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THE 71-SECOND VARIATION OF DQ HERCULIS

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ABSTRACT

Photoelectric observations of the old nova DQ Her obtained at the Princeton University Observatory and Kitt Peak National Observatory have been used to determine an accurate period for the 71^s variation. By using the autocorrelation technique and a least squares method we find the period in 1969 to be $71^s.06547 \pm 0^s.00002$. This represents a significant change in the period since Walker's 1961 observations, as expected from the rate of period change we detected using data spanning the period 1967–1971.

Allowing for the rate of period change, it has been possible to combine all the observations from 1954 to 1971 ($\sim 10^7$ cycles) to obtain accurate values of P and $\dot{P}$ and an estimate of $\ddot{P}$. We find that the period has been decreasing steadily on a time scale $(P/\dot{P})$ of 2.64×10^6 years; error or phase jitter as measured by the rms value of the $(O - C)$ timing is only $4^s.3$. The extreme stability of the period compared with expected damping or evolutionary time scales makes plausible a rotational interpretation of the origin of the periodic light variations.

Subject headings: novae — pulsation — stars, individual

I. INTRODUCTION

DQ Herculis (1934) is probably the best observed of all novae, and it may present the best opportunity to understand novae and one type of evolution in old close binary systems. There are two distinct periodic phenomena shown, a 71^s sinusoidal variation superposed on an Algol-type light curve having period 4^h34^m. Thus there are four "good numbers" determinable in principle for this system: two periods and two rates of period change. Three of these numbers are known. The purpose of this paper is to derive, from our own and others' observations, the fourth number and to present some interpretation of the system.

The 71^s period is the shortest period of regular variation known in any star except for the pulsars and compact X-ray sources. The physical origin of this light variation, discovered and studied by Walker (1954, 1956, 1958, 1961), is still not understood. During 1967 and 1969 we obtained observations of DQ Her aimed at studying the 71^s variation and, in particular, redetermining its period, which had not been done since Walker's observations. Herbst (1970) used these observations to determine a period for the variation on 1969 April 11 of $71^s.06550 \pm 0^s.00005$. When combined with Walker's (1961) period P_s, this implied a change in period $\dot{P}_s$ at the rate of -2.0×10^{-5} s yr^{-1} or $(P/\dot{P})_s \sim 10^{6.5}$ years.

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Nather and Warner (1969) and Warner *et al.* (1972) have investigated both the eclipse and the short-period light variations. The eclipse period is increasing nonuniformly on a time scale of $(P/\dot{P})_e = +7.86 \times 10^6$ years; this is interpreted as due to systemic mass loss. Those authors find that the 71^s variation is diminished in amplitude during eclipse and also changes in phase by precisely 2π near eclipse. We think that the data are consistent with no net phase change per orbit (cf. § VI); in any case, phase changes of exactly $2\pi N$ per orbit will not affect our derived value of $(P/\dot{P})_s$. Warner *et al.* have also given times of maxima for the 71^s variation on several nights, mainly in 1969.

We have been able to combine all the available data, spanning 17 years and 7.5 million cycles, to determine accurate values of P_s and $\dot{P}_s$ and to estimate the acceleration $\ddot{P}_s$.

II. OBSERVATIONS

The observations were obtained with the 91-cm telescope at the Princeton University Observatory, except for the night of 1969 April 11 for which the 127-cm telescope at Kitt Peak National Observatory was used. The latter observations were generously taken for us by Dr. Barry Lasker, who used the University of Michigan data system described briefly elsewhere (Hesser and Lasker 1971). The observing equipment and techniques have been described by Lawrence, Ostriker, and Hesser (1967), Hesser, Ostriker, and Lawrence (1969), and Ostriker and Hesser (1969). In short, on each night of observation

TABLE 1

LOG OF OBSERVATIONS

Date	Starting Time UT	Starting Time JD$_{\odot}$	Record Length (s)	ϕ
1967 May 18	06:27:49	2,439,628.77097	6480	0.083
1967 September 25	02:00:55	2,439,758.58404	7200	0.534
1967 October 1	00:43:54	2,439,764.53035	1800	0.245
1967 October 2	02:23:39	2,439,765.59959	3480	0.767
1967 October 3	00:04:40	2,439,766.50303	5340	0.433
1969 April 11	08:51:30	2,440,322.86972	10752	0.921
1969 August 23	01:32:27	2,440,456.56534	8760	0.424
1969 September 29	23:59:59	2,440,494.49987	7500	0.345
1969 October 4	23:51:58	2,440,499.49413	9460	0.139
1969 October 5	23:56:27	2,440,500.49728	7520	0.321

DQ Her was monitored continuously with a pulse-counting photometer for periods up to 3 hours. Observations were obtained in white light, and the counts were recorded at intervals of 1^s or 2^s (KPNO). The full amplitude of the 71^s variation with the Princeton system was ~ 200 counts per minute.

