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THE REDSHIFT-DISTANCE RELATION. VII. ABSOLUTE MAGNITUDES OF THE FIRST THREE RANKED CLUSTER GALAXIES AS FUNCTIONS OF CLUSTER RICHNESS AND BAUTZ-MORGAN CLUSTER TYPE: THE EFFECT ON q_0

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ABSTRACT

The ratio of angular diameters of the first three ranked galaxies in E and S0 aggregates define contrast parameters that correlate well with Bautz-Morgan (BM) cluster types. Interpreted as an *apparent* magnitude difference, 5 log θ_1/θ_2 varies from 1.3 mag for Bautz-Morgan class I clusters to 0.4 mag for class III.

The *absolute* magnitudes also change with BM class. Magnitude residuals from the Hubble diagram show that the first-ranked galaxy is absolutely brighter in class I clusters than in class III by $\langle \Delta M_v \rangle = 0.6$ mag. However, the second and third ranked are *fainter* by 0.5 mag in class I compared to class III clusters.

This startling, but well-determined, inverse effect suggests that the dominance of first-ranked galaxies in clusters occurs *at the expense* of the fainter members.

Because both the Bautz-Morgan *class* and the first-ranked absolute *magnitudes* are independent of cluster richness, we argue that the Bautz-Morgan effect is more likely to be related to an initial condition of cluster formation than to later evolution by processes that depend on the rate of interaction of cluster members.

New data, obtained by extensive counting, are given for the galaxy population (N) of all groups and clusters in our sample. There is no correlation of first-ranked absolute magnitudes with N at a level more significant than 1 σ, but a significant correlation does exist for second- and third-ranked galaxies.

The Hubble diagram using the V magnitudes of first-ranked galaxies in 98 E and S0 groups, corrected for aperture effect, K-dimming, galactic absorption, Bautz-Morgan effect, and cluster richness shows good agreement with a linear redshift-distance relation, and has the small dispersion of $\sigma(M_v) = 0.28$ mag for the distribution of horizontal residuals.

The effect of the BM and the richness corrections on the value of q_0 in our sample is negligible compared with the large errors of the current determinations. Only the grossest alternatives to q_0 (apparent) $= 1 \pm 1$ (such as $q_0 = -1$ or $q_0 > 3$) can be discarded from the data now available, no matter how the material is analyzed. New data for many clusters with large redshift ($z > 0.4$) are needed for a finer solution. However, the prediction of steady-state cosmology ($q_0 = -1$) is clearly at variance even with the present data.

Subject headings: galaxies, clusters of — galaxies, photometry of

I. INTRODUCTION

The Bautz-Morgan (1970, hereafter called BM) system for classifying clusters of galaxies appears to be fundamental to the problems of understanding the distribution of absolute magnitudes $\langle M \rangle_1$ of brightest cluster members, and of the dependence of $\langle M \rangle_1$ on cluster richness. The classification is based on the apparent contrast of the

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brightest galaxy relative to fainter cluster members. But is the first-ranked galaxy *absolutely* brighter in clusters where the contrast is greatest (BM class I), or is it that the remainder of the cluster galaxies are fainter than average? The point is clearly important for questions of galaxy and cluster formation; but more practically, we need the data to correct for the Bautz-Morgan effect in the Hubble diagram to test for a possible systematic effect on the determination of q_0.

Bautz and Abell (1972*a*, *b*) have studied this question by estimating the absolute magnitude of the first-ranked cluster galaxy from certain assumed properties of the cluster luminosity functions $\phi(M)$. Taking the absolute magnitude M^* of a particular break in $\phi(M)$ to be constant, and measuring the difference in the apparent magnitudes, $m^* - m(1)$, between the break and the first brightest galaxy, gives the absolute magnitude of the brightest as $M(1) = M^* + m(1) - m^*$. From the data available to them, Bautz and Abell concluded that $M(1)$ determined this way is brighter for galaxies in BM class I clusters than for first-ranked members of later BM-type clusters.

It is possible to test this important conclusion without reference to properties of $\phi(M)$, or to assumptions about M^*, by applying the redshift-distance relation to the clusters directly. We have analyzed the magnitude residuals from the Hubble diagram of Paper VI (Sandage 1973, table 4) as functions of BM cluster type (§ III), and subsequently of cluster richness (§ IV), and confirm the existence of the effect discovered by Bautz and Abell. The investigation is extended to second- and third-ranked cluster members (§ III) with the startling result that the sense of the correlation is reversed for them.

The BM effect is statistically removed from the (m, z) data in § IV. The resulting Hubble diagram and the effect of the BM and the richness corrections on the determination of q_0 is discussed in § V.

II. PHOTOMETRIC DATA AND A QUANTITATIVE DEFINITION OF THE BAUTZ-MORGAN CLUSTER TYPE

a) *The Data*

To permit a more refined search for a richness effect than that of Paper II (Sandage 1972*b*), we determined several characteristics of the groups and clusters in our sample by measurement of either the original plates or of the *Palomar Sky Survey* prints. The results, listed in table 1, require various explanations.

The Bautz-Morgan cluster type in column (4) was taken either from their original paper (BM 1970), from Bautz (1972), or from Bautz and Abell (1972*b*) as denoted by a superscript 1 in column (4). Using the types defined in these three papers as standards, we classified the remaining clusters (tables 4 and 5 of Paper VI). The superscripts in column (4) refer to our classification made from Palomar Schmidt plates if the number is 2; if 3, we used prints of the *Palomar Sky Survey*; and if 4, we used plates taken with the 200-inch (508-cm) telescope.

Although it was easier to classify the BM cluster type from plates, we found no significant difference between plates and paper prints once we realized how to recognize an extended envelope on the high-contrast paper. The mean accuracy of our classification, relative to clusters already classified by Bautz, Morgan, and Abell, is better than half a class. The largest differences occur in the difficult region of BM classes II and II–III.

Column (5) of table 1 gives the Abell (1958) richness class (if in parentheses, it was estimated by us). Column (6) identifies the plate material (PP for *Palomar Sky Survey* prints; 48, 100, and 200 for plates from the Mount Wilson and Palomar telescopes).

Measurements of the angular diameters of many galaxies in each cluster were made

TABLE 1

ABSOLUTE V MAGNITUDES AND MAGNITUDE RESIDUALS OF FIRST THREE BRIGHTEST GALAXIES IN CLUSTERS AND GROUPS OF GIVEN RICHNESS AND BAUTZ-MORGAN TYPE

