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OPTICAL POLARIZATION OF THE CRAB NEBULA PULSAR. III. NEW OBSERVATIONS, PREDICTIONS, AND THE POSSIBILITY OF VARIABILITY

D. C. FERGUSON, W. J. COCKE, AND T. GEHRELS
University of Arizona, Tucson, Arizona 85721
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ABSTRACT

New and very precise measurements of the optical polarization of the Crab Nebula pulsar are presented. Better fits by the relativistic vector model than were previously reported are possible due to an improved method of plotting the data. A prediction of the true polarization arising outside the pulsar is made. The possibility of secular changes in the pulsar's polarization is discussed.

Subject headings: polarization — pulsars

I. THE OBSERVATIONS

Our observational procedure has been previously described by Cocke *et al.* (1970) and Cocke, Ferguson, and Muncaster (1973). A two-channel polarimeter was used at the Cassegrain focus of the Steward Observatory 2.3-m telescope, in conjunction with a multiscalar (hereafter referred to as the CAT, for computer of average transients) and this time also rigged in parallel with a NOVA 1200 digital computer (hereafter referred to as the NOVA). The two polarimeter outputs were cycled simultaneously through two separate memory banks in the CAT and two in the NOVA. The total sweep time for the memory banks was set equal to the appropriate Doppler-shifted pulsar frequency. Pulse amplifiers and photon-counting techniques were used to record the fluxes.

Channel width for the CAT was 0.179 ms and for the NOVA was 0.069 ms. The measurements were made for a total of 1018^{m} (17^{h}) on the nights of the following UT dates in 1972: November 29 for 203^{m}, November 30 for 229^{m}, December 1 for 297^{m}, and December 2 for 289^{m}. All measurements were made with a focal-plane diaphragm of either 5″ or 7″ diameter and unfiltered photocathodes having S-13 response. The CAT readings were logged on magnetic tape. Those of the NOVA were punched on paper tape. All data reduction was done with the University of Arizona CDC 6400 computer.

Each channel of the CAT and the NOVA contained light from the pulsar, nebular background, and the north-following star (the pulsar's optical companion). It was assumed that the light from the pulsar itself was negligible during a time interval 2.67 ms (15 CAT channels or 38 NOVA channels) in width. This was centered at times 6.5 ms before the main pulse peak for the CAT data and 9.1 ms before the main pulse peak for the NOVA data, and the averages of these channels were subtracted from the raw data. The CAT reductions are consistent with the reduction of our 1969, 1970–71, and 1971 data as presented in Cocke *et al.* (1973, hereinafter referred to as Paper II).

In tables 1 and 2 we present the latest results, obtained with our two systems of data logging. In the case of the NOVA data, we include both the raw data and the data corrected for extrinsic[1] polarization, in this case taken to be 3.6 percent at 167°.5 (a value predicted by the relativistic vector model as discussed in § III). The errors given are rms statistical variances. It can be seen that the errors are significantly increased

