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ELEMENT IDENTIFICATIONS IN FIVE Ap STARS

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ABSTRACT

We have examined 2 Å mm^{-1} spectra of β CrB, 73 Dra, 78 Vir, HR 4816, and HR 4072 for the presence of some 74 elements. The technique combines traditional line-identification methods with a Monte Carlo analysis of wavelength coincidences. The identifications are divided into three categories: definite, good, and possible. A majority of the discussion is devoted to the latter category. In many instances, the lines in question arise from more than 10 eV so that a positive identification alone would imply a large abundance excess. Elements in this category which may be present in one or more of the five stars are Al, S, Kr, and a few others. A high significance level was found for Pt in HR 4072. The *statistical* results for Pt and U were negative or weak in all other cases but the tracings provide some evidence for the presence of these elements. No evidence was found for Pm, or any of the transuranic elements.

Subject headings: abundances, stellar — line identifications — peculiar A stars

I. INTRODUCTION

All theories of the origin of the chemical abundances in stars are severely handicapped by our lack of observational data in two areas. First, we have no knowledge of the isotopic abundances for most of the elements. Second, the abundances of the elements themselves are known only in certain regions of the periodic table.

Iron itself, and a number of the elements shortward of the iron peak have been studied in some detail. There are many missing elements between hydrogen and iron in most stellar-abundance studies, but it has been possible to make some meaningful comparisons of the observational data with theoretical predictions. Longward of the iron peak, knowledge of stellar abundances is appallingly incomplete. The first hiatus usually stretches from gallium ($Z = 31$) to rubidium ($Z = 37$), and after the appearances of strontium, yttrium, zirconium, and sometimes niobium and molybdenum, a void continues from technetium ($Z = 43$) through cesium ($Z = 55$). Abundances can usually be found for barium, and frequently for lanthanum, but abundances are rarely given for more than three or four of the 14 lanthanide rare earths ($Z = 58$–71), and fragmentary information for a few of the heavier elements is only found in a few peculiar stars.

In a previous paper (Hartoog, Cowley, and Cowley 1973, henceforth called Paper I) we introduced a statistical method of line identification based on a Monte Carlo type determination of the probability of chance occurrence. The present paper represents a further application of this technique. In § III we present the results for five peculiar A stars: β CrB, 73 Dra, 78 Vir, HR 4816, and HR 4072. In § IV we briefly discuss these results vis-à-vis current theories of Ap stars. In the Appendix we compare the results of our Monte Carlo technique with the formula of Russell and Bowen (1929).

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II. PROCEDURE

Coudé spectra at 2.3 Å mm^{-1} were obtained for each star in table 1 with the 2.1-m telescope of the Kitt Peak National Observatory. The spectra were measured on the Grant engine in the offices of the observatory in Tucson, as described in Paper I for HR 7575. The stellar-line lists thus generated contain measurements of all features that appeared to be real, that is, above the level of the grain noise. If a blended feature could reasonably be attributed to two lines, two measurements were usually made.

TABLE 1
OBSERVATIONAL DATA

Star	Date	Time of Mid-Exposure (UT)	Wavelength Range Covered (Å)	No. Lines Measured
β CrB	1971 May 6	4^h13^m	3601–4944	3109
73 Dra	1971 May 5	10 45	3738–4800	1844
78 Vir	1971 May 3	7 42	3706–4737	1511
HR 4072.......	1971 May 4	3 49	3706–4731	985
HR 4816.......	1971 May 5	5 49	3748–4738	1969

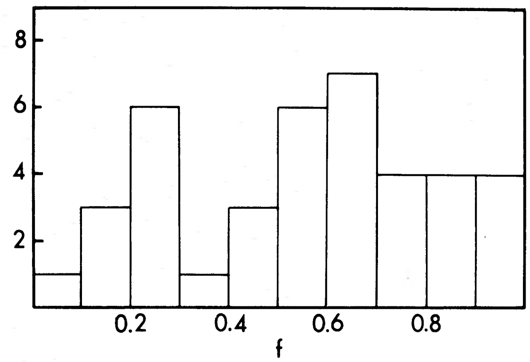

FIG. 1.—Histogram of the parameter f for transuranic elements in the five Ap stars in the current study plus the results for HR 465 and 7575 from Paper I.

The details of the statistical procedure were given in Paper I. We compute a quantity S, given by

$$S = (H - \langle H \rangle)/\sigma\,. \tag{1}$$

H is the number of coincidences or "hits" on the laboratory wavelengths while $\langle H \rangle$ is the average number of coincidences on $\mathcal{N}$ sets of nonsense wavelengths. The quantity σ^2 is the variance of the distribution of H. Crudely, S is the number of standard deviations that the coincidences of stellar and laboratory wavelengths are from the expected number of chance coincidences. We also compute a quantity f, given by

$$f = n/\mathcal{N}\,, \tag{2}$$

where n is the number of times there were an equal number or more coincidences on the nonsense sets of wavelengths than on the laboratory set. This then is an estimate of the probability of obtaining H coincidences by chance.

Most of the results discussed here used $\mathcal{N} = 100$ and a coincidence tolerance of ± 0.06 Å. For the majority of the trace elements, tolerances of ± 0.08 and ± 0.04 Å were also used, and the test at ± 0.06 was repeated with $\mathcal{N} = 1000$. The wavelength references are the same as those given in Paper I, with the exception of As II, where we used the new wavelengths of Li and Andrew (1971). Cr I, Mn I, and Ce III wavelengths were taken from the tables of Zaidel' *et al.* (1970). *We emphasize here that statistical analysis is based purely on wavelength coincidences, and must therefore be supplemented by standard identification processes. It is a refinement of the standard methods, not a replacement for them, and all of our studies of trace elements include the usual line-by-line comparison of stellar tracings with laboratory data.*

In order to obtain some kind of empirical check on our results, we have examined the results of our tests on the transuranic elements Pu, Am, Cm, Bk, and Es. A histogram of the parameter f is shown in figure 1. (We have added the results from Paper I for HD 465 and HR 7575.) There are 42 cases examined, one of which has $f = 0.05$. As was explained in Paper I this case was regarded as due to chance; statistically, we would actually expect *two* results out of 42 cases, if *none* of the above transuranic elements were present. The histogram of figure 1 shows a distribution that is weighted toward values of f *greater* than 0.5, while there appears to be a paucity of small values of f. This means that in the actual trials more coincidences were found, on the average, with the nonsense wavelengths than with the laboratory wavelengths. We shall not attempt to answer the interesting question of whether this result is in any way "significant." Instead, we note that, at least for these elements, our technique has been conservative.

If $f \leqslant 0.05$ we feel that the coincidence test is sufficiently "positive" so that a detailed examination of tracings of the spectra is worthwhile. In a few special cases, such as uranium and platinum, we have examined the spectral data even when the statistical evidence was less favorable. Whenever $S \geqslant 4$, the evidence for a positive identification is so strong that there is no need for further efforts. Indeed, most of these elements have already been identified, and their presence may be taken for granted.