Cluster (1)	Abell (2)	z (3)	$B-M$ (4)	R_{AB} (5)	Plate (6)	$\langle\theta_1\rangle$ (7) sec	v_2-v_1 $5\log\frac{\theta_1}{\theta_2}$ (8) mag	v_3-v_1 $5\log\frac{\theta_1}{\theta_3}$ (9) mag	$M_v^c(1)$ (10)	$M_v^c(2)$ (11)	$M_v^c(3)$ (12)	ΔM_1 (v) (13)	ΔM_2 (v) (14)	ΔM_3 (v) (15)	N (16)	B (17)	N_c^{48} (18)
Virgo	...	0.0038	III^1	(1)	PP	...	...	...	-23.35	...	...	-0.05	...	...	...	...	~60
Fornax	...	0.0051	II-III^2	(1)	PP	...	...	...	-23.52	...	...	-0.22	...	...	...	...	~50
Peg I	...	0.0128	II^3	(0)	PP	161.0	1.69	2.01	-23.17	-21.16	-21.16	+0.13	+1.11	+1.19	59	0	59
0122 + 3305	...	0.0170	II^3	(1)	48	101.6	1.65	1.65	-23.28	-21.63	-21.63	+0.02	+0.96	+0.72	61	0	61
Perseus	426	0.0181	II-III^1	2	48	87.2	0.57	0.57	-23.68	-23.11	-23.11	-0.38	-0.52	-0.76	63	0	63
Coma	1656	0.0222	II^1	2	48	89.2	0.22	0.62	-24.11	-23.89	-23.49	-0.81	-1.30	-1.14	115	0	115
Abell 1213	1213	0.0287	III^1	1	48	31.5	0.19	0.35	-22.46	-22.27	-22.11	+0.84	+0.34	+0.24	84	15	69
Hercules	2151	0.0341	III^1	2	48	...	...	...	-23.46	...	...	-0.16	...	...	...	...	~85
Abell 119	119	0.0387	II-III^1	1	48	36.9	0.69	0.69	-23.08	-22.39	-22.39	+0.22	+0.20	-0.04	96	30	66
2308 + 0720	Peg II	0.0428	II-III^2	(0)	48	33.6	0.33	0.33	-23.38	-23.05	-23.05	-0.08	-0.46	-0.70	33	10	23
2322 + 1425	2589	0.0440	I^1	0	48	33.6	1.34	2.34	-22.99	-21.55	-20.65	+0.31	+1.04	+1.70	70	21	49
1145 + 5559	1377	0.0516	II-III^1	1	48	20.1	0.23	0.58	-23.06	-22.83	-22.48	+0.24	-0.24	-0.13	75	20	55
0106 - 1536	151	0.0526	II^1	1	48	22.1	0.78	0.78	-23.37	-22.59	-22.59	-0.07	0.00	-0.24	106	20	86
1024 + 1039	1020	0.0649	III^1	1	48	16.8	0.48	0.71	-23.14	-22.66	-22.43	+0.16	-0.07	-0.08	100	20	80
1239 + 1852	1589	0.0718	II-III^2	0	48	23.5	0.48	0.91	-23.66	-23.18	-22.75	-0.36	-0.59	-0.40	66	20	46
1520 + 2754	2065	0.0722	III^1	2	48	16.8	0.48	0.48	-22.92	-22.44	-22.44	+0.38	+0.15	-0.09	109	20	89
Abell 2670	2670	0.0775	I^1	3	48	33.6	0.77	1.50	-23.56	-22.79	-22.06	-0.26	-0.20	+0.29	218	34	184
Abell 2029	2029	0.0777	I^1	2	PP	(25.0)	(1.73)	(2.44)	-24.28	(-22.55)	(-21.84)	-0.98	+0.04	+0.51	...	...	...
0705 + 3506	568	0.0779	II-III^1	0	48	15.4	0.30	0.53	-23.38	-23.08	-22.85	-0.08	-0.49	-0.50	61	34	27
1513 + 0433	2048	0.0944	III^1	1	200	13.3	0.62	1.02	-23.29	-22.67	-22.27	+0.01	-0.08	+0.08	127	55	55
Abell 98	98	0.1028	II-III^1	3	48	30.2	0.26	0.88	-23.39	-23.13	-22.51	-0.09	-0.54	-0.16	244	15	229
Abell 274	274	0.1289	II-III^1	3	48	13.4	0.48	0.48	-23.35	-22.87	-22.87	-0.05	-0.28	-0.52	...	...	...
1431 + 3146	1930	0.1312	III^1	1	200	11.1	1.10	1.10	-23.41	-22.31	-22.31	-0.11	+0.38	+0.14	180	100	61
1055 + 5702	1132	0.1345	II-III^1	1	48	13.4	0.62	0.62	-23.60	-22.98	-22.98	-0.30	-0.39	-0.63	51	16	35
1153 + 2341	1413	0.1426	I^4	3	200	13.3	1.50	1.69	-24.19	-22.69	-22.50	-0.89	-0.10	-0.15	261	60	155
1641 + 1327	2224	0.1499	III^1	3	...	...	...	...	-23.08	...	...	+0.22	...	...	...	...	...
1534 + 3749	2100	0.1532	III^1	3	200	11.1	0.16	0.87	-23.26	-23.10	-22.39	+0.04	-0.51	-0.04	387	180	159
0025 + 2223	31	0.1594	III^1	2	48	11.4	0.27	0.27	-23.24	-22.97	-22.97	+0.06	-0.38	-0.62	80	12	64
1228 + 1050	1553	0.1651	II-III^1	2	48	18.1	0.35	0.65	-23.08	-22.73	-22.43	+0.22	-0.14	-0.08	135	16	119
0138 + 1832	234	0.1730	II^2	1	48	16.8	0.48	0.48	-22.73	-22.25	-22.25	+0.57	+0.34	+0.10	36	3	33
1309 - 0105	1689	0.1745	I-II^1	4	100	8.5	0.45	0.61	-23.16	-22.71	-22.55	+0.14	-0.12	-0.20	328	100	228
1304 + 3110	1677	0.1831	III^1	2	200	7.4	0.24	0.24	-23.30	-23.06	-23.06	0.00	-0.47	-0.71	205	60	111
0925 + 2044	801	0.1917	I-II^1	2	200	8.9	0.38	0.74	-23.80	-23.42	-23.06	-0.50	-0.83	-0.71	140	47	95
1253 + 4422	1643	0.1979	III^4	1	200	6.7	0.27	0.53	-22.71	-22.44	-22.18	+0.59	+0.15	+0.17	226	90	136
0855 + 0321	732	0.2018	II-III^1	1	200	6.7	0.53	0.53	-23.21	-22.68	-22.68	+0.09	-0.09	-0.33	132	40	71
1447 + 2617	...	0.36	II-III^1	(2)	200	5.9	0.26	0.61	-22.88	-22.62	-22.27	+0.42	-0.03	+0.08	152	60	71
0024 + 1654	...	0.38	III^1	(2)	200	4.4	0.42	0.62	-23.43	-23.01	-22.81	-0.13	-0.42	-0.46	218	70	113
3C31	N383	0.0169	III^3	(0)	48	60.4	0.88	0.88	-23.00	-22.12	-22.12	+0.30	+0.47	+0.23	13	0	13
3C40	194	0.0180	II^1	0	48	57.0	0.27	0.42	-22.87	-22.60	-22.45	+0.43	-0.01	-0.10	39	0	39
3C66	347	0.0215	II-III^1	0	48	43.6	0.36	1.34	-22.97	-22.61	-21.63	+0.33	-0.02	+0.72	51	12	39
3C465	2634	0.0301	I-II^1	1	48	53.7	1.02	1.02	-23.13	-22.11	-22.11	+0.17	+0.48	+0.24	70	12	58
3C338	2199	0.0303	I^1	2	PP	120.8	1.63	1.63	-23.77	-22.14	-22.14	-0.47	+0.45	+0.21	89	0	89
3C317	2052	0.0351	II^3	0	PP	75.8	1.77	2.10	-23.21	-21.44	-21.11	+0.09	+1.15	+1.24	78	15	63
M23 - 112	2638	0.0825	III^3	2	PP	13.4	0.62	0.93	-23.09	-22.47	-22.16	+0.21	+0.12	+0.19	...	...	...
3C219	...	0.1745	II^4	(2)	200	8.1	0.19	0.19	-23.20	-23.01	-23.01	+0.10	-0.42	-0.66	191	75	89
3C28	115	0.1959	III^1	3	48	11.4	0.58	0.58	-23.18	-22.60	-22.60	+0.12	-0.01	-0.25	104	7	97
3C295	...	0.461	I^1	(1)	200	3.9	0.73	1.22	-23.56	-22.83	-22.34	-0.26	-0.24	+0.01	95	~20	58
0037 + 0630	76	0.0377	I^1	0	PP	73.8	0.98	1.32	-23.44	-22.46	-22.12	-0.14	+0.13	+0.23	59	9	50
