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SOFT X-RAY SPECTROSCOPY OF THREE EXTRAGALACTIC SOURCES

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ABSTRACT

Spectra for 3C 273, M87, and NGC 4151 are presented in the 0.5–5 keV band. The bandwidth of our rocket-borne instrumentation permits information to be derived on absorption in the low energy spectra without the need to normalize to a nonsimultaneous hard X-ray result of a different experiment. None of our spectra require strong self-absorption at a statistically significant level. General techniques for error estimation in parameters of X-ray spectra are discussed, and the results are applied to the spectra previously reported for these objects. It is shown that the $\chi^2 + 1$ technique which has been used in these analyses can drastically underestimate the magnitude of these errors. We conclude that the suggested division of extragalactic X-ray sources into two groups characterized by the magnitude of absorption in their spectra is premature.

Subject headings: quasi-stellar sources or objects — Seyfert galaxies — spectra, X-ray — X-ray sources

I. INTRODUCTION

It has been suggested that extragalactic X-ray sources may be fundamentally divided into two groups characterized by the magnitude of absorption in their spectra (Giacconi 1973; Kellogg 1973). These authors have cited *Uhuru* results which indicate that sources contained within rich clusters of galaxies, such as Virgo, Coma, and Perseus, show no evidence for soft X-ray self-abosrption, while individual objects lying outside of cluster, such as Seyfert galaxies and QSOs, show soft X-ray absorption far in excess of that attributable to the path length through material in our own Galaxy. Several workers have attempted to study these spectra with instrumentation sensitive exclusively in the soft X-ray band (e.g. Zaumen, Catura, and Fisher 1973; Gorenstein, Harris, and Gursky 1972). Since the magnitude of soft X-ray absorption derived depends heavily on the assumed shape of the hard X-ray spectrum, interpretation of such data requires normalization to the results of a separate, nonsimultaneous experiment, thus subjecting the final interpretation to the statistical and systematic uncertainties of both experiments.

In this paper we report data acquired with a rocket-borne argon-methane proportional counter equipped with a thin (260 μg cm^{-2}) Kimfol window and sensitive to celestial X-rays in the 0.2–5.0 keV band. This sensitivity range permits us to derive simultaneous information on the magnitude of soft X-ray absorption and the slope of the hard X-ray spectrum. Observations were made of M87, 3C 273, and NGC 4151, providing examples of a cluster source, a QSO, and a Seyfert galaxy. The column density of neutral galactic hydrogen in the direction of each of these sources is 1–2 $\times 10^{20}$ atoms cm^{-2} (e.g., Radhakrishnan *et al.* 1971; Wesselius 1973), thus leaving the low energy spectra of these objects unattenuated by material in our own Galaxy and permitting a search for self-absorption.

The instrumentation employed is similar to that we have described previously (Bowyer *et al.* 1974; Margon *et al.* 1974), with the exception of the effective collecting area (491 cm^2) and collimation (2.86° FWHM circular). The payload was flown aboard an Aerobee 170A sounding rocket launched from White Sands Missile Range, New Mexico, on 1973 January 12, and reached a peak altitude of 229 km. The experiment was pointed by an attitude control system at 3C 273 for 79 s, M87 for 37 s, NGC 4151 for 65 s, and two additional background points, located 5° south of 3C 273 and 4° south of NGC 4151, for 47 and 24 s, respectively.

II. ANALYSIS

Our detector provides 31 channels of pulse-height information, and we have used these data to derive the energy spectrum of each of the observed sources, utilizing the reduction method described by Cruddace *et al.* (1972), and the photoelectric absorption cross sections of Cruddace *et al.* (1974). We have, however, reconsidered the problem of error est mation for the derived free parameters. In our previous work, we have accepted all sets of values which have a χ^2 confidence above $e^{-1/2} = 0.607$ of the peak confidence, which is analagous to 1 standard deviation of a Gaussian variate. The size of this contour of constant probability in parameter space provides an effective indication of the steepness of the gradient of the likelihood function associated with one set of data; i.e., it shows how the goodness of fit is affected by a change in free parameters. However, this technique does not provide a statistically precise answer to the question of how the values of the free parameters would vary, due to counting statistics only, if the experiment were repeated a very large number of times with no change in the systematic errors.

