A POSSIBLE WIDTH-LUMINOSITY CORRELATION OF THE Ca II K_1 AND Mg II k_1 FEATURES

T. R. AYRES, J. L. LINSKY,* AND R. A. SHINE
Joint Institute for Laboratory Astrophysics, University of Colorado and National Bureau of Standards, Boulder, Colorado
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ABSTRACT

Existing high resolution stellar profiles of the Ca II and Mg II resonance lines suggest a possible width-luminosity correlation of the K_1 minimum features. We show that such a correlation can be simply understood if the continuum optical depth of the stellar temperature minimum, τ_c^*, is relatively independent of surface gravity as suggested by three stars studied in detail.

Subject headings: atmospheres, stellar — Ca II emission — chromospheres, stellar — line profiles

I. INTRODUCTION

Several strong resonance and subordinate lines in the spectra of late-type stars show width-luminosity correlations. The most notable of these correlations are the Wilson-Bappu effect obeyed by the Ca II 3934 Å K-line emission core (Wilson and Bappu 1957); an analogous effect shown by the ultraviolet Mg II 2796 Å k-line (Kondo *et al.* 1972; Moos *et al.* 1974); and a weaker correlation seen in the Hα absorption core (Kraft, Preston, and Wolff 1964; LoPresto 1971; Fosbury 1973). In addition, Lutz, Furenlid, and Lutz (1973) have shown that the extensive damping wings of the Ca II H and K lines broaden with increasing luminosity in much the same way as the line cores.

In this letter we present evidence for a width-luminosity relation obeyed by the K_1 and k_1 minimum features (see, e.g., fig. 1*a*) in the inner wings of the Ca II and Mg II resonance lines. The K_1 width, $\Delta\lambda_{K_1}$, is physically significant because it is a rough measure of the location of the stellar temperature minimum in line optical depth units. We suggest that the observed correlation of $\Delta\lambda_{K_1}$ with luminosity can be attributed to the same mechanism that broadens the line wings—the well-known variation of photospheric mass column

* Staff Member, Laboratory Astrophysics Division, National Bureau of Standards.

density (g cm^{-2}) with gravity (Aller 1963; Avrett 1972; Peytremann 1972; Lutz *et al.* 1973)—if the continuum optical depth of the stellar temperature minimum, τ_c^*, is roughly independent of gravity.

II. OBSERVATIONAL CORRELATIONS

The observations discussed here include all the high-resolution data presently available. Although these observations were made with different instruments and techniques, some photoelectric and some photographic, the inhomogeneity of the data sources is not a serious drawback because the K_1 and k_1 widths depend only on the *relative intensities* in each profile and on the instrumental functions. Hopefully we have minimized the latter influence by limiting our sample to high-resolution data. Furthermore, we consider only spectral types F–K and luminosity classes III–V in order to avoid possible misinterpretation of distorted profiles, such as those produced by mass loss and bulk motions in the extended envelopes of late-type supergiants (Liller 1968).

The K_1 widths for seven stars and the analogous k_1 widths for six stars are given in table 1, and are plotted against absolute visual magnitude in figure 1*b*. The K_1 widths for β Gem and α Tau were measured at lower dispersion than the other five stars, but the α Tau emission width W_0, measured from the same profile, is in good agreement with the value given by Fosbury (1973), so instrumental corrections for α Tau and β Gem are probably unnecessary.

We find that the data in figure 1*b* can be represented by a relationship of the form

$$M_V \approx (-12 \pm 2)\log\Delta\lambda_{K_1} + \text{const}, \qquad (1)$$

which is consistent with the six k_1 widths for

$$\log(\Delta\lambda_{k_1}/\Delta\lambda_{K_1}) \approx 0.40\text{--}0.45 . \qquad (2)$$

We can convert this empirical dependence of $\Delta\lambda_{K_1}$ on M_V to a more useful form using the method of Reimers (1973), which applies a bolometric correction to M_V to obtain $\Delta\lambda_{K_1}$ as a function of M_{bol}, and then inverts $L \propto 10^{-(M_{\text{bol}}/2.5)} \propto R^2 T_{\text{eff}}^4$ via a mass-luminosity relation to get

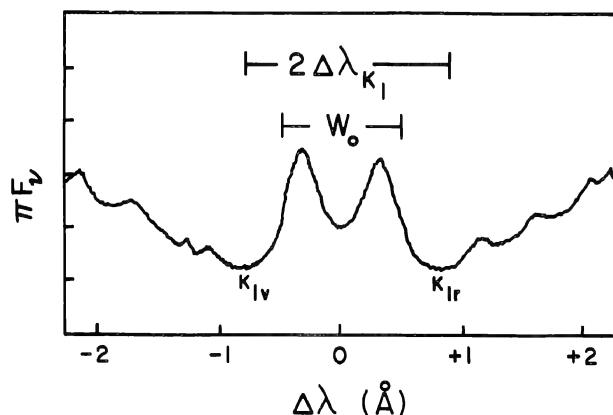$$\Delta\lambda_{K_1} = \Delta\lambda_{K_1}(g, T_{\text{eff}}) ,$$

FIG. 1*a*.—Schematic Ca II K-line profile illustrating the $K_{1\text{red}}$ and $K_{1\text{violet}}$ minimum features, $\Delta\lambda_{K_1}$, and the half-intensity emission width, W_0.

