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THE LONG-TERM BEHAVIOR OF THE MAGNETIC FIELD OF β CORONAE BOREALIS

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ABSTRACT

Because of conflicting interpretations of its magnetic field variations, we have examined all of the Zeeman analyzed plates of β CrB in the files of the Hale Observatories. We measured all of the plates not previously measured and also all of the plates taken close to south polarity extremum. Our measurements confirm some of the small values of H_e at the south polarity extremum reported by Babcock. However, we find some observational evidence that at south polarity extremum there is a correlation between measured field strength and photographic density of the spectroscopic plate. Numerical models show that the effect should be present in an oblique-rotator model with a displaced dipole configuration of the magnetic field. This effect could be responsible for the small fields reported by Babcock and casts further doubts on the reality of nonperiodic variations in β CrB.

Subject headings: magnetic stars — peculiar A stars — stars, individual

I. INTRODUCTION

After examining a decade of magnetic field measurements of β Coronae Borealis, Preston and Sturch (1967) suggested the possibility that the negative extremum of the 18.5-day magnetic field variation is itself variable, with a period of about 10.5 years. Severny (1970), using a photoelectric technique, found zero field or crossover at phases when the extremum of negative polarity was expected, thus apparently confirming Preston and Sturch's suspicion.

Wolff and Bonsack (1972) observed the magnetic field of β CrB in 1971. The negative polarity extremum, according to the 10.5-year period, should have been at minimum (i.e., nearest to zero). They found the negative extremum to be at -700 gauss. Therefore, it appears that there is no 10-year periodic variation in the south polarity extremum of the star. However, figure 6 in Preston (1967) provides evidence for a variation, perhaps irregular, of the south polarity extremum superposed on the well-established 18.5-day period.

It is important to determine whether these variations really exist, as they would indicate either a failure of the oblique-rotator model or another mechanism that acts in conjunction with it. Consequently, we have reexamined Babcock's earlier Zeeman analyzed plates of β CrB.

II. RESULTS

a) *Measurements*

Few of the plates of β CrB taken before 1952 were ever measured, and only eye estimates of the field signs appear in Babcock's (1958) catalog. We inspected all of those plates and measured all but a few which were taken with lower dispersion (10 Å mm^{-1}) or which were hopelessly underexposed. Moreover, since (Preston and Sturch 1967, table 3) the south polarity extremum supposedly tends toward zero field

TABLE 1

H_e MEASUREMENTS

Plate # (1)	Date (2)	Phase (3)	H_e (4)	H_e (5)
Ce 4305*	1946 June 18	0.50	−22	0:
Ce 4326*	1946 July 16	0.01	−70	
Ce 4329*	1946 July 17	0.07	+143	
Ce 4334*	1946 July 18	0.12	+20	
Ce 4694*	1947 May 12	0.25	+560	+ ?
Ce 4723*	1947 June 28	0.78	−176	− ?
Ce 5239	1948 July 23	0.93	−334	− ?
Ce 5636	1949 May 10	0.67	−221	+ ?
Ce 5755	1949 July 16	0.30	+726	+
Ce 5937	1949 Sept. 13	0.48	+610	+
Ce 6221	1950 Apr. 7	0.64	−217	0:
Ce 6323	1950 June 29	0.12	+51	+
Ce 6490	1950 Aug. 31	0.50	+428	0:
Ce 7065	1951 May 28	0.14	+567	+
Ce 8261	1952 Aug. 8	0.82	−455	−70
Pb 939	1953 May 30	0.79	+6	−170
Pb 948	1953 June 1	0.89	−111	−280
Pb 1049	1953 July 25	0.81	−94	−280
Pb 1058	1953 July 27	0.90	−303	−250
Pb 1344	1954 Jan. 26	0.83	−750	−725
Pb 1575	1954 July 10	0.74	−950	−710
Pb 1580	1954 July 11	0.79	−127	−290
Pb 1584	1954 July 12	0.85	−670	−340
Pb 1904	1955 Jan. 13	0.86	−264	−375
Pb 2066	1955 May 3	0.80	−421	−420
Pb 2179	1955 Aug. 2	0.72	−685	−660
Pb 2555	1956 Apr. 19	0.85	−622	−645

* The phase shift compensator was not used.

in 1952 and 1953, we remeasured the plates taken during these years close to the phase of negative polarity extremum.

