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# STEPS TOWARD THE HUBBLE CONSTANT. IV. DISTANCES TO 39 GALAXIES IN THE GENERAL FIELD LEADING TO A CALIBRATION OF THE GALAXY LUMINOSITY CLASSES AND A FIRST HINT OF THE VALUE OF $H_0$

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#### ABSTRACT

Distances to 39 late-type (Sc–Sd–Sm–Ir) field galaxies have been obtained from the sizes of their H II regions as measured on red-sensitive plates taken with the Mount Wilson and the Palomar reflectors. The distances follow from the calibration in Paper I, and are combined with apparent magnitudes to give absolute magnitudes. The sample is increased to 48 galaxies by adding members of the Local and NGC 2403 groups, whose distances are known from Cepheids. The distribution of absolute magnitudes among the van den Bergh luminosity classes ( $L_c$ ) is determined, and a new absolute calibration of the mean total galaxian luminosity with luminosity class is obtained.

The derived  $\langle M_{pg}^{0,i} \rangle = f(L_c)$  relation is tested for shape by comparing it with the relative calibration for Virgo cluster spirals of different  $L_c$ , and with van den Bergh's initial calibration that used the ratio of redshifts to obtain relative distances of field galaxies in Humason, Mayall, and Sandage.

The adopted absolute magnitudes, combined with the apparent magnitudes of Virgo cluster spirals, gives the distance to the Virgo cluster as  $(m - M)_0 = 31.45 \pm 0.09$ , or  $D = 19.5 \pm 0.8$  Mpc. Two methods (the Hubble diagram for first-ranked E galaxies in clusters, and the color-magnitude relation for E galaxies in the Coma cluster compared with Virgo) show that the Virgo cluster has no detectable peculiar velocity. The systemic velocity is  $\langle v_0 \rangle = 1111 \pm 75$  (rms) km s<sup>-1</sup>. This, combined with D, gives a first hint at the value for the Hubble constant as  $H_0 = 57 \pm 6$  km s<sup>-1</sup> Mpc<sup>-1</sup>. The value agrees well with that obtained in Paper VI which follows, where many field spirals are used at large redshifts beyond any possible local velocity anisotropy.

The value of  $H_0$  obtained here via the Virgo cluster depends not on the data for the Virgo cluster alone, but on the total statistical sample of 84 brightest E galaxies that make up the Hubble-diagram formulation for clusters.

Subject headings: cosmology — galaxies — redshifts

#### I. INTRODUCTION

Methods of determining distances to late-type galaxies using sizes of H II regions and brightest resolved stars were calibrated in Papers I and II (Sandage and Tammann 1974*a*, *b*), and were used to find the distance to the M101 group in Paper III (Sandage and Tammann 1974*c*) of this series. The step from NGC 2403 to the M101 group is fundamental in two respects: (1) a distance beyond the realm of Cepheid variables is reached by using precision indicators; (2) a single point at luminosity class Sc I is established in the calibrations of H II regions and brightest stars.

In the absence of a well-determined distance to M101, we were forced in Papers I and II to *extrapolate* the calibrations to Sc I luminosities, and in Paper III to accept only a single calibrated galaxy at Sc I. However, the distance indicators  $\langle D \rangle_3$  for H II regions and  $\langle M(\text{first}) \rangle$  for brightest stars have nonnegligible dispersions, and it is important to test if the resulting single M101 calibration is *representative*.

We make this test in the present paper by determining H II distances to many late-type galaxies in the general field (including Sc II, Sc I–II, and Sc I systems) by using the adopted and extrapolated parts of the calibration of Paper I (eq. [7] and fig. 11). Total galaxian *absolute* magnitudes are determined by using these distances, and the results provide a calibration of  $M_B$ (galaxy) =  $f(L_c)$  for luminosity classes Sc I to Ir V.

If the extrapolation at the bright end of equation (7) (Paper I) had been incorrect, the *shape* of the  $f(L_c)$  function would then differ from that obtained from the relative function  $m(\text{galaxy}) = f(L_c)$  for Virgo cluster spirals, and from van den Bergh's (1960a) original relative calibration (based on the Hubble diagram for various luminosity classes). The comparison of the three functions constitutes the tests, and provides an absolute calibration for total magnitudes of galaxies. This is the principal result of the paper.

Ancillary results include H II distances to 39 field galaxies, and the distance modulus to the Virgo cluster, based on data for its spiral members. These distances are not without interest themselves, as many of the galaxies in the sample are bright, and have often been observed for rotational velocities, masses, absolute luminosities, etc., where knowledge of the distance is required.

Finally, precise distances to many field galaxies are needed as an auxiliary parameter in the mapping

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of the velocity field. This mapping is, of course, crucial to the central problem of this series because the nature of the velocity field is tied closely to the problem of the Hubble constant itself. Naturally, the presence of perturbations will affect its "local" value; but of more importance theoretically, the presence or absence of velocity anomalies, if they are caused by local inhomogeneities in the galaxian distribution such as the Ursa Major-Virgo complex, is a direct measure of the local ratio of the gravitational potential energy to the kinetic energy of the expansion, provided that all the mass is clumped like the visible galaxies. Hence, the strength of the perturbations says something of the need (or lack thereof) for missing mass to explain the energy ratio, and hence something about the deceleration parameter. An accurate determination of these perturbations can be expected to provide this crucial number, but again, accurate distances must first be known.

The extent to which the data in this paper can be used to test for velocity anomalies such as those often discussed by de Vaucouleurs (1958, 1966, 1972) will be treated in Paper V of this series. However, it is already known (Sandage, Tammann, and Hardy 1972) that data on *nearby* E groups do not show large measurable velocity perturbations.

We treat in this paper the distances to 39 field galaxies from H II regions in § II; the resulting absolute magnitudes in § III; the calibration of the luminosity classes and a test of the extrapolation of H II region sizes to Sc II and Sc I galaxies of Paper I in § IV and V; the errors in § VI; and finally the distance to the Virgo cluster (and hence a value of the Hubble constant by an earlier method [Sandage 1968]) in § VII.

## II. DISTANCES TO 39 FIELD GALAXIES FROM H II REGION SIZES

The angular sizes of the three largest H II regions were measured in 41 spiral field galaxies on 65 plates suitable for the detection of H II regions (29 103aE +H $\alpha$  filter [80 Å FWHM]; 15 103aE + RG filter; 21 image tube plates with H $\alpha$  filter). Most of the plates were obtained at the prime focus of the Hale 5-meter telescope, although a few were taken with the 2.5-m Hooker reflector.

The three largest H II regions, as defined by the precepts of Paper I, were identified and measured. Photographs of a few of the galaxies are given in figures 1–4 (plates 9–12), where H $\alpha$  and matching blue plates are paired, illustrating the ease with which the H II regions could generally be isolated and measured. The circumstances of the plates are given in the figure legends. The plate prefixes are PH for the 5-m Palomar Hale telescope; H for the 2.5-m Mount Wilson Hooker telescope; S for the Mount Wilson 1.5-m reflector; and PS for the 1.2-m Palomar Schmidt: The suffices for observer's initials represent B for Baade, H for Hubble, P for Plaut, and S for Sandage.