Since we were interested in the 71^s period variation, we avoided or removed observations recorded during eclipse. Hence, we have no information concerning the eclipse light curve or the peculiarities which the 71^s variation reveals during eclipse.

Table 1 is a log of the observations. In addition to the date and starting times (UT and JD$_{\odot}$) we give the record length (number of seconds of continuous observation) and the binary phase (ϕ) at the beginning of each run computed from Walker's (1961) elements. It should be noted that the 1969 April 11 run at Kitt Peak is of higher quality than the Princeton observations, presumably for three reasons: (1) superior sky conditions, (2) use of a larger telescope, and (3) a longer record length. Because of this we sometimes consider the results from that night separately.

III. REDUCTIONS

The data for each night were first condensed into 5^s time units. A mean light curve for each night was then formed by stacking the data into 15 bins using a period of $71^s.06$. The difference between this stacking period and the actual period was small enough that no error in phase large enough to change the light curve accumulated over the length of a night's run. The light curve from each night also had associated with it a heliocentric time corresponding to the beginning of the light curve. A further small correction ($\pm 1^s$ at most) was added to some of the time lags obtained in the autocorrelation analysis to approximately account for the binary motion of the pulsating star.

The autocorrelation technique, described by Ostriker and Hesser (1968), has been used to determine the period accurately. When this technique is used, there is an inherent ambiguity in the period caused by the fact that the exact integral number of cycles which occurred during the calculated time lag is unknown. This means that, for any pair of nights, there are a number of different period possibilities. To choose the proper period (i.e., proper number of cycles elapsed between observed maxima), one plots the possibilities determined for different pairs of nights and looks for the period which is consistent with all the data; for a well-spaced series of observations, the period can be determined unambiguously.

IV. RESULTS

a) The Period of the 71-Second Variation in 1969

For use in determining an initial estimate for the period and as a matter of course, a power spectrum was obtained for each data set in table 1. A peak in the power spectrum corresponding to a period of $71^s.1$ was always present, although it was not always the strongest feature. This is presumably due to the irregular "flickering" noted by several observers. The location of the peaks provided us with an initial estimate for the period of the 71^s variation. No other statistically significant peaks were found.[1]

We have plotted in figure 1 (1) the period and associated probable error determined from a record length weighted average of the location of the peaks in the individual power spectra; (2) the location of the peak of the power spectrum obtained on 1969 April 11; (3) Walker's (1961) period. The power spectra data indicate a period roughly in the range $71^s.05$–

[1] Except possibly one at $\sim 89^s$.

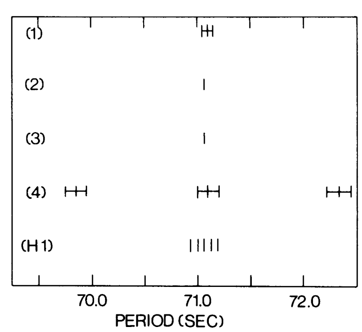

FIG. 1.—Initial estimates for P_s. (1) Mean of power spectra peaks, (2) peak of April 11 power spectrum, (3) Walkers' (1961) period, (4) April 11 run split in two parts, (H1) see table 2.