TABLE 1

CAT DATA; NP0532 POLARIZATION

BINS	MEAN TIME (ms)	UNCORRECTED	
		P(%)	Θ(°)
25–27	− 3.04	15.4 ± 1.4	56.5 ± 2.5
28–30	− 2.50	15.5 ± 0.8	63.5 ± 1.5
31–33	− 1.97	15.7 ± 0.3	72.5 ± 0.5
34–36	− 1.43	15.6 ± 0.2	83.1 ± 0.5
37–39	− 0.89	13.3 ± 0.1	95.1 ± 0.3
40–42	− 0.36	10.0 ± 0.2	109.6 ± 0.4
43–45	+ 0.18	2.8 ± 0.3	140.2 ± 2.7
46–48	+ 0.72	3.5 ± 0.2	170.9 ± 1.8
49–51	+ 1.25	2.8 ± 0.3	162.8 ± 3.2
52–54	+ 1.79	2.8 ± 0.5	152.0 ± 4.7
55–57	+ 2.33	4.3 ± 0.6	136.2 ± 4.2
58–60	+ 2.86	3.9 ± 0.8	135.7 ± 6.3
94–96	+ 9.30	13.5 ± 2.4	84.4 ± 5.4
97–99	+ 9.85	15.7 ± 1.7	90.5 ± 3.1
100–102 ...	+10.38	13.9 ± 1.1	93.2 ± 2.4
103–105 ...	+10.91	12.0 ± 0.7	85.4 ± 1.6
106–108 ...	+11.45	12.9 ± 0.6	91.5 ± 1.3
109–111 ...	+11.99	11.5 ± 0.5	93.1 ± 1.2
112–114 ...	+12.52	10.8 ± 0.4	97.3 ± 1.0
115–117 ...	+13.06	8.1 ± 0.3	102.2 ± 1.0
118–120 ...	+13.60	5.0 ± 0.3	111.3 ± 1.3
121–123 ...	+14.13	2.8 ± 0.3	132.2 ± 3.4
124–126 ...	+14.67	2.6 ± 0.3	149.6 ± 2.9
127–129 ...	+15.21	2.7 ± 0.4	167.1 ± 4.4
130–132 ...	+15.74	2.6 ± 0.4	177.5 ± 5.0
133–135 ...	+16.28	2.5 ± 0.5	177.8 ± 5.4
136–138 ...	+16.82	4.7 ± 0.7	176.9 ± 4.2
*139–141 ...	+17.35	3.8 ± 1.0	9.7 ± 7.6
142–144 ...	+17.89	5.4 + 1.2	15.3 ± 6.6
145–147 ...	+18.43	8.7 ± 1.4	13.6 ± 4.7

* $P < 4\sigma$.

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[1] Extrinsic polarization here means polarization arising outside the source, either in the interstellar medium or in the neighborhood of the pulsar, or both.