The major and minor axes of both the halo and the core were measured for the first three H II regions and

were reduced to a uniform system according to the methods of Paper I. The reduced measurements are listed in table 1, where the mean angular size of the halo, the core, and the mean for halo and core for the largest H II region  $(\langle \theta_H \rangle_1; \langle \theta_C \rangle_1; \langle \theta_H, \theta_C \rangle_1)$  are given in columns (5), (6), and (7), and the same for the three largest H II regions in columns (8), (9), and (10). Columns (11)–(13) list the kind and number of plates used for each galaxy; relatively poor plates are indicated by a colon, and marginal plates by parentheses. Also listed in columns (2) and (3) are the Hubble galaxy type and the luminosity class from van den Bergh (1960a), or in the case of IC 10 from van den Bergh (1965); luminosity classes by the present authors are shown in brackets.

The linear size in parsecs of  $\langle D_H, D_G \rangle_3$  is shown in column (4) as determined from the luminosity class using equation (7) of Paper I, which is our final calibration. The relation cannot be expected to hold for two Sb galaxies in the sample (NGC 1068 and NGC 2841), and it may be less reliable for the galaxies NGC 2903 and NGC 7640 which are classified as Sb<sup>+</sup> by van den Bergh (1960*a*), albeit Sc by Hubble. Retaining tentatively the latter two leaves 39 galaxies for a distance determination. (The anonymous galaxy A 0103 is de Vaucouleurs and de Vaucouleurs's [1964] notation for the Shapley-Ames [1932] galaxy New 1). Distances follow directly by combining the data in columns (4) and (10) as

$$r (\text{pc}) = \frac{206265 \langle D_H, D_C \rangle_3}{\langle \theta_H, \theta_C \rangle_3} \,. \tag{1}$$

#### III. ABSOLUTE MAGNITUDES

The calculated distances and the resulting absolute magnitudes are summarized in table 2. The true distance moduli shown in column (3) are corrected for a galactic absorption suggested by other data (Sandage 1973c [Paper V]), listed in column (5) to give the apparent blue distance modulus  $(m - M)_{AB}$  tabulated in column (6). (The second decimals shown in cols. [3] and [6] have, of course, no significance; they are used only to avoid an accumulation of rounding off errors.)

Holmberg's (1958)  $m_{pg}$  magnitudes are given in column (7). For those galaxies not studied by Holmberg we calculate an equivalent magnitude using  $m_{pg} = B(0) + 0.149(B - V) - 0.22$  (de Vaucouleurs and de Vaucouleurs 1964), and list the result in parentheses. The true absolute magnitudes,  $M_{pg}^0$ , given in column (8) follow from these  $m_{pg}$  values, combined with  $(m - M)_{AB}$  listed in column (6).

The most uncertain values in table 2 are for the highly obscured galaxies IC 10 and IC 342. The true distances are not particularly uncertain, but the absolute luminosities are almost unknown due to the highly speculative  $A_{pg}$  values. The absorption and apparent magnitude for IC 10 are taken from de Vaucouleurs and Ables (1965) after correcting some numerical errors. Their  $m_{total}(B)$  was transformed to the B(0) system and hence to Holmberg's  $m_{pg}$  system by relations in de Vaucouleurs and de Vaucouleurs

#### STEPS TOWARD THE HUBBLE CONSTANT

#### TABLE 1

#### ANGULAR SIZES OF THE FIRST LARGEST AND THE MEAN OF THE THREE LARGEST

H II REGIONS IN 41 PROGRAM FIELD GALAXIES REDUCED TO STANDARD CONDITIONS AS IN PAPER I

Galaxy (1)	Туре (2)	L <sub>c</sub> (3)	⟨D <sub>H</sub> , D <sub>C</sub> ⟩ <sub>3</sub> (4)	$\left< \theta_{\rm H} \right>_{\rm l}$ (5)	<θ <sub>C</sub> > <sub>1</sub> (6)	$\langle \theta_{\rm H}, \theta_{\rm C} \rangle_{\rm I}$ (7)	<θ <sub>H</sub> > <sub>3</sub> (8)	<θ <sub>C</sub> > <sub>3</sub> (9)	$\langle \theta_{\rm H}, \theta_{\rm C} \rangle_3$ (10)	103 a-E + Hα (11)	103 a-E + RG (12)	Image Tube (13)
NGC 247	s <sup>-</sup>	IV	171	25!'60	2!'80	14!'20	18!'55	1!'95	10!'25	1	- -	
428	Scp	III-IV	219	5.50	1.10	3.30	4.77	0.52	2.64	1		
628	Sc	I	461	10.83	0.71	5.77	7.98	1.74	4.86	1	1	
672 925	SBc S(B)c	III II-III	268	10.30	1.11	5.70	8.89	1.05	4.97	1	1	1
925	Sc*	III-III III-IV:	316	11.66	0.83	6.25	7.36	1.45	4.41		1	
1058	Sbp	111-1V:	219	3.50 5.38	1.81 2.10	2.65 3.74	3.32	1.28	2.30	1:	1	
1073	S(B)c	II	364	10.62	1.31	5.97	4.73 8.09	1.50 1.01	3.12 4.55	2:	1	(1)
1232	Sc	I	461	12.15	1.10	6.63	8.09 7.27	1.93	4.55	1	1	(1)
2500	s+	īv	171	10.25	2.25	6.25	7.92	1.28	4.60	1		
2841	Sb-	I		2.80	1.70	2.25	3.52	0.57	2.04	1		
2903	$Sb^+$	I-II	(461)	8.85	3.85	6.35	7.73	2.85	5.29	1		
3027	Sc(t)	[111]	2.68	8.23	2.07	5.15	5.57	1.54	3.55	1	1	
3184	Sc	II	364	12.20	2.75	7.48	8.20	1.93	5.07	1		
3486	$\mathbf{Sc}$	II	364	7.28	1.81	4.55	6.32	1.70	4.01	-		2
3631	Sc	I	461	6.95	2.75	4.85	5.83	2.40	4.12	1		-
3726	Sc(*)	I-II	412	5.80	2.75	4.28	5.23	2.58	3.91	1		
3810	Sc	I	461	10.88	2.35	6.62	8.59	2.31	5.45			2
3938	Sc	I	461	8.05	3.05	5.55	7.55	2.22	4.88	1		
4214 4321	Ir+ Sc	III-IV I	219	14.02	8.02	11.02	10.70	3.77	7.23		1	2
4321	sc st	IV-V	461 123	7.52 22.55	1.62	4.57	6.70	1.85	4.28		1	
4449	Ir	III	268	22.55	4.46 13.55	13.50	17.69	2.20	9.94	(1)		2
4559	Sc	п-ш	316	25.45 11.35	4.15	19.50 7.75	20.47 9.87	7.85	14.16	1		
4631	Sc*	III?	268	21.00	4.40	12.70	9.87	3.13 3.85	6.50	1		
4656		[IV]	171	11.65	3.85	7.75	11.00	2.58	10.59 6.79	(1)		
5068	S(B)c	III-IV	219	6.94	3.93	5.43	8.04	3.18	5.61	1		2
5194	Sc(t)	I	461	21,55	3.30	12.43	15.50	4.33	9.92	1		2
5236	Sc	I-II	412	18.25	5,50	11.88	13,85	5.32	9.58	1:		
5248	Sc	I	461	8.85	1.65	5.25	7.45	1.67	4.56	Ĩ:		
5486	Scp	[III-IV:]	219	4.87	1.70	3.28	4.13	1.54	2.83			2
6015	Sc	II	364	4.02	0.94	2.48	3.54	0.91	2.22			2 2
6412	Sc	II	364	9.15	2.80	5.98	8.38	2.45	5.42	1:		
6643	Sc	I-II	412	5,85	2.70	4.28	3.81	2.06	2.94		1:	
6946	Sc G(D)) +	I:	461	19.56	0	9.78	15.28	2.77	9.03	1:	2	
7640 7741	S(B)b <sup>+</sup> SBc	II: II	(364)	8.25	3.24	5.74	5.98	2.48	4.23		2	(1) 2
IC 10	S BC	(IV)	364	5.19 24.35	2.37	3.78	3.35	1.55	2.45		1	2
IC 10 IC 342	Sc	[I-II]	171 412	24.35	0 3.55	12.17	20.60	3.15	11.87	1	1	
IC 1727	Sc	[III-IV]	219	9.40	1.10	12.34 5.25	17.08 7.73	4.18 0.65	10.63	2		
Anon 0103	Sc		268	7.14	0.69	3.92	4.83	0.65	4.19 2.72	1		2
		J			••••	3.75	1.05	0.01	4.14			4