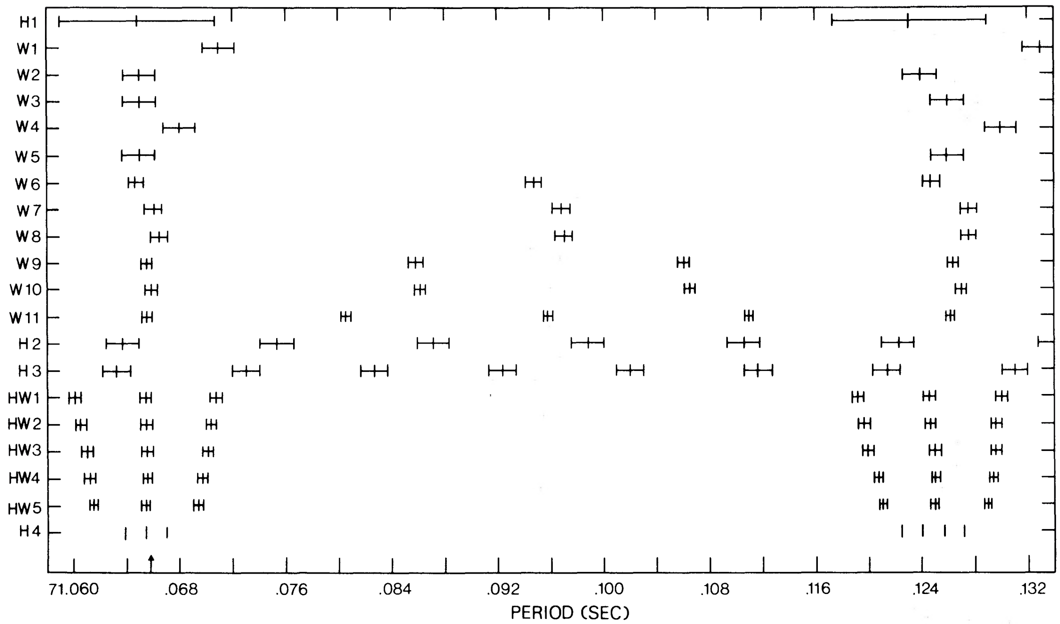

FIG. 2.—Period possibilities for pairs of nights with increasing time intervals. See table 2 for night pair codes and intervals. The arrow indicates Walker's (1961) period.

71ˢ15. Also shown in figure 1 are the results of autocorrelation analysis of the April 11 run (split in two equal segments) and the 1969 October 4–5 pair. On the basis of these data we expect a period around 71ˢ06 (although 71ˢ12 is not ruled out).

Figure 2 shows the results of autocorrelation analysis (for method, see Ostriker and Hesser 1968) of pairs of nights from the 1969 observing season, where we plot on each horizontal line the period possibilities obtained for a given pair of nights. Table 2 gives the code for night pairs and the associated time intervals. H numbers refer to runs given in table 1; W numbers, to runs

TABLE 2

NIGHT PAIR CODES

Code	Runs	Interval (days)	Code	Runs	Interval (days)
H1........	Oct. 4/Oct. 5	1	HW7.......	1003/Aug. 23	39
H2........	Sep. 29/Oct. 4	5	HW8.......	1036/Sep. 29	49
H3........	Sep. 29/Oct. 5	6	HW9.......	1028/Sep. 29	50
H4........	Aug. 23/Sep. 29	38	HW10......	1020/Sep. 29	51
H5........	Aug. 23/Oct. 4	43	HW11......	1015/Sep. 29	52
H6........	Aug. 23/Oct. 5	44	HW12......	1013/Sep. 29	53
H7........	Apr. 11/Aug. 23	134	HW13......	1036/Oct. 4	54
H8........	Apr. 11/Sep. 29	172	HW14......	1036/Oct. 5	55
H9........	Apr. 11/Oct. 4	177	HW15......	1028/Oct. 4	55
H10.......	Apr. 11/Oct. 5	178	HW16......	1028/Oct. 5	56
			HW17......	1020/Oct. 4	56
W1........	1003/1004	1	HW18......	1020/Oct. 5	57
W2........	1013/1015	1	HW19......	1015/Oct. 4	57
W3........	1015/1020	1	HW20......	1015/Oct. 5	58
W4........	1020/1028	1	HW21......	1013/Oct. 4	58
W5........	1028/1036	1	HW22......	1013/Oct. 5	59
W6........	1013/1020	2	HW23......	1004/Sep. 29	76
W7........	1015/1028	2	HW24......	1003/Sep. 29	77
W8........	1020/1036	2	HW25......	1004/Oct. 4	81
W9........	1013/1028	3	HW26......	1004/Oct. 5	82
W10.......	1015/1036	3	HW27......	1003/Oct. 4	82
W11.......	1013/1036	4	HW28......	1003/Oct. 5	83
			HW29......	Apr. 11/1003	95
HW1......	1036/Aug. 23	11	HW30......	Apr. 11/1004	96
HW2......	1028/Aug. 23	12	HW31......	Apr. 11/1013	119
HW3......	1020/Aug. 23	13	HW32......	Apr. 11/1015	120
HW4......	1015/Aug. 23	14	HW33......	Apr. 11/1020	121
HW5......	1013/Aug. 23	15	HW34......	Apr. 11/1028	122
HW6......	1004/Aug. 23	38	HW35......	Apr. 11/1036	123