The Zeeman analyzed spectrograms were measured on a Grant measuring engine, mostly by one of us (M. M. D.). We obtained the values of the longitudinal magnetic field H_e from the Zeeman displacement in microns, using the usual formula (Babcock 1958). All of the plates taken before 1954 at phases of south polarity were measured by both of us, with good agreement between the two measures of each plate. As a final check, we remeasured plates taken in 1954 and 1955 both at negative and positive extrema. Our values of H_e agree well with the ones reported by Babcock (1958).

The results of the measurements are shown in table 1. Column (1) gives the plate number, column (2) the date, column (3) the phase of the 18.5-day magnetic variation according to the ephemeris of Preston and Sturch (1967), column (4) the values of H_e from our measurements, and column (5) the magnetic field or sign estimate given by Babcock (1958). The plates marked with an asterisk in column (1) were taken without the phase shift compensator (Babcock 1962).

At the declination of β CrB and for a wavelength of 4500 Å the phase shift introduced by the flat mirror in the coudé beam is $\sim\pi/4$. Because of a large and variable cross-talk between linear and circular polarizations which affects the measured Zeeman shifts in an unknown way and because of a decrease in the circular polarization signal (see Appendix), the values of H_e derived from uncompensated plates are to be considered dubious.

b) *Magnetic Variation Curve*

In figure 1 we show the values of H_e from the plates taken before 1952. The magnetic variation curve from Preston and Sturch (1967) is indicated as a continuous line. When the magnetic field is positive, the plates agree reasonably well with the continuous curve, although there is a tendency for our measurements to give smaller values of H_e. This discrepancy between values of H_e for Mount Wilson and Palomar plates on the one hand and Lick plates on the other is attributable to the different choice of lines governed by the dispersions used (Preston and Pyper 1965).

Unfortunately, few of the early Babcock plates were taken close to the phase of negative extremum. Plate Ce 5636 taken at phase $\varphi = 0.67$ and estimated (+ ?) by Babcock gives a magnetic field of -220 gauss. Plate Ce 4723 was taken at $\varphi = 0.78$, very close to negative extremum, but without the use of a compensator; therefore, the measured field (-176 gauss) is not reliable. Moreover, this plate is very dense (see the discussion of exposure effects in § III*c*), and probably represents an upper limit only. This value of H_e is not plotted in figure 2*a*. Among the 1952 and 1953 plates only three were taken near negative extremum. Plate Ce 8261 ($\phi = 0.82$) has markedly tilted comparison lines. Repeated measurements by both of us, after careful alignment of the analyzing slit, consistently gave fields of about -500 gauss, in sharp disagreement with the previous measurement (-70) given by Babcock (1958). The remaining two plates, Pb 939 ($\phi = 0.79$) and Pb 1049 ($\phi = 0.81$) were found to give fields somewhat weaker than those in Babcock's catalog.

III. DISCUSSION

We consider three possible interpretations of the measurements.

a) *Irregular Variations of the South Polarity Extremum*

Our measurements of Pb 939 and Pb 1049 confirm the small fields in Babcock's catalog and therefore the suspicion of an irregular behavior. If there are no systematic errors in the measurements, then it is conceivable that something akin to sunspot