(1964). The absorption of IC 342 is almost totally unknown. Its value in parentheses is from the cosecant law alone, but the true value may be considerably larger, judged by the low apparent surface brightness on the *Sky Survey* plates. The system is important because it is an Sc I–II galaxy, and the absorption should eventually be determined by fundamental methods.

#### IV. ABSOLUTE MAGNITUDE CALIBRATION OF THE LUMINOSITY CLASSES

The galaxies in table 2 have been sorted according to luminosity classes in table 3 so as to obtain mean values for the absolute magnitudes. IC 10 and IC 342 are omitted for reasons previously mentioned; three galaxies are omitted (NGC 3027, NGC 5486, and Anon 0103) because of our uncertain luminosity classification. For internal consistency, we have also omitted NGC 2903 and NGC 7640 (Sb<sup>+</sup> by van den Bergh, Sc by Hubble and Sandage) but with no effect on our results.

The 32 remaining galaxies from table 2 are combined in table 3 with 11 late-type galaxies in the Local Group and the M81 group (Paper I, table 1) and the five members of the M101 group (Paper III; NGC 5477 is omitted because its  $m_{\rm pg}$  is not well determined) to give 48 galaxies available for the calibration.

However, calibration of the absolute magnitude is not entirely straightforward because of appreciable internal absorption in the brighter luminosity classes. A correction to face-on magnitudes is evidently necessary because it happens that the brightest luminosity classes in our sample are, on the average, seen more nearly face-on than are galaxies in classes II–III and III.

Holmberg (1958) has shown that spiral galaxies

561

#### ALLAN SANDAGE AND G. A. TAMMANN

TABLE 2

DISTANCES AND ABSORPTION-FREE ABSOLUTE MAGNITUDES

FOR THE PROGRAM GALAXIES IN TABLE 1

Gal	laxy	r(Mpc)	(m-M) <sup>0</sup>	bII	A pg	(m-M) <sub>AB</sub>	mpg	M <sup>o</sup> pg
NGC	247	3.4	27.68	-83.5	0.00	27.68	9.47	-18.21
	428	17.1	31.17	-61.4	0.00	31.17	11.74	-19.43
	628	19.6	31.46	-45.7	0.03	31.49	9.74	-21.72
	672	11.1	30.23	-33.8	0.10	30.33	11.31	-19.02
	925	14.8	30.85	-23.3	0.20	31.05	10.53	-20, 52
	1058	19.6:	31.47:	-20.4	0.24	31.71:	11.74	-19.97:
	1073	16.5	31.09	-51.9	0.00	31.09	11.43	-19.66
	1232	20.7	31,58	-57.8	0.00	31.58	10.46	-21,12
	2500	7.7	29.42	31.6	0.12	29.54	12.13	-17.41
	2903	(18.0)	(31.27)	44.5	0.04	(31, 31)	9.48	(-21.83)
	3027	15.6	30.96	39.1	0.07	31.03	12.31	-18.72
	3184	14.8	30.85	55.6	0.00	30.85	10.28	-20.57
	3486	18.7	31,36	65.5	0.00	31.36	11.00	-20.36
	3631	23.1	31.82	59.0	0.00	31.82	10.91	-20.91
	3726	21.7	31.69	64.9	0.00	31.69	10.84	-20.85
	3810	17.5	31.21	67.2	0.00	31.21	11.30	-19.91
	3938	19.5	31.45	69.3	0.00	31.45	10.79	-20,66
	4214	6.3	28.98	78.1	0.00	28.98	10.12	-18,86
	4321	22.2	31.73	76.9	0.00	31.73	10.07	-21.66
	4395	2.6	27.03	81.6	0.00	27.03	10.66	-16.37
	4449	3.9	27.96	72.4	0.00	27.96	9.90	-18.06
	4559	10.0	30,01	86.5	0.00	30.01	10.26	-19.75
	4631	5.2:	28.59:	84.2	0.00	28.59:	9.71	-18.88:
	4656/7	5.2	28.58	84.7	0.00	28.58	10.74	-17.84
	5068	8.1	29.53	41.4	0.06	29.59	(10.92)	-18.67
	5194	9.6	29,91	68.6	0.00	29.91	8.88	-21.03
	5236	8.9	29.74	32.0	0,12	29.86	(8.11)	-21,75
	5248	20.9	31.60	68.7	0.00	31,60	10.36	-21,24
	5486	16.0:	31.01:	58.8	0.00	31.01:		
	6015	33.8	32.65	44.1	0.04	32.69	11.69	-21.00
	6412	13.9	30.71	31.3	0.12	30.83	12.19	-18,64
	6643	28.9	32, 30	28.2	0.15	32.45	11.61	-20.84
	6946	10.5:	30.11:	11.7	0.52	31.63:	9.67	-20.96
	7640	(17.8)	(31.25)	-18.9	0.27	(31.52)	11.31	(-20, 21)
	7741	30.7	32.43	-34.4	0.10	32.53	11.63	-20.90
IC	10	3.0	27.36	- 3.3	3.1	30.5	(12.0)	(-18.5)
IC	342	8.0:	29.51:	10.6	(0.59)	30.10:		
IC	1727	10.8	30.16	-33.9	0.10	30.26	12.10	-18,16
	0103	20.3	31.54	-68.8	0.00	31.54	(12, 2)	-19.3

suffer intrinsic reddening that depends on the inclination; the path length through the disk increases as the ratio of major to minor axis becomes larger. He finds the intrinsic absorption correction for Sc galaxies to face-on orientation to be

$$A_{\rm ng}{}^{i} = 0.28(\csc i - 1), \qquad (2)$$

where the inclination angle i between the equatorial plane and the line of sight should be calculated from Hubble's (1926) equation

$$\cos^2 i = (1 - q^2)/(1 - q_0^2)$$
. (2a)

Here q is the ratio of the apparent major and minor axes of the projected image, and  $q_0$  is the intrinsic flattening of the galaxy.