TABLE 3

OBSERVED MAXIMA OF THE 71-SECOND VARIATION

Date	Time of Maximum ($JD_\odot$ − 2,439,000)
1967 May 18	628.77135
1967 September 25	758.58441
1967 October 1	764.53034
1967 October 2	765.59970
1967 October 3	766.50370
1969 April 11	1322.86973
1969 August 23	1456.56498
1969 September 29	1494.50031
1969 October 4	1499.49378
1969 October 5	1500.49718

given in table 3 of Warner *et al.* (1972); HW, to combinations of the two. For our data the time intervals were determined by autocorrelation. For the Warner *et al.* data, or when combining our data with theirs, time lags were determined solely from the estimated times of maxima. The times of maxima for our individual light curves are given in table 3. The error bars on each period possibility reflect the uncertainty in determining these times of maxima (estimated at $\pm 5^s$ for our light curves).

In figure 2 we restrict ourselves to the period range around 71^{s}1 indicated by the power spectra. Note that for pairs of nights with time lags greater than that of HW1, we plot for clarity only those period possibilities of interest. The arrow, at a period of 71^{s}06579, indicates Walker's (1961) period. As we go toward longer time separation between the pairs of nights, it becomes clear that no period near 71^{s}12 is consistent with all the data, leaving only 71^{s}064 < P_s < 71^{s}067.

Further refinement of the period is shown in figure 3; again the arrow without an error bar indicates Walker's period (his error bar is less than the width of the arrow). The other arrow indicates the adopted period and associated standard error determined as follows.

From figure 3 we determine the period accurately enough that the exact integral number of cycles between the April 11 maximum and all successive