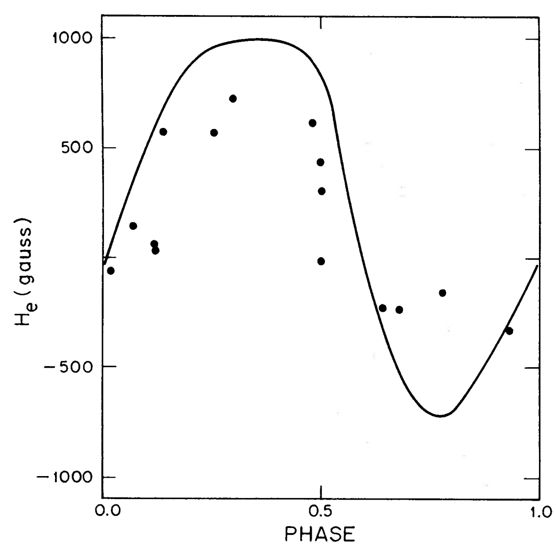

FIG. 1.—The values of H_e from spectrograms taken before 1952 are plotted in the 18.5-day cycle. The full curve is taken from Preston and Sturch (1967). The observation at phase 0.78 is dubious and probably is an upper limit.

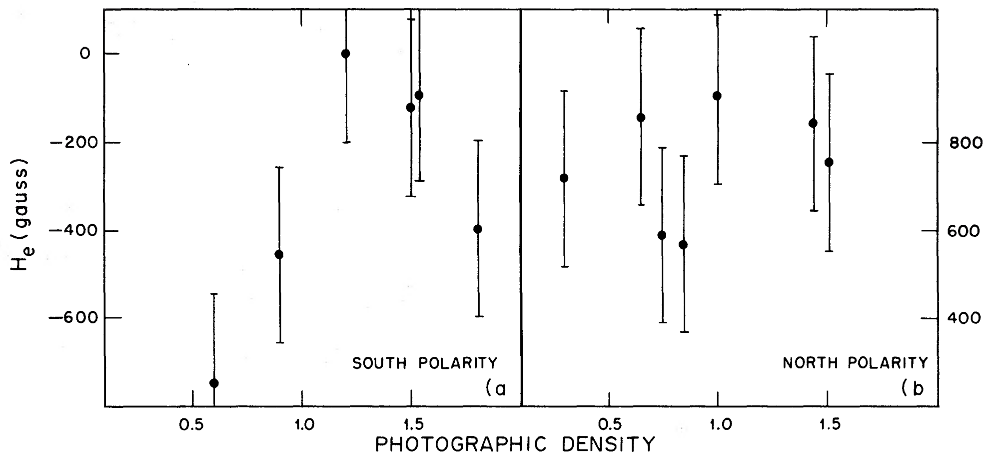

FIG. 2.—(*a*) The observed values of H_e near negative extremum (the phases range from 0.79 to 0.83) plotted against the photographic density of the continuum 4180 Å. (*b*) A similar plot for values of H_e near positive extremum. The error bars represent $\pm 1\ \sigma$.

activity affects the south polarity extremum. One has then to explain why the north polarity extremum is not affected.

b) *Errors in the Compensation*

Errors in the setting of the compensator used to cancel the phase shifts introduced by the flat mirror (Babcock 1962) might have destroyed the polarization signal and therefore the Zeeman shifts. It seems improbable that so many errors occurred, always near negative extremum.

c) *Exposure Effects on the Zeeman Displacements*

A third explanation can reconcile the scatter at south polarity with the oblique rotator model. Previous numerical models of magnetic stars (Borra 1972) show that for displaced dipole configurations of the magnetic field, different parts of the Zeeman analyzed line profiles can give widely differing values of the magnetic field. We therefore decided to investigate exposure effects on the Zeeman displacements. Microphotometer tracings of the region near 4180 Å for the plates of β CrB in the plate files of the Hale Observatories yielded photographic densities near the continuum from 0.2 to 1.9.