TABLE 2

SUMMARY OF IDENTIFICATIONS

Star	Definite	Good	Possible
β CrB	Mg II*, Ca II*, Sc II, Ti I, Ti II, Cr I, Cr II, Fe I*, Fe II, Sr II, Ce II, Gd II, Eu II	Si II, V II, Mn I, Y II, Dy II	(P II), (Zn II), Zr II, Nb II, Mo II, La II, Sm II, (Os II), U II*
73 Dra	Mg II*, Ca II*, Ti I*, Ti II, Cr I, Cr II, Mn I, Fe I, Fe II, Sr II*, Y II	Si II, Eu II	(Al II), Sc II, V II, Mn II, (Kr II), Pt II*, U II*
78 Vir	Mg II*, Ca II*, Ti II, Cr I, Cr II, Fe I, Fe II, Sr II*, Eu II	Ce II, Ni II	(Al II), Mn II, Pt II*
HR 4816.......	Mg II*, Ca II*, Ti II, Cr I, Cr II, Mn I, Mn II, Fe I, Fe II, Sr II*	Si II, Ce II	Sc II, Zr II, Nb II, (In II), La II, V II*
HR 4072.......	Mg II*, Si II*, Ca II, Ti II, Cr I*, Cr II, Mn I, Mn II, Fe I, Fe II, Sr II*, Y II, Zr II, Pt II		(S II), Ar I, Kr I, (Te II)

TABLE 3

WAVELENGTH COINCIDENCE STATISTICS FOR FIVE Ap STARS

	β CrB			73 Dra			78 Vir			HR 4816			HR 4072		
	H/N	S	f	H/N	S	f	H/N	S	f	H/N	S	f	H/N	S	f
Ne I	9/33	-.62	0.82	2/25	-1.38	0.97	0/21	-1.85	1.00	4/21	-.24	0.66	0/21	-.97	1.00
Ne II	9/18	2.14*	0.032*	2/14	-.80	0.90	2/15	-.45	0.80	7/14	2.37*	0.025*	1/15	-.40	0.83
Al II	3/10	0.49	0.43	4/8	2.08*	0.066*	4/8	2.54*	0.028*	2/8	0.04	0.63	2/8	1.25	0.23
Si II	5/5	2.60	<.01	4/5	2.93	0.01	3/5	1.81	0.10	5/5	2.90	0.01	3/5	2.74	0.03
P II	14/33	1.85*	0.057*	9/31	1.13	0.20	7/31	-.62	0.35	6/31	-.54	0.74	4/31	1.38	0.15
S I	1/3	-.24	0.80	0/3	-.66	1.00	0/3	-0.70	1.00	0/3	-.84	1.00	0/3	0.0	1.00
S II	9/28	0.29	0.45	6/24	-.02	0.54	0/24	-2.22	1.00	6/24	-.43	0.75	7/24	2.35*	0.027*
Cl II	13/36	0.61	0.35	8/36	0.14	0.52	7/33	0.23	0.47	10/32	1.08	0.18	8/33	1.38	0.14
Ar I	8/33	-.28	0.72	3/27	-1.33	0.94	4/27	-.22	0.62	3/27	-1.69	0.99	7/27	2.01*	0.046*
Ar II	8/25	0.27	0.47	6/24	0.46	0.43	5/25	0.43	0.43	5/24	-.17	0.62	2/24	-.35	0.72
Sc II	22/30	7.10	<.01	9/21	2.23*	0.033*	6/21	1.08	0.20	10/21	2.45*	0.014*	2/21	-.68	0.85
Ti I	29/60	3.33	<.01	34/51	6.76	<.01	9/54	-.37	0.72	15/49	0.90	0.22	6/54	-.59	0.76
Ti II	96/73	12.42	<.01	65/82	12.81	<.01	56/86	10.49	<.01	60/78	10.81	<.01	50/86	14.37	<.01
V II	47/125	2.66	<.01	31/101	2.07*	0.035*	23/103	1.00	0.18	29/94	1.47	0.08	10/103	-1.39	0.96
Cr I	18/25	4.92	<.01	18/22	8.64	<.01	20/24	8.06	<.01	18/22	6.91	<.01	6/24	1.76	0.06
Cr II	55/83	7.72	<.01	44/55	10.71	<.01	42/59	12.30	<.01	47/52	11.56	<.01	41/59	13.41	<.01
Mn I	16/25	3.96	<.01	14/24	4.33	<.01	7/21	1.66	0.10	14/21	4.22	<.01	9/21	3.82	<.01
Mn II	23/74	1.08	0.18	15/43	2.25	0.03	17/51	2.95*	0.007*	23/39	5.47	<.01	19/51	4.73	<.01
Fe I	(see footnote†)			18/33	4.91	<.01	26/42	7.46	<.01	19/30	4.43	<.01	32/42	10.04	<.01
Fe II	46/59	7.63	<.01	41/54	8.99	<.01	42/54	12.75	<.01	43/50	10.42	<.01	34/54	15.10	<.01
Co I	3/7	0.74	0.38	2/7	0.46	0.42	1/7	-.45	0.78	3/7	1.04	0.24	1/7	0.02	0.64
Co II	0/1	-.33	1.00												
Ni I	1/2	1.09	0.32	1/1	1.72	0.25	0/1	-.68	1.00	0/1	-.71	1.00	0/1	-.40	1.00
Ni II	4/10	0.46	0.45	2/10	-.21	0.69	6/10	3.28*	0.002*	5/10	1.63	0.13	3/10	1.39	0.17
Cu II	4/7	1.80	0.10	1/6	-.22	0.76	2/6	0.95	0.26	3/6	1.61	0.12	1/6	2.36	0.15
Zn I	4/10	.42	0.47	1/9	-.62	0.85	2/9	0.47	0.41	3/9	1.02	0.23	4/8	2.60*	0.025*
Zn II	6/12	2.05*	0.044*	1/8	-.62	0.88	2/9	0.30	0.49	4/8	1.60	0.12	0/9	-1.20	1.00
Ga II	1/8	-.79	0.91	1/5	-.38	0.78	1/6	-.16	0.75	1/5	-.44	0.82	0/6	-1.23	1.00
Ge I	0/2	-1.14	1.00	1/2	1.06	0.31	0/2	-.48	1.00	0/2	-.67	1.00	1/2	2.28	0.16
Ge II	2/4	0.53	0.41	0/3	-.72	1.00	1/2	2.09*	0.153*	0/2	-.48	1.00	0/2	0.00	1.00