0106 + 0155	147	0.0441	III^1	0	PP	50.3	0.31	0.31	-22.88	-22.57	-22.57	+0.42	+0.02	-0.22	35	0	35
0150 + 3555	262	0.0168	III^1	0	PP	114.1	0.58	0.58	-23.08	-22.50	-22.50	+0.22	+0.09	-0.15	47	0	47
0154 + 3159	278	0.0904	III^3	0	PP	23.5	0.13	0.13	-23.41	-23.28	-23.28	-0.11	-0.69	-0.93	45	0	35
0243 + 3640	376	0.0487	I-II^3	0	PP	43.6	1.34	1.34	-23.31	-21.97	-21.97	-0.01	+0.62	+0.38	86	40	46
0451 + 7957	505	0.0543	I^3	0	PP	67.1	1.73	2.61	-23.82	-22.09	-21.21	-0.52	+0.50	+1.14	79	36	43
0514 + 0625	539	0.0267	III^1	1	48	38.2	0.42	0.42	-22.98	-22.56	-22.56	+0.32	+0.03	-0.21	111	25	86
0545 - 2538	548	0.0391	III^1	1	48	35.6	0.13	0.26	-23.36	-23.23	-23.10	-0.06	-0.64	-0.75	82	12	70
0609 + 4838	553	0.0670	II^1	0	48	16.8	0.48	0.48	-23.44	-22.96	-22.96	-0.14	-0.37	-0.61	114	46	68
0705 + 4843	569	0.0193	II^2	0	48	57.0	0.95	1.15	-23.24	-22.29	-22.29	+0.06	+0.30	+0.26	64	17	47
0717 + 5551	576	0.0404	III^1	1	48	26.8	0.00	0.00	-23.18	-23.18	-22.09	+0.12	-0.59	-0.83	60	0	60
0740 + 0930	592	0.0621	III^2	1	48	25.5	0.18	0.51	-23.46	-23.28	-22.95	-0.16	-0.69	-0.60	48	15	33
0810 + 5813	634	0.0266	III^1	0	48	45.0	0.35	0.35	-22.96	-22.61	-22.61	+0.34	-0.02	-0.26	53	9	44
0825 + 3036	671	0.0497	II^1	0	48	45.0	1.54	1.97	-23.74	-22.20	-21.77	-0.44	+0.39	+0.58	70	24	46
0906 - 0927	754	0.0537	I^1	2	PP	67.1	1.99	1.99	-23.86	-21.87	-21.87	-0.56	+0.72	+0.48	89	15	74
1019 - 0442	993	0.0530	III^1	0	PP	43.6	0.00	0.36	-23.60	-23.60	-23.24	-0.30	-1.01	-0.89	35	0	35
1034 - 2716	1060	0.0115	III^1	1	PP	100.6	0.31	0.40	-23.18	-22.87	-22.78	+0.12	-0.28	-0.43	51	0	51
1056 + 0147	1139	0.0376	II-III^3	0	PP	43.6	0.80	0.90	-23.06	-22.26	-22.16	+0.24	+0.33	+0.19	50	9	41
1119 + 3437	1228	0.0344	III^1	1	48	42.3	0.11	0.38	-22.98	-22.87	-22.60	+0.32	-0.28	-0.25	41	0	41
1123 + 3537	1257	0.0339	III^1	0	48	28.8	0.16	0.16	-22.51	-22.35	-22.35	+0.79	+0.24	0.00	76	26	50
1132 + 4920	1314	0.0335	III^2	0	48	45.0	0.35	0.77	-23.36	-23.01	-22.59	-0.06	-0.42	-0.24	98	29	69
1134 + 5515	1318	0.0189	III^2	1	48	45.0	0.24	0.35	-22.99	-22.75	-22.64	+0.31	-0.16	-0.29	105	0	105
1142 + 2007	1367	0.0204	II-III^1	2	PP	85.2	0.12	0.52	-23.35	-23.23	-22.83	-0.05	-0.64	-0.48	76	0	76
1324 - 2652	1736	0.0431	III^1	0	PP	49.0	0.19	0.19	-23.32	-23.13	-23.13	-0.02	-0.54	-0.78	50	0	50
1600 + 1603	2147	0.0351	II^3	1	PP	77.2	0.20	1.41	-23.28	-23.08	-21.87	+0.02	-0.49	+0.48	103	0	103
1603 + 1635	2152	0.0440	III^1	1	PP	57.0	1.02	1.15	-23.26	-22.24	-22.11	+0.04	+0.35	+0.24	93	8	85
1610 + 2940	2162	0.0318	II-III^3	0	PP	85.2	0.12	0.31	-23.44	-23.32	-23.13	-0.14	-0.73	-0.78	17	0	17
1627 + 4101	2197	0.0322	II^1	1	PP	112.1	0.49	0.85	-23.76	-23.27	-22.91	-0.46	-0.68	-0.56	39	0	39
1919 + 4352	2319	0.0549	I-II^3	1	PP	75.8	1.64	1.64	-23.65	-22.01	-22.01	-0.35	+0.58	+0.34	57	0	57
2342 + 0853	2657	0.0414	II-III^1	1	PP	50.3	0.88	1.37	-22.72	-21.84	-21.35	+0.58	+0.75	+1.00	61	10	51
2348 + 2653	2666	0.0273	I-II^3	0	PP	80.5	1.26	2.03	-23.68	-22.42	-21.65	-0.38	+0.17	+0.70	68	15	53
0131 - 36	...	0.0298	(III)	(0)	PP	(30.2)	(0.26)	(1.27)	-23.06	(-22.80)	(-21.79)	+0.24	-0.21	+0.56	~(14)	0	~(14)
0915 - 11	3C218	0.0522	I^3	(0)	PP	53.7	1.62	1.80	-23.74	-22.12	-21.94	-0.44	+0.47	+0.41	40	0	40
1245 - 41	N4696	0.0113	(II)	(1)	PP	(90.6)	(1.42)	(2.15)	-23.10	(-21.68)	(-20.95)	+0.20	+0.91	+1.40	~(50)	0	~(50)
1332 - 33	I4296	0.0114	(I)	(0)	PP	(80.5)	(2.39)	(2.67)	-23.40	(-21.01)	(-20.73)	-0.10	+1.58	+1.62	(15)	0	(15)
1400 - 33	N5419	0.0138	(I)	(0)	PP	(75.1)	(2.52)	(2.86)	-23.54	(-21.02)	(-20.68)	-0.24	+1.57	+1.67	(18)	0	(18)
N68	HMS	0.0226	III^2	(0)	48	53.7	0.29	0.29	-22.50	-22.21	-22.21	+0.80	+0.38	+0.14	17	0	17
N80	HMS	0.0209	III^2	(0)	48	67.1	1.11	1.51	-23.10	-21.99	-21.59	+0.20	+0.60	+0.76	19	0	19
N128	HMS	0.0155	III^3	(0)	PP	(67.1)	(1.99)	(3.00)	-23.16	(-21.17)	(-20.16)	+0.14	(+1.42)	(+2.19)	5	0	5
N194	HMS	0.0177	III^2	(0)	PP	(47.0)	(0.00)	(1.11)	-22.92	(-22.92)	(-21.81)	+0.38	(-0.33)	(+0.54)	14	0	14
N741	HMS	0.0188	II^3	(0)	PP	(73.8)	(2.49)	(3.70)	-23.26	(-20.77)	(-19.56)	+0.04	(+1.82)	(+2.79)	10	0	10
N1600	HMS	0.0160	I^3	(0)	PP	107.3	(2.52)	(3.30)	-23.44	(-20.92)	(-20.14)	-0.14	(+1.67)	(+2.21)	10	0	10
N2563	HMS	0.0159	III^2	(0)	PP	(50.3)	(0.48)	(1.21)	-22.78	(-22.30)	(-21.57)	+0.52	(+0.29)	(+0.78)	17	0	17
N2832	HMS	0.0200	I-II^2	(0)	PP	(57.0)	(2.26)	(2.41)	-23.23	(-20.97)	(-20.82)	+0.07	(+1.61)	(+1.53)	23	0	23
N3158	HMS	0.0234	I-II^2	(0)	PP	(63.7)	(1.88)	(2.17)	-23.50	(-21.62)	(-21.33)	+0.20	(+0.97)	(+1.02)	18	0	18
N5044	HMS	0.0087	III^3	(0)	PP	(100.6)	(0.98)	(1.65)	-22.66	(-21.68)	(-21.01)	+0.64	(+0.91)	(+1.34)	29	0	29
N5077	HMS	0.0084	III^3	(0)	PP	(60.4)	(2.05)	(2.53)	-22.58	(-20.53)	(-20.05)	+0.72	(+2.06)	(+2.30)	6	0	6
N5353	HMS	0.0076	III^3	(0)	PP	(63.7)	(0.00)	(0.00)	-22.78	(-22.78)	(-22.78)	+0.52	(-0.19)	(-0.43)	5	0	5
N7242	HMS	0.0204	II-III	(0)	PP	(50.3)	(2.87)	(3.50)	-22.98	(-20.11)	(-19.48)	+0.32	(+2.48)	(+2.87)	15	0	15
N7385	HMS	0.0258	III^3	(0)	PP	(53.7)	(0.14)	(1.65)	-23.16	(-23.02)	(-21.51)	+0.14	(-0.43)	(+0.84)	15	0	15