In order to have a uniform set of measurements we remeasured all of the remaining plates taken between phases 0.7 and 0.9 with the Grant comparator. Our measurements agree reasonably well with the values reported in Babcock's catalog. A plot of the values of H_e as a function of photographic density for all of the points at south polarity shows a tendency for thin plates to give larger fields than strongly exposed plates, although the scatter is large. This scatter is caused partly by the fact that there is a spread in the values of H_e due to the magnetic variations. Moreover, crossover is present during most of the cycle and the shapes of the Zeeman analyzed line profiles are variable throughout the cycle, complicating any simple relation between exposure and Zeeman displacements. Figure 2*a* shows the values of H_e as a function of photographic density in the continuum when we restrict ourselves to a narrow range of the cycle near negative extremum. The figure shows some evidence supporting a dependence of the measured Zeeman shifts on photographic exposure. The same trend is present when we use the values of H_e from Babcock's catalog.

Figure 2*b* shows a similar plot for values of H_e near positive extremum. There is no dependence of H_e on the exposure.

It remains to be explained why the effect is present at south polarity only. We believe that this can be understood in terms of the geometry of the magnetic field. The field geometry of β CrB can be approximated by a displaced dipole (Wolff and Wolff 1970). At south polarity the observer views the strongest pole. The Zeeman analyzed line profiles result from the contributions of regions of greatly differing magnetic field strengths. A region of weak field gives a smaller displacement and contributes more to the core of the line than a region of strong field. Thus during measurement of Zeeman analyzed spectrograms, weighting the core more heavily results in smaller displacements. A strong exposure tends to emphasize the core of the lines at the expense of the wings, thereby producing a smaller displacement. At north polarity, the star presents the weaker pole to the observer. The local field strength does not vary as greatly over the visible disk, the field is more nearly purely longitudinal, the line profiles simpler and narrower, and the measurements less dependent on the part of the line used.

We have tested this qualitative model numerically, using the computer program described by Borra (1972, 1973). The wavelength dependence of the absorption coefficients is here taken to be Gaussian, the saturated line is assumed to have a simple triplet splitting, a g-value of 1.2, and a wavelength of 4200 Å. The geometry of the magnetic field is as given by Wolff and Wolff (1970). The theoretical Zeeman analyzed line profiles are then convolved with a Gaussian instrumental profile of 0.07 Å half-width, and a characteristic curve is applied to the profiles. The resulting direct "photographic" transmission profiles are then reduced to yield a longitudinal magnetic field.

Figure 3*a* shows the theoretical longitudinal field versus photographic density at the continuum at south polarity, when only the lower 30, 50, or 70 percent of the lines is used to compute their effective wavelength. The figure shows that H_e depends both on photographic exposure and on the part of the line used to measure the shifts. Figure 3*b* shows the theoretical relationship between H_e and photographic density at north polarity. There is a totally negligible dependence on exposure and only a slight (5%) dependence on the part of the line used.

It is difficult to reproduce exactly the weighting that the eye gives to different parts of the line profile, but there is probably a tendency to emphasize the core of the line. The displaced dipole configuration of the magnetic field is probably only an idealization of the actual geometry, and if the above arguments are correct the value of H_e

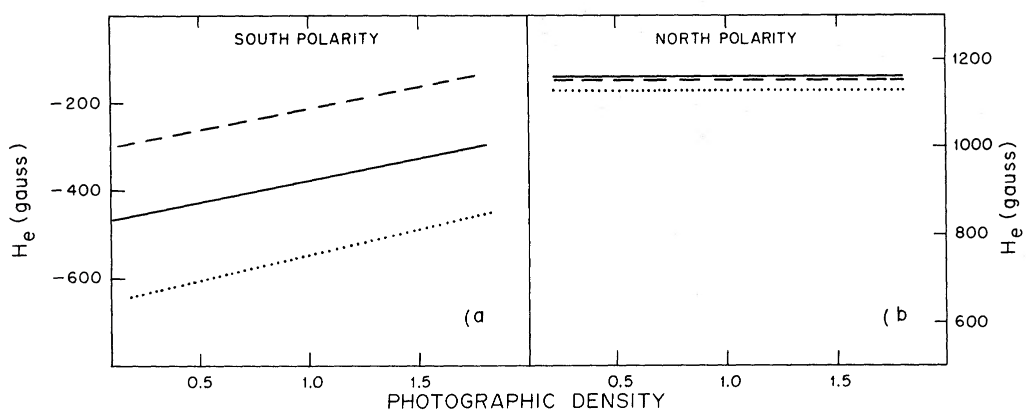

FIG. 3.—(*a*) The theoretical relations between exposure and photographic density of the continuum at negative extremum. In (*b*) the relations are at positive extremum. The dashed, full, and dotted lines show respectively the relations obtained when we consider only the lower 30%, 50%, and 70% of the Zeeman analyzed line profiles when computing the Zeeman displacements.