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We briefly reconsider this analysis from first principles. When N independent spectral data D_i are employed to test a theoretical trial spectrum F_i having p adjustable parameters, it is customary to estimate the goodness of fit by means of the statistic

$$S \equiv \sum_{i=1}^{N} \frac{(D_i - F_i)^2}{\sigma_i^2} .$$

If the data are normally distributed with predictable variances σ_i^2, and if the F_i are linearly dependent on the parameters, then the following statements are applicable: (*a*) For the true (but unknown) adjustment of parameters, S will be χ^2 distributed with N degrees of freedom. (*b*) For the best-fitting adjustment of the parameters, its minimum value $S_{\min}$ will be χ^2 distributed with $N - p$ degrees of freedom. (*c*) The decomposition theorem of χ^2 variates (e.g., Jenkins and Watts 1968) establishes that the difference of these statistics will be χ^2 distributed with p degrees of freedom, independent of $S_{\min}$.

We wish to emphasize that the confidence in the class of models being tested may be judged by the rank of $S_{\min}$ among all χ^2_{N-p}, i.e., by

$$Q(S_{\min}; N - p) \equiv \int_{S_{\min}}^{\infty} f(\chi^2; N - p) d\chi^2 .$$

Furthermore, the range of parameter values not rejected at a given significance level can be set using item (*c*) above; the critical limiting parameter space contour outside which the adjustment is rejected with significance α is

$$\chi_L^2 = \chi^2_{\min} + \chi_p^2(\alpha) , \qquad (1)$$

where $\chi_p^2(\alpha)$ is the tabulated χ^2 value for p degrees of freedom and probability $Q = \alpha$.

It is important to consider the degree to which the assumptions of normal errors, known variances, and linear parameter dependence are satisfied. The Poisson distribution, which governs accumulation of X-ray signals, has $\sigma_i^2 = F_i$ and exhibits an asymptotically normal form for large means. The assumption that the bin variances σ_i^2 are exactly known from theory can be rigorously justified only if they are not themselves adjustable; however, in practical cases having more than a few counts per bin, the use of intensities minimizing S leads only to a small upward bias in the derived means, and negligibly affects spectral parameters. The assumption of linear dependence of the F_i upon the parameters has been discussed in detail by Beale (1960), who recommends the use of the simple S statistic to define a confidence region even in the nonlinear case.

We have further verified that equation (1) is appropriate to our analysis by conducting extensive numerical simulations which fit spectra to data generated by power-law and bremsstrahlung functions and perturbed with a random number generator. The results of one phase of this simulation are given in table 1. The function

$$F(x) = ax^{-c} \exp(-bx^{-3})$$

was evaluated for $x = 1, 2, \ldots, 7$ with $a = 2500$, $b = 4.60$, and $c = 1.50$. This corresponds to the case $N = 7, p = 3$, and we would therefore expect $S_{\text{true}} \sim \chi_7^2$, $S_{\min} \sim \chi_4^2$, and $S_{\text{true}} - S_{\min} \sim \chi_3^2$. Eighty-three iterations of perturbation and fitting were performed, resulting in the "observed" values listed in the table. The agreement with the predicted values (which are based solely on the known mean and standard deviations of the χ^2 distribution) is excellent, thereby confirming the conclusion of Beale (1960) that the linear confidence region is appropriate to this nonlinear case. Therefore, in this and future work we will use equation (1) to determine the range of free parameters permitted by the data.

While 1 σ errors ($\alpha = 0.32$) are often reported, these may not be the most useful since there is then an appreciable probability that the true parameter values lie outside of the stated range, and the reader has no way of quantitatively reducing this probability. We will use the 90 percent probability level ($\alpha = 0.1$) to reflect this fact.