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TABLE 2

NOVA DATA: NP 0532 POLARIZATION

WINDOW NO.	BINS	MEAN TIME (ms)	UNCORRECTED		CORRECTED	
			P(%)	Θ(°)	P(%)	Θ(°)
1	69–74	− 2.65	13.92 ± 0.79	59.6 ± 2.3	16.97 ± 0.79	63.2 ± 1.3
2	75–80	− 2.24	16.98 ± 0.64	65.8 ± 1.1	20.33 ± 0.64	67.8 ± 0.9
3	81–86	− 1.82	16.04 ± 0.26	72.8 ± 0.5	19.60 ± 0.26	73.6 ± 0.4
4	87–92	− 1.41	16.19 ± 0.22	82.5 ± 0.4	19.75 ± 0.22	81.6 ± 0.3
5	93–98	− 1.00	14.41 ± 0.19	92.1 ± 0.4	17.64 ± 0.19	89.2 ± 0.3
6	99–104	− 0.58	12.38 ± 0.14	104.0 ± 0.3	14.82 ± 0.14	98.4 ± 0.3
7	105–110	− 0.17	9.14 ± 0.11	115.8 ± 0.4	10.57 ± 0.11	106.1 ± 0.3
8	111–116	+ 0.24	3.80 ± 0.17	147.2 ± 1.3	2.57 ± 0.17	114.4 ± 1.9
9	117–122	+ 0.65	4.10 ± 0.20	172.1 ± 1.4	0.79 ± 0.20	15.3 ± 7.1
10	123–128	+ 1.07	3.88 ± 0.26	165.1 ± 2.0	0.42 ± 0.26	142.4 ± 16.0
11	129–134	+ 1.48	4.10 ± 0.45	156.0 ± 3.2	1.60 ± 0.45	125.0 ± 7.9
12	135–140	+ 1.89	2.64 ± 0.56	144.4 ± 5.8	2.60 ± 0.56	101.1 ± 6.1
13	141–146	+ 2.30	4.99 ± 0.76	131.0 ± 4.3	5.23 ± 0.76	110.4 ± 4.1
	237–242	+ 8.91	12.14 ± 1.94	57.5 ± 4.4	15.08 ± 1.94	61.9 ± 3.7
	243–248	+ 9.32	15.34 ± 2.20	74.9 ± 4.2	18.93 ± 2.20	75.4 ± 3.3
	249–254	+ 9.74	11.14 ± 1.90	96.6 ± 4.8	14.15 ± 1.90	92.1 ± 3.8
	255–260	+10.15	13.41 ± 1.58	88.4 ± 3.4	16.80 ± 1.58	86.1 ± 2.7
	261–266	+10.56	12.24 ± 1.15	91.4 ± 2.7	15.52 ± 1.15	88.3 ± 2.1
14	267–272	+10.97	12.18 ± 0.73	92.6 ± 1.8	15.40 ± 0.73	89.2 ± 1.4
15	273–278	+11.39	13.41 ± 0.69	93.2 ± 1.5	16.59 ± 0.69	89.9 ± 1.2
16	279–284	+11.80	12.52 ± 0.45	91.1 ± 1.0	15.80 ± 0.45	88.1 ± 0.8
17	285–290	+12.21	10.67 ± 0.31	92.5 ± 1.0	13.90 ± 0.31	88.8 ± 0.6
18	291–296	+12.62	9.61 ± 0.32	97.6 ± 1.0	12.58 ± 0.32	92.3 ± 0.7
19	297–302	+13.04	8.35 ± 0.31	105.9 ± 1.1	10.75 ± 0.31	97.8 ± 0.8
20	303–308	+13.45	6.16 ± 0.30	108.0 ± 1.4	8.51 ± 0.30	97.1 ± 1.0
21	309–314	+13.86	3.36 ± 0.32	131.3 ± 2.7	4.12 ± 0.32	103.1 ± 2.2
22	315–320	+14.28	2.12 ± 0.29	139.6 ± 3.8	2.98 ± 0.29	95.5 ± 2.8
23	321–326	+14.69	2.70 ± 0.41	150.7 ± 4.3	2.02 ± 0.41	101.4 ± 5.7
24	327–332	+15.10	3.37 ± 0.43	165.8 ± 3.6	0.31 ± 0.43	97.8 ± 27.1
25	333–338	+15.51	3.67 ± 0.45	2.2 ± 3.6	1.84 ± 0.45	38.8 ± 6.9
26	*339–344	+15.93	2.65 ± 0.68	2.3 ± 7.7	1.84 ± 0.68	54.9 ± 10.1
27	345–350	+16.34	3.92 ± 0.56	176.6 ± 4.0	1.23 ± 0.56	29.6 ± 12.3
28	*351–356	+16.75	2.01 ± 0.93	178.0 ± 13.1	1.87 ± 0.93	66.2 ± 13.2
	357–362	+17.17	4.53 ± 1.07	8.4 ± 6.8	3.03 ± 1.07	34.6 ± 9.7
	363–368	+17.58	3.73 ± 1.02	14.6 ± 8.1	3.34 ± 1.02	45.1 ± 8.5
	*369–374	+17.99	3.16 ± 1.32	24.6 ± 11.9	4.09 ± 1.32	53.5 ± 8.9
	375–380	+18.40	10.35 ± 1.48	12.2 ± 4.1	8.46 ± 1.48	21.6 ± 5.0

* $P < 4\sigma$.

in Θ (the position angle of the electric vector maximum relative to celestial north) near the polarization minima by the removal of the extrinsic polarization. The reason for this is apparent in figures 1 and 2 which show much of the NOVA data in the following manner somewhat more useful than presented previously. In these figures (fig. 1 shows the main optical pulse, fig. 2 the secondary) the amount of polarization of the raw data is plotted radially and twice the position angle is plotted longitudinally. In such plots, the extrinsic polarization is a constant vector to be subtracted from the measured values. Also, one can see that since the relative Stokes parameters are defined as $Q/I = P \sin 2\Theta$ and $U/I = P \cos 2\Theta$ for a source with no circular polarization, these parameters may be read directly off such a plot as distances from the zero lines in two orthogonal (x, y)-directions. Removal of the extrinsic polarization amounts to a translation (without rotation) of the axes of the plot so that the new origin lies at the end of the extrinsic polarization vector. When this is done (for any reasonable value of the extrinsic polarization in a direction similar to that removed here or in Paper II) the position angle errors of measurements near polarization minimum must increase, as these measurements are then nearer to, and the error boxes subtend larger angles at, the origin.