Instead, we have assumed here that

$$\csc i = a/b \tag{2b}$$

as for an infinitely thin plate. The individual axis ratios a/b were taken from de Vaucouleurs and de

Vaucouleurs (1964). The method simplifies the procedure, and also has two other advantages: (1) It avoids the problem of Holmberg's (1958) major axis *a* not being corrected for projection effects. As pointed out by Öpik (1923*a*, *b*) and Holmberg (1946), the major axis appears to be too large in inclined spirals. The axis ratios in the RCBG contain a correction for the effect. (2) It is not necessary to make a decision on the true value of  $q_0$ . Holmberg (1958) had adopted an average value  $q_0 = 0.2$  for all spirals, while Heidmann, Heidmann, and de Vaucouleurs (1971*a*) suggest that a considerable change occurs in  $q_0$  for different spiral subtypes.

The justifications for our simplified method are: (1) The differences between equations (2a) and (2b) are very small for galaxies that are nearly face-on ( $i \simeq 90^{\circ}$ ) —the majority of our sample for which luminosity classification is possible. (2) Our adopted absorption corrections  $A_{pg}^{i}$  agree well with those from Holmberg's (1958) procedure in cases where  $A_{pg}^{i} \le 0.15$  mag. For  $A_{pg}^{i} > 0.15$  mag, the present corrections are smaller by ~0.1 mag; consequently they agree more closely

TABLE 3

FACE-ON ABSOLUTE MAGNITUDES OF THE PROGRAM GALAXIES

ARRANGED BY LUMINOSITY CLASSES

				Weight	22 23 53 53 53 53 53 53 53 53 53 53 53 53 53		55 3 86 3 16:	8-
1		3		M pg	-15.82 -16.37 -16.69 -16.53 -16.35	0.	-14.55 -14.36 -14.46:	
Weight	3 1 2 2 2 2 2 4	8-1253	± 0.29	bg bg	8.13 8.13 10.66 10.87 11.03 11.22 12.95 12.95 mean =	b	9.88 13.20 mean =	
M <sup>o, i</sup> pg	-19.17 -19.01 -18.06 -19.49 -19.07 0.67	-17.86 -18.86 -18.70 -20.27 -19.50		Galaxy	NGC 6822 4395 IC 2574 Ho II NGC 2366 Ho IV		IC 1613 Ho I	
0.28 × cosec i (cosec i-1)	0.17 1.03* 0.00 0.47 mean = σ =	0.00 0.00 0.03 0.02 0.02	0.31 mean = σ =	Type	S IV-V and Ir IV-V		Ir V	
cosec i	1. 62 5. 50 Ir 2. 69	Ir Ir 1.10 1.07 1.26	2.09	Weight	~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~	-+1		8)
M <sup>o</sup> pg	-19.00 -18.88 -18.06 -19.02	-17.86 -18.86 -18.67 -19.97 -19.43	-18.16	M <sup>o</sup> pg	-18.21 -17.53 -17.84 -17.84 -18.08 -18.05 -17.68		-16.42	nberg 195
pg Pg	8.56 9.71 9.90 11.21	0.73 10.12 10.86 11.50		pg pg	9.47 9.47 10.03 11.22 11.25 11.62 11.62	mean = 0 = 1	2.85	lue ( Holn
Galaxy	NGC 2403 4631 4449 672	LMC NGC 4214 5068 1058 428	IC 1727	Galaxy	NGC 247 4236 4656/7 5474 5585 5585 5204	0 0 1	SMC	* Maximun possible value ( Holmberg 1958 )
Type	Sc III and Ir III	Sc III-IV LMC and NGC Ir III-IV		Type	S IV and Ir IV		Ir IV to IV-V	* Maximu
Weight	2012210		1 2 1 0.21	~ ~	2 1 1 0.27 0.27	~ ~ ~	2 0.38	0
M <sup>o, i</sup> pg	-21.11 -21.18 -20.99 -21.74 -21.68 -21.35	$\begin{array}{c} -20.69\\ -20.96\\ -20.01\\ -21.18\\ 0.40\\ \end{array}$	$\begin{array}{c} -21.78\\ -20.96\\ -21.11\\ -21.20\pm 0\\ 0.42 \end{array}$	-20.58	-20.44 -19.67 -21.01 -21.38 -21.38 -18.68 -18.68 -0.81	-18.66 -20 73	$\frac{-20.23}{-19.70} \pm 0$	
0.28 × (cosec i-1)	0.01 0.15 0.03 0.02 0.11 0.04	0.03 0.05 0.10 mean = <b>0</b> =	0.03 0.11 0.27 mean = <b>0</b> =	0.01	0.08 0.01 0.11 0.38 0.05 mean = 0.05	0.17	11 11	
cosec i	1.05 1.55 1.12 1.07 1.41 1.15	1.12 1.18 1.35	1.12 1.41 1.95	1.02	$1.29 \\ 1.05 \\ 1.38 \\ 2.35 \\ 1.18 \\ 1.18 $	1.62 1.74	2.40	
M <sup>o</sup> pg	-21.10 -21.03 -20.96 -21.72 -21.66 -21.24 -21.12	-20. 66 -20. 91 -19. 91	-21.75 -20.85 -20.84	-20.57	-20.36 -19.66 -20.90 -21.00 -18.64	-18.49	-19.75	
no Pg	8.20 8.88 9.15 9.71 9.71 10.07 10.36	10.79 10.91 11.30	7.99 10.84 11.46	10.28	11.00 111.43 111.53 111.65 112.07	6.07 10.33	10.26	
Galaxy	NGC 5457 5194 6946 628 4321 5243 5243 1232	3938 3631 3810	NGC 5236 3726 6643	NGC 3184	3486 1073 7741 6015 6412	M 33 MGC 925	4559	
Type	Sc I		Sc I-II	Sc II		Sc II-III		1

with the corrections indicated by Heidmann, Heidmann, and de Vaucouleurs (1971b). Applying their unusual equation  $A_{pg}{}^i = 0.72 \log \csc i$  (where *i* is determined as set out in their Paper I [Heidmann *et al.* 1971*a*]) to all galaxies in table 3, leads to face-on corrections that are again practically identical with our present values for  $A_{pg}{}^i \le 0.15$  mag.

rections that are again practically identical with our present values for  $A_{pg}^i \le 0.15$  mag. Hence, equation (2), using (2b), is a reasonable compromise between Holmberg (1958) and Heidmann *et al.* (1971*b*). In table 3, the values of csc i = a/b for each galaxy are listed in column (5); the resulting corrections  $A_{pg}^i$  and the corrected magnitudes  $M_{pg}^{0,i}$ are given in columns (6) and (7).

However, these corrections for internal absorption for inclined spirals are still not quite satisfactory. Holmberg's (1958) corrections obey an intuitively satisfactory cosecant law, but they are based on uncorrected axis ratios, and lead to surprisingly high  $A_{pg}/E_{pg-v}$  values. On the other hand, the corrections by Heidmann *et al.* (1971*b*) are consistent with  $A_{pg}/E_{pg-v} \simeq 4.0$ , and they are based on more nearly correct axis ratios, but they are unusual (1) because of their form ( $A^i \propto \log \csc i$ ), and (2) because  $A^i$  is independent of the type of spiral in *their* analysis.