on the plates or the prints with a visual micrometer to begin a study of the diameter function[1] (Hardy 1973).

Listed in column (7) of table 1 is the measured angular diameter (θ_1 in arc sec) of the first-ranked galaxy. The quantities $5 \log \theta_1/\theta_2$ and $5 \log \theta_1/\theta_3$ in columns (8) and (9) should closely be the difference in apparent magnitude between the first three ranked members (Sandage 1972*a* [Paper I], fig. 3).[2]

Columns (10), (11), and (12) give the absolute V_C magnitudes of the first three ranked cluster galaxies. The value for the brightest galaxy (col. [10]) is calculated from $M_{V_c} = V_c - 5 \log cz - 16.50$, where the Hubble constant is taken to be $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The magnitudes for the second- and third-ranked galaxies (cols. [11] and [12]) are calculated by applying the magnitude differences of columns (8) and (9) to $M_v^c(1)$.

Residuals from the mean absolute magnitudes are listed in columns (13)–(15), where $\langle M_{v_1}{}^c \rangle = -23.30$, $\langle M_{v_2}{}^c \rangle = -22.59$, and $\langle M_{v_3}{}^c \rangle = -22.35$. The negative sign means brighter than average.

So as to replace Abell's quantized richness classes by a continuous variable with a larger range, we made counts for the population of each cluster, as follows. The angular diameters were estimated for all galaxies that occur within a circle of fixed linear size at the cluster. In particular, the angular diameter was taken to be $\theta = 137(1 + z)^2 z^{-1}$ arc sec, which corresponds to $D \simeq 4$ Mpc ($H_0 = 54 \text{ km s}^{-1}$ if $q_0 = +1$).[3] The total number of galaxies within our standard area that were within $\log \theta_3/\theta_n = 0.5$ (i.e., 2.5 mag) in angular ratio from the third brightest member were counted. This number N is listed in column (16) of table 1.

A background correction, determined by counting adjacent areas, was applied to each cluster. The estimated background numbers are listed in column (17), and the difference (in col. [18]) represents the estimate of cluster population within 2.5 mag of the third brightest within a radius of 2 Mpc from the center as counted on 48-inch Schmidt plates.[4] The *internal* accuracy of the number is ~ 15 percent as judged by repeated counts of several clusters. But the main source of error is traced to the presence of many galaxies of low surface brightness in some clusters. *Magnitude* and *surface brightness* are related differently for these than for the majority of the galaxies, and their measured diameters will not give magnitudes on the same scale. A reasonable estimate of the external error for data in column (18) is about 30 percent.

b) *Contrast and Bautz-Morgan Type*

The correlation between BM type and the magnitude *differences* in columns (8) and (9) in table 1 is strong. Clusters in the first part of table 1 (excluding the Westerlund and Wall, and the HMS groups) were sorted into BM classes, within which mean

[1] No attempt was made to homogenize the estimates by accounting for the differences among the plates and between the plates and the prints; hence no useful cosmological information on the (θ, z) relation is contained in these data. The estimates were used here only to obtain magnitude *differences* between the first three ranked cluster galaxies in a given cluster. Systematic differences among the photographic materials do not enter this particular problem.

[2] These angular diameters are taken to be the average of the major and minor axes, which is near enough the areal proportional value of $(\theta_m \theta_{mi})^{1/2}$.

[3] This size is smaller than Abell's standard diameter ($D \simeq 6$ Mpc) in the $z \rightarrow 0$ limit, but the Abell radius shrinks relative to a constant metric size by $(1 + z)^{-2}$ because it does not include the effect of aberration on the angular measurements. However, the effect on estimates of cluster population is very small because both our radius and the Abell radius are quite large compared with the main body of typical clusters. Small changes in the peripheral area have almost negligible influence on the counts. For the same reason, the counts are largely independent of q_0.

[4] A systematic difference exists between our counts using 200-inch plates and Schmidt plates. From five clusters counted in both series, a factor $\langle N \rangle_{200} = (1.3 \pm 0.3)\langle N \rangle_{48}$ was obtained, and all numbers in columns (16)–(18) of table 1 were reduced to the system of the Schmidt.

TABLE 2

MAGNITUDE DIFFERENCES BETWEEN FIRST THREE CLUSTER GALAXIES AS FUNCTION OF BAUTZ-MORGAN CLASS

Class	V_2-V_1 mag	RMS mag	V_3-V_1 mag	RMS mag	N
I........	1.33	±0.16	1.79	±0.17	8
I-II......	1.02	0.21	1.23	0.22	6
II........	0.82	0.17	1.08	0.19	13
II-III.....	0.43	0.05	0.70	0.07	17
III.......	0.37	0.05	0.51	0.06	29

values were calculated for the magnitude difference between the second- and third-ranked galaxies from the first. The results are listed in table 2 and plotted in figure 1. Least-squares solutions give $V_2 - V_1 = -0.50(\mathrm{BM}) + 1.80$ with a correlation of $r = 0.98$, and $V_3 - V_1 = -0.62(\mathrm{BM}) + 2.30$ with the same correlation. The Bautz-Morgan effect is clearly confirmed quantitatively. The *difference* in contrasts between BM classes I to III is $V_2 - V_1 \simeq 0.9$ mag. It is $V_3 - V_1 \simeq 1.2$ mag for the third- compared with the first-ranked.

III. ABSOLUTE MAGNITUDES OF FIRST THREE RANKED-CLUSTER MEMBERS CORRELATED WITH BAUTZ-MORGAN TYPE

The distribution of absolute magnitudes as a function of BM type was found by studying the residuals from the mean Hubble line (cols. (13)–(15) of table 1). The results, listed in table 3 and shown in figure 2, are startling. The absolute magnitudes of first-ranked galaxies in clusters of large contrast (BM classes I and I–II) *are* brighter than average, in agreement with the conclusion of Bautz and Abell. But the second- and third-ranked are *fainter* than average absolutely. This result appears to be well

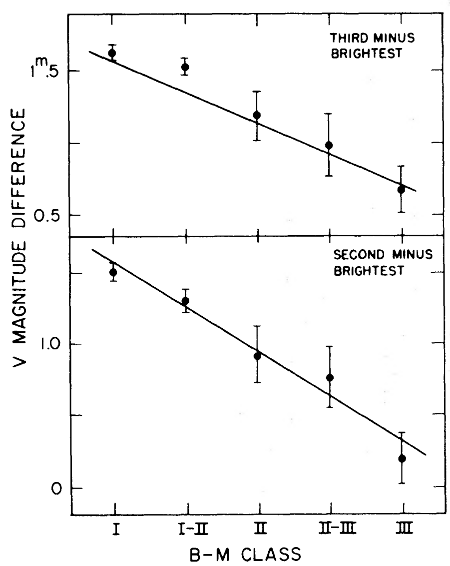

FIG. 1.—Difference in *apparent* magnitude between the first brightest cluster member and the second and third, respectively, as a function of Bautz-Morgan class. Data are from columns (8) and (9) of table 1, and depend on measurement of angular diameters rather than magnitudes. Summary is from table 2.