* Based on 1000 sets of nonsense wavelengths
† Statistics for FeI not computed

TABLE 3—Continued

	β CrB			73 Dra			78 Vir			HR 4816			HR 4072		
	H/N	S	f	H/N	S	f	H/N	S	f	H/N	S	f	H/N	S	f
As II	12/47	-0.42	0.70	12/46	0.78	0.27	8/44	-0.09	0.60	12/44	0.57	0.31	2/44	-1.26	0.96
Se II	10/28	0.47	0.36	8/28	-.71	0.30	7/26	1.12	0.21	10/26	1.45	0.12	2/26	-.57	0.84
Br I	7/12	2.31*	0.025*	5/12	1.36	0.16	4/12	1.37	0.16	4/12	0.61	0.41	1/12	0.08	0.65
Br II	5/8	1.52	0.12	1/8	-.36	0.78	2/7	0.76	0.38	2/7	0.16	0.55	0/7	-.91	1.00
Kr I	5/12	0.82	0.29	2/12	-.86	0.87	3/12	0.57	0.37	2/12	-.68	0.81	5/12	3.18*	0.010*
Kr II	14/57	-.84	0.86	17/48	2.54*	0.011*	8/49	-.33	0.64	10/44	-.28	0.67	4/49	-0.70	0.84
Y II	16/31	2.77*	0.005*	12/23	3.96	<.01	8/25	1.82	0.06	5/23	-.30	0.69	23/25	11.04	<.01
Zr II	14/32	2.43*	0.021*	7/18	1.85	0.06	4/20	-0.20	0.61	8/17	2.07*	0.040*	9/20	4.07	<.01
Nb II	8/14	2.57	0.016*	3/10	0.52	0.43	2/11	0.10	0.61	5/9	2.26*	0.039*	2/11	0.05	0.58
Mo I	3/18	-1.03	0.88	3/14	-.24	0.69	1/14	-1.15	0.96	2/14	-.83	0.86	3/14	0.60	0.36
Mo II	9/19	2.37*	0.021*	1/10	-.86	0.83	2/12	-.38	0.75	3/9	0.44	0.42	4/12	1.56	0.13
Ru I	11/34	0.34	0.40	4/27	-.84	0.84	7/32	0.44	0.45	8/26	0.95	0.24	4/32	-.07	0.63
Ru II	0/2	-.65	1.00												
Rh I	5/23	-.62	0.80	3/15	-.11	0.62	3/16	-.21	0.70	3/15	-.52	0.80	4/16	0.80	0.32
Pd I	3/8	0.80	0.32	2/5	1.21	0.23	1/6	-.13	0.71	1/5	-.16	0.79	2/6	1.25	0.28
Pd II															
Ag I	3/5	1.49	0.15	2/5	1.04	0.22	2/5	1.12	0.24	1/5	-.39	0.82	1/5	0.64	0.44
Ag II	1/6	-.47	0.84	2/4	1.48	0.20	1/3	0.86	0.39	1/3	-.04	0.66	0/3	-.44	1.00
Cd II	1/3	-.01	0.72	0/3	-1.09	1.00	1/3	0.84	0.45	0/3	-1.00	1.00	0/3	-.63	1.00
In I	1/2	0.87	0.46	0/2	-.66	1.00	1/2	1.36	0.29	0/2	-.76	1.00	0/2	-.14	1.00
In II	4/11	0.48	0.41	1/9	-.51	0.83	1/9	-.29	0.72	4/9	2.01*	0.064*	1/10	0.60	0.43
Sn I	2/3	1.76	0.16	1/2	0.91	0.36	1/2	1.12	0.28	0/2	-.71	1.00	0/2	-.43	1.00
Sn II															
Sb II	8/26	-.01	0.60	3/26	-.96	0.87	5/23	0.77	0.30	7/23	0.97	0.20	1/23	-.62	0.81
Te II	6/25	-.54	0.78	4/23	-.09	0.62	3/22	-.29	0.69	6/21	0.64	0.37	5/22	1.94*	0.067*
Xe I	5/18	-.36	0.75	3/14	0.11	0.57	3/13	0.84	0.25	1/13	-1.30	0.94	1/13	0.13	0.58
Xe II	7/27	0.11	0.59	4/22	-.68	0.84	4/22	0.06	0.63	6/22	0.17	0.53	4/22	1.31	0.18
Cs II	14/38	0.74	0.33	12/38	1.11	0.14	5/37	-.62	0.79	14/37	1.83	0.07	4/37	-.21	0.70
La II	26/60	2.49	<.01	17/53	1.81	0.07	13/53	1.05	0.16	20/52	2.33*	0.017*	6/53	-.43	0.74
Ce II	74/95	11.52	<.01	24/86	1.12	0.15	30/88	3.43	<.01	34/85	3.09	<.01	12/88	-.11	0.63
Ce III	18/47	1.41	0.12	14/85	1.23	0.15	10/45	0.70	0.29	11/45	0.19	0.46	5/45	-.09	0.63
Pr II	11/35	0.10	0.52	9/35	0.62	0.33	9/35	0.91	0.25	8/35	-.43	0.72	6/35	0.28	0.45
Pr III	5/24	-0.75	0.80	6/23	0.16	0.50	4/23	-.15	0.64	9/23	1.76	0.08	5/23	2.03*	0.056*
Nd II	23/57	1.34	0.09	17/54	1.70	0.08	15/56	1.57	0.10	17/52	1.18	0.15	10/56	0.50	0.38
Pm II	11/43	-.37	0.71	6/33	-.64	0.79	8/36	0.55	0.32	9/32	0.46	0.40	2/36	-1.33	0.96