TABLE 1

STELLAR PARAMETERS, K_1 AND k_1 WIDTHS

Star	Spectral Type*	M_V*	$\log K_1 (\mathrm{km\ s^{-1}})$	Approximate Spectral Resolution (Å)	$\log k_1 (\mathrm{km\ s^{-1}})$	Approximate Spectral Resolution (Å)
Sun	G2 V	+4.83	1.37 ± 0.02[1]	$\lesssim 0.04$	1.89 ± 0.05[2] 1.93[3] 1.96[4]	∼0.025 ∼0.10 $\lesssim 0.05$
α Boo	K2 II	−0.20	1.82 ± 0.03[5]	$\lesssim 0.04$	2.29[6]	≈0.4
α Cen A	G2 V (G2 V)[8]	+4.35 (+4.38)[8]	1.47 ± 0.05[7] 1.43[8]	≈0.06 ∼0.04	...	...
α Cen B	K5 V (K1 V)[8]	+5.69 (+5.76)[8]	1.50 ± 0.02[7] 1.46 ± 0.03[8]	≈0.06 ∼0.04	...	...
α CMi	F5 IV–V	+2.65	1.60 ± 0.02[9]	∼0.04	1.90 ± 0.10[10]	0.25–0.50
α Tau	K5 III	−0.70	1.92[11]	∼0.10	2.27[12]	∼0.4
β Cas	F2 IV	+1.5	...	...	2.15 ± 0.07[10]	0.25–0.50
β Gem	K0 III	+0.95	1.71 ± 0.02[13]	∼0.10–0.20	2.12[12]	∼0.4

* Allen 1973, except as noted.

NOTES.—(1) Linsky and Avrett 1970; flux profile.
(2) Lemaire and Skumanich 1973.
(3) Brinkmann, Green, and Barth 1966.
(4) Tousey et al. 1974.
(5) Griffin 1968.
(6) Moos et al. 1974.
(7) Boesgaard and Hagan 1974.
(8) Rodgers 1972.
(9) Ayres et al. 1974.
(10) Kondo et al. 1972.
(11) Ayres and Linsky 1973.
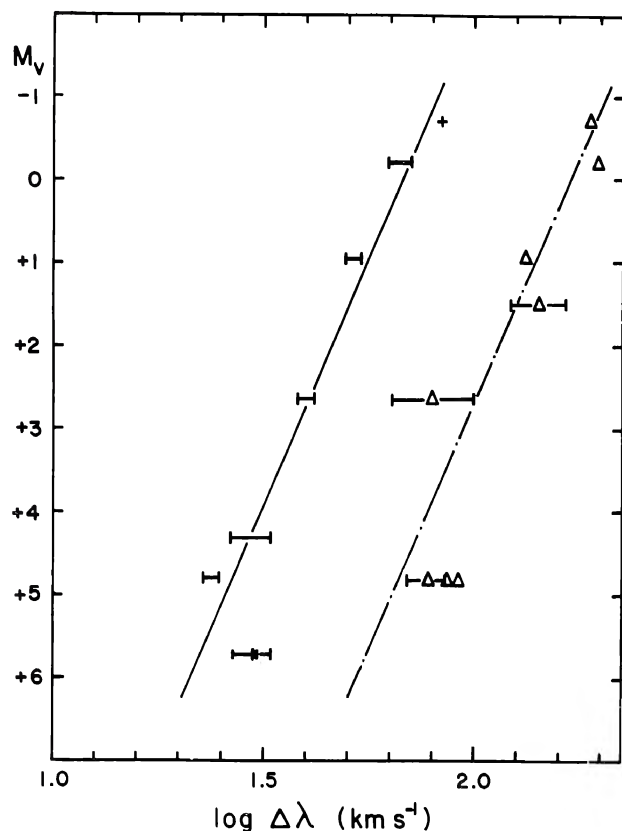(12) Linsky et al. 1975.
(13) Liller 1968.