From the tables it can be seen that, while the polarization angles from the uncorrected CAT and NOVA data are identical at identical phases of the light curve (to within the errors of the observations), the polarization amounts from the CAT data are often slightly smaller than those from the NOVA data. This is attributed to the fact that shorter time windows were used in the NOVA logged reductions, and thus whenever the intrinsic position angle was sweeping, less variation in polarization angle took place within each NOVA time window than in the corresponding CAT window. Also, the subtracted backgrounds were centered on different times for the two data sets, which might account for a slight amount of difference. The same general behavior is exhibited in each case, however, and in all the polarization measurements we have made since 1969. In both pulses the amount of

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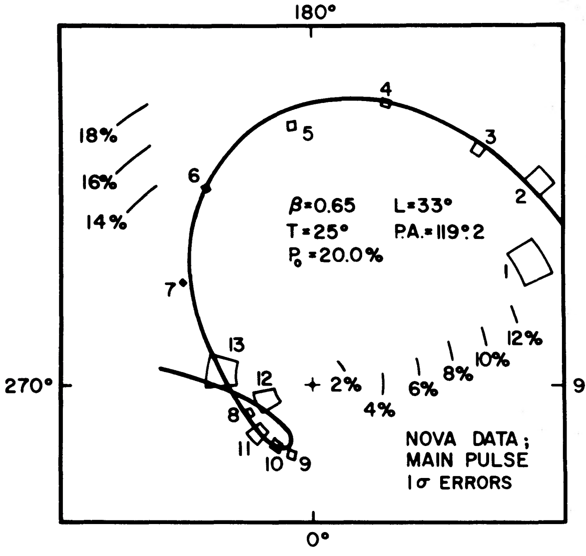

FIG. 1

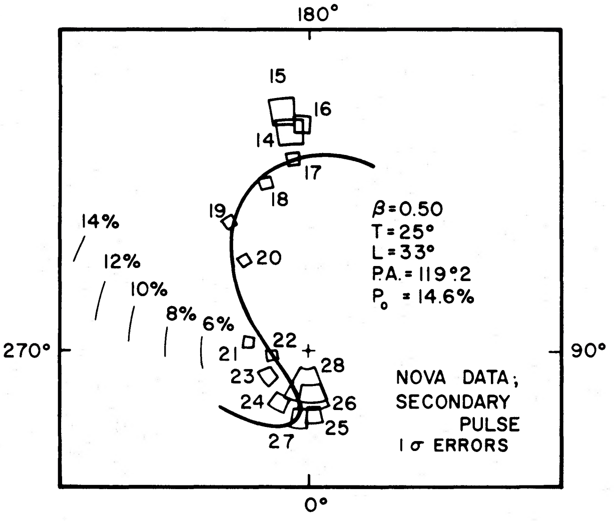

FIG. 2

FIG. 1.—The behavior of the position angle versus the amount of polarization for the main optical pulse. The polarization amount is plotted radially; twice the position angle of the electric vector is plotted longitudinally. The numbers by the error boxes indicate the window numbers of table 2. The heavy line is a theoretical fit by the relativistic vector model, as discussed in § III.

FIG. 2.—Same as fig. 1, but for the secondary pulse.

polarization decreases to a minimum near the time of maximum sweep rate of the polarization angle, which always occurs at the same angle and phase, and then slowly rises.

II. SECULAR CHANGES

The question of the possible secular variability of the polarization of the Crab Nebula pulsar which was raised in Paper II is still open, despite extensive checks for possible systematic errors in the measurements. As can be seen from examination of table 3 of Paper II, and the tables of this paper, the uncorrected data obtained in the fall of 1972 have almost exactly twice the polarization amounts, but identical position angles, as those obtained for corresponding phases in the fall of 1971. Such behavior of the data, uncorrected for the effects of interstellar polarization, could be explained in one of the following four ways:

1. The extrinsic polarization is zero and the intrinsic polarization at all phases of the light curve varies by the same factor over a period of years. Such an interpretation is unlikely, because the *interstellar* polarization has been estimated by several authors to be a few percent at the distance of the pulsar (see § III). However, if this interpretation is correct, the relativistic vector model cannot be valid.