TABLE 3—Continued

	β CrB			73 Dra			78 Vir			HR 4816			HR 4072		
	H/N	S	f	H/N	S	f	H/N	S	f	H/N	S	f	H/N	S	f
Sm II	36/104	1.74	0.04	8/86	-2.92	1.00	23/95	1.70	0.06	17/81	-0.60	0.74	13/95	0.12	0.50
Eu II	6/10	2.23	0.04	6/8	3.82	<.01	8/9	4.74	<.01	4/8	1.39	0.18	2/9	0.80	0.35
Gd II	40/54	7.15	<.01	7/38	-.47	0.72	5/42	-1.18	0.91	12/37	1.00	0.19	7/42	0.37	0.43
Tb II	10/48	-.46	0.73	5/26	-.56	0.74	7/29	0.76	0.29	5/26	-.94	0.90	6/29	0.91	0.26
Dy II	25/49	3.67	<.01	6/35	-.57	0.79	8/37	0.37	0.35	10/32	1.34	0.12	7/37	0.59	0.32
Ho II	5/20	-.46	0.74	1/16	-1.57	0.97	4/17	0.05	0.58	3/16	-.50	0.79	2/17	-.58	0.80
Er II	8/23	0.72	0.28	2/15	-.89	0.88	4/17	0.42	0.43	3/15	-.22	0.69	3/17	0.09	0.55
Tm II	4/17	-.36	0.74	2/10	0.18	0.62	2/12	-.12	0.63	2/8	0.23	0.45	4/12	1.81	0.08
Yb II	23/62	1.76	0.07	9/47	-.35	0.71	9/49	0.01	0.53	13/45	0.62	0.29	5/49	-.72	0.80
Yb III	10/27	1.01	0.22	5/25	-.36	0.72	5/25	0.26	0.50	9/25	1.09	0.17	5/25	0.58	0.38
Lu I	12/47	-.43	0.73	11/43	0.74	0.32	12/43	1.48	0.11	13/41	1.19	0.14	7/42	1.00	0.16
Lu II	3/17	-.98	0.92	2/14	-.75	0.86	0/14	-1.73	1.00	3/14	-.31	0.71	2/14	-.08	0.62
Hf II	9/37	-.45	0.76	5/25	-.14	0.60	5/28	-.14	0.66	4/25	-.94	0.86	5/28	0.77	0.34
Ta II	7/29	0.15	0.51	1/17	-1.78	1.00	1/18	-1.41	0.97	5/16	0.67	0.38	1/18	-1.01	0.92
Os I	2/8	-.53	0.80	2/8	0.03	0.57	2/8	0.70	0.38	3/7	0.96	0.29	0/8	-1.14	1.00
Os II	6/9	2.59*	0.015*	2/7	0.62	0.36	2/7	0.26	0.51	3/7	0.82	0.33	0/7	-1.08	1.00
Ir I	0/1	-.67	1.00	0/1	-.59	1.00	0/1	-.57	1.00	0/1	-.35	1.00	1/1	3.37	0.08
Ir II	0/1	-.75	1.00				0/1	-.43	1.00				0/1	-.20	1.00
Pt II	6/17	0.79	0.30	5/15	1.12	0.21	5/15	1.74*	0.083*	5/15	0.89	0.28	10/15	6.03*	<.001*
Au II	3/10	0.02	0.63	2/8	0.13	0.60	1/7	-0.42	0.78	2/7	0.28	0.50	1/7	0.43	0.49
Tl I	0/1	-.70	1.00	0/1	-.45	1.00	0/1	-.47	1.00	0/1	-.45	1.00	0/1	-.38	1.00
Tl II	2/7	-.13	0.70	1/7	-.42	0.86	3/6	1.76	0.10	1/6	-.40	0.80	3/6	1.85	0.10
Pb I	3/9	0.79	0.31	1/5	-.29	0.75	2/6	1.01	0.30	2/5	0.95	0.29	1/6	0.81	0.42
Pb II	1/8	-1.01	0.95	3/7	1.54	0.16	4/7	2.75*	0.017*	0/7	-1.55	1.00	1/7	0.07	0.59
Bi I	1/3	-.04	0.73	1/3	0.63	0.46	0/3	-.86	1.00	1/3	0.00	0.70	0/3	-1.08	1.00
Ac II	6/16	0.84	0.29	2/15	-.58	0.78	0/15	-2.27	1.00	1/14	-1.75	0.99	3/15	1.29	0.20
Th II	4/20	-.73	0.85	2/15	-.73	0.89	3/17	-.18	0.65	4/14	0.44	0.44	0/17	-1.72	1.00
Pa II	2/12	-1.00	0.94	0/11	-1.88	1.00	1/11	-.84	0.92	2/11	-.63	0.84	0/11	-1.25	1.00
U II	12/30	1.09*	0.175*	8/26	1.32*	0.141*	6/28	0.32*	0.456*	9/25	1.55*	0.104*	6/28	1.18	0.23
Pu II	26/93	-.31	0.62	17/93	-.81	0.80	17/93	0.05	0.52	22/93	-.23	0.67	10/93	0.17	0.45
Am II	10/27	1.25	0.16	6/20	1.08	0.24	3/20	-.11	0.62	3/17	-.51	0.78	4/20	0.86	0.23
Cm	36/127	-.10	0.57	23/108	-.15	0.55	20/113	-.02	0.60	26/105	-.06	0.56	19/113	1.27	0.13
Bk	6/14	1.18	0.16	1/5	-.10	0.68	0/9	-1.31	1.00	1/3	0.29	0.57	1/9	-.12	0.69
Cf	5/14	0.45	0.40	1/10	-.85	0.93	2/13	-0.39	0.77	3/10	0.39	0.45	3/13	0.55	0.38
Es	1/5	-.35	0.80	1/3	0.63	0.51	1/4	0.41	0.51	1/3	0.34	0.63	0/4	-.73	1.00

In an ideal case one makes identifications on the basis of a correlation of intensities, especially within multiplets between the laboratory and stellar spectrum. It is our experience that this kind of clear correlation is only possible for spectra with large values of S. For the trace elements, workers have frequently used the concept of "unblended lines," i.e., a wavelength of a trace element that does not coincide with any other possible elements to within the tolerance of the wavelength measurements. If features are present at these "unblended" wavelengths (as well as at the positions of any stronger laboratory feature), a tentative identification may be made. The serious drawback of this method is that it assumes completeness of the wavelength sources used to find possible blends. Such sources may be very incomplete. Under these conditions, our confidence in an identification is considerably strengthened if in addition to the presence of an "unblended" line, our statistical analysis tells us that the wavelength coincidences are significant.

III. RESULTS

A summary of the statistical results is given in table 2. Following Paper I, we have listed three categories for each star. Elements in the "definite" category have $S > 4$, except for Ca II and Sr II, which are identified on the basis of their resonance doublets, and Mg II, for which $\lambda 4481$ is an obvious indicator. We did not attempt to establish the presence of $\lambda 4554$ of Ba II. The "good" category usually has $4 \geqslant S \geqslant 3$. For both of these cases we rarely obtain as many coincidences on any of the nonsense sets of wavelengths as on the laboratory set, so the parameter f is not meaningful. The "possible" category is best defined in terms of f rather than S, and has $0.05 \geqslant f \geqslant 0.01$. The elements for which our overall evidence is not encouraging are put in parentheses. Cases where the assignment to a category is made on the basis of the tracing rather than the wavelength statistics are marked with an asterisk. The presence of those elements in the "possible" category with an asterisk or parentheses is highly speculative.

In some instances we have not obtained a significant result on elements that are undoubtedly present. The referee of this paper has cited Si II and Sc II in 78 Vir as examples. Undoubtedly our laboratory lists have too many faint lines for this particular star. The conclusion that such spectra are only weakly represented in the spectrum of these stars is inescapable. The "presence" of the element, as determined by some different technique is, of course, not excluded.

Table 3 gives the ratio H/N of coincidences, H, to the number of lines N in the laboratory list along with the parameters S and f.

Let us now turn to a discussion of some of the marginal cases in the individual stars. In this discussion, we shall indicate measured stellar wavelengths which have been corrected for the radial velocity of the star by λ^*. The laboratory wavelengths, obtained from the references given in Paper I, will be indicated by λ.

a) *β CrB = HR 5747 = HD 137909*

This well-known cool Ap star was analyzed by Adelman (1973*a*, *b*), and is the subject of an extensive analysis by Shore and Hardorp (1974, see also Adelman and Shore 1973). If we examine the five stars included in the present study we find that the line density is highest in β CrB. Although the star has a periodic magnetic variation (Preston and Sturch 1967) there is little or no evidence of spectrum variability that could not be associated with the Zeeman effect. The results for Ne II and Br I were found to be entirely due to chance and will not be discussed.

i) P II: $Z = 15$ ($f = 0.057$, $S = 1.85$, $H/N = 14/33$)

For the most part, the stronger P II lines were included among the 14 hits. An examination of the tracing shows features at the positions of the eight strongest lines that were not measured because of nearby blends or weakness. It is interesting that the best alternate of one of the "Ne II" lines, $\lambda^*3664.16$, is P II $\lambda 3664.19$.