FIG. 1b.—K_1 widths versus absolute visual magnitude. Crosses and error bars (left of figure) are for Ca II K; triangles and error bars (right of figure) are for Mg II k. Solid line is least-squares fit to K-line data, $\log \Delta\lambda_{K_1} = (-12 \pm 2)M_V +$ const; dot-dashed line is same relationship for k-line data with $\log (\Delta\lambda_{K_1}/\Delta\lambda_{k_1}) = +0.40$.

where $g \propto M/R^2$ is the stellar surface gravity and T_{eff} is the effective temperature. We find that our data are consistent with

$$\Delta\lambda_{K_1} \propto g^{-0.27 \pm 0.04} T_{\mathrm{eff}}^{1.4 \pm 0.2} . \qquad (3)$$

III. ANALYSIS

For simplicity we ignore the dependence of $\Delta\lambda_{K_1}$ on T_{eff}, because the variation of $T_{\mathrm{eff}}^{1.4}$ (∼40 percent) over spectral types F–K is small compared with the factor of ∼5 variation of $g^{-0.27}$ between main-sequence stars and giants. We now consider the consequences of our premise that the mean continuum optical depth of the stellar temperature minimum is relatively independent of surface gravity.

a) The Continuum Optical Depth of $T_{\min}$

The dominant source of photospheric continuous opacity at the wavelengths of maximum emergent flux for normal late-type stars is the H^- ion. The frequency-integrated H^- b-f opacity is proportional to the electron pressure (Gingerich 1964); i.e.,

$$\kappa^{H^-}(\mathrm{cm^2\ g^{-1}}) \propto P_e . \qquad (4)$$

Over the temperature range we are considering, $3000° \mathrm{K} \lesssim T_{\min} \lesssim 5000° \mathrm{K}$, hydrogen is predominantly neutral and P_e is dominated by metal ionization. Hence (e.g., Mihalas 1970),

$$P_e = \mathrm{const}\ P_g A_{\mathrm{met}} = \mathrm{const} \cdot g m A_{\mathrm{met}} , \qquad (5)$$

where P_g is the gas pressure, A_{met} is the abundance of easily ionized metals relative to hydrogen, g is the surface gravity, and m is the mass column density (increasing inward). The second equality above is ob-

tained from hydrostatic equilibrium. Integrating equation (4) over m, we find that the continuum optical depth of the stellar temperature minimum τ_c^* scales as

$$\tau_c^* \propto A_{\mathrm{met}} m^{*2} g , \qquad (6)$$

where m^* is the column density at T_{min}.

If τ_c^* is roughly independent of gravity, we have

$$m^*(\mathrm{g\ cm^{-2}}) \approx \mathrm{const}\ A_{\mathrm{met}}^{-1/2} g^{-1/2} . \qquad (7)$$

In physical terms, low gravity stars require more mass above their photospheres than high gravity stars for similar continuum optical depths, because of the approximately quadratic pressure dependence of H^- formation.

b) The Formation of the K_1 Feature

Detailed studies of the Ca II line formation problem for a variety of model atmospheres (Ayres, Linsky, and Shine 1974; Shine and Linsky 1974) usually show that the K_1 minimum feature is formed in the radiation damping dominated portion of the absorption profile, and that the location of K_1 in $\Delta\lambda$ is often a good indicator of the location of the temperature minimum in physical depth. In fact, the K_1 minimum appears roughly in that portion of the inner wings where monochromatic optical depth $\tau_{\Delta\lambda} \approx 2/3$ occurs at the temperature minimum (e.g., an Eddington-Barbier relation for the flux).

The numerical models on which the above notions are based used the heretofore almost universal assumption of complete redistribution (CRD) in the line transfer calculation. Some recent work using the more physical "partial redistribution" approximation (PRD) has shown significant departures from CRD, especially at the low densities characteristic of stellar temperature minima (Milkey and Mihalas 1973, 1974; Milkey, Ayres, and Shine 1974). However, our experience with a two-level representation of Ca II and the Milkey, Ayres, and Shine results for Mg II suggest that even within the framework of PRD, K_1 retains its sensitivity to the location of T_{min}. Furthermore, the scaling with m^* is essentially unchanged from the CRD result derived below. Hence, for clarity, we assume CRD in what follows, recognizing that the resulting scaling laws for $\Delta\lambda_{K_1}$ also should be applicable in the PRD approximation.

In order to exploit the Eddington-Barbier relation above, we must calculate the monochromatic optical depth at T_{min}. Ignoring van der Waals broadening compared with radiative damping we find:

$$\tau_{\Delta\lambda}^* = \mathrm{const}\ N_{\mathrm{Ca}^+}^* / \Delta\lambda^2$$
$$\propto A_{\mathrm{Ca}} m^* / \Delta\lambda^2 . \qquad (8)$$

Here N_{Ca^+} is the column density of Ca^+ ions at T_{min}, A_{Ca} is the calcium abundance relative to hydrogen, and $\Delta\lambda$ is the wavelength (Å) from line center. The proportionality constant contains various atomic parameters such as the oscillator strength and radiative damping width. In the second line of equation (8) we have

assumed that all of the calcium atoms are in the ground state of Ca^+.

