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BLACK-HOLE–NEUTRON-STAR COLLISIONS

JAMES M. LATTIMER AND DAVID N. SCHRAMM
Departments of Astronomy and Physics, The University of Texas at Austin
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ABSTRACT

The tidal breakup of a neutron star near a black hole is examined. A simple model for the interaction is calculated, and the results show that the amount of neutron-star material ejected into the interstellar medium may be significant. Using reasonable stellar statistics, the estimated quantity of ejected material is found to be roughly comparable to the abundance of r-process material.

Subject headings: black holes — hydrodynamics — mass loss — neutron stars

The tidal encounter of an object with a black hole, with the resultant "tube of toothpaste" effect, has been elucidated by Wheeler (1971) and examined by Mashhoon (1973) and Bardeen, Press, and Teukolsky (1972). We examine here the specific case of black-hole–neutron-star collisions. Although these circumstances would appear to be relatively improbable, it is demonstrated below that a significant amount of neutron-star material may suffer tidal ejection into the interstellar medium via such collision processes.

Let a neutron star of mass m_{ns} be moving in the Schwarzschild gravitational field of a black hole of mass M_{bh}. The components $F_T{}^i$ of the tidal force exerted by the black hole in the Fermi frame (Manasse and Misner 1963) of the neutron star are given in the equation of geodesic deviation (Misner 1969)

$$F_T{}^i \text{ per unit mass} = -R^{(i)}{}_{(0)(j)(0)}x^{(j)}, \qquad (1)$$

where $x^{(j)}$ gives the coordinates in the Fermi frame, and $R^{(i)}{}_{(0)(j)(0)}$ lists the components of the Riemann tensor as evaluated in the Fermi frame. This equation is valid only when applied to distances much less than (average value of Riemann tensor)$^{-1/2}$. When the neutron star approaches the black hole too closely, the neutron star grows dynamically unstable and breaks up. Call the distance from the black hole at which this occurs the "Roche limit" (Fishbone 1972*a*, *b*). For a first approximation to this limit one can refer to the analysis performed by Fishbone (1972*b*), displayed in figure 1, for the motion of an incompressible Newtonian fluid on a circular geodesic trajectory. It is clearly apparent that the Roche limit lies well outside the horizon of the black hole.

Because the Roche limit is outside the event horizon, the possibility exists that some of the disrupted material may be ejected to infinity. Of course, unbound hyperbolic orbits which penetrate the Roche limit can yield such ejection, but the probability of such events is low. This *Letter* will concentrate on the most plausible astrophysically important case: a neutron star in a bound circular orbit decaying due to the emission of gravitational radiation. The rate of decay in the weak-field approximation is given in Zel'dovich and Novikov (1971).

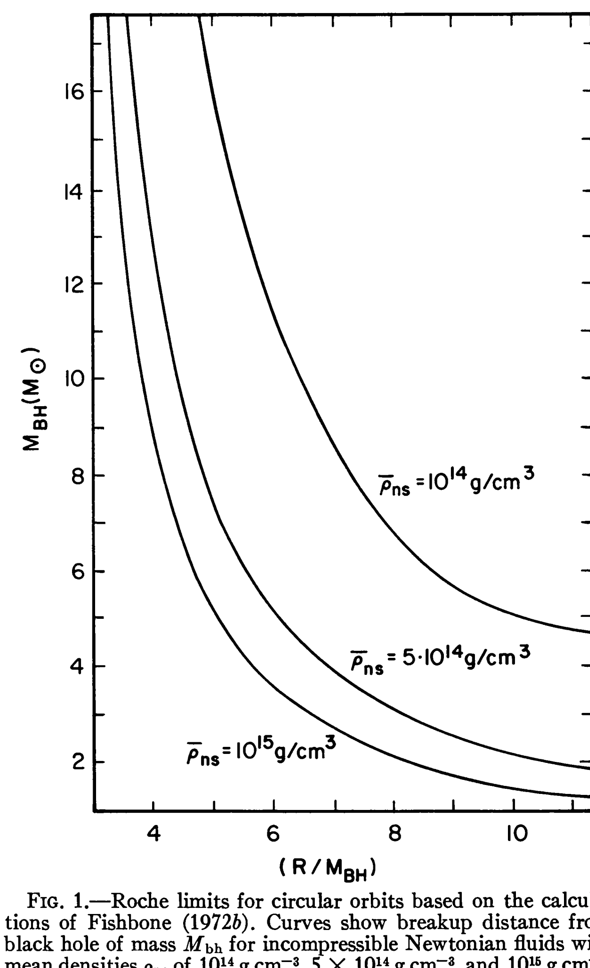

FIG. 1.—Roche limits for circular orbits based on the calculations of Fishbone (1972*b*). Curves show breakup distance from black hole of mass M_{bh} for incompressible Newtonian fluids with mean densities ρ_{ns} of 10^{14} g cm^{-3}, 5×10^{14} g cm^{-3}, and 10^{15} g cm^{-3}. The distance from the black hole is given in units of the geometrized mass of the black hole.