The referee of this paper states that "I could find no evidence for P II in this star on the basis of Martin's (1959) spectroscopic analysis." Our wavelength list was also based on Martin's study. A reconsideration of this case shows that most of the lines in question *may* be given alternate identifications that are more or less plausible. The situation is not a simple one because the number of coincidences at ± 0.06 was 14, and it is difficult to establish this many alternate identifications with confidence. Nevertheless, it appears that, statistics notwithstanding, the case for these alternates is at least as good in β CrB as for P II itself. Under these circumstances, we conclude that it would be impossible to identify P II in a star like β CrB *even if the lines were present*. This would not hold for hotter stars, where weak lines of Cr I, Fe I, and Sc II are no longer present to obfuscate the identification.

ii) Zn II: $Z = 30$ ($f = 0.044$, $S = 2.05$, $H/N = 6/12$)

The atomic-level structure is extremely simple for both Zn I and Zn II. The strong lines all lie far in the ultraviolet, and the lines that are considered come from highly excited levels (e.g., 12.6 eV). There is a systematic shift of up to ± 0.04 Å in the wavelengths that can be derived from the *Atomic Energy Levels* (Moore 1952) and those listed in the Multiplet Tables (Moore 1945). Alternate identifications to Zn II cannot be found for $\lambda\lambda^*3739.94$, 3806.35, and $\lambda 3840.34$. The latter line was not measured, but is clearly present in the wing of $\lambda 3840.44$ of Fe I. A case can be made that all three lines in the multiplet $5p\ ^2P^o$–$6d\ ^2D$ are present. Zn should primarily be first ionized by a factor of ~ 100. However, if lines are present from 12 eV in the Zn II spectrum, it is reasonable to expect some evidence for Zn I. The statistical evidence is entirely negative, but the tracings show that some features are weakly present at the wavelengths of the stronger Zn I lines $\lambda\lambda 4722.16$ and 4810.53. These fragments of information are insufficient to resolve the question of the presence or absence of Zn II, but they do show that the identification cannot be dismissed out of hand.

iii) Nb II: $Z = 41$ ($f = 0.016$, $S = 2.57$, $H/N = 8/14$)

Adelman (1972) has suggested that Nb II is present in 9 Tau and γ Equ, and mentions λ4527.65 as usable for line identification. A weak, unmeasured feature is present at this wavelength on our tracing. This identification seems possible, although it is disturbing that there is very weak evidence for λ3818.86, which should be slightly stronger than λ4527.65.

iv) Mo II: $Z = 42$ ($f = 0.021$, $S = 2.37$, $H/N = 9/19$)

Good alternate identifications cannot be found for $\lambda\lambda$*3688.32, 3742.34, 4209.65, and λ4363.64. The latter feature was not measured but is present on the tracings. The overall evidence for the presence of Mo II appears rather good.

v) Os II: $Z = 76$ ($f = 0.015$, $S = 2.59$, $H/N = 6/9$)

Results at other wavelength tolerances are not as good: $f(\pm 0.04) = 0.08$, and $f(\pm 0.08) = 0.08$. Brandi and Jaschek (1970) suggested Os II is present in β CrB on the basis of λ3894.89, an "unblended" line. We did not measure this line, but there is a weak feature present near this wavelength on our tracings. Consideration of all evidence indicates that Os II, if present at all, is only weakly present.

vi) Pt II: $Z = 78$ ($f = 0.30$, $S = 0.79$, $H/N = 6/17$)

Brandi and Jaschek (1970) identified Pt II in β CrB using the "unblended" lines $\lambda\lambda$4046.45, 4062.66, 4148.30, and 4288.40. The first three of these lines were not measured and if they are present on the tracing, they are very weak and/or badly blended. The last line is present as a weak measured feature but on the basis of the Pt II line intensities in HR 4072 (Dworetsky 1969 and this paper) it should be weaker than the first two lines. We find no evidence for Pt II.

vii) U II: $Z = 92$ ($f = 0.19$, $S = 1.16$, $H/N = 12/30$)

Brandi and Jaschek (1970) identified U II in β CrB and this identification has been confirmed by Hardorp and Shore (1971) and Adelman and Shore (1973). The last authors discussed this identification in detail on the basis of more extensive observational material than ours, and concluded that U II lines were weakly present. Although most of the features discussed by Adelman and Shore are weaker than the lines measured on our plate, they are all weakly present on our tracings. Even though we question whether any "statistical" significance can be attached to these coincidences, we agree that U II is probably weakly present.

b) 73Dra = AF Dra = HR 7879 = HD 196502

This well-known variable was classified by Cowley *et al.* (1969, henceforth called C^2J^2) as A0p (Sr, Cr, Eu). Preston (1967) has studied the spectral and magnetic variations. According to his ephemeris, our plate was taken at a phase of 0.84 of the $20^{d}2754$ period. The zero point is defined with respect to the europium (chromium and titanium) maximum. Thus the chromium-europium "patch" should have been approaching at the time our plate was taken, and our $\Delta\lambda$ histograms, in fact, show negative radial velocities as they should. Abundance studies have been made by Faraggiana and Hack (1962) and Galkin (1964). At the phase of our plate the rare earths are remarkably weak; only europium is well identified.

Interesting fragments of information on this star are available from a variety of sources: Short-term variations in the strengths of the hydrogen lines (Wood 1964) and Ca II K-line (Honeycutt 1966) have been reported. The light variations of this star have been studied by Rakos (1963). Preston (1967) discusses possible secular variations in the spectrum, while Rakos's variable period for the light has been recently discussed by Renson (1972), who attributes the apparent changes in period to small irregular fluctuations in light, as well as observational uncertainties.

i) Kr II: $Z = 36$ ($f = 0.011$, $S = 2.54$, $H/N = 17/48$)

The lines in question have high excitation potentials $\chi_{\rm lower}$ 13–15 eV, and consequently, enormous Boltzmann factors. This in itself does not seem sufficient to exclude the presence of Kr II, since Cl II and Se II, etc., which have similar Boltzmann factors, have been identified in Ap stars. The absence of statistical evidence for Kr I makes an identification extremely dubious. Nevertheless, an examination of the tracing shows that features are present at the wavelengths of the five strongest Kr II lines, and no acceptable alternate identifications can be found for any of these lines to within ± 0.08 Å. Ordinarily, this would be considered quite strong evidence for an identification.

