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Coherent Scattering of Cosmic Neutrinos

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Summary. It is shown that cosmic neutrino scattering can be non-negligible when coherence effects previously neglected are taken into account. The coherent neutrino scattering cross section is derived and the neutrino index of refraction evaluated. As an example of coherent neutrino scattering, a detector using critical reflection is described which in principle can detect the low energy cosmic neutrino background allowed by the measured cosmological red shift.

Key words: neutrinos — scattering

I. Introduction

Various theories predict high densities of low-energy astrophysical neutrinos, yet little is known experimentally concerning them. The primary limit on the density of very low-energy astrophysical neutrinos is that the gravitational field they produce should not curve the universe more than is allowed by the measured cosmological red shift. Thus,

$$\int_0^\infty P(E_\nu)\, dE_\nu < \bar{\varrho} c^2 , \tag{1}$$

where $\bar{\varrho} \simeq 10^{-29}$ g/cm^3, and $P(E_\nu)$ is the energy density (erg/cm^3-eV) at a neutrino energy E_ν. (A zero neutrino rest mass is assumed.) For a mean neutrino energy $\bar{E}_\nu$, the limit on the energy density and energy flux $\mathscr{F}(E_\nu)$ (erg/cm^2-s-eV) is

$$P(\bar{E}_\nu) < \frac{\bar{\varrho} c^2}{\bar{E}_\nu} \tag{2}$$

and

$$\mathscr{F}(\bar{E}_\nu) < \frac{\bar{\varrho} c^3}{\bar{E}_\nu} . \tag{3}$$

If the above neutrinos are not degenerate then $\bar{E}_\nu$ is at least 10^{-2} eV. Evaluating Eqs. (2) and (3) for $\bar{E}_\nu = 10^{-2}$ eV yield

$$\begin{aligned} &P(10^{-2}\,\mathrm{eV}) < 10^{-6} \\ &\mathscr{F}(10^{-2}\,\mathrm{eV}) < 3 \times 10^{4} . \end{aligned} \tag{4}$$

Although the flux limits of Eq. (4) are very large, essentially no experimental effort has been made for their detection.

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All previous neutrino analyses concentrated upon interactions which result in a high energy charged particle being produced or a nuclear transformation occurring in the final state. The wave-like properties of neutrinos resulting in coherence phenomena have, however, until now been neglected.

In the present paper neutrino coherence phenomena are analyzed and some possible applications discussed. In § II the coherent neutrino scattering cross section is derived and the neutrino index of refraction evaluated. In § III, as an example of coherent neutrino scattering, a low energy cosmic neutrino detector using critical reflection is described.

II. The Coherent Neutrino Scattering Cross Section

The interaction Hamiltonian H_I for the scattering of a neutrino from a macroscopic object, $\mathcal{O}$, can be written as

$$H_I = H_I^e + H_I^n \tag{5}$$

where H_I^e, H_I^n is the weak interaction Hamiltonian for the electrons and nuclei of $\mathcal{O}$, respectively. The weak interactions are presently understood in terms of a current-current interaction. Observed phenomena such as β decay, μ capture, strange-particle decay, and high-energy neutrino interactions require only charge-changing currents such as $(e^-, \bar{\nu}_e)$, (e^+, ν_e), $(n, \bar{p})$, etc. For $H_I^n \neq 0$ in Eq. (5), neutral currents such as $(p, \bar{p})$ and $(n, \bar{n})$ are needed. There are theoretical indications that neutral currents exist. For example, the approximate $\Delta I = 1/2$ rule observed in strangeness-changing nonleptonic decays may arise if the $\Delta I = 3/2$ part of the charged current interaction is cancelled by a neutral current contribution (Gell-Mann and Rosenfeld, 1957). The approximate octet dominance may result through a partial cancellation of the 27 part of the weak Hamiltonian (Gell-Mann, 1964). Several recently proposed models of the weak interactions that show promise of being renormalizable, such as that of Weinberg (1972), predict the existence of neutral currents.