2. The intrinsic polarization of the pulsar at every phase varies in the same ratio as a variable extrinsic polarization between the pulsar and observer. Since no evidence has ever been found for a variable *interstellar* polarization, this possible explanation seems very unlikely. However, some type of plasma process in the vicinity of the pulsar could be envisioned which might have such an effect.

3. The extrinsic polarization is a nonzero constant, but the intrinsic polarization of the pulsar varies over the years by different factors at different phases of the light curve, in just such a way as to make the total observed polarization at every phase seem to vary by a single factor at unchanged position angle. It is hard to imagine a mechanism for such a change, and it is just short of incredible that the uncorrected position angles could remain unchanged in such a complicated variation. In this case, also, the simple relativistic vector model could not be valid.

4. Some systematic error in either the 1969 and 1972 data or in the 1970/71 and 1971 data made the uncorrected amounts for those years a factor of 2 different from the correct values. The possibility of systematic errors is always present, although the checks for them described below have not revealed them.

The data taken in the year 1969 (Cocke *et al.* 1970) agree to within the errors with those of Kristian *et al.* (1970) in both angle and amount, and also agree with the data taken in the year 1972 and presented in this paper. A standard star (HD 50064) was measured on the same date and with the same equipment and reduction routines as those of the pulsar in 1972 and the results agree with measurements previously published by Serkowski, Gehrels, and Wisniewski (1969). Thus the case is made for the absence of systematic errors in the measurements of 1969 and 1972.

The data taken with *U*, *B*, and *V* filters in 1970/71 agree in angle and amount with those obtained in unfiltered light in the fall of 1971 (Paper II). In 1971 a check of the equipment used that year and the previous year was carried out using Polaroid filters on the Steward Observatory 21-inch (53-cm) telescope in Tucson, and the system gave identical polarizations to

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those measured independently by Gehrels (private communication). Also, the light curve formed by the sum of the two perpendicularly polarized channels was compared with the light curve formed by the sum of the two unpolarized channels (runs made with the depolarizer in the beam) at corresponding instrumental angles, and all such light curves were intercompared and compared with other published light curves. No significant differences (aside from those due to different time resolutions) between our data and other published light curves were found. Thus the case is made for the absence of important systematic errors in the 1970/71 and 1971 data.

A single flaw in a circuit of the electronics used in 1970 and 1971 has been found (see Muncaster and Cocke 1972) which lowered the number of counts in one channel of the polarimeter preferentially at times of high count rate, such as those of the main pulse peak, by a maximum of 3 percent. As this flaw was only important near the main pulse peak, however, it could not account for a lowering of the polarization by a constant factor in both the main and secondary pulses. Thus, the statement we made in Paper II that a thorough check of the electronics yielded no flaw which could be responsible for the discrepancy was correct.

On account of the independent confirmation by other groups of our polarizations in 1969 and 1972, we believe that the measurements made in those years are correct and lead to a maximum uncorrected polarization amount of about 16 percent for the main pulse. Perhaps a systematic error, or a combination of errors, led to our differing results in 1970 and 1971, but the possibility of a true variability of the pulsar, as in the first three cases above, cannot be excluded.

III. FITS BY THE RELATIVISTIC VECTOR MODEL

As can be seen from figure 1, the polarization of the entire main pulse seems to behave in a simple fashion. Figure 2, however, reveals that the polarization of the secondary pulse appears to vary in an orderly fashion only between times 11.50 and 16.54 ms after the main-pulse peak. We will try to fit only these intervals with the relativistic vector model since it predicts a highly ordered behavior. Note however that at least 83 percent of the light from the secondary pulse comes within the specified interval, that at earlier times in the secondary pulse the data may be confused by some light from the primary pulse, and that the relativistic vector model, as it stands, makes no allowance for an emission region of nonzero size.

The relativistic vector model, as described by Ferguson (1971) and in Papers I and II of this series, treats the polarization variation expected from a corotating localized emission region containing a rigid particle alignment vector (such as a magnetic field line), if the amount and angle of the polarization are proportional, respectively, to the apparent projected length and orientation of that vector on the plane of the sky. The aberration of light introduces sizable effects for corotation distances greater than about one-tenth the radius of the speed-of-light cylinder. In the present paper, the alignment vector is constrained to lie in the meridian plane of the emission region. The more general case has been solved by Ferguson (1974).