A line-by-line search for Kr I does reveal a number of weak, coincident features, but the density of such features is so high that it is difficult to draw any conclusions from this. The high density of weak features in this spectrum is itself puzzling, since we find that the line-rich rare earths are rather weak. This situation certainly requires clarification.

ii) Os I and Os II: $Z = 76$ (I: $f = 0.57$, $S = 0.03$, $H/N = 2/8$; II: $f = 0.36$, $S = 0.618$, $H/N = 2/7$)

Guthrie (1969) identified both Os I and Os II in 73 Dra and this was confirmed by Jaschek and Malaroda (1970). The statistical results above do not support this identification and an examination of the tracings did not lend much support either. Particularly disturbing is the apparent absence of the Os II line λ3894.89. However, in light of the fact that this star is a spectrum variable and that the lines being discussed are very weak, we simply indicate that this identification requires careful study on additional plate material.

iii) Pt II: $Z = 78$ ($f = 0.21$, $S = 1.12$, $H/N = 5/15$)

Jaschek and Malaroda (1970) identified Pt II in 73 Dra. The statistical results do not support this identification, but an examination of the tracing does show that the three strongest lines of Pt II appear to be present. Two of these, λλ4046.45 and 4514.17, were measured at λ*4046.48 and λ*4514.13. The third line λ4061.66 appears to be present, but is badly blended. The weaker Pt II lines which are usually found in stars (Dworetsky 1969 and present work) may be present, but additional plates are required to confirm this. At least tentatively we consider this a possible identification.

iv) Au I: $Z = 79$ ($f = 0.56$, $S = 0.09$, $H/N = 3/15$)

Jaschek and Malaroda (1970) suggested Au I as a probable, but not certain, identification in 73 Dra. The statistical results and a search of the tracings do not support this identification. Of the five strongest lines in the wavelength range studied, only λ4607.51 is clearly present and has no good alternate identification. The line was not measured, presumably because it is a part of a moderate strength blend. However, λλ3897.86 and 4065.07, if present at all, are badly blended and λλ4040.93 and 4792.58 are both absent, although the plate is weakly exposed at this last wavelength. No features were measured within ±0.06 Å of any of these laboratory wavelengths. Again, because of the spectrum variability, this identification should be checked on additional plate material, but we find little support for it. It should be noted that there is considerable disagreement in the wavelength of Au lines between various sources. We have used wavelengths from Ehrhardt and Davis (1971).

v) U II: $Z = 92$ ($f = 0.141$, $S = 1.32$, $H/N = 8/26$)

Guthrie (1969) suggested U II as a possible identification in 73 Dra and this was confirmed by Jaschek and Malaroda (1970). The statistical results are not what one would call good; they are quite similar to the results for β CrB. An examination of the tracing does lend some support to the identification. The strongest U II line, λ3859.58, is clearly present and no reasonable alternate identification is available for this line. While the reality of many of the weaker features, especially λ4241.67, should be checked on additional plates, we at least tentatively consider this a possible identification.

c) 78 Vir = HR 5105 = HD 118022

This star was classified by C^2J^2 as A1p (Cr, Sr, Eu). It was included in the studies of Adelman (1972, 1973*a*, *b*) who made both line identifications and abundance determinations. The rare earths are somewhat more apparent than in 73 Dra, but are still much weaker than in β CrB. According to Roman (1949) 78 Vir is a probable member of the Ursa Major stream, which means that its age can be fixed at approximately 3×10^8 years (Schmidt 1963). The magnetic period has been determined by Preston (1969) to be $3^d.7220$. Although the Ca II K-line varies periodically in strength, there is little evidence for a general spectrum variability such as is found in 73 Dra. The Ge II and Pb II results shown in table 3, which appear to be of marginal statistical significance, were judged to be entirely due to chance because more logical alternate identifications could be found for almost all of the wavelength coincidences.

i) Al II: $Z = 13$ ($f = 0.03$, $S = 2.68$, $H/N = 4/8$)

The four coincidences were λλ*4026.54, 4585.82, 4588.20, and 4666.74. The latter two wavelengths are almost certainly Cr II and Fe II, respectively. The lower excitation potentials of the first two lines are 13.6 and 15 eV, respectively, so the overabundance of Al would be similar to overabundances of P, S, or Cl in Ap stars where the second spectra of these elements are found. There is a Ti I line in multiplet 185 at λ4026.53, which does not seem likely in 78 Vir in view of the weakness of Ti I. The precise wavelength of the Al II line is open to doubt, since the laboratory study only quotes one decimal. A modern laboratory study of Al II is certainly needed. In υ Sag, Greenstein and Adams (1947) measured a feature at λ*4026.55 which they attribute to Al II. This υ Sag identification itself seems none too certain. If Al were present, and greatly overabundant, we would expect to see the resonance lines of Al I λλ3961.52 and 3944.01. This statement is based on the appearance of these lines in ι CrB (Aller and Ross 1970) and HR 465 (Aller 1972). The stronger of these lines is absent, although a faint, unmeasured feature appears near λ3944.01. This evidence is not encouraging.

ii) Ni II: $Z = 28$ ($f = < 0.01$, $S = 3.31$, $H/N = 6/10$)

This is the only star in which enough Ni II lines were measured to give a significant number of coincidences. Our inability to make the identification in other stars is some indication that the variation in Ni abundance from star to star is erratic.

iii) Pt II: $Z = 78$ ($f = 0.09$, $S = 1.88$, $H/N = 5/15$)

Two of the three strong Pt II lines λλ4515.17 and 4046.45, were measured at λλ*4514.13 and 4046.41. The third line, λ4061.66, could be present in the wing of a strong feature at λ*4061.81. A check of some of the fainter lines is encouraging. Adelman (1972) has already concluded that Pt may be present in this star, and we can confirm this. The case is better than in 73 Dra.

iv) U II: $Z = 92$ ($f = 0.47$, $S = 0.39$, $H/N = 6/28$)

If uranium is present in 78 Vir, the identification must be based on the presence of the strongest line λ3859.58 which was measured at λ*3859.57. This is one of the "unblended" lines. The other, λ4241.67, is lost in the noise on our tracing, although Adelman (1972) measured an 11.2 mÅ feature. The case for the identification of uranium in this star is not as good as in β CrB.

d) HR 4816 = HD 110066

C^2J^2 classified this star at A0p (Sr, Cr, Eu) and remarked that the K-line had a sharp feature, superposed on a broad shallow feature. This is similar to Preston's (1969) description of the K-line in 78 Vir, at least at some phases. A rather weak (≤ 300 gauss) magnetic field was measured by Babcock (1958), who remarked upon the abnormality of some of the line profiles. Preston (1971) determined a surface field H_s of 3.6 kilogauss, and a $v \sin i \leq 6$ km s. A complete line-identification list is currently being prepared by Adelman. This star is one of the 21 studied by Adelman (1972, 1973*a*, *b*). The Ne II result shown in table 3 was judged to be due to chance and will not be discussed.

i) Zr II: $Z = 40$ ($f = 0.06$, $S = 2.02$, $H/N = 8/17$)

The strong Zr II lines λλ*3958.23, 3998.93, and 4149.16 are present and do not have alternate identifications, although λ*3998.93 is probably blended with Cr I (307). The presence of Zr II in HR 4816 is quite likely.

ii) Nb II: $Z = 41$ ($f = 0.02$, $S = 2.4$, $H/N = 5/9$)