One can estimate the relative velocities imparted to the disrupted material by the time the Roche limit is reached. If part of the material has velocities which correspond to escape velocities (i.e., energies adequate to cause that part of the material to escape from the system to infinity), then that part of the material would escape if considered free from then on. Since the tidal force dominates the self-gravity of the neutron star at the Roche limit, one can make the approximation that the self-gravity is negligible after the neutron star reaches the Roche limit.

Notice that we do not assume an unbound free-particle (self-gravity neglected) model until after the neutron star has broken up. Obviously if one assumes a free-particle model throughout, then all the particles are on bound orbits and there can be no ejecta. In the model considered, particles near the center of mass will, of course, continue to follow the center of mass trajectory (into the black hole), but the rest of the material may, in some instances, have received tidal and hydrodynamic boosts sufficient to expel them from the system. (In general, tidal forces alone are insufficient to eject matter to infinity.) Thus the problem reduces to that of estimating the internal velocities of the neutron-star material at breakup.

To estimate these velocities we have assumed the neutron star to be an incompressible homogeneous Newtonian ellipsoid. One can derive from the Euler hydrodynamical equations the matrix equation describing the semiaxes of the ellipsoid (Chandrasekhar 1969; Mashhoon 1972). Using the formalism of Chandrasekhar (1969) and Manasse and Misner (1963), one can find the velocities (in the Fermi frame) for each position in the neutron star. It is then easy to obtain the energy at infinity per unit rest mass, γ, and the angular momentum per unit rest mass, l, for each position in the star following Misner, Thorne, and Wheeler (1973).

Positions in the neutron star with escape velocities at the Roche limit have $\gamma > 1$ and periastron $> 2\ M_{\rm bh}$. We have plotted in figure 2 the typical magnitude of the velocity required in the Fermi frame for escape from the system to infinity. Also displayed is $\gamma_{\rm CIRC}$, the energy of a particle in a circular orbit at the indicated R. Notice, for large R, that as the energy of the orbit decreases a higher escape velocity is required. When $R < 6\ M_{\rm bh}$, the approximations used begin to break down and $\gamma_{\rm CIRC}$ may no longer be considered the energy of a particle at rest in the Fermi frame.

We have shown in figure 2 some of the results of this model for $M_{\rm bh} = 10\ M_\odot$, $m_{\rm ns} = 1\ M_\odot$, and $\bar{\rho}_{\rm ns} = 10^{14}$ g cm^{-3}. The average surface velocities of the ellipsoid actually obtained are graphed: note the rapid rise of this velocity at the apparent Roche limit around 6.3–6.5 $M_{\rm bh}$ (compare this with fig. 1, for $\bar{\rho}_{\rm ns} = 10^{14}$ g cm^{-3}). These velocities quickly exceed the escape velocity. Since the ellipsoid calculation is Newtonian, the results should not be taken seriously when the resultant velocities approach c; however, escape velocities at these radii are nonrelativistic.

We have also determined the amount of matter ejected to infinity. Since the results are sensitive to where the transition from ellipsoid to free-particle model takes place, we have carried the calculation out for a number of assumed Roche limits ($R_{\rm R}$) in the interesting range $R_{\rm R} = 6$–$7\ M_{\rm bh}$. The instantaneous velocities of 60 equally spaced points within the ellipsoid are computed. Using the transformation equations, orbits for each of these points are obtained on the assumption that they will now follow ordinary Schwarzschild geodesics. From this, the mass fraction of material that would escape if $R_{\rm R}$ were the real Roche limit is obtained. The result of this calculation is that in the region $R_{\rm R} = 6.6$–$7\ M_{\rm bh}$ the mass ejection is 0, while in the region $R_{\rm R} = 6.25$–$6.6\ M_{\rm bh}$ the mass ejection varies from 0.00 to 0.20 $m_{\rm ns}$. Since the actual breakup is not instantaneous (though it occurs quickly as can be seen in fig. 2), it is reasonable to take an average over several values of $R_{\rm R}$. We then obtain an average amount of mass ejection of $\sim 0.05 \pm 0.05\ m_{\rm ns}$.