In figures 1, 2, 3, and 4 the solid lines represent the adopted best fits of the relativistic vector model to the data corrected for 3.6 percent extrinsic polarization at 167°.5. In figures 1 and 2, the statistical probability that the correct value lies within each error box is only 46 percent, so even a perfect theory would average only this percentage of hits on the boxes plotted. In figures 3 and 4, where the time behavior is shown to better advantage, a 68 percent hit rate is expected. The parameters of the best fits are shown in the figures, where T is the tilt of the rotation axis with respect to the plane of the sky, P.A. is the position angle of the rotation axis on the plane of the sky, β is the corotation velocity in speed of light units, L is the angle between the magnetic vector and the rotation equator in the emission frame, and P_0 is the maximum amount of polarization.

Figure 5 shows the possible qualitative behaviors of the polarization in this model. At polarization minimum, there is either a loop or a sharp bend around the origin. The extrinsic polarization, if the model is correct, must be such as to displace the center of the bend or loop to its position in the uncorrected data. On this basis, a best-fit value of the extrinsic polarization at the Crab pulsar of 3.6 percent at 167°.5 was obtained. The extrinsic polarization is the most poorly known quantity in the analysis of the polarization data of the Crab pulsar. Previous measurements of stars *around* the nebula give an interstellar polarization of about 2 percent at 147° (Cocke *et al.* 1970). The true extrinsic polarization at the pulsar may be higher, however, for the possibility that the pulsar's surroundings might somehow affect the polarization of light passing through them cannot be excluded.

The relativistic vector model can only be valid if the extrinsic polarization at the pulsar is in the range 2.4–4.1 percent at 145°–180° position angle (or if the extrinsic polarization is smaller by a factor of 2 and the amounts of pulsar polarization given in Paper II and measured in the fall of 1971 are correct). Thus, one could say that the model *predicts* the extrinsic polarization to be in the above range.

A comment about the polarization minima is in order here. The intrinsic minima, if the model is correct, are so small that the magnetic vectors must be within a few degrees of our line of sight at minima. In the relativistic vector model as it stands, this implies that for each emission region $\sin L = \gamma \sin T$ almost exactly [where $\gamma = (1 - \beta^2)^{-1/2}$]. It also implies that the direction of sweep of intrinsic polarization at the minima, to the east or west of north, is nearly indeterminate (at $\sin L = \gamma \sin T$ the loop or bend becomes a cusp in fig. 5). If $\sin L < \gamma \sin T$ it will be in one direction, whereas if $\sin L > \gamma \sin T$ it will be the opposite. Thus, the statement by Lyne *et al.* (1971) that the direction of sweep is an important consideration about the nature of the emission region is incorrect.