The four lines that were not measured can be found on the tracings, and are either in close blends or, in the case of λ3781.38, there is a weak feature that was not measured. Of the five measured lines there are no good alternate identifications for λλ*3863.01 and 3898.31. The feature λ*4579.49 may be attributed in part to an unclassified Fe II line, λ4579.52, but this feature looks like a blend of two lines, and there is no good alternate to λ4549.45 of Nb II for the feature in the violet portion of this blend. Since the meteoritic Nb/Zr ratio is -0.34 dex, the possible presence of Zr increases the likelihood of Nb. Thus, the spectroscopic and general-plausibility arguments support the statistical evidence for Nb II in HR 4816.

iii) In II: $Z = 49$ ($f = 0.02$, $S = 2.55$, $H/N = 4/9$)

Of the four hits, only one, λ*4655.49, has no good alternate identification. This weak feature coincides fairly closely with λ4655.51 of In II, which happens to be the strongest of the In II lines in our list. The lower excitation potential is 12.65 eV. The evidence presented here for indium is weak, but this identification merits additional study. If it were correct, a large and puzzling overabundance of indium is implied.

iv) Pt II: $Z = 78$ ($f = 0.28$, $S = 0.89$, $H/N = 5/15$)

Clearly, there is no statistical evidence for Pt II. Of the three strongest lines, only one, λ4514.17 was measured, at λ*4514.21. This coincides closely with a faint Fe I line in multiplet 514, λ4514.19, but it is difficult to establish that Fe I actually is the principal contributor. Since multiplet 514 has only two lines, we examined a total of six lines in multiplets 513–516. Only one had a measured wavelength, but the tracing revealed some feature, best described as "loud noise" at each wavelength. Similarly, something can be found at the position of the two strong platinum lines that were not measured. It does not appear to be possible to draw any support for a platinum identification from these disparate bits of evidence.

v) U II: $Z = 92$ ($f = 0.10$, $S = 1.70$, $H/N = 9/25$)

An examination of the tracings shows something present at the position of all the strong U II lines. More significantly, the two "unblended" lines λλ3859.58 and 4241.67 were both measured (at λλ*3859.63 and 4241.63). Adelman (1972) previously suggested this identification and has derived an abundance for U from the two unblended lines of ~ -9.2 [$=\log(U/H)$].

e) HR 4072 = HD 89822

C²J² call this star A0p [Si, (Sr, Hg)]. It is a double-lined spectroscopic binary. Guthrie (1966) studied the chemical composition of both components. It was in this star that Dworetsky (1969) announced the identification of Pt II lines. Both Hg I and Hg II lines have been identified; Dworetsky, Ross, and Aller (1970) derived a Hg/H ratio of -6.0 dex. Babcock's (1958) work shows a weak, variable magnetic field $H_e <$ 350 gauss. Conti (1970) gives a more recent discussion of the Zeeman measurements as well as the spectroscopic orbit. He finds $|H_e| \leq 100 \pm 50$ gauss for the primary. The secondary appeared to have had a sizable field, at least at one phase: $|H_e| = 2200 \pm 450$ gauss. The Zn I and Pr III results shown in table 3 were judged to be due to chance and will not be discussed.

i) S II: $Z = 16$ ($f = 0.027$, $S = 2.35$ $H/N = 7/24$)

Three of the seven coincidences have plausible alternate identifications; λλ*3923.40, 4028.74, 4525.01, and 4145.10 do not. If S II is present in this star, it is only weakly so, since only one line in multiplet 44 was measured. It is not possible to confirm or deny the marginal presence of this multiplet from the tracings. The presence of S II in the Mn star π^1 Boo has been noted by Jaschek, Jaschek, and Gonzáles (1965), while Aller and Ross (1970) measured S II λ4162.70 in ι CrB. Thus the presence of S in at least some Mn stars is established, although it is a moot question whether the present study has found it in HR 4072.

ii) Kr I: $Z = 36$ ($f = 0.01$, $S = 3.18$, $H/N = 5/12$)

The tables of Zaidel' *et al.* (1970) give λ4318.55 as having an intensity of 1400, the largest of any of the lines which would fall on our plate. Since this line was not measured, some effort was made to identify the five measured lines as something other than Kr I. These lines are λλ*4273.92, 4282.98, 4319.56, 4362.70, and 4376.18. Convincing alternate identifications were not easy to find; a check of the basic reference on Kr I (Meggers *et al.* 1931) showed the Zaidel' *et al.* intensity of λ4318.55 to be a misprint. The intensity should be 400. The measured features are quite weak, and some of the wavelengths are undoubtedly perturbed by other lines. With a wavelength tolerance of ± 0.04 Å, $f = 0.24$, but at ± 0.08, $f < 0.01$, $S = 3.92$; a quite respectable significance. Although confirmation is needed, the spectroscopic evidence for Kr I is encouraging.

iii) Te II: $Z = 52$ ($f = 0.067$, $S = 1.94$, $H/N = 5/22$)

Of the five "hits" three, λλ3905.64, 3918.51, and 4048.83 may be attributed to Cr II, Fe II, and Fe II. Good alternate identification to Te II are not available for λλ*4261.15 and 4478.67. A Ce II line λ4261.16 is not a possible alternate to the first of these lines, since the strongest Ce II line, λ4186.60, was not measured and is weak or absent on the tracing. There is an unidentified feature at this wavelength in HR 465, where we have suggested Te II is present. Sm II λ4478.66 coincides closely with the second line, but it too is an unlikely alternate because of the weakness of Sm II in HR 4072. Although the evidence for Te II is rather slim, a further investigation of the identification is merited. Since platinum is clearly present (see below), and krypton possibly so, there is the potential that all three *r*-process peaks are present in this star.

iv) Pt II: $Z = 78$ ($f = < 0.01$, $S = 6.90$, $H/N = 10/15$)

The statistical results show that the confidence given to the platinum identification in this star is amply justified. We have used our own tracings of HR 4072 for estimates of Pt II intensities. For stellar identifications they seemed preferable to the laboratory eye estimates.

Table 4 gives a comparison of our measured wavelengths for Pt II in HR 4072 with the laboratory wavelengths of Shenstone (1938) and the measured wavelengths in HR 4072 by Dworetsky (1969). This shows that our measurements and those of Dworetsky agree for the three strongest lines, λλ4046.45, 4061.66, and 4514.17 within the expected errors. However, for the weaker lines, our wavelengths are systematically shorter than those of Dworetsky and in good agreement with the laboratory wavelengths of Shenstone. We offer no explanation of this, but in light of the interpretation which has been given to the Pt II wavelengths (Dworetsky and Vaughan 1973) we suggest the matter requires further investigation and clarification.

v) U II: $Z = 92$ ($f = 0.23$, $S = 1.18$, $H/N = 6/28$)

There is little evidence for the presence of U II in HR 4072. A weak feature—hardly above the noise—was measured at λ*3859.60. There is only noise at the position of the next strongest line λ3854.66.