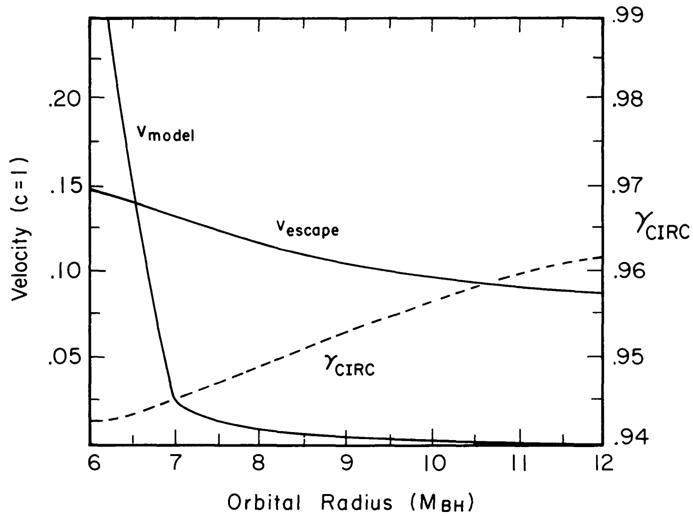

FIG. 2.—Results of the ellipsoidal model described in this paper. The rest (Fermi) frame of the neutron star (ellipsoid) is in a decaying circular orbit (Zel'dovich and Novikov 1971) at the indicated radius. The energy per unit rest mass of a particle stationary in the Fermi frame is plotted as $\gamma_{\rm CIRC}$. The magnitude of the velocity (in the Fermi frame) required for a particle to escape to infinity is $v_{\rm escape}$, while $v_{\rm model}$ is the typical surface velocity of the ellipsoid at R. The orbital radius is in units of $M_{\rm bh}$, and $c = 1$. For this case $M_{\rm bh} = 10\ M_\odot$, $m_{\rm ns} = 1\ M_\odot$, and $\bar{\rho}_{\rm ns} = 10^{14}$ g cm^{-3}.

These findings are not in conflict with the results of an earlier more naive model (Lattimer and Schramm 1974) which also indicated that roughly $\sim 0.05\ m_{\rm ns}$ would be ejected to infinity. However, the possibility of zero mass ejection cannot be totally excluded, and work on more advanced models is currently being undertaken to more accurately estimate the mass ejected.

Several qualifications should be noted here. When $R \sim 6\ M_{\rm bh}$, the equation of geodesic deviation is valid only when applied to distances much less than $(M_{\rm bh}/R^3)^{-1/2}$, or about 15 $M_{\rm bh}$. In the case considered, the ellipsoid was not deformed to within 0.1 of this value until $R \lesssim 6.2\ M_{\rm bh}$ was reached. Also note that when the Roche limit is less than 6 $M_{\rm bh}$, the weak-field approximation of the orbital decay rate can no longer be considered valid. For the neutron-star densities we are considering, this occurs when $M_{\rm bh} \gtrsim 12\ M_\odot$. Finally, these calculations lose a degree of validity when $M_{bh} \lesssim 5\ m_{ns}$, since then the geometrical effects of the gravitational mass of the neutron star are not to be overlooked.

Believing the estimate of mass loss per black-hole–neutron-star encounter to be ~5 percent, we can estimate the amount of such mass ejection per square parsec in the disk of the Galaxy. The models of massive stars discussed by Arnett and Schramm (1973) indicate that stars $\gtrsim 8\ M_\odot$ eventually evolve into neutron stars or black holes. For the sake of simplicity, assume that, say, one-half of all stars greater than 8 $M_\odot$ form black holes in the proper range, and one-half form neutron stars. Black-hole–neutron-star pairs that begin with a separation of $\lesssim 0.1$ a.u. decay gravitationally in a time less than the age of the Universe. (In fact, if effects other than gravitational radiation are taken into account, binaries with initial separations significantly greater than 0.1 a.u. can decay within 10^{10} years—see, e.g., van den Huevel and de Loore 1973.) Thus, using the statistics of Heintz (1969), ~6 percent of all binaries which have been estimated to be composed of black-hole–neutron-star pairs are in the proper range.

Heintz also estimates that ~60 percent of the stars in the upper regions of the main sequence are in binary systems. Ostriker, Richstone, and Thuan (1974) give the rate of formation of stars greater than 8 $M_\odot$ in the galactic plane as 1.1×10^{-11} pc^{-2} yr^{-1}. A conservative correction for the higher stellar formation rate earlier in the history of the Galaxy can be obtained by multiplying this rate by ~2 (Tinsley 1974). Assuming the age of the Galaxy to be ~10^{10} years (Schramm 1973), and combining the above factors, one sees that an accumulation of $2 \times 10^{-4}\ M_\odot$ pc^{-2} of ejected neutron-star material is implied. Since the total surface density in the solar neighborhood is 75 $M_\odot$ pc^{-2} (Oort 1965), the mass fraction estimated for the ejected material is ~3×10^{-6}. Note that this is approximately the mass fraction of the *r*-process elements (Burbridge *et al.* 1957), and only an order of magnitude less than the observed deuterium abundance (Rogerson and York 1974).

Thus, to recapitulate, black holes of less than about 20 $M_\odot$ are capable of tidally disrupting a neutron star. This breakup occurs well outside the event horizon, in contrast to the order-of-magnitude estimate of Bardeen *et al.* (1972). Furthermore, it appears that the ejection is violent enough for a certain amount of neutron-star matter to be ejected into the interstellar medium. Initially being very neutronized, this matter may be converted into deuterium or even *r*-process material. In fact, to rough orders of magnitude, stellar statistics indicate that enough of this matter can be ejected to be of comparable magnitude to the observed solar-system *r*-process abundances.

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James M. Lattimer: Department of Astronomy, University of Texas, 15.212 RH Moore Building, Austin, TX 78712

David N. Schramm: Enrico Fermi Institute, University of Chicago, Chicago, IL 60637