The combination of our narrow experiment collimation (which causes the corrected source intensity to be a very sharp function of the position of the source in the field of view), a large limit cycle on the vehicle attitude control system, and the failure of one of the two onboard aspect cameras, has resulted

TABLE 1

RESULTS OF NUMERICAL SIMULATION

	Predicted	Observed
Mean S_{true}	7.00	7.44 ± 0.40
S.D. S_{true}	3.74	3.60 ± 0.28
Mean $S_{\min}$	4.00	4.17 ± 0.29
S.D. $S_{\min}$	2.38	2.64 ± 0.21
Mean $S_{\text{true}} - S_{\min}$	3.00	3.27 ± 0.26
S.D. $S_{\text{true}} - S_{\min}$	2.45	2.38 ± 0.19
Mean $a_{\min}$	2500	2546 ± 26
Mean $b_{\min}$	4.60	4.67 ± 0.03
Mean $c_{\min}$	1.50	1.50 ± 0.01
Fraction of $(a, b, c)_{\text{true}}$ within $S_{\min} + 6.25$	0.90	0.84 ± 0.04
Fraction of $(a, b, c)_{\text{true}}$ within $S_{\min} + 3.5$	0.68	0.64 ± 0.07
Fraction of $(a, b, c)_{\text{true}}$ within $S_{\min} + 1$	0.20	0.17 ± 0.10

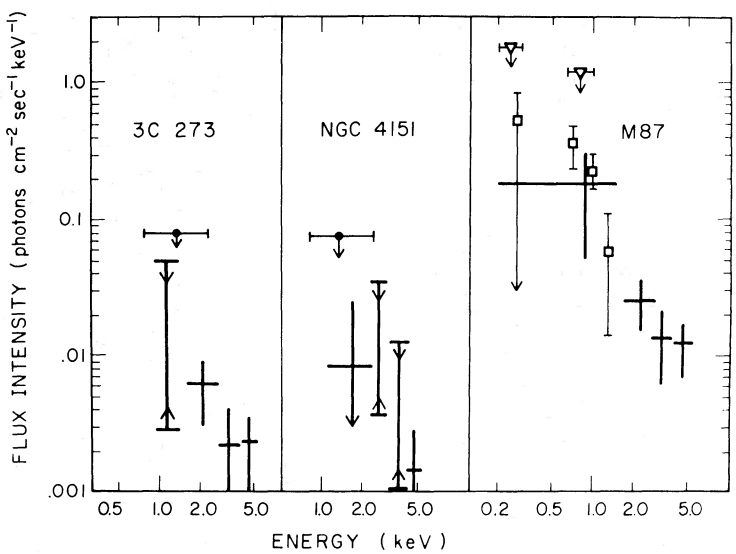

FIG. 1.—*Solid lines*: Energy spectra of the three observed extragalactic sources. Error bars are $\pm 1\ \sigma$ statistical errors. Each point represents data from 4 to 8 pulse-height-analyzer channels, which have been summed for clarity. Points where the derived intensity depends heavily on assumed spectral shape are shown as 1 σ upper and lower limits, although they represent positive detections of flux. Upper limits of other experimenters are shown at the 3 σ level. *Circles*, Zaumen *et al.* (1973); *squares*, Catura *et al.* (1974); *triangles*, Gorenstein *et al.* (1972).

in a large formal uncertainty in our derived absolute source intensities, estimated to be ± 50 percent. This is not of particular concern because satellite-borne instrumentation has already defined these intensities better than even optimally possible in our short exposures. The parameters of interest in this experiment concern the shape of the observed pulse-height spectrum, and these are independent of the overall intensity calibration.