TABLE 4

PLATINUM WAVELENGTHS IN HR 4072

Laboratory	Intensity	Dworetsky	This Paper	Remarks
3447.78.....	10	47.80	...	our plate
3577.20.....	10	77.23	...	too faint
3766.40.....	10	66.46	66.387	deep in H II
3806.91.....	5	07.00	...	very weak
3970.06.....	15	...	...	Hϵ
4023.81.....	3	23.85	23.827	...
4034.17.....	5	34.20	34.178	...
4046.45.....	20	46.51	46.497	...
4061.66.....	10	61.68	61.687	...
4105.45.....	0	...	05.452	...
4148.30.....	5	48.33	48.302	...
4288.40.....	5	88.45	88.397	...
4514.17.....	10	14.20	14.191	...

IV. DISCUSSION

The current work was undertaken as an attempt to fill in some of the gaps in the periodic table that existed in the line-identification and abundance studies of Ap stars. Hopefully, the more complete the abundances are, the better chance we have of understanding how the peculiar abundances have come about in the Ap stars. At this time, however, no clear-cut conclusions can be drawn.

First, in attempting to attack this problem it seems important to emphasize that it is of the utmost importance to examine the identification of trace elements with great care and to indicate clearly when some possibility of error exists. In Paper I and the present paper we have attempted to do this. While we realize that uncertain results are much less satisfying, incorrect identifications will only confuse the situation and make any solution more difficult. Ultimately, the hope of a solution to this problem must lie in better laboratory data. Both the observational and theoretical ends of the problem are presently held up to a great extent by the lack of good laboratory wavelengths, classifications, and intensities. With these problems in mind, we can make a few comments based on the results of our present work.

1) Fowler, Burbidge, and Burbidge (1955) and Brancazio and Cameron (1967) proposed surface nuclear reactions to explain the peculiar abundances found in Ap stars. These mechanisms are able to produce a wide range of heavy elements and might be able to reproduce many of the known overabundances. However, they are not able to produce enhanced abundances of uranium. It appears possible that U is overabundant in many Ap stars (Adelman 1973*a*, *b*) and the abundance is enormously enhanced in at least one Ap star (Cowley and Hartoog 1972). This, along with other problems (see Adelman 1973*a*) makes surface nuclear reactions unlikely to be the main source of heavy element production.

2) Many people have suggested that the *r*-process may be responsible for the abundance peculiarities. The *r*-process is particularly attractive because it makes specific predictions of the enhancement of characteristic elements (e.g., Se, Te, Pt), which invite observational tests. Its drawback is that many of the "peak" elements are extremely difficult to identify since all the lines in the visible spectrum arise from very highly excited levels, and thus in general are weak. In these cases, a positive identification would immediately imply an abundance excess because of the enormous Boltzmann factors involved. We have made possible identifications for some elements on or near the peaks in the expected *r*-process distribution such as Os, Pt, Se, Kr, and Te. The *r*-process is the only classical process that has been suggested for the production of U (see, however, Amiet and Zeh 1968). This evidence, however, is not conclusive, since we have also made some marginal identifications of

elements with equally high excitation potential which fall well away from the r-process peaks, such as Zn. Similarly, the identification of gallium and its large overabundance in κ Cnc (Aller 1970) is widely accepted, and this cannot be explained by the r-process. It is not clear if other theories could also lead to enhancement of some of the r-process peak elements, greatly confusing the observational tests.

3) Michaud (1970) suggested that radiative-driven diffusion may explain the abundance anomalies of the Ap stars. It is very difficult to make any comparison of our results with this theory, since a lack of laboratory data (specifically for the third and fourth spectra) has prevented any predictions of the expected abundance ratios except for the lighter elements. More laboratory work must be done and more detailed theoretical calculations need to be made for the heavier elements before any meaningful comparison can be made with our results.

4) Comments similar to those above on the diffusion process also apply to the magnetic accretion theory of Havnes and Conti (1971). This theory predicts the enhancement of the heavy elements, but makes no clear statement about which elements are expected nor by how much each will be overabundant. It is probably not applicable to the (Hg) Mn stars such as HR 4072, since the latter do not have strong magnetic fields.

The main conclusion that can be drawn from this is that there is no clear explanation at present of the Ap abundance peculiarities. It should be kept in mind that a combination of processes may be operative in these stars so that we should not completely discard a theory because it fails to predict every aspect of all Ap spectra. It is clear that more observational data (both laboratory and stellar) are needed in addition to refinements and predictions for the competing theories.

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APPENDIX

A COMPARISON OF THE RUSSELL-BOWEN FORMULAE WITH MONTE CARLO RESULTS

Russell and Bowen (1929) showed that if the measured wavelengths on a stellar line list are distributed at random, then the probability, $P(\Delta\lambda)$, of finding a line within $\pm\Delta\lambda$ of some arbitrary wavelength is given by

$$P(\Delta\lambda) = 1 - \exp\left[-2\rho\Delta\lambda\right], \qquad \text{(A1)}$$

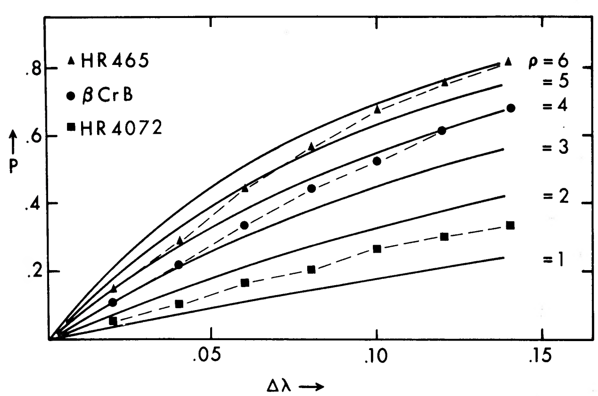

FIG. 2.—Probability of a chance coincidence within a tolerance window $\pm\Delta\lambda$. The line density ρ (in lines per Å) is given as a parameter. The solid lines were computed with the Russell-Bowen formula; the dashed lines give the Monte Carlo results for three stars.

where ρ is the line density. The assumption of a random distribution of lines, however, has been questioned by Cowley and Aller (1972) and Havnes and van den Heuvel (1972) who presented evidence that the Russell-Bowen formulae underestimated the probability of coincidence because of systematic effects in the spacing of lines on lists such as those considered in this paper. We now have a large body of data on which to test this.

Figure 2 shows the probability of coincidence as a function of $\Delta\lambda$ computed from equation (A1) for a series of line densities. The points connected by dashed lines show comparable probabilities computed by our Monte Carlo technique using line lists for three different stars between 3800 and 4200 Å. The HR 465 line list was described in Paper I. The β CrB and HR 4072 lists are from the present work. The line densities on the lists between 3800 and 4200 Å are 3.50 for HR 465, 2.51 for β CrB, and 1.27 for HR 4072. We see from figure 2 for the two highest density cases that equation (A1) with a single line density is not able to predict the probability of coincidence as a function of $\Delta\lambda$. Furthermore, in all cases the probability of coincidence is higher than that predicted from equation (A1) using the density of lines on the lists, although this disagreement becomes smaller as one goes to small tolerances and low line densities. These results clearly show the desirability of using a Monte Carlo technique and indicate that the assumption upon which the Russell-Bowen formulae rests (that the lines are distributed randomly in wavelength) is not justified for line lists of the type used here.

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