In view of these new unexpected points, we attempted to derive the inclination corrections anew from Virgo cluster spirals alone. For Sc galaxies, the suggested corrections are somewhat larger than those of Holmberg (1958). However, we attach little weight to the result because even a *slight* dependence of the luminosity classification on the inclination would disqualify the method.

However, because the galaxies in table 3 are predominantly close to face-on, the average difference of various possibilities for the inclination correction are smaller than 0.1 mag, and are therefore negligible for our present purposes. For the aforementioned reasons we believe that the problem of the internal absorption for inclined spirals of types Sb and Sa appears yet to be solved (see also Tully 1972).

The near lack of internal absorption for most Sm–Ir galaxies, the high transparency of the Magellanic Clouds themselves, and the uncertainty of the inclination determinations for Ir's (but see Hodge and Hitchcock 1966) suggest that no inclination corrections should be applied for Ir galaxies. However, our lack of correction for this type alone imposes an artificial step in the  $M_{pg}^{0,i}$  values in the present sample, which was solved by leaving all galaxies (Sc and Ir) of class IV or fainter uncorrected. The three Ir galaxies brighter than class IV in our sample were also assumed to have zero tilt corrections.

The values of  $M_{pg}^{0,i}$  were assigned weights (in the final column of table 3) ranging from  $\frac{1}{2}$  to 3, according to the reliability of the distance determination and of the luminosity classification. The weighted mean luminosity for each class is shown in table 3, together with the standard error of the mean, and the standard error of a single determination. Although the errors are poorly determined due to the small number of galaxies in each group, we note that if the luminosity spread for each class is, in fact, this small, then distances to

*individual* galaxies can be determined with errors of  $\sigma$   $(\Delta D/D) = 0.46\Delta m \simeq 0.2$  for Sc I and Sc I-II systems, and to about twice this value for all later classes, which appears to be a most satisfactory result.

#### V. INDEPENDENT TESTS OF THE CALIBRATION

The mean values of  $\langle M_{pg}^{0,t} \rangle$  from table 3 are plotted against luminosity class in figure 5 as closed circles with error bars. The progression with  $L_c$  is unmistakable, and, if judged by the errors, well determined.

So that there can be no misunderstanding, we wish again to emphasize that the points for  $L_c < II-III$  depend on the extrapolation of equation (7) (fig. 11) of Paper I, and that if the extrapolation had been made differently, the closed circles in figure 5 here would have differed correspondingly.

The importance of testing the calibration is seen from the fact that  $\langle M_{pg} \rangle$  for Sc I and Sc I–II is the basis for our calculation of the Hubble constant in Paper VI. A difference of  $\Delta M_{pg}^{0,i}$  of 0.5 mag causes a difference in H by the nontrivial factor of 1.26. Or stated differently, to obtain the Hubble constant systematically to within 10 percent requires that we know  $\langle M_{pg} \rangle$  to within 0.2 mag. We have two tests of the present calibration.

A. Many late-type spirals of all luminosity classes exist in the Virgo cluster. A first listing was made many years ago by Hubble (1936, table 3), and has often been rediscussed (Humason, Mayall, and Sandage 1956, table 2 [called hereafter HMS]; de Vaucouleurs 1961; van den Bergh 1960a). A correlation of the apparent magnitudes with the luminosity classes for these spirals tests the *shape* of the calibration over the range from Sc I to Sc or Ir III.

The available data for Virgo cluster spirals are listed in table 4. Columns (2) and (3) give the type and class from van den Bergh (1960*a*); column (4) is the magnitude from Holmberg (1958) or from de Vaucouleurs and de Vaucouleurs's (1964) B(0) transformed to Holmberg's system. Column (5) and (6) give the tilt to the line of sight (csc *i*) as previously defined, and column (7) is the tilt-corrected magnitude  $m_{pg}^{i}$ . The weights in column (8) indicate the reliability of the luminosity classification.

The mean apparent magnitude, and the standard error of the mean, are given in table 4 for each interval of the luminosity classification.<sup>1</sup> To compare with the absolute calibration of table 3, we fitted the table 4 values in figure 5 to the absolute values for classes II, II–III, and III, giving half weight to II because this class was already an extrapolation of equation (7) (Paper I). The fit, shown by open squares, indicates a slightly steeper brightening in  $M_{pg}$  for the brightest (I–II, I) luminosity class than equation (7) (which is essentially the interpolation line in fig. 1).

<sup>1</sup> Note that the standard error of a single determination in column (9) of table 4 for Sc I, Sc II–III, and Sc III is about half of the corresponding values in table 3. Clearly the values in table 4 represent more nearly the true cosmic scatter of the luminosity functions themselves, while those in table 3 are a combination of this true scatter with our error in determining the distances by the H II region method.

No. 3, 1974

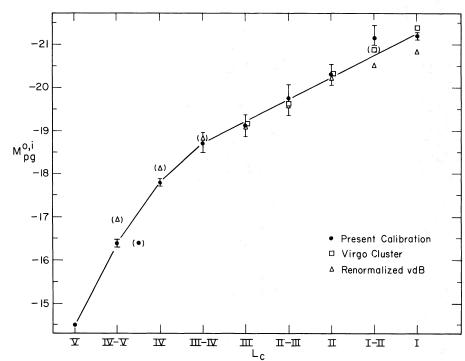


FIG. 5.—Calibration of the absolute magnitude for various luminosity classes. Dots with error bars, the present results from table 3; open squares, the relative calibration using Virgo cluster spirals (table 4) normalized to  $(m - M)_0 = 31.48$ ; triangles, the van den Bergh relative calibration normalized to the dots at the mean of classes II, II–III, III, and III–IV; the line is eq. (3) of the text for  $L_c \ge 3.5$ , and made to pass through the dots for fainter classes.

The effect is small  $(\Delta M_{pg} \simeq 0.20 \text{ mag})$ . Fitted at classes III, II–III, and II, the Virgo cluster Sc I cluster members have  $\langle M_{pg}^{0,i} \rangle = -21.38$  instead of -21.18 for our field Sc I galaxies (table 3).

B. If one assumes that there is no local velocity anisotropy, van den Bergh's (1960a) relative distances derived on the basis of the ratio of the redshifts also provides the *shape* of the  $M_{pg} = f(L_c)$  relation. We have fitted his results into figure 5 by normalizing his calibration to the mean of the closed dots (table 3) in the interval II to III-IV, giving half-weight to class II. We also give half-weight to class III-IV, which is uncertain in van den Bergh's own calibration. The necessary mean shift from van den Bergh's tabulated luminosities is  $0.82 \pm 0.16$  mag brighter. The resulting comparison is shown as open triangles in figure 5.

Van den Bergh's renormalized calibration is now fainter than ours for classes I-II and II; this is the opposite sense from the Virgo cluster comparison just discussed. The open triangle for Sc I in figure 5 is at  $M_{pg}^{0,t} = -20.82$  compared with our calibration of -21.18. However, van den Bergh did not consider inclination effects in his analysis. We give his value half-weight for this reason and for the reason that the method suffers in principle from any local non-uniformities in the velocity field. Averaging the Virgo cluster results with van den Bergh's renormalized calibration at half-weight gives results that agree with our calibration in table 3 (-21.18 for Sc I) to within 0.01 mag.