at south extremum used by Wolff and Wolff is somewhat in error; therefore, the parameters of the displaced dipole they give cannot be exact. It is therefore not surprising that figure 3*a* does not reproduce exactly figure 2*a*.

The theoretical results shown in figure 3 provide a reasonable interpretation of two features seen in the magnetic variation plot of Preston (1967, fig. 6). First, the smaller scatter at north polarity is explained, since exposure effects are minimal at these phases, and different parts of the line give similar magnetic fields. Second, the scatter at south polarity is explained, since more heavily exposed plates at these phases are expected to give weaker fields; the net effect should be to produce scatter in one direction only, which "fills in" the south polarity branch of the magnetic curve. This expectation is supported by the trend seen in figure 2.

The displaced dipole model also can explain why Severny (1971) did not detect the south polarity of β CrB. Models by Borra (1972) show a simple S-shaped polarization profile at north polarity and a double-S at south polarity as the very core of the line gives a different field polarity than the wings. It is possible that with the 0.2 Å band-pass and step size used by Severny either zero polarization or a complicated signal will be seen.

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APPENDIX

UNCOMPENSATED ZEEMAN SPECTROGRAMS

Unno (1956) gives for the intensities of the ordinary and extraordinary rays of the Zeeman analyzer:

$$I_{x,\delta} = \tfrac{1}{2}[I - Q\sin 2\phi\cos\delta + U\cos\delta\cos 2\phi + V\sin\delta]\,,$$

$$I_{y,\delta} = \tfrac{1}{2}[I + Q\sin 2\phi\cos\delta - U\cos 2\phi\cos\delta - V\sin\delta]\,,$$

where I, Q, U, and V are the four Stokes parameters of the incident light, δ the retardation of the analyzer, and ϕ the azimuth of the fast axis of the retarder with respect to the "horizontal" direction. For a Zeeman analyzer in the longitudinal mode $\delta = \pi/2$ and therefore

$$I_{x,\delta} = \tfrac{1}{2}[I + V]\,, \qquad I_{y,\delta} = \tfrac{1}{2}[I - V]\,.$$

The flat mirror acts both as a dichroic partial polarizer and as a linear retarder. For simplicity and clarity in the following, we will consider it to act only as a retarder. For an uncompensated beam, the analyzer to be considered is a combination of the Zeeman analyzer and flat mirror. The retardation δ then depends also on the retardation of the flat and on the relative orientation of the eigenaxes of the mirror and of the quarter-wave plate. The retardation therefore will vary with hour angle.

Let us consider the case $\phi = \pi/4$; and for an estimate of the order-of-magnitude of the effects introduced by the lack of compensation, let us consider the case where the eigenaxes of mirror and quarter-wave plate coincide.

We have then (for β CrB and $\lambda = 4500$ Å) $\delta = 3\pi/4$ and

$$I_{x,\delta} = \tfrac{1}{2}[I + 0.707Q + 0.707V]\,,$$

$$I_{y,\delta} = \tfrac{1}{2}[I - 0.707Q - 0.707V]\,.$$

We have thus a decrease by 30 percent of the circular polarization signal (longitudinal field) and a large (70%) cross-talk from the linear polarization (transverse field). Numerical models (Borra 1972) based on the magnetic geometry of β CrB determined by Wolff and Wolff (1970) show that the magnitude of the linear polarization in the spectral lines is comparable to that of the circular polarization, at all phases of the magnetic variation.

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