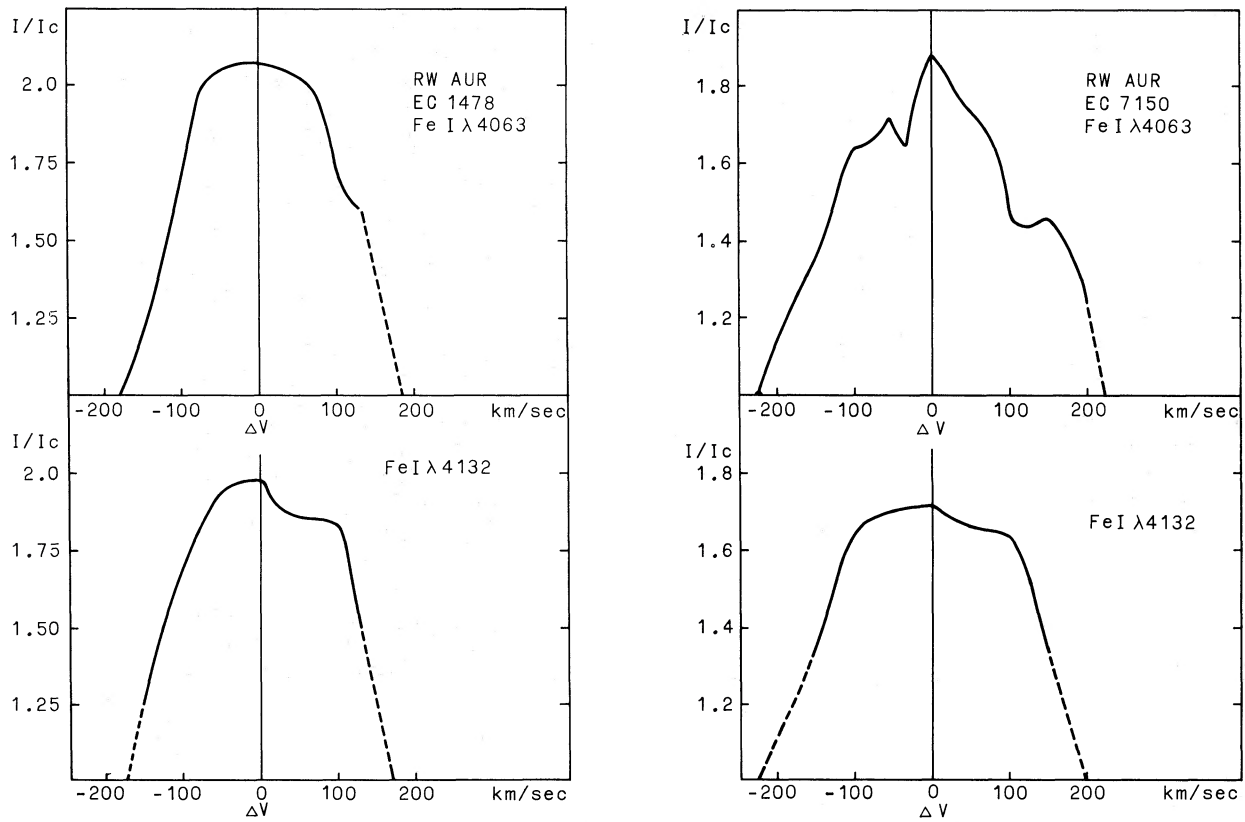
The lower limit on N_{H} is more interesting. This limit is imposed by the requirement that $\lambda 4132$ be enhanced relative to $\lambda 4005$. At intermediate densities at these temperatures, collisions with neutral hydrogen dominate the fine-structure transitions. If the density is too low, these collisions cease to be effective, and the lower levels of the fluorescent lines become highly overpopulated relative to the lower level of $\lambda 3969$. This in turn enhances the population of the upper level of $\lambda 4005$. (For a more detailed discussion of this point, see Paper I.) Since in RW Aur, $\lambda 4005$ is weak (the line ratio $\lambda 4132/\lambda 4005$ is large), therefore $N_{\text{H}}/W \gtrsim 10^{10}$. This lower limit is also quite independent of the temperature of the region. Kuhl's fit for RW Aur has $N_{\text{H}} \sim 10^{11}$ at the inner edge of the envelope—if the outflow is spherically symmetric, then at $\geq 3 R_*$ we expect $N_{\text{H}} \leq 10^{10} \text{ cm}^{-3}$. Thus the density is barely sufficient to produce the fluorescent lines if the material in the envelope is distributed uniformly. It is interesting to note that most of the other stars examined by Kuhl had densities at the inner edge of $\sim 10^{10} \text{ cm}^{-3}$; these stars also show little or no fluorescent emission.

Thus the interaction of two effects determines the strength of fluorescent iron lines: the velocity field and the density of the envelope. These effects may not be unrelated; stars showing more extreme mass-loss rates are also more likely to have massive, extended envelopes. In either case, strong fluorescent lines are signs of an extended, active circumstellar envelope.

V. INTERPRETATION

We now have enough information to write a tentative description of what is happening in the region surrounding T Tauri stars. Matter is being ejected from the surface at velocities which may approach the velocity of escape and with a tangential component of at least $20\text{--}50 \text{ km s}^{-1}$ from the rotation of the star. This matter is at about 10^4 K and emits hydrogen and Ca II lines, most of which are produced within $0.5 R_*$ of the surface of the star. The matter then cools until, at around $5 R_*$ or so, it has reached a temperature $\leq 5000 \text{ K}$. If the density is sufficiently high, the neutral iron in this region absorbs H ϵ and Ca II H, and reradiates the light at $\lambda\lambda 4063, 4132$.