But the calibration clearly still requires some smoothing, as indicated by the very bright magnitude for class I–II. Our value depends on only three galaxies; the mean cannot be reliable due to  $\sigma(\Delta M) \simeq 0.5$  mag for a single observation. Our final adopted calibration of  $\langle M_{pg}^{0,i} \rangle$  is listed in table 5.

The size of the dispersions for the classes is generally as listed in table 3 for any practical applications, or in table 4 in nature. The results are well represented from class I to III-IV by the linear relation

$$M_{\rm pg}^{0,i} = 1.02L_c - 22.27$$
  $(L_c \ge 3.5),$  (3)

as shown by the straight line in figure 5.

 TABLE 5

 Adopted  $M_{pg}$  Calibration of the Luminosity Classes

Class	$M_{ m pg}$	Remarks
<u> </u>	-21.25	*
I–II	-20.74	*
ÎI	-20.23	*
II–III	-19.72	*
ÎII	- 19.21	*
III-IV	-18.70	*
ĪV	-17.79	+
IV-V	-16.35	+
<b>V</b>	-14.46:	ŕ

\* After inclination correction to faceon configuration.

† No inclination correction.

#### ALLAN SANDAGE AND G. A. TAMMANN

TABLE 4

#### RELATIVE CALIBRATION OF THE LUMINOSITY CLASSES

FROM SC GALAXIES ASSUMED TO BE IN THE VIRGO CLUSTER

Gal	axy	Туре	L <sub>c</sub>	m <sub>pg</sub>	cosec i	0.28 × (cosec i-1)	$m_{pg}^{i}$	Weight	σ
NGC	4254	Sc	I	10.37	1.10	0.03	10.34	2	
	4303	Sc	I	10.01	1.10	0.03	9.98	2	
	4321	Sc	I	10.07	1.07	0.02	10.05	2	
	4535	S(B)c	I:	10.38	1.45	0.13	10.25	1	
						mean:		± 0.06	0.1
	4666	Sc	I - II:	11.40	3.72	0.76	10.64	1	
						mean:	(10.64)	_	
	4178	SBc	II:	11.75	2.69	0.47	11.28	1	
	4189	Sc	II:	12.51	1,15	0.04	12.47	1	
	4536	Sc(t)	11:	10.94	2.34	0.38	10.56	ī	
	4602	Scn*	11:	12.13	2.82	0.51	11.62	ī	
	4651	Scp		11.21	1.48	0.13	11.08	1	
	4654	Sc(*)	II	11.03	1.74	0.21	10.82	2	
	001	50( )			1.1.	mean:		± 0.24	0.6
	4152	Sc(n?)	II - III:	13.25	1.29	0.08	13.17	1	0.0
	4487	Sc	II - III	(11.82)	1.41	0.11	11.71	2	
	4504	Scn	II - III:	(12.01)	1.55	0.15	11.86	ĩ	
	4632	Sc*	II - III	(12.38)	2,57	0.44	11.94	2	
	4731	SBcp	II - III:	(11.63)	1.91	0.25	11.38	ī	
	4781	Sc*	II - III	(12.10)	2,14	0.32	11.78	2	
				(		mean:		$\pm 0.17$	0.5
	4116	SBc	III:	12.29	1.70	0.20	12.09	1	0.0
	4212	Sc	III	12.08	1.62	0.17	11.91	2	
	4273	Sc*t	III:	12.51	1.48	0.13	12.38	1	
	4420	Sc*	ші:	(12.94)	2.09	0.31	12.63	î	
	4428	Sc*	III:	(13, 31)	2.19	0.33	12.98	ī	
	4496	S(B)c	III:	(11.99)	1.29	0.08	11.91	ī	
	4519	Sc	ші	12.47	1.35	0.10	12.37	2	
	4595	Sc*	III:	(13.24)	1.41	0.11	13.13	ī	
	4597	SBc*	III:	(12.68)	2.09	0.31	12.37	1	
	4658	Sc(*)	III	(12.86)	2.09	0.31	12.55	2	
	4713	Sc*	III	12.31	1,51	0.14	12.17	2	
	4775	Sc(*)	III	(12.23)	1.05	0.01	12.22	2	
	4790	Sc*	III:	(12.94)	1.48	0.13	12.81	1	
	4808	Sc*	III	(12.64)	2,24	0.35	12.29	2	
	4900	Sc* or Ir	ш	12.17	1.10	0.03	12.14	2	
	1,00	50 01 II	***			mean:		± 0.07	0.3

#### VI. TRUE ERRORS

A real danger for the present calibration is that the measurement of the sizes of H II regions is distance dependent. The apparent magnitude of the galaxies within a given luminosity class is a good relative distance indicator to within the dispersion  $\sigma (M_{pg}^{0,t})$ , and should then correlate with  $M_{pg}^{0,t}$  if a systematic error with distance were present. Inspection of table 3 shows that this is not the case. The only strongly deviating very distant galaxy is NGC 3810 of luminosity class I. However, we believe that the actual luminosity class of this galaxy is I–II, or even II, and that it should properly be placed in a later section of table 3. In class II, NGC 6412 is the faintest galaxy, but the next faintest (distant) galaxy is NGC 6015, which is the intrinsically brightest in its class.

A test of the *random* accuracy of the H II region distances is afforded by six pairs, groups, or clusters for which more than one distance could be determined in table 3. These data are listed in table 6, where the assignment of specified galaxies to groups follows

HMS, de Vaucouleurs (1974), and Paper V which follows.

The agreement of the distances within a given aggregate is generally very satisfactory. The average deviation of a single galaxy from the group mean is  $\langle \Delta D/D \rangle = 0.16$ , although the number of cases is not sufficient to make a rigorous error analysis. However, the agreement for the few cases here available is gratifying because it suggests that the expectations set out in Paper I for the method of H II region distances are fulfilled. The largest errors in the table occur for the CVn I cloud, whose reality is the least certain (cf. Paper V).

#### VII. DISTANCE TO THE VIRGO CLUSTER AND A FIRST INDICATION OF THE VALUE OF THE HUBBLE CONSTANT

The two tests of the absolute calibration in  $\S$  V give hints toward a value for the Hubble constant.