III. RESULTS

In figure 1 we present spectra for the three sources, as well as relevant previous work of other experimenters. Each of our data points is corrected for instrumental response by the technique described previously, and represents our best estimate of the flux incident at Earth. The position of the individual points is dependent upon the assumed spectral shape when using this technique; for the purposes of the figure we have used the highest-confidence power-law fit for all of the sources. In a few cases the measured net count rates, although nonzero at a statistically significant level, are small enough that the derived flux intensity is strongly dependent on the assumed incident spectrum. For these points we have plotted the largest 1 σ upper limit and smallest lower limit for any spectrum consistent with the data.

In figure 2 we show contours of constant probability in parameter space for the fitting procedure for both the power-law and thermal bremsstrahlung mechanisms for each source. While in principle one might wish to plot three-space contours for all the free parameters, or two-dimensional cross sections through such contours, the large error bars on our intensity parameters create a very broad maximum of probability, and the majority of such contours in intensity space would be identical. We have therefore displayed contours for the two spectral parameters only, with the intensity set to yield the minimum S at

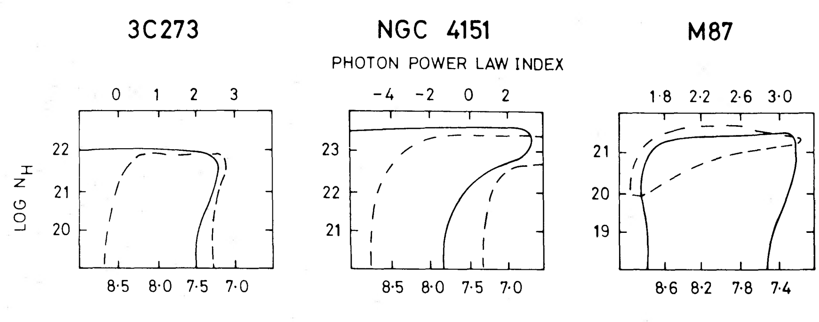

FIG. 2.—Contours of constant probability at the $\alpha = 0.1$ level for the fits to the data in fig. 1. *Solid line*, thermal bremsstrahlung model with Born-approximation Gaunt factor (read lower scale); *broken line*, power-law model (read upper scale; numerically larger indices represent steeper spectra).

each point on the mesh. These contours illustrate how the free parameter values are constrained for each model, and all such values within the contours should be considered viable solutions. In the following we discuss individual source spectra.

a) *3C 273*

The data presented here are in excellent agreement with our previous work on this source (Margon, Bowyer, and Lampton 1972). Also shown in figure 1 is the three-sigma upper limit of Zaumen *et al.* (1973), which is a factor of 2–10 above our detections. Figure 2 indicates that our spectra are compatible with no self-absorption in the 3C 273 spectrum. Kellogg (1973) and Giacconi (1973) have presented a preliminary unnormalized spectrum of 3C 273 in the 2–10 keV band showing absorption with unit optical depth at 2.7 $\pm$ 0.7 keV, which lies marginally outside our error contours. However, these authors employ an error analysis procedure in which the 1 σ errors are calculated for each parameter by allowing χ^2 to increase by one while all other parameters are held constant (Kellogg 1974). Although this procedure has been employed in general discussions of analysis of X-ray spectra (Gorenstein, Gursky, and Garmire 1968) and is now commonly used by many workers in X-ray astronomy, it is incorrect for two reasons. First, any technique which varies the free parameters separately neglects the strong correlation between these parameters, and thus underestimates the errors. Second, even for a linear fitting procedure this $\chi^2 + 1$ approach is valid only in the limiting case where $p = 1$, as has been noted by Riegler (1969) and by Cline and Lesser (1970). This may also be seen by noting that for $p = 1$, $\alpha = 0.32$, equation (1) yields $\chi_L{}^2 = \chi^2{}_{\min} + 1.00$. Using the *Uhuru* observations of 3C 273, where $p = 3$, $N = 7$, we derive $\chi_L{}^2 = \chi^2{}_{\min} + 3.5$ for 32 percent significance (1 σ). Working to the more realistic level of significance of 10 percent, we find $\chi_L{}^2 = \chi^2{}_{\min} + 6.25$ as the proper error estimate, rather than $\chi^2{}_{\min} + 1$ which has been used. In table 1, using the results of our numerical simulation, we have indicated the fraction of true parameter values which fall within each of these contours. These results further verify that equation (1) correctly describes the errors for the specific nonlinear analysis of interest, and that the $\chi^2 + 1$ approach drastically underestimates these errors. It seems unlikely that the *Uhuru* data with errors properly assessed will be incompatible with our results at a statistically significant level.