TABLE 3

RESIDUALS OF M_V^c MAGNITUDES FROM MEAN HUBBLE LINE FOR FIRST, SECOND, AND THIRD RANKED CLUSTER GALAXIES RELATED TO BAUTZ-MORGAN CLASS

BM Class	First Ranked N	$\langle \Delta M_V^c(1) \rangle$	RMS	Second Ranked N	$\langle \Delta M_V^c(2) \rangle$	RMS	Third Ranked N	$\langle \Delta M_V^c(3) \rangle$	RMS
I........	13	-0.36	±0.09	8	+0.29	±0.16	8	+0.49	±0.22
I-II......	8	-0.13	±0.09	6	+0.15	±0.23	6	+0.12	±0.20
II........	15	-0.02	±0.09	13	+0.08	±0.20	13	+0.10	±0.20
II-III.....	19	-0.05	±0.06	17	-0.23	±0.09	17	-0.21	±0.12
III.......	42	+0.22	±0.04	29	-0.17	±0.07	29	-0.27	±0.07

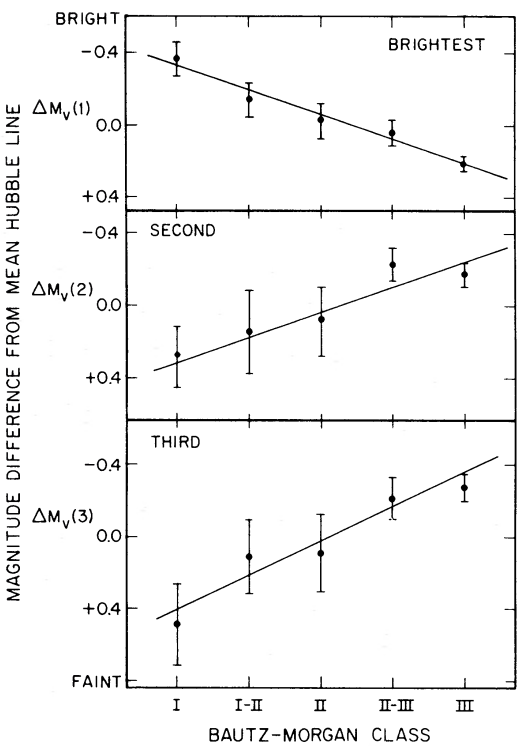

FIG. 2

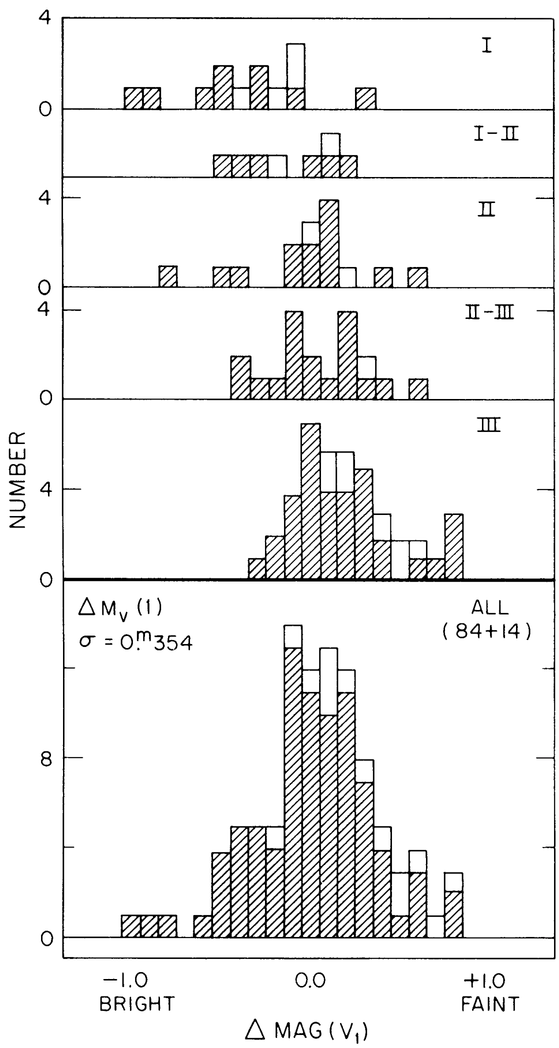

FIG. 3

FIG. 2.—Difference in *absolute* magnitude for the first three-ranked cluster members relative to the mean line of the Hubble diagram, as a function of Bautz-Morgan class. Data are from columns (13)–(15) of table 1, sorted into BM groups and averaged. Summary is from table 3.

FIG. 3.—Histogram of the residuals of absolute magnitude of first-ranked galaxies from the mean, sorted into Bautz-Morgan classes. *Open bars*, HMS groups; *hatched bars* are for the larger clusters. The summed distribution in the bottom panel is from fig. 6 of Paper VI.

determined because the errors (rms) shown in figure 2 are small. The least-squares correlations are $\Delta M_v(1) = 0.268(\text{BM}) - 0.584$; $\Delta M_v(2) = -0.260(\text{BM}) + 0.544$; $\Delta M_v(3) = -0.370(\text{BM}) + 0.786$, with correlation coefficients of about 0.95 for each solution.

The distribution of magnitude residuals within each BM class is shown in the first five panels of figure 3; their sum is given in the sixth. Open bars are the 14 HMS groups listed at the end of table 1 (note that they occur in all BM classes); hatched areas are for all remaining aggregates in table 1. The progressive faintward march with increasing BM class is shown clearly. Because of the systematic nature of the correlation, a systematic error in a determination of q_0 could occur by a special form of the Scott (1957) effect. That such selection does not occur in the present sample is shown in § V. In this regard, it is interesting to note that the average of the magnitudes of the first- and second-ranked galaxies is closely independent of BM effects; a near null equation results from adding the equations for $\Delta M_v(1)$ and $\Delta M_v(2)$.

Preliminary to an eventual understanding of the effect itself and of the opposite sense for the brightest and the next brightest galaxies (fig. 2), two of perhaps many possibilities suggest themselves. The dominance of the first-ranked galaxy at the expense of fainter members was caused by either (1) an early dominance at the time of formation, due to some initial condition, or (2) a result of much later events such as tidal stripping, with subsequent matter transfer to the dominant galaxy (Gallagher and Ostriker 1972; see also Gunn and Gott 1972 and Oemler 1973). We are inclined to believe the first possibility because the BM effect appears to be independent of cluster richness (Bautz and Morgan 1970, and the data of table 1 here with a clear null correlation of BM class with N_c^{48}), whereas the stripping efficiency should depend on population density (Gallagher and Ostriker 1972).

IV. RICHNESS CORRECTION AFTER REMOVING THE BAUTZ-MORGAN EFFECT

a) *Richness Correlation*

The mean values for the Bautz-Morgan effect (table 3 and fig. 2) were applied to the ΔM_i^c residuals listed in columns (13)–(15) of table 1 to remove the BM correlation statistically. The new residuals, $\delta M_{v(i)}^{\text{BM}}$, so corrected, were sorted by richness classes and averaged, with the results given in table 4 and plotted in figure 4.

No strong correlation of first-ranked residuals with richness is shown by these data (the correlation line in the upper panel of figure 4 is practically flat). The present data are, then, in essential agreement with the conclusions of Paper II, and with the prior discussion by Peach (1969). Inclusion of the BM correction has, however, improved the analysis by reducing the internal dispersion.