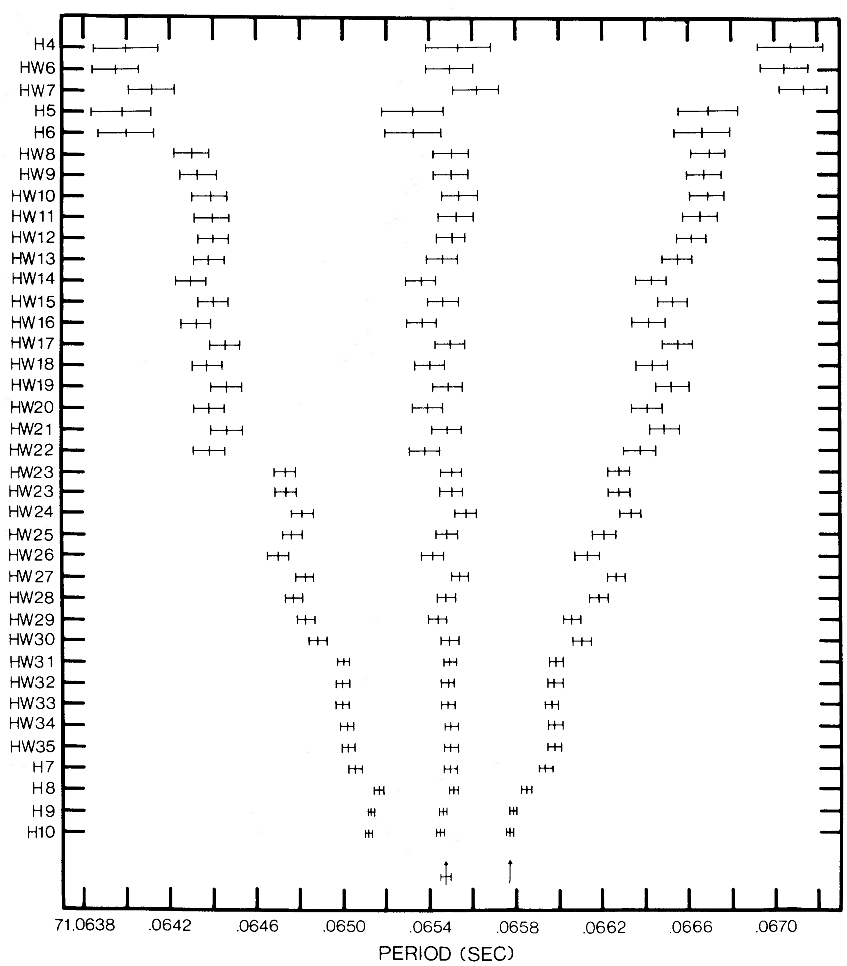

FIG. 3.—Same as fig. 2 but for larger time intervals. The arrow at 71^{s}06579 indicates Walker's (1961) period. The other arrow indicates the period and associated standard error for 1969 determined by a least-squares method described in the text.

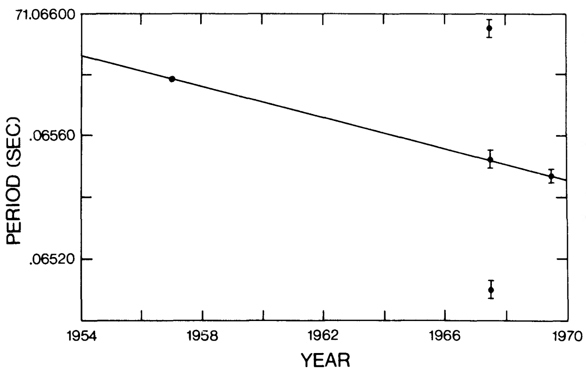

FIG. 4.—P_s as a function of time. Solid line connects 1954–1959 and 1969 periods. Points in 1967 represent period possibilities determined independently for that year.

maxima can be determined. These numbers were then used with the corresponding time lags in a least-squares fit. The result of this was $P_s = 71^s.06547 \pm 0^s.00002$ (s.e.), a significant change in the period since Walker's determination.

b) *Rate of Change of Period*

In addition to the 1969 observations there are times of maxima for the 71^s variation on five nights in 1967 (see table 1) and one night each in 1970 and 1971 (Warner *et al.* 1972). Although the five nights in 1967 are not properly spaced to allow an unambiguous determination of the period in that year, they do allow us to restrict the period to a set of discrete possibilities. Some of these are plotted in figure 4 along with the independent determinations in 1969 and 1954–1959. Clearly these data are consistent with an approximately linear decrease in the period from 1954 to 1969 at a rate of $\dot{P}_s \sim -2.4 \times 10^{-5}$ s yr^{-1}. From the period in 1969 and this rough knowledge of the rate of change of period, it is possible to determine the integral number of cycles elapsed between each observed maximum from 1967 to 1971. These numbers (which are "error-free" quantities) and the associated time lags allow us to determine the period and rate of change of period over this interval. A least-squares fit gives the following results:

$$\begin{aligned} \text{Epoch} &= \text{JD}_\odot\ 2{,}440{,}322.86977 \pm 0^d.00003\,, \\ P_s &= 71^s.065459 \pm 0^s.000002\,, \\ \dot{P}_s &= -2.66 \pm 0.22 \times 10^{-5}\ \text{s yr}^{-1} \\ & \qquad (1967\text{–}1971)\,. \end{aligned} \tag{1}$$

The quoted errors are standard errors. The standard deviation of the fit is $4^s.5$. There is no reason to introduce a higher-order fit to the data at this point.

One remaining problem is Walker's (1961) result that the period of the 71^s variation remained constant over the 5-year span of his observations. This is clearly inconsistent with the values for $\dot{P}_s$ that we give above if we suppose that the rate of period change has been constant. We have found, however, that it is possible to reinterpret Walker's data simply by assuming one less cycle between his 1954 observation and the others. That is, his cycle number 0 becomes our cycle 1, while all other cycle numbers are unchanged. A least-squares fit to his data with this change yields the following results:

$$\begin{aligned} \text{Epoch} &= \text{JD}_\odot\ 2{,}436{,}407.89657 \pm 0^d.00003\,, \\ P_s &= 71^s.065772 \pm 0^s.000004\,, \\ \dot{P}_s &= -2.91 \pm 0.20 \times 10^{-5}\ \text{s yr}^{-1} \\ & \qquad (1954\text{–}1959)\,. \end{aligned} \tag{2}$$

The standard deviation of this fit is $4^s.3$, identical to the standard deviation which Walker's analysis yields. Comparison of the $O - C$ diagram for this solution (fig. 5) with Walker's (1961) figure 4 shows that there is no reason to prefer one solution to the other, except that ours gives a period change which is consistent with the recent observations, while a constant-period solution is not compatible with them. Of course, the possibility remains that the period began to change sensibly only in 1959.