b) *NGC 4151*

The points in figure 1 represent the first soft X-ray detection of this source. The upper limit of Zauman *et al.* (1973) is indicated, and is again compatible with our data, lying about a factor of 10 above our points. Figure 2 indicates that due to our small exposure to this source, we are unable to comment on the self-absorption; both unattenuated and attenuated spectra are compatible with our data. Again comparison with the Kellogg (1973) and Giacconi (1973) results on this source is made difficult by lack of proper error estimation for the *Uhuru* data; these authors quote unit optical depth at 5.7 $\pm$ 1.1 keV, which does not seem compatible with our detection of the source in the 2–5 keV band at the 99 percent confidence level.

c) *M87*

The field of view of our instrument encompassed the entire source Virgo X-1, which is known to be extended and centered on M87. Our soft X-ray results for this source are in agreement with and complementary to those of Catura *et al.* (1974), shown in figure 1. Our data are also compatible with the upper limits of Gorenstein *et al.* (1972), which lie a factor of 10 above our detections. Numerous observations of M87 in the 2–10 keV band have been summarized by Lampton *et al.* (1971). Constraints on the M87 spectral parameters from the current data are shown in figure 2. The thermal fit of 3.6×10^7 K noted by Catura *et al.* (1974) does lie within our contour, although detailed comparison is difficult since these authors quote no errors.

Both Catura *et al.* (1974) and Gorenstein *et al.* (1972) note that unless self-absorption is invoked, a thermal model provides a considerably better fit to their data. However, both of these results rely on normalization to *Uhuru* data at higher energies, and in view of the error analysis uncertainties discussed here, we feel these conclusions must be regarded as tentative. Our own power-law models permit acceptable fits without invoking self-absorption.

There have been two reports of variability of intensity and spectral shape of the M87 X-ray source (Byram, Chubb, and Friedman 1971; Janes *et al.* 1971). Lampton *et al.* (1971) demonstrated that the statistics of the X-ray data involved did not support the conclusion of variability, but it is desirable to supplement this result with further observational data. Kellogg *et al.* (1972) present 5 months of data which show no evidence for X-ray intensity variability. Our current spectral data are in good agreement with our previous results obtained in 1969 (Lampton *et al.* 1971), and thus represent observational evidence for no significant spectral variability over a 4-year baseline.

IV. CONCLUSION

We have obtained spectra for three extragalactic objects which represent both of the distinct classes suggested by Kellogg (1973) and Giacconi (1973). Our results are consistent with and improve upon previous X-ray data for these objects. A possible exception is the preliminary *Uhuru* results on these sources, but the extent of the differences, if any, between the data cannot be determined since the *Uhuru* error limits have been systematically underestimated. In addition, the conclusions from the *Uhuru* data were based only on power-law models; the limits on self-absorption inferred from thermal models will always be much weaker.

Our data do not require self-absorption in any of these sources, although in some cases they permit such an effect. A critical review of the total existing data on these sources leads us to conclude that these data are not sufficiently precise to allow us to distinguish between and classify these objects on the basis of absorption in their low-energy spectra. Consequently we suggest that the division of extragalactic X-ray sources into two classes characterized by the magnitude of this absorption is premature. We note that the inability of experiments to easily distinguish between thermal and nonthermal emission mechanisms may always make such classification schemes ambiguous.

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