Figure 4, however, shows a new result. A significant correlation of absolute magnitude with cluster richness does exist for second- and third-ranked galaxies. The

TABLE 4

CORRELATION OF MAGNITUDE RESIDUAL FROM HUBBLE LINE WITH RICHNESS AFTER BAUTZ-MORGAN CORRECTION FOR FIRST THREE RANKED CLUSTER GALAXIES

R	$\delta M_v^{\text{BM}}(1)$	RMS	N	$\delta M_v^{\text{BM}}(2)$	RMS	N	$\delta M_v^{\text{BM}}(3)$	RMS	N
	First Ranked			Second Ranked			Third Ranked		
0.....	+0.10	±0.04	44	+0.45	±0.12	44	+0.62	±0.14	44
1.....	+0.04	0.05	29	+0.10	0.09	27	+0.11	0.09	27
2.....	-0.16	0.08	16	-0.21	0.13	15	-0.26	0.12	15
3.....	-0.12	0.08	7	-0.24	0.10	6	-0.14	0.13	6
4.....	+0.28	...	1	-0.27	...	1	-0.32	...	1

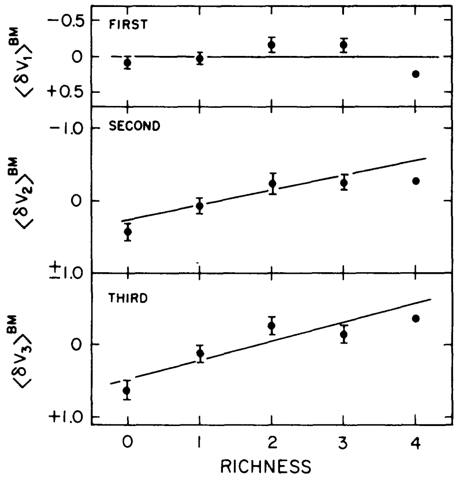

FIG. 4.—The mean *absolute V* magnitude residuals from the Hubble line corrected for Bautz-Morgan effect by table 3, correlated with Abell richness class for the first three-ranked cluster galaxies. Summary data are from table 4.

absolute luminosity of galaxies fainter than the first is then determined by the number of galaxies present in the group. Why this should be so for faint members but not for the first-ranked is not presently understood. Again, the otherwise attractive tidal stripping model of Gallagher and Ostriker (1972) would seem to require that the dominant galaxy would become progressively brighter for increasing population density of the cluster, which evidently does not occur.

One of the goals of this investigation was to extend the richness correlation to small groups. For this, the Abell richness group *zero* had to be subdivided to provide discrimination between groups of 5 and aggregates of say 30 members. It was to this end that the count program was undertaken to obtain the N_c^{48} data.

The result is shown in figure 5, where the absolute-magnitude residuals (after Bautz-Morgan correction) for first-ranked galaxies is shown as the ordinate (brighter than average magnitudes are negative) plotted against log N_c^{48}. A very shallow correlation may exist, but the range is only 0.35 mag over the population interval $5 < N_c^{48} < 220$. Hence, variation occurs at about the 1 sigma level only, and is therefore only marginally significant.

The formal solution for the 98 points in figure 5 is $\delta M_v^{BM}(1) = -0.213 \log N_c^{48} + 0.384$, with a correlation coefficient of only 0.25.

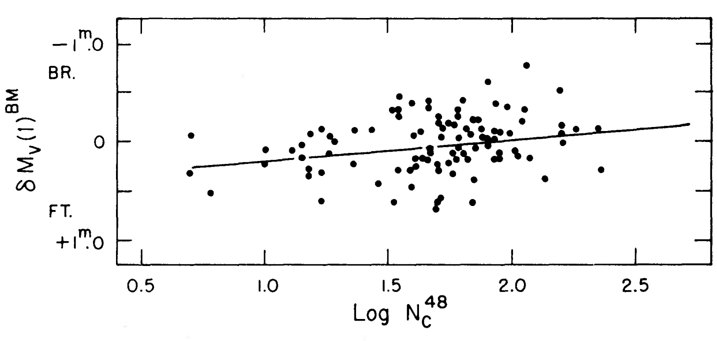

FIG. 5.—Individual absolute *V*-magnitude residuals for first-ranked cluster members as corrected for Bautz-Morgan effect as a function of cluster population. The line is the least-squares solution $\delta M_V^{BM}(1) = -0.213 \log N_c^{48} + 0.384$.

TABLE 5

COMPARISON OF MEAN DEVIATIONS AND SIGMA VALUES FOR THE 97 GROUPS AND CLUSTERS AFTER VARIOUS CORRECTIONS (UNIT IS ABSOLUTE MAGNITUDE)*

Residual	Corrections	Including HMS Groups (97)	Without HMS Groups (83)	HMS Groups Alone (14)	Basic Data
$\langle \Delta V_C \rangle$	Aperture,	$+0^m.046$	$0^m.004$	$+0^m.30 \pm 0^m.08$	Tables 4, 5
$\sigma(\Delta V_C)$	K_V and A_V	$0^m.354$	$0^m.344$	$0^m.313$	of paper VI
$\langle \delta V_1^{BM} \rangle$	Above plus	-0.002	-0.036	+0.20 ± 0.06	Tables 1, 3
$\sigma(\delta V_1^{BM})$	BM effect	0.293	0.292	0.216	here
$\langle \delta V_1^{T} \rangle$	Above plus	+0.012	0.000	+0.08 ± 0.06	Tables 1, 3 and
$\sigma(\delta V_1^{T})$	richness	0.285	0.293	0.226	fig. 5 here

* $\delta V_1^{BM} = \Delta V_1$ (Table 1) $- \langle \Delta V_1 \rangle^{BM}_{\text{Table 3}}$

* $\delta V_1^{T} = \delta V_1^{BM}$ - Fig. 5 (richness).

b) Residuals before and after Corrections

The distributions of first-ranked residuals, before and after the various corrections, are given in table 5. Listed are (1) the distribution of "uncorrected" magnitudes M_v^c (tables 4 and 5 of Paper VI), (2) residuals corrected for Bautz-Morgan effect alone δM_v^{BM}, and (3) residuals δM_V^T after the richness correction was applied as well, where T denotes total correction. Three subsamples are given in table 5: the 98 aggregates including the HMS groups; 83 aggregates which is the total sample minus the HMS groups; and the HMS group alone.

The sigma of the magnitude distribution generally decreases with each correction ($\sigma = 0.35$ mag for the raw residuals; $\sigma = 0.28$ after the BM and the richness corrections). Note also that the mean residuals for the *HMS groups* decline from $\langle \Delta M_v^c \rangle = +0.30$ mag without BM and richness corrections, to 0.08 ± 0.06 mag after the total correction. The slight faintness of the HMS group mean magnitudes is evidently largely removed by the population correction.

Figure 6 shows the effect of the BM and richness corrections on the distributions themselves. The lower panel, copied from Paper VI, is noticeably wider in σ than the upper panel.

V. THE HUBBLE DIAGRAM AND THE EFFECT OF THE BAUTZ-MORGAN AND RICHNESS CORRECTIONS ON DETERMINATIONS OF q_0

A summary of the final magnitudes, after various corrections, is given in table 6 for the total sample. The basic magnitudes B_c, V_c, and R_c of Paper VI (tables 4 and 5), corrected for Bautz-Morgan effect with the use of table 3, are listed in columns (8)–(10). Magnitudes further corrected for cluster richness (fig. 5) are given in columns (11)–(13).

The Hubble diagram using V_c^T magnitudes is plotted in figure 7 for the entire sample. The residuals, taken relative to the line $V_c^T = 5 \log cz - 6.83$, are distributed as in the top panel of figure 6 (§ IV).

The fit to Hubble's linear law $cz = Hr$ locally[5] is excellent, as shown by no systematic deviation of the observations from the line. The conclusion to be drawn from these data is the same as given previously (Sandage, Tammann, and Hardy 1972, table 1), where a formal least-squares comparison gave agreement between the observations and the Hubble linear expansion law to within $\sigma/2$.