From the two periods and epochs given above we can determine a third and independent estimate of the rate of period change

$$\dot{P}_s = -2.8 \pm 0.2 \times 10^{-5}\ \text{s yr}^{-1} \quad (1958\text{–}1969) \tag{3}$$

in good agreement with the other two estimates.

We now inquire whether it is possible to tie all the data from 1954 to 1971 together (i.e., to determine the number of cycles ΔN elapsed between the two epochs above). If the time interval between the two epochs is Δt, then we have, to third order (neglecting the $\dot{P}^2$ term which is very small):

$$\Delta N = \frac{\Delta t}{P_s} - \tfrac{1}{2}\dot{P}_s\left(\frac{\Delta t}{P_s}\right)^2 - \tfrac{1}{6}P_s\ddot{P}_s\left(\frac{\Delta t}{P_s}\right)^3 . \tag{4}$$

We can estimate $\ddot{P}_s$ from the differences in $\dot{P}_s$ for the two epochs, and find $\ddot{P}_s = +2.3 \pm 2.8 \times 10^{-7}$ s yr^{-2}.

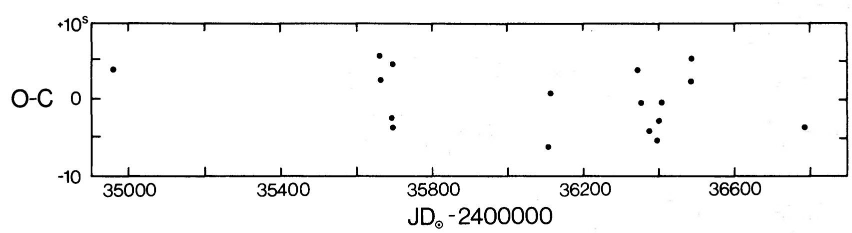

FIG. 5.—$O - C$ diagram for Walker's 1954–1959 data. P_s and $\dot{P}_s$ are given in eq. (2).

Calculating ΔN by using the values found for the 1958 epoch, we find $\Delta N = 4759737.3 \pm 0.8$. Working backwards from the 1969 epoch, we find $-\Delta N = 4759738.3 \pm 0.9$. The only integer falling within both error bars is 4759738. [This result is unchanged if we neglect the $(t/P_s)^3$ term.] Adopting this value for ΔN, we find that the best fit to all the data is

$$T_{\max} = 2440322.87022 + 8.2251691 \times 10^{-4}E - 3.51 \times 10^{-16}E^2 + 3.5 \times 10^{-24}E^3(\mathrm{JED}_\odot)\,. \tag{5}$$

This result was derived allowing for the difference between ET and UT (39^s at the epoch above). Therefore, $T_{\max}$ is given in Julian ephemeris days (JED). Alternatively we may write the above result as

$$\begin{aligned} P_s &= 71\overset{s}{.}065461 \pm 0\overset{s}{.}000002\,, \\ \dot{P}_s &= -2.69 \pm 0.08 \times 10^{-5}\ \mathrm{s\ yr^{-1}}\,, \\ |P/\dot{P}|_s &= 2.64 \times 10^6\ \mathrm{yr}\,, \\ \ddot{P}_s &= +3.6 \pm 1.1 \times 10^{-7}\ \mathrm{s\ yr^{-2}} \end{aligned} \tag{6}$$

at the above $\mathrm{JED}_\odot$. The second derivative appears to be relatively too big, indicating that, as is so often the case, 3 σ results are of doubtful significance. The first derivative, being derived independently from three sets of data, is almost certainly correct. The standard deviation of the fit is $4\overset{s}{.}3$; individual values of $O - C$ are shown in figure 6.