Taking $\tau_{\Delta\lambda_{K_1}}^* \approx 2/3$, we obtain

$$\Delta\lambda_{K_1} = \mathrm{const}\ (A_{\mathrm{met}} m^*)^{1/2} , \qquad (9)$$

where we have assumed that the abundance of the other easily ionized metals scales as A_{Ca}.

Finally, substituting equation (7) for m^* above, we obtain

$$\Delta\lambda_{K_1} \approx \mathrm{const}\ A_{\mathrm{met}}^{1/4} g^{-1/4} , \qquad (10)$$

which shows essentially the same dependence on gravity as the empirical result of equation (3) in § II, as well as a weak dependence on metal abundance.

We note that a completely analogous argument applies to the k_1 features of Mg II k, except that the k_1 width in ($\mathrm{km\ s^{-1}}$) will be larger than the K_1 width by roughly the factor

$$\left(\frac{\Delta\lambda_{k_1}}{\Delta\lambda_{K_1}}\right) \approx \frac{\lambda_k}{\lambda_K}\left(\frac{A_{\mathrm{Mg}}}{A_{\mathrm{Ca}}}\right)^{1/2}\left(\frac{f_k}{f_K}\right)^{1/2}\left(\frac{\Gamma^k}{\Gamma^K}\right)^{1/2} , \quad (11)$$

where the f's are oscillator strengths and the Γ's are the radiation damping parameters. Using the values of the atomic parameters given by Shine and Linsky (1974) and the solar abundance ratio (~ 14; Lambert and Warner 1968) we find:

$$\left(\frac{\Delta\lambda_{k_1}}{\Delta\lambda_{K_1}}\right) \approx 3.5 , \qquad (12)$$

in reasonably good agreement with the empirical value of 2.5–2.8 derived in § II and the solar value of 3.2 $\pm$ 0.5.

We suspect that our apparent overestimate of $(\Delta\lambda_{k_1}/\Delta\lambda_{K_1})$ is caused by the density dependence of the relative departure of Mg II and Ca II from CRD. This dependence arises from the effect of the Ca II metastable states which tend to mix a fixed fraction of incoherence (i.e., CRD) into an essentially pure coherent scattering problem. Since the incoherence fraction, λ_ν, for Mg II has no such lower bound, and since the departure from CRD is a sensitive function of λ_ν, the $(\Delta\lambda_{k_1}/\Delta\lambda_{K_1})$ ratio is likely to show an observable dependence on gravity.

IV. DISCUSSION

We have demonstrated that the simple assumption that $\tau_c^* \approx$ constant independent of gravity can reproduce the observed increase of $\Delta\lambda_{K_1}$ (and hence also $\Delta\lambda_{k_1}$) with decreasing M_V. If our assumption is not correct and we say $\tau_c^* \approx g^n$, then the effect on equation (10) is to introduce a multiplicative factor of $g^{n/4}$. However, our choice of $\tau_c^* \approx$ constant is not completely ad hoc, but is motivated by semi-empirical upper photosphere–low chromosphere models for the Sun (Linsky and Avrett 1970; Vernazza, Avrett, and Loeser 1973), Procyon (Ayres et al. 1974), and Arcturus (Ayres and Linsky 1975). These models show a slow increase in τ_c^* with decreasing g, amounting to possibly a factor

of 6 as the gravity decreases by a factor of 600. Thus n might be ≈ -0.28 and the power of g in equation (10) -0.32 instead of -0.25, but our results are not substantially altered.

Finally, since the empirical $\Delta\lambda_{K_1}-M_V$ correlation is quantitatively similar to the width–M_V relation obeyed by the K-line emission half-width, W_0 (e.g. Reimers 1973), we can speculate that either:

(1) W_0 measures a point in the K-line profile that may be controlled by radiation damping rather than Doppler broadening (e.g. Chipman 1972); or (2) W_0 is broadened by the Doppler effect, and the variation of chromospheric turbulence with gravity is coincidentally the same as the "mass" effect which broadens K_1 and the entire damping wings of K.

Although we must admit the possibility of the latter situation, it would be a remarkable coincidence indeed. If the former situation is true, then the many attempts to link the Wilson-Bappu effect to an increase in chromospheric turbulence with increasing luminosity (e.g., Hoyle and Wilson 1958; Kraft 1959; Reimers 1973) may be based on an oversimplified physical picture of the formation of the K-line emission core and inner wings.

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T. R. Ayres, J. L. Linsky, and R. A. Shine: J.I.L.A., University of Colorado, Boulder, CO 80302