A graphical comparison is given in figure 8, which shows the magnitude residuals about the $q_0 = +1$ line as a function of redshift. The envelopes of the distribution are

[5] "Locally" is here taken to mean redshifts larger than 3000 km s^{-1} so as to avoid any nearby anisotropy such as suggested by de Vaucouleurs (1959), but less than $cz \simeq 30,000$ km s^{-1} to avoid the effects of deceleration and light travel time on the meaning of cz and r.

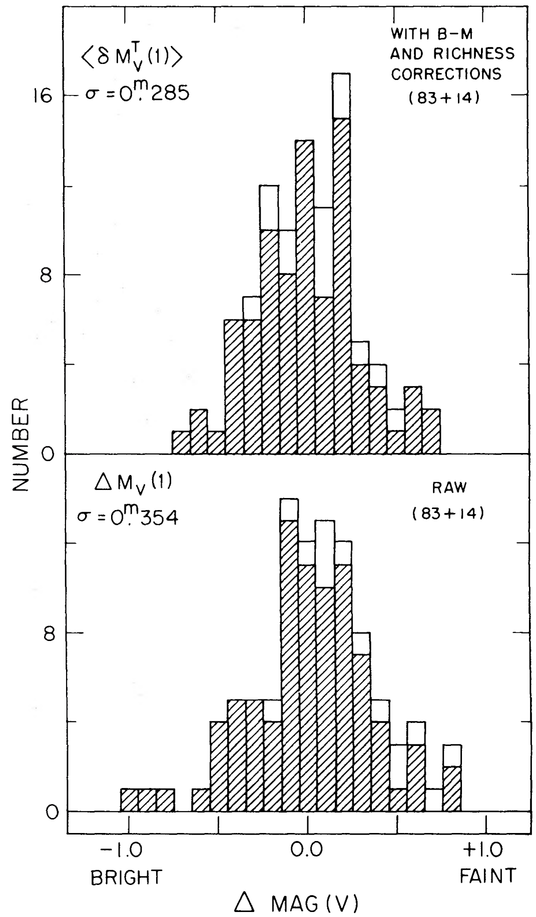

FIG. 6.—Histogram of absolute V-magnitude residuals for first-ranked cluster galaxies corrected for Bautz-Morgan and richness effects (*upper*), compared with the distribution without these corrections (*lower*) taken from fig. 6 of Paper VI. *Open bars*, HMS groups; *hatched bars*, all others.

two lines *parallel* to the ordinate, similar to the diagram drawn from earlier data in Paper II (fig. 5). A stringent limit to any noncosmological redshift component Δz from this constancy of $\Delta z/z$ is $\sigma(\Delta z/z) \lesssim 0.1$, or $c\Delta z \simeq 10^2$ km s^{-1} in order of magnitude, by the same argument and by the same amount as in Paper II (§ VI).

Theoretical lines for particular values of the apparent deceleration parameter are drawn in figure 8 from the standard Mattig (1958) (m, z) equation. Clearly, by inspection, q_0(apparent) $\simeq 1 \pm 1$ from these data, even after the BM correction.

To assess the effect of the corrections more precisely, we made 10 formal least-squares solutions for q_0 using the method of Paper II (§ VIII). Various combinations of the data in three colors (*BVR*) were used. Three solutions were made using the B_c, V_c, and R_c magnitudes of Paper VI, tables 4 and 5; six solutions used the corrected magnitudes in columns (8)–(13) of table 6 for the great clusess alone (no HMS or Westerlund and Wall groups); and one solution was made for all 97 aggregates in the sample, using V_c^T magnitudes.

A summary of results is given in table 7. The $\sigma(M_v)$ values listed in the final column show the magnitude dispersions at the minimum. Recall that the solution from Paper II was q_0(apparent) $= +0.96 \pm 1$ ($2\,\sigma$) using V_c magnitudes for 39 clusters, giving $\sigma(\Delta M_v{}^c) = 0.25$ mag (Paper II, fig. 10). These values are the same as those in table 7, when the errors are considered.

TABLE 6

PHOTOMETRIC DATA FOR FIRST RANKED CLUSTER AND GROUP GALAXIES (TABLES 4 AND 5 OF PAPER VI)
CORRECTED FOR RICHNESS AND BAUTZ-MORGAN CLUSTER TYPE*