1. The normalization of van den Bergh's data to table 3 required that his tabulated values (van den

GA	LAXIES IN THE SAM	ie Gro	OUP OR CLUSTE	R	
Galaxy	Group	1	r (Mpc)	$\langle r \rangle$	$\Delta r/\langle r \rangle$
NGC 672 IC 1727 }······	Pair		${11.1 \\ 10.8}$	10.95	${0.01 \\ 0.01}$
NGC 925 NGC 1058	N1023 group		${14.8 \\ 19.6}$	17.2	${0.14 \\ 0.14}$
NGC 4214 NGC 4395 NGC 4449	CVn I cloud		$ \left\{\begin{array}{c} 6.3\\ 2.6\\ 3.9 \end{array}\right\} $	4.27	$\begin{cases} 0.48 \\ 0.39 \\ 0.09 \end{cases}$
NGC 4631 NGC 4656/7	CVn II cloud		$\begin{cases} 5.2 \\ 5.2 \\ 5.2 \end{cases}$	5.2	{0.00 {0.00
NGC 5068 NGC 5236	N5128 group		$\left\{\begin{array}{c} 8.1\\ 8.9 \end{array}\right\}$	8.5	${0.05 \\ 0.05}$
NGC 6412 NGC 6643	N6643 group		${13.9 \\ 28.9}$	21.4	${0.35 \\ 0.35}$
NGC 4321 Virgo cluster	Virgo cluster		${22.2 \\ 19.5}$	19.5	{0.12 

Individual Distances and Deviations from the Mean for Galaxies in the Same Group or Cluster

Bergh 1960*a*, table 1) be made brighter by  $-0.79 \pm 0.03$  mag. However, van den Bergh's sample is apparent-magnitude limited because he was constrained to work within the galaxies in HMS, which were essentially so chosen. To convert to a distance-limited sample, his calibration must be corrected for at least part of the Eddington (1914)-Malmquist (1920) correction. Van den Bergh (1960*b*) proposed that his calibration be made fainter by 0.17 mag for distance-limited samples.

=

Our sample is distance limited (the galaxies must be near enough to have measurable angular sizes for their H II regions), and the faint correction to van den Bergh clearly applies for the comparison. Hence, the normalization shift from van den Bergh's 1960 calibration to ours is  $-0.79 - 0.17 = -0.96 \pm 0.1$  mag. Because his calibration rests on the *assumption* that  $H_0 = 100$  km s<sup>-1</sup> Mpc<sup>-1</sup>, we would require by this route that

or

$$-0.96 = 5 \log (H/100)$$
$$H_0 = 64.2 \pm 3 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

2. The distance to the Virgo cluster on the H II scale of Paper I follows from the normalization constant required to fit the data in table 4 with those of table 3 (or fig. 5), or after smoothing, with the adopted calibration of table 5. The two fits are equivalent at the 0.05 mag level and give a mean of

$$(m - M)_0 = 31.45 \pm 0.09$$

for a distance of 19.5  $\pm$  0.8 Mpc. This should be com-

pared with  $(m - M)_0 = 26.8$  of Hubble (1936) and an early lower limit value of 31.1 by Sandage (1968).

The value for the systemic velocity of the Virgo cluster is now known with better accuracy than in recent years. A new discussion (Sandage and Tammann 1975) of additional redshift data shows that earlier suggestions by de Vaucouleurs (1961) and by de Vaucouleurs and de Vaucouleurs (1974) of systematically different mean velocities of the spiral and elliptical members cannot be maintained. But even before the new radial velocities became available, the older evidence itself did not, in fact, support this view (Tammann 1973). The new redshift determinations confirm that the ellipticals and the spirals have the same redshift to within the statistics (Sandage and Tammann 1975). Based on a preliminary discussion, the mean velocity from 95 galaxies within a radius of  $6^{\circ}$  from the center of the E cloud of the cluster is  $\langle v_0 \rangle = 1111 \pm 75 \text{ (rms) km s}^{-1}.$ 

However, the problem of using the Virgo cluster for a determination of  $H_0$  has always involved the question of a possible peculiar velocity component due to local virial motions, or other causes. We believe the evidence is now strong that there is no such component to within the accuracy of two tests.

1. The Virgo cluster data fit the Hubble diagram for bright E cluster galaxies to better than  $\sigma/5$ . It is shown elsewhere (Sandage 1973b, fig. 2) that  $V_c =$  $5 \log cz - 6.76$  for every brightest cluster E galaxy in 84 clusters to within  $\sigma = 0.30$  mag. The brightest E galaxy in the Virgo cluster is NGC 4472 with observed  $V_c = 8.45$  (Sandage 1973b, table 2) which agrees with the calculated value of  $V_c = 8.47$  using  $\langle v_0 \rangle = 1111$ km s<sup>-1</sup>. This leaves no margin for a peculiar velocity, to within the statistics of  $\sigma = 0.3$  mag ( $\equiv 0.3v_0/2.17 =$ 134 km s<sup>-1</sup> at the Virgo cluster). We discount the ad hoc suggestion that a large peculiar velocity just compensates a large peculiar absolute magnitude for NGC 4472, which would leave fortuitous agreement of the Virgo cluster with the general Hubble diagram for E galaxy clusters. Rather, we believe the agreement is real, and that no peculiar velocity exists.

2. The Virgo and Coma clusters have the same colormagnitude diagram (Sandage 1973a) if the data are shifted in magnitude by  $5 \log z_1/z_2$ , where  $z_1$  and  $z_2$ are the observed redshifts.

In view of these facts, we see no reason to maintain that the Virgo cluster velocity is suspect at a level that is larger than the observational statistics of these two tests (Abell 1972; de Vaucouleurs 1972; Gudehus 1973).

The formal value of  $H_0$  from the Virgo cluster alone is, then,

$$H_0 = 57.0 \pm 6 \,\mathrm{km \, s^{-1} \, Mpc^{-1}}$$
,

where the errors are rms values and represent the combined effect of the formal rms uncertainty in the distance ( $\pm 0.8$  Mpc) and the rms uncertainty in the mean velocity ( $\pm$ 75 km s<sup>-1</sup>). The true uncertainty in this value of  $H_0$  is, of course, much greater because of possible systematic errors in our adopted distance scale (eq. [7], Paper I; eq. [3], fig. 5, and table 5 here). We note only in passing that this value of  $H_0$  is almost identical with that we shall obtain in Paper VI, where the method is free from problems of local velocity

<sup>2</sup> Neither of these tests support Abell's (1972, fig. 1) conclusion that an anomaly of  $\Delta cz \simeq 400$  km s<sup>-1</sup> exists in the Virgo cluster redshift, an anomaly which he requires from his assumption that the magnitude of a break in certain cluster luminosity functions has a stable mean absolute magnitude. The comparison of the different views is most clearly seen by comparing two Hubble diagrams (Abell 1972, fig. 1; Sandage 1973b, fig. 2).

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anisotropies. Again, we take this agreement to mean that no such effect occurs for the Virgo cluster itself.

We wish to note finally that the value of  $H_0$  via this route through the Virgo cluster is *not* a determination by the use of one cluster alone, because the distance to Virgo provides the calibration straightaway of the absolute magnitude of the ridge line in the Hubble diagram for 84 clusters (Sandage 1973b, fig. 2 and eq. [4]). This is done by noting that the point  $V_c = 8.45$ ,  $\log cz = 3.046$  for NGC 4472 lies 0.02 mag from this line. In the absence of a velocity perturbation (justified above and in Paper V), this shows that the point  $V_c =$ 8.47 at log cz = 3.046 on the ridge line corresponds to  $\langle M_{\rm vc} \rangle = -23.02$ , found by using our distance to the Virgo cluster. Note that the total cluster sample is involved in the placing of the ridge line which, with 84 clusters, can be done with an accuracy of  $\sigma \simeq 0.3\sqrt{83}$ = 0.03 mag. Hence, we believe that the value of  $H_0$ obtained in this way has some weight.