We are particularly interested in the case of negligible momentum transfer where the scatterer, $\mathcal{O}$, is left in its initial state. This is coherent scattering. Using a general Hamiltonian in Eq. (5) which includes the form of the weak interaction for the possible existence of neutral currents, and assuming that the neutrino interacts independently with protons and neutrons in a nucleus, we obtain for the average coherent scattering cross section per atom

$$\bar{\sigma}_0 = \frac{2G^2 E_\nu^2 N}{\pi} |Zg_e + Zg_p + (A-Z)\, g_n|^2 \tag{6}$$

where G is the weak coupling constant, A is the atomic weight of the atoms of $\mathcal{O}$, g_e, g_p and g_n are coupling constants of order unity, N is the number of atoms in $\mathcal{O}$, Z is their atomic number, and for simplicity of notation, $\hbar = c = 1$. If neutral currents do not exist and $V-A$ is the only interaction at low energies, we have, in Eq. (6), $|g_e|^2 = 1$ and $g_p = g_n = 0$.

We can also evaluate the index of refraction for neutrinos for the case $R \gg ƛ$, where R is the dimension of $\mathcal{O}$. Similar to interference phenomena for slow neutrons (Fermi and Marshall, 1947), the index of refraction for neutrinos is given by

$$n = 1 - 2\pi ƛ^2 \mathscr{A}, \tag{7}$$

where $\mathscr{A}$ is the total real part of the forward scattering amplitude per cm^3. The wave function of a neutrino at some distance from a scattering atom in the forward direction may be written as the sum of a term $\exp(ikx)$ representing the primary wave and a term $-(\bar{\sigma}_0/4\pi)^{\frac{1}{2}}$ $[\exp(i\phi)\exp(ikr)]/r$ representing the scattered wave, where ϕ is the phase angle of scattering. We obtain from equation (6) for the index of refraction for neutrinos

$$n = 1 - \frac{2^{1/2} \mathscr{N} G |Zg_e + Zg_p + (A-Z)\, g_n| \cos\phi}{E_\nu} \tag{8}$$

where $\mathscr{N}$ is the density of atoms (cm^{-3}).

III. A Low Energy Cosmic Neutrino Detector

We use Eq. (8) to describe a detector sensitive to the flux of low energy cosmic neutrinos predicted by Eq. (4). Similar to the critical scattering of slow neutrons (Fermi and Marshall, 1947), the critical angle for scattering neutrinos from the surface separating the vacuum from the medium is, from Eq. (8)

$$\theta_c \simeq \left[\frac{\mathscr{N} G}{E_\nu}\right]^{1/2} |Zg_e + Zg_p + (A-Z) g_n|^{1/2} \times 2^{1/4} |\cos\phi| \,. \tag{9}$$

The critical scattering takes place in the vacuum if $\cos\phi > 0$ and in the medium if $\cos\phi < 0$. (Of course since the density of matter such as iron is very much greater than the density of air, the expression (9) also is valid if air is substituted for vacuum.)

Taking the medium as iron,

$|\cos\phi| \simeq 1$, $|Zg_e + Zg_p + (A-Z) g_n|^{1/2} \simeq (A+Z)^{1/2}$,

$E_\nu \simeq 10^{-2}$ eV, we obtain

$$\theta_c \simeq 3\text{s}\,. \tag{10}$$

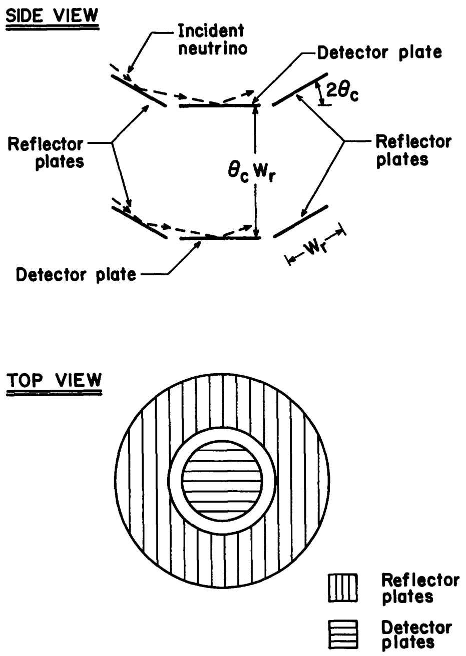

Fig. 1. Schematic diagram of a low energy cosmic neutrino detector

A schematic diagram for a low energy cosmic neutrino detector, based on critical scattering of cosmic neutrinos, is shown in Fig. 1. Cosmic neutrinos are concentrated by reflector plates onto detector plates. Each neutrino scattered from a detector plate transfers a momentum $\sim 2E_\nu\theta_c$. Since the wavelength $ƛ$ of a $E_\nu \simeq 10^{-2}$ eV neutrino is $\sim 20\,\mu$, individual detector and reflector plates need not be much thicker than this.