From the profiles of the Fe I lines in figure 2, there is a hint that the occultation, if present, is at $\Delta V = 0$ to $\Delta V = 100 \text{ km s}^{-1}$. This suggests that most of the velocity around $5 R_*$ is rotational, and very little is expansion. From the width of the profile, then, $V_\theta \sin i \simeq 200 \text{ km s}^{-1}$ at about $5 R_*$, and $V_r \simeq 50 \text{ km s}^{-1}$. One way in which such a large rotational velocity may be produced is if the region closest to the star is threaded by a magnetic field and is approximately corotating. If this persists to $5 R_*$, then from 50 km s^{-1} at the star we obtain $V \sin i \simeq 250$



FIGS. 2a (left), 2b (right).—The fluorescent lines $\lambda\lambda 4063, 4132$ for two plates of RW Aur. The dashed portions of the profile have been produced by extrapolation due to the presence of blends in the wings of these lines.

km s^{-1} at $5 R_*$ in the equatorial plane. Mestel (1968) developed the solar-wind theory to include the case of a rapidly rotating star with a strong magnetic field. One of the parameters describing his model is $l =$ ratio of gravitational energy to thermal energy; for RW Aur, if the circumstellar envelope is at $\sim 10^4$ K, l is very large. This suggests that the stellar wind is not thermally driven. However, Mestel's model allows for stars with cool "coronae" to have centrifugally driven stellar winds. A comparison of his model with our deductions about the velocity field from the line profiles yields some interesting results.

If we assume that corotation ceases between 3 and $5 R_*$ in the equatorial plane of RW Aur, then the surface magnetic field strengths required may be deduced from Mestel's equation (12) and the requirement that the Alfvén velocity equal the flow velocity at the critical point. This gives a surface magnetic field on the order of 10^2 gauss—not an impossibly high value for a young star.

Again using Mestel's results, we can estimate the angular momentum loss from a star like RW Aur. Adapting his equation (49), we can write

$$\frac{1}{L} \frac{dL}{dt} = \Gamma \frac{1}{M_*} \frac{dM}{dt}, \quad (3)$$

where $\Gamma = \frac{2}{3}$ for spherically symmetric mass loss from

a nonmagnetic star. For rapidly rotating stars with strong magnetic fields, Γ can be quite large—some of the cases Mestel calculated had $\Gamma \geq 10^3$. If we assume $\Gamma \approx 10$ for RW Aur, which should prove to be a very conservative estimate, and also take Kuhl's mass-loss estimate of $1.1 \times 10^{-7} M_\odot \text{ yr}^{-1}$, we find that the e -folding time for angular momentum loss is $\sim 10^5$ years. Even if the mass loss has been overestimated by a factor of 10, this predicts that most of the star's angular momentum will be lost easily during its T Tauri stage.

Another way to produce 200 km s^{-1} at $5 R_*$ is by Keplerian orbital motion. If we assume that the entire width of the Ca II and hydrogen lines is due to rotation, then $V_0 = 450 \text{ km s}^{-1}$ for RW Aur. At $5 R_*$ this becomes 200 km s^{-1} , just the width observed for the fluorescent lines. The disadvantage of this approach is that it requires the star to be very rapidly rotating—at or beyond the "stable" circular velocity. This may, of course, explain the irregular variability of T Tauri stars, and their mass loss. However, the stellar absorption spectra that are seen for some T Tauri stars indicate much smaller rotational velocities, on the order of 50 km s^{-1} . Since the larger velocities required by this model would have the effect of rendering the underlying stellar spectrum invisible, that is not conclusive evidence against it. It is interesting to note (see, e.g., Paper I, table 3) that the most energetic

T Tauri stars do not show underlying absorption spectra, possibly because they are most rapidly rotating. From purely qualitative arguments it is not possible to distinguish between "solid" rotation and other rotation laws for the inner envelopes using line profiles; this distinction is one of the goals of detailed profile fitting.

An advantage to including rotation is that we do not require the large ejection velocity at the surface postulated by the ballistic models; most of the kinetic energy is provided by the rotation. If our interpretation of the fluorescent line profiles is correct, moreover, such rapid ejection at the surface is not likely. A pure expansion model with $V_0 \sim V_{\text{escape}}$ predicts $V_r \approx 200 \text{ km s}^{-1}$ at $5 R_*$ for RW Aur—clearly much larger than the expansion velocity suggested by the occultation in the Fe I lines. If $V_r \approx 50 \text{ km s}^{-1}$ at $5 R_*$, then the expansion velocity near the surface must be $\sim 100 \text{ km s}^{-1}$ or less—at least a factor of 3 different from the velocity required by the nonrotating model. Thus a model which includes rotation in generating line profiles will result in substantially lower mass loss rates.

Kuhi (1964, 1966) found $V_0 \approx V_{\text{escape}}$ for most of the stars he considered, with the lowest velocity $\approx 0.7 V_{\text{escape}}$. This tendency for stars to have profiles of width on the order of their escape velocities may

be explained in two ways. One interpretation is that it is a peculiarity of the simple model, which requires flat-topped emission lines if $V > V_{\text{escape}}$, and a "pileup" if $V < V_{\text{escape}}$. It is possible that careful introduction of temperature gradients, finite optical depth, or other such reasonable extra parameters, or adoption of a different model, will remove this coincidence. A more physical explanation is that $V \approx V_{\text{escape}}$ is selected because that is also roughly the surface velocity at which a rotating object becomes unstable, and centrifugally driven mass loss occurs. If most of the line width is rotation, not expansion, then $V_0 \approx V_{\text{escape}}$ is expected for stars which are rotating near breakup velocity. Thus the large mass-loss rates of T Tauri stars could be not only a cure for excess angular momentum but also caused by it.

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