As always, it is a pleasure to thank the night assistants over the past 20 years at the Palomar Hale telescope and the Mount Wilson Hooker reflector for their help in obtaining the plate material upon which table 1 is based. John Bedke and Felice Woodworth prepared the illustrations for press in a most excellent way. Finally, it is a pleasure to again thank the U.S. National Science Foundation for grant GP-14801 for two years during the analysis, which made the papers in this series possible. One of us (G. A. T.) thanks the Swiss National Science Foundation for subsequent support.

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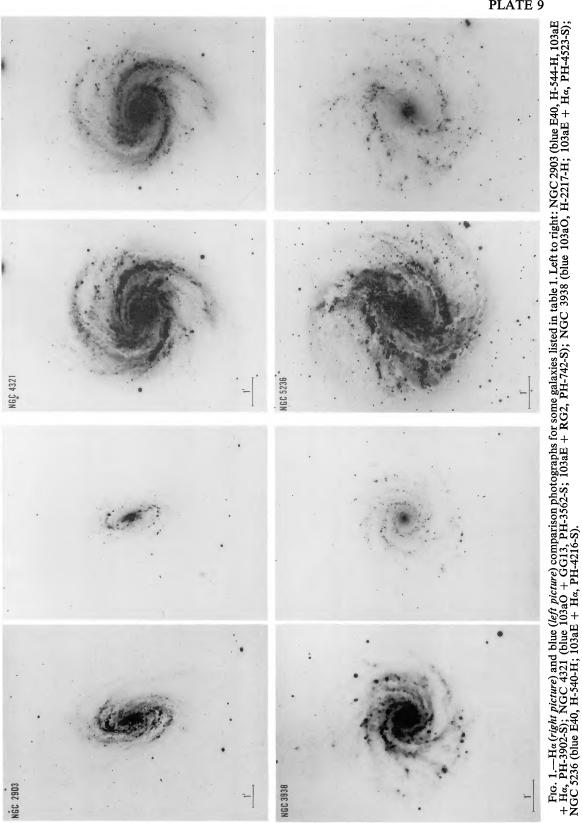
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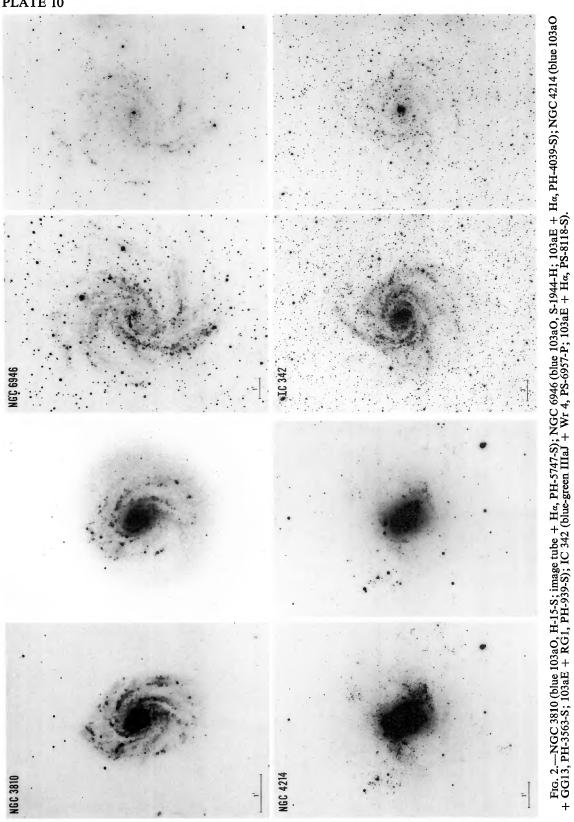
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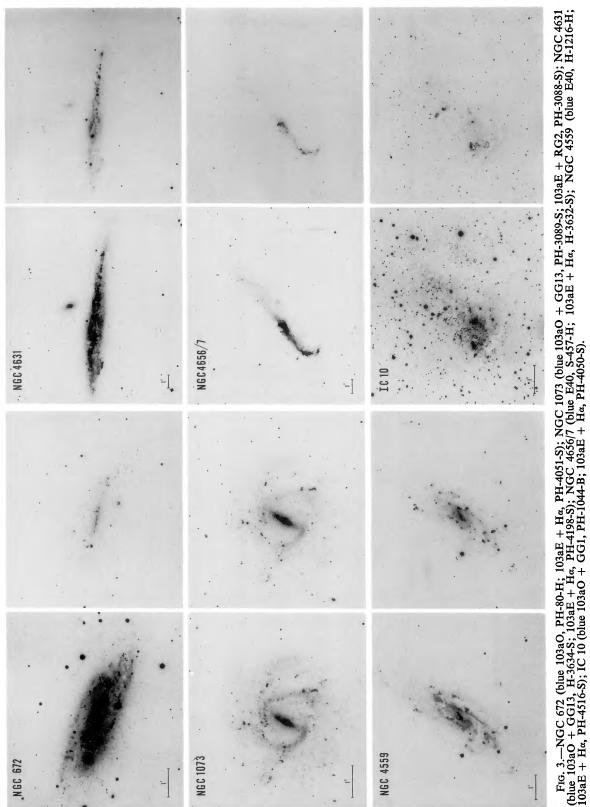
ALLAN SANDAGE: Hale Observatories, 813 Santa Barbara Street, Pasadena, CA 91101 G. A. TAMMANN: Hamburger Sternwart, D-2050 Hamburg 80, Germany



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### PLATE 12

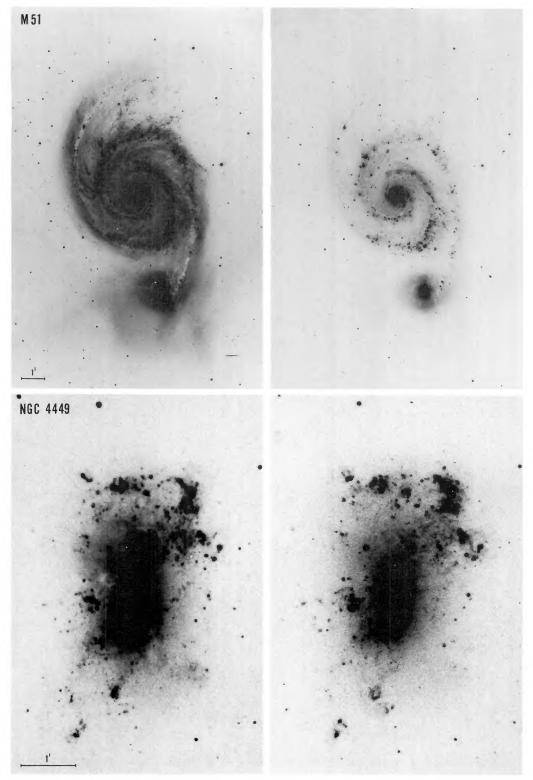


Fig. 4.—M51 (blue 103aO, PH-201-MH; 103aE + H $\alpha$ , PH-3922-S); NGC 4449 (blue E40, H-549-H; 103aE + H $\alpha$ , PH-4524-S). SANDAGE AND TAMMANN (see page 560)