We can check the consistency of this interpretation by comparing this P_s and $\dot{P}_s$ with those determined for the same epoch but using only the recent data. They are in agreement. Similarly, we can calculate the P_s and $\dot{P}_s$ that we would expect at the 1958 epoch, and, once again, these numbers fall within the error bars of the quantities determined solely from the 1954–1959 data. If we adopt different values for ΔN (e.g., 4759737 or 4759739), we do not get this consistency and the standard deviations of the fits are also higher. It is interesting to note that the derived time scale for change in the pulsation period is of the same order as Nather and Warner (1969) found for the change in the eclipse period. In fact, to a high degree of accuracy,

$$|P/\dot{P}|_e \simeq 3|P/\dot{P}|_s\,. \tag{7}$$

The significance of this result is doubtful since $\dot{P}_e$ is not constant.

V. INTERPRETATIONS OF P_s

There are, as usual, two immediately obvious astronomical clock mechanisms available for producing the periodic variations: genuine pulsation, and rotation of a nonaxisymmetric star. On the basis of the usual arguments, a short period requires a dense star. If the variations arise from radial pulsations, then the source is a degenerate dwarf with mass 0.1–0.2 $M_\odot$ (cf. Kraft 1963), with the larger values appropriate if the outer hot layers distend the star significantly. If the clock is provided by rotation, any degenerate dwarf more massive than 0.2 $M_\odot$ would be suitable.

Both the observations of old novae and the theories available for understanding nova explosions indicate the probable presence of degenerate stars in nova systems as required by the proposed clock mechanism. The models usually envision (cf. Kraft 1963 and Paczynski 1971 for reviews) mass transfer from a red dwarf (mass M_R) to a white dwarf (mass M_W) separated by a distance a; the white dwarf is surrounded by a gaseous ring (radius $\sim 0.1a$) which in turn is often supplied by a gas stream emanating from the red companion.

It is quite conceivable that pulsations could be driven by unstable nuclear burning near the surface of the white dwarf. If thermal effects do not inflate the star too much, then adding mass to it will tend to increase its mean density and decrease its pulsation period. Nauenberg's (1972) approximate mass-radius relation for cold stars gives for mean polytropic index $n = 3/(2 - m)$; here

$$(d \log R/d \log M) \equiv (n - 1)/(n - 3)$$

and m is defined below.

Substituting into an approximate interpolation formula derived from the work of Hurley, Roberts, and Wright (1966),

$$\omega_{\mathrm{puls}} = (4\pi G\bar{\rho})^{1/2}(0.71 - 0.06n + 0.14n^2)$$

for $\gamma = 5/3$, we would roughly expect

$$\left(\frac{\Delta P}{P}\right)_s \approx -\frac{\Delta M_W}{M_W}(1 + 1.4m) \qquad (\text{for } m \ll 1)\,, \tag{8}$$

where

$$m \equiv (0.69 M_W)^{4/3}\,.$$

Following this interpretation, the mass transfer rate implied by the observed value of $\dot{P}_s$ is $\sim 5 \times 10^{-8}\ M_\odot$

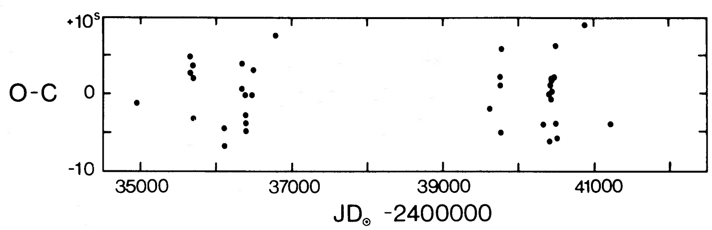

FIG. 6.—$O - C$ diagram for all data, 1954–1971. P_s, $\dot{P}_s$, and $\ddot{P}_s$ are given in eq. (6).

yr^{-1}, rather large considering that the low-mass companion would have an extremely long nuclear time scale.

The implied orbital change if there were no mass loss from the system is

$$\left(\frac{\Delta P}{P}\right)_{\rm orb} \approx -3\frac{\Delta M_W}{M_W}\left(1 - \frac{M_W}{M_R}\right), \qquad (9)$$

which has the wrong sign if the white dwarf is the less massive star, as required by the observations according to the pulsation hypothesis.[2] In any case, it is larger than the change in pulsation period (instead of smaller as observed) unless the two stars have nearly equal mass. Mass *loss* from the system at a judiciously chosen rate combined with mass *transfer* to the white dwarf can be arranged in the appropriate ratios to give both period changes their observed values, but the result seems somewhat contrived.