The separation of consecutive plates is $\theta_c W_r$, as shown in Fig. 1, where W_r is the radial dimension of the reflector plate. This condition ensures that consecutive plates do not interfere with one another. In a stack of plates of thickness d there are $d/W_r\theta_c$ plates. For example, for θ_c given by Eq. (10), $d \sim 60$ cm, $W_r \sim 2$ m, we have $d/W_r\theta_c \sim 2000$.

Let the integration time be τ. The total momentum transferred to the detector plates is

$$\Delta p \simeq (2E_\nu \theta_c) \times (\theta_c/2\pi) \times (2\pi R_r W_r) \times \mathscr{F} \times (d/W_r \theta_c) \times \tau \simeq 2E_\nu \theta_c R_r \mathscr{F} d\tau\,, \tag{11}$$

where $\mathscr{F}$ is given by Eq. (4), and the area of each reflector plate is $2\pi R_r W_r$.
Substituting reasonable values in Eq. (11), such as $R_r \simeq 3\,\text{m}$, $\tau \simeq 10\,\text{h}$, $d \simeq 60\,\text{cm}$, and $E_\nu \simeq 10^{-2}\,\text{eV}$ with $\mathscr{F}$ given by Eq. (4), we obtain

$$\Delta p \simeq 4 \times 10^{18}\,\text{eV}/c\,. \tag{12}$$

There are a number of ways to detect a momentum transfer of the above magnitude. We immediately note that Weber's detector (Weber, 1969, 1970 a,b) is sensitive to momentum transfers of magnitude less than this (Weber, 1971). The characteristics of a possible detector are the following: 1) The detector plates of mass M_d are supported by a cable in a vacuum chamber so that it forms a pendulum. 2) The vacuum chamber is isolated from all outside acoustic noise. 3) The reflector plates (outside the vacuum chamber) are moved in a period equal to the pendulum, so that during half its motion the reflected neutrinos scatter from the detector plates, while during the other half, they do not. 4) The system is made insensitive to the noise reaching the vacuum chamber by supporting a reference-pendulum, of the same period of the detector-pendulum, close to the detector-pendulum. 5) The motion of the detector-pendulum is measured with respect to the reference-pendulum by piezo-electric crystals, for example. Any motion of the chamber will induce exactly equal motions in both pendulums.
The thermal noise content of the detector-pendulum is kT. The ratio, $\mathscr{R}$, of signal to thermal noise is then

$$\mathscr{R} = \frac{(\Delta p)^2}{2M_d kT}\,. \tag{13}$$

Taking T as room temperature, $M_d \simeq 100\,\text{kg}$, we obtain from Eq. (13)

$$\mathscr{R} \simeq 5\,. \tag{14}$$

Of particular interest, Eq. (14) indicates that the low energy cosmic neutrino background is detectable.
The momentum transfer of Eq. (12) can also be measured, in principle, in a manner similar to the experiments of Fox and Shamir (1971). The detector plates can be attached to one end of a bar that is floating in mercury. Optical techniques ensure that the reflector plates are always aligned with the detector plates. The momentum transfer of Eq. (12) induces a rotation which is then measurable.

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Note added in proof: It is to be noted that chemical binding effects at low energies are negligible in equation (6), but are appreciable for a similar equation for photon scattering. The scattering amplitude for photon scattering from a singly charged scatterer (e.g. electron) is inversely proportional to the mass of the scatterer. Chemical binding causes the scatterer to appear more massive than it actually is, thereby decreasing the scattering cross section. The scattering amplitude for neutrino scattering, however, is approximately independent of the mass of the scatterer, making chemical binding effects small. Interestingly enough, it can be shown that chemical binding tends actually to *increase* the cross section of equation (6), as in the case of neutron scattering.