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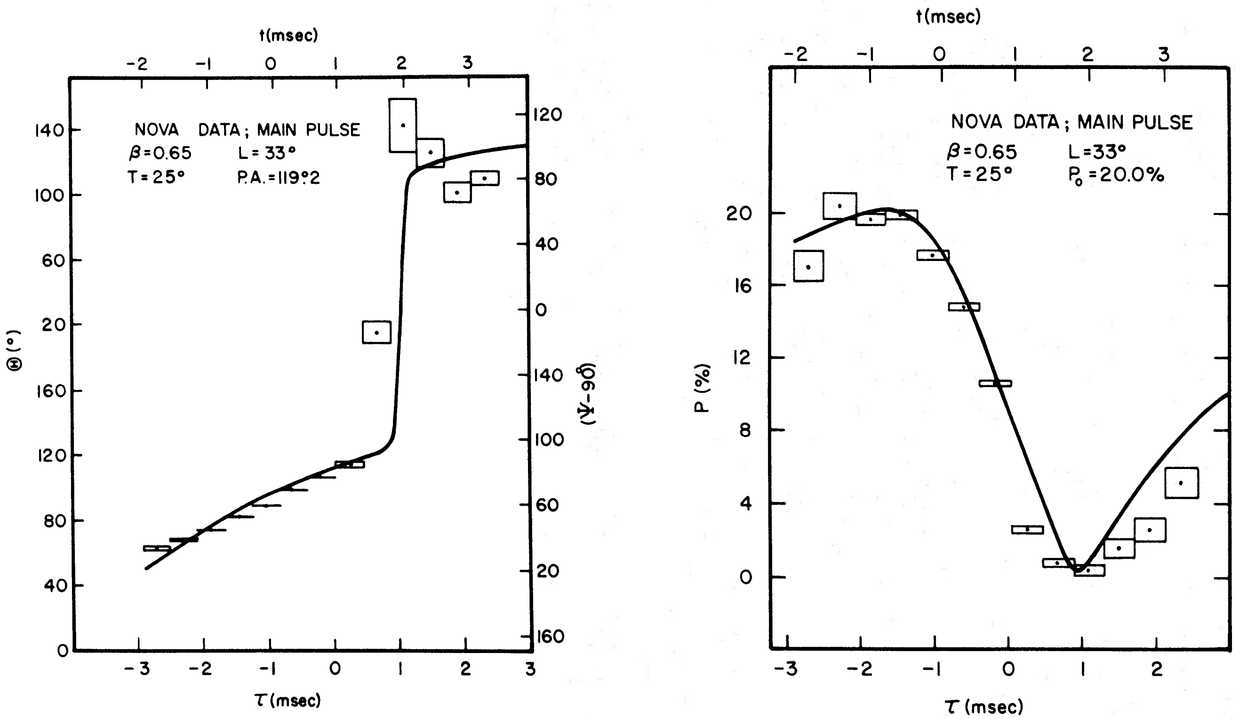

FIG. 3*a*

FIG. 3*b*

FIG. 3.—The polarization position angle and amount for the main pulse versus time. The quantity τ in ms is the time since the main-pulse peak. The time since the emission region was most nearly traveling toward us is t. The quantity Θ is the position angle of the electric vector. The quantity Ψ' is a theoretical angle. The heavy line is a theoretical fit by the relativistic vector model.