Furthermore, the long time scale for period change, the stability of the phase, and the stability of the period rate of change (itself changing by less than 10 percent since 1957) are difficult to understand according to the pulsation hypothesis, since the nova outburst occurred in 1934 and the thermal time scales in the blue star are relatively short. Thus, consider rotation.

The change in rotational period due to matter infall on M_W is easily calculated if one assumes cold, uniformly rotating, low-mass stars:

$$\left(\frac{\Delta P}{P}\right)_{\rm rot} = +\frac{\Delta M}{M_W}\left[-4.9(1 + 0.3m)\frac{\Omega_R}{\Omega} + 0.33(1 - 7.6m)\right], \qquad (10)$$

where

$$\Omega_R \equiv (GM_W/R_W{}^3)^{1/2}. \qquad (11)$$

The second term is due to the change in the moment of inertia; the first, to the change in angular momentum. The numerical coefficient of Ω_R/Ω is approximate; its sign follows from the assumption of prograde rotation. Now one can consider a moderately massive white dwarf of, say, 1 $M_\odot$. Then $\Omega_R/\Omega \approx 10$ and the mass transfer rate would be only $5 \times 10^{-9}\ M_\odot\ {\rm yr}^{-1}$. The companion star would now be less massive (from Kraft's 1963 table 1) so that $(\Delta P/P)_{\rm orb}$ would have the correct sign and approximately the correct value. Finally, the red dwarf would be sufficiently massive to fill its Roche lobe and evolve at the required rate to drive the mass transfer. A more detailed discussion of this model is reserved for a later communication.

VI. VARIATIONS OF PHASE

Warner *et al.* (1972) present evidence for a rapid increase in phase as the pulsating star passes through eclipse, the total change being 2π per orbit. Since all observations of phase are modulo 2π, we may equally well consider the phase to vary from 0 to $+\pi$ as the eclipse is entered and from $-\pi$ to 0 during exit with no net change. The above authors propose nonradial pulsations as the source of the variability.

However, there is a serious objection to any interpretation wherein the light variations are assumed to come from the white dwarf itself. The ingress to and egress from eclipse of the pulsating component each appears (from figs. 1 and 5 of Warner *et al.* 1972) to last longer than $\sim 400^s$. The relative velocity of the two stars is in excess of 150 km s^{-1} (the value of K_1, Kraft 1963). Thus the pulsating component is physically larger than 6×10^9 cm and is probably several times this value. This is large compared with the diameter of even a low-mass degenerate dwarf. Thus it is more attractive to consider the possibility that the pulsating component arises from a circumstellar disk. There are several possible models one can construct wherein some disturbance propagates around the disk with a period of 71^s.[3] One can show then that the gradual eclipse of the disk would give an appropriate phase change. As an example, consider the case in which the back side of a disk is brightly illuminated (reflection) by a disturbance having the character $\cos(2\theta - \omega t)$, where θ is longitude on the disk and $\omega = 2\pi/P_s$. Let Q be the amplitude of the variable component and S the steady light from the disk. Then one can show that, when the fraction of the disk eclipsed is η, the disk emission, $I(t)$, is

$$I(t) = S(1-\eta) + Q(1-\cos\omega t), \quad 0 < \frac{t\omega}{2\pi} < 1 - \eta,$$
$$I(t) = S(1-\eta), \quad (1-\eta) < \frac{t\omega}{2\pi} < 1. \qquad (12)$$

Taking the sine and cosine transforms of $I(t)$, we find that phase ϕ is given by

$$\tan\phi = \frac{1 - \cos 2\pi\eta + (1 - \cos 4\pi\eta)/4}{+\sin 2\pi\eta + \pi(1-\eta) - (\sin 4\pi\eta)/4}, \qquad (13)$$

and varies steadily from 0 to π as the eclipse proceeds. On egress the other edge of the disk is seen first and the phase varies from $-\pi$ to 0.

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[2] Pulsation ($l = 2$) implies $M_W \approx 0.15\ M_\odot$; then the mass function (Kraft 1963) gives $M_R \approx 0.18$.

[3] A similar interpretation has been recently and independently suggested by M. Rees in a paper by Bath, Evans, and Pringle (1974).

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