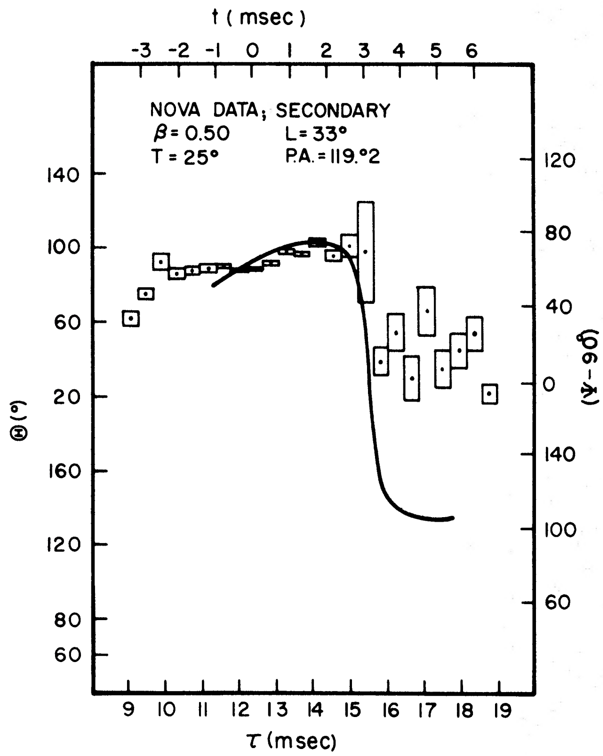

FIG. 4*a*

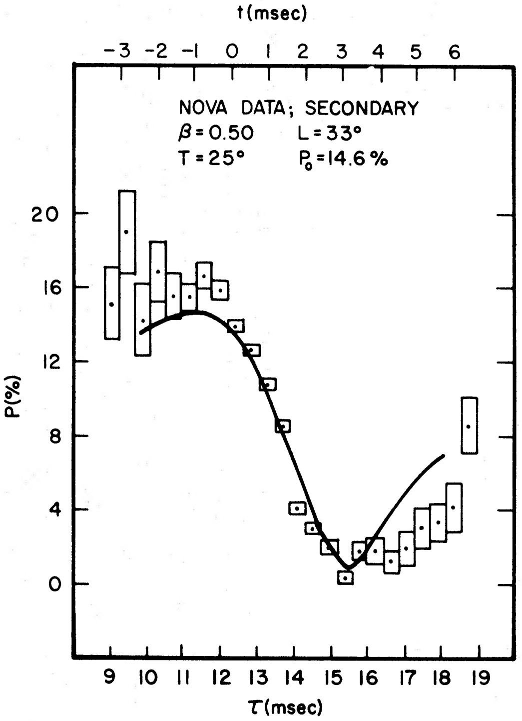

FIG. 4*b*

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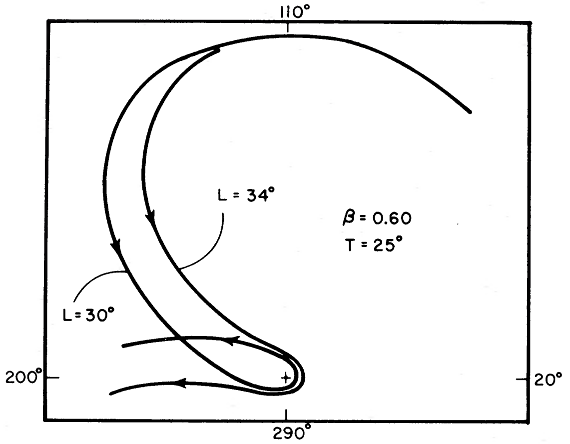

FIG. 5.—The possible qualitative behaviors predicted by the relativistic vector model.

IV. COMMENTS ON THE MODEL FITTING

In obtaining fits by the relativistic vector model, the parameter T is mainly dependent on the relative rates of change of amount and position angle at early phases in the main pulse, shown in figure 1. β for each region is obtained from the behavior of the amount and angle of polarization with time (figs. 3 and 4). Then L is obtained from the relation $\sin L = \gamma \sin T$ for each region, taking into account the behavior near polarization minimum in figures 1 and 2. The P.A. is found from figure 1 and verified by figures 3 and 4, while P_0 for each emission region is simply the proper scale factor to make the curves in figures 1 and 2 go through the maximum number of error boxes.

It is intriguing that $\sin L = \gamma \sin T$ so exactly for each emission region. Either this is a remarkable coincidence, or some property of the emission regions makes the net polarization at minimum nearly zero regardless of orientation.

Note added 1974 February 12.—J. R. P. Angel and P. Martin (private communication) have advised us that from recent measurements of the polarizations of filaments in the Crab Nebula they have found the interstellar polarization to be 2.0 ± 0.2 percent at 160°. It is noteworthy that the position angle they find agrees well with that predicted by the relativistic vector model. However, the amount of interstellar polarization they measure is smaller than the extrinsic polarization predicted by the relativistic vector model if the measurements reported in this paper are correct. This possibility was discussed in § III, and does not change the conclusions of this paper.

In conclusion, we thank George Coyne, Jack Frecker and Kristof Serkowski for their assistance. This work was supported by the National Science Foundation.

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ERRATUM FOR PAPER I

Ferguson would like to rectify an error in his previous exposition of the relativistic vector model. Table 1 of Ferguson (1973) is valid only for certain values of L and θ. The correct values for all L and θ are given in the table.

VALUES OF ($\Psi' - 90°$)

θ	$(90° + L) < M$		$(90° + L) > M$	
	$\delta' > 90°$	$\delta' < 90°$	$\delta' > 90°$	$\delta' < 90°$
$0° < \theta < 90°$	$-\chi - N$	$\chi - N$	$-\chi - N$	$\chi - N$
$90° < \theta < 180°$	$-\chi + N$	...	$-\chi + N$	...
θ	$(90° - L) < M$		$(90° - L) > M$	
	$\delta' > 90°$	$\delta' < 90°$	$\delta' > 90°$	$\delta' < 90°$
$180° < \theta < 270°$	$\chi - N$	...	$-\chi - N$	...
$270° < \theta < 360°$	$\chi + N$	$-\chi + N$	$-\chi + N$	$\chi + N$

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