Cluster (1)	l^{II} (2)	b^{II} (3)	z (4)	log cz (5)	$B-M$ (6)	N_C^{48} (7)	B_C^{BM} (8)	V_C^{BM} (9)	R_C^{BM} (10)	B_C^T (11)	V_C^T (12)	R_C^T (13)	$\delta V_{C,1}^{T*}$ (14)
Clusters													
Virgo	287	+70	0.00381	3.058	III	~60	9.22	8.22	7.38	9.21	8.21	7.37	-0.25
Fornax	240	-57	0.00509	3.184	II-III	~50	9.85	8.85	...	9.83	8.83	...	-0.26
Peg I	88	-48	0.0128	3.584	II	59	12.28	11.27	10.39	12.27	11.26	10.38	+0.17
0122 + 3305	131	-29	0.0170	3.710	II	61	12.74	11.79	10.92	12.73	11.78	10.91	+0.06
Perseus	151	-13	0.0181	3.735	II-III	63	12.03	11.45	10.69	12.03	11.45	10.69	-0.40
Coma	57	+88	0.0222	3.824	II	115	(12.52)	(11.53)	...	12.57	11.58	...	-0.71
Abell 1213	201	+69	0.0287	3.935	III	69	14.51	13.50	12.65	14.52	13.51	12.66	+0.66
Hercules	32	+44	0.0341	4.017	III	~85	13.68	12.90	...	13.71	12.93	...	-0.32
Abell 119	126	-64	0.0387	4.065	II-III	66	14.67	13.69	12.82	14.67	13.69	12.82	+0.20
2308 + 0720	84	-48	0.0428	4.109	II-III	23	14.51	13.61	12.92	14.42	13.52	12.83	-0.20
2322 + 1425	93	-43	0.0440	4.120	I	49	15.36	14.47	13.65	15.34	14.45	13.63	+0.68
1145 + 5559	141	+59	0.0516	4.191	II-III	55	15.28	14.34	...	15.27	14.33	...	+0.20
0106 - 1536	143	-78	0.0526	4.198	II	86	15.05	14.14	13.37	15.08	14.17	13.40	+0.01
1024 + 1039	233	+52	0.0649	4.290	III	80	15.56	14.59	...	15.58	14.61	...	-0.01
1239 + 1852	287	+81	0.0718	4.333	II-III	46	15.42	14.45	13.58	15.39	14.42	13.55	-0.42
1520 + 2754	43	+57	0.0722	4.334	III	89	15.97	15.03	14.14	16.00	15.06	14.17	+0.22
Abell 2670	81	-69	0.0775	4.366	I	184	16.10	15.13	14.28	16.20	15.23	14.38	+0.23
Abell 2029	7	+50	0.0777	4.367	I	...	15.37	14.41	13.55	15.39	14.43	13.57	-0.58
0705 + 3506	182	+18	0.0779	4.369	II-III	27	15.85	14.91	13.96	15.77	14.83	13.88	-0.18
1513 + 0433	5	+49	0.0944	4.452	III	55	16.14	15.25	14.40	16.13	15.24	14.39	-0.19
Abell 98	121	-42	0.1028	4.489	II-III	229	16.45	15.50	14.63	16.57	15.62	14.75	0.00
Abell 274	161	-64	0.1289	4.587	II-III	...	17.01	16.03	15.12	17.09	16.11	15.20	0.00
1431 + 3146	51	+67	0.1312	4.595	III	61	16.69	15.84	...	16.69	15.84	...	-0.30
1055 + 5702	149	+54	0.1345	4.606	II-III	35	16.81	15.88	...	16.75	15.82	...	-0.38
1153 + 2341	225	+77	0.1426	4.631	I	155	16.76	15.82	...	16.68	15.74	...	-0.58
1641 + 1327	358	+34	0.1499	4.653	III	...	17.50	16.47	...	17.58	16.39	...	-0.04
1534 + 3749	61	+54	0.1532	4.662	III	159	17.29	16.33	...	17.37	16.41	...	-0.07
0025 + 2223	115	-40	0.1594	4.680	III	64	17.37	16.44	15.65	17.37	16.44	15.65	-0.13
1228 + 1050	285	+73	0.1651	4.695	II-III	119	17.79	16.85	...	17.85	16.91	...	+0.26
0138 + 1832	139	-43	0.1730	4.714	II	33	18.35	17.36	16.65	18.29	17.30	16.59	+0.56
1309 - 0105	313	+61	0.1745	4.719	I-II	228	17.86	17.06	...	17.98	17.18	...	+0.42
1304 + 3110	82	+85	0.1831	4.740	III	111	(17.63)	(16.68)	...	17.68	16.73	...	-0.14
0925 + 2044	209	+43	0.1917	4.760	I-II	95	17.62	16.63	15.63	17.66	16.67	15.67	-0.30
1253 + 4422	121	+73	0.1979	4.774	III	136	18.41	17.44	...	18.48	17.51	...	+0.47
0855 + 0321	226	+29	0.2018	4.782	II-III	71	18.11	17.15	16.30	18.10	17.14	16.29	+0.06
1447 + 2617	37	+63	0.36	5.033	II-III	71	19.73	18.73	17.79	19.72	18.72	17.78	+0.38
0024 + 1654	115	-45	0.38	5.057	III	113	19.10	18.13	...	19.15	18.18	...	-0.28
Radio Clusters													
3C31	127	-30	0.0169	3.706	III	13	12.86	11.81	10.98	12.71	11.66	10.83	-0.04
3C40	142	-63	0.0180	3.732	II	39	13.30	12.31	11.49	13.26	12.27	11.45	+0.44
3C66	140	-17	0.0215	3.810	II-III	39	13.36	12.53	11.67	13.32	12.49	11.63	+0.27
3C465	104	-33	0.0301	3.956	I-II	58	14.25	13.28	12.41	14.24	13.27	12.40	+0.32
3C338	63	+44	0.0303	3.958	I	89	13.70	12.88	12.05	13.73	12.91	12.08	-0.05
3C317	9	+50	0.0351	4.022	II	63	14.38	13.42	12.55	14.38	13.42	12.55	+0.14
M23-112	66	-64	0.0825	4.394	III	...	15.95	15.16	14.34	15.97	15.18	14.36	+0.04
3C219	174	+45	0.1745	4.719	II	89	17.76	16.92	16.09	17.79	16.95	16.12	+0.18
3C28	124	-37	0.1959	4.769	III	97	17.72	16.94	16.00	17.76	16.98	16.04	-0.04
3C295	97	+61	0.461	5.141	I	58	19.99	19.00	18.15	19.98	18.99	18.14	+0.12
Westerlund and Wall													
0131 - 36	261	-77	0.0298	3.951	III	~14	13.89	12.97	...	13.75	12.83	...	-0.10
0915 - 11	243	+25	0.0522	4.194	I	40	14.94	14.09	...	14.90	14.05	...	-0.09
1245 - 41	302	+22	0.0113	3.498	II	~50	11.87	10.91	...	11.85	10.89	...	+0.23
1332 - 33	313	+28	0.0114	3.535	I	15	12.09	11.13	...	11.96	11.00	...	+0.16
1400 - 33	320	+27	0.0138	3.617	I	18	12.37	11.40	...	12.25	11.28	...	+0.02
Peterson's Abell List													
								V_C''	R_C''				
A76	117	-56	0.0377	4.053	I	50	...	13.73	12.90	...	13.71	12.88	+0.28
A147	131	-60	0.0441	4.122	III	35	...	14.07	13.23	...	14.01	13.17	+0.23
A262	137	-25	0.0168	3.702	III	47	...	11.76	10.96	...	11.73	10.93	+0.05
A278	139	-29	0.0904	4.433	III	35	...	15.08	14.28	...	15.02	14.22	-0.32

TABLE 6 -continued

Cluster (1)	l^{II} (2)	b^{II} (3)	z (4)	log cz (5)	$B-M$ (6)	N_C^{48} (7)	B_C^{BM} (8)	V_C^{BM} (9)	R_C^{BM} (10)	B_C^T (11)	V_C^T (12)	R_C^T (13)	$\delta V_{C,1}^{T*}$ (14)
A376	147	-21	0.0487	4.165	I-II	46	...	14.19	13.39	...	14.16	13.36	+0.16
A505	132	+22	0.0543	4.212	I	43	...	14.14	13.32	...	14.10	13.28	-0.13
A539	196	-18	0.0267	3.904	III	86	...	12.86	12.05	...	12.89	12.08	+0.20
A548	230	-24	0.0391	4.069	III	70	...	13.31	12.51	...	13.32	12.52	-0.20
A553	165	+14	0.0670	4.303	II	68	...	14.64	13.85	...	14.65	13.86	-0.04
A569	169	+23	0.0193	3.763	II	47	...	12.14	11.31	...	12.11	11.28	+0.12
A576	161	+26	0.0404	4.084	III	60	...	13.56	12.75	...	13.55	12.74	-0.04
A592	210	+16	0.0621	4.270	III	33	...	14.22	13.54	...	14.16	13.48	-0.36
A634	159	+34	0.0266	3.902	III	44	...	12.87	12.04	...	12.84	12.01	+0.16
A671	193	+33	0.0497	4.173	II	46	...	13.69	12.88	...	13.66	12.85	-0.38
A754	239	+25	0.0537	4.207	I	74	...	14.08	13.30	...	14.09	13.31	-0.12
A993	249	+42	0.0530	4.201	III	35	...	13.73	12.93	...	13.67	12.87	-0.50
A1060	270	+27	0.0115	3.538	III	51	...	10.84	10.03	...	10.82	10.01	-0.04
A1139	251	+53	0.0376	4.052	II-III	41	...	13.69	12.91	...	13.65	12.87	+0.22
A1228	187	+69	0.0344	4.014	III	41	...	13.42	12.65	...	13.38	12.61	+0.14
A1257	183	+70	0.0339	4.007	III	50	...	13.85	13.03	...	13.83	13.01	+0.62
A1314	152	+64	0.0335	4.002	III	69	...	12.98	12.18	...	12.99	12.19	-0.19
A1318	144	+59	0.0189	3.753	III	105	...	12.10	11.35	...	12.15	11.40	+0.22
A1367	234	+73	0.0204	3.787	II-III	76	...	12.08	11.31	...	12.10	11.33	0.00
A1736	313	+35	0.0431	4.112	III	50	...	13.56	12.71	...	13.54	12.69	-0.19
A2147	29	+45	0.0351	4.022	II	103	...	13.39	12.60	...	13.43	12.64	+0.15
A2152	30	+44	0.0440	4.121	III	85	...	13.67	12.90	...	13.70	12.93	-0.08
A2162	49	+46	0.0318	3.980	II-III	17	...	12.96	12.18	...	12.84	12.06	-0.23
A2197	65	+44	0.0322	3.985	II	39	...	12.73	11.93	...	12.69	11.89	-0.40
A2319	76	+14	0.0549	4.217	I-II	57	...	14.11	13.34	...	14.10	13.33	-0.16
A2657	97	-50	0.0414	4.094	II-III	51	...	14.25	13.40	...	14.23	13.38	+0.59
A2666	107	-34	0.0273	3.913	I-II	53	...	12.56	11.80	...	12.54	11.78	-0.20
HMS Groups													
N68	114	-32	0.0226	3.831	III	17	13.90	12.94	12.14	13.78	12.82	12.02	+0.50
N80	114	-40	0.0209	3.797	III	19	13.16	12.16	11.33	13.05	12.05	11.22	-0.10
N128	112	-60	0.0155	3.667	III	5	12.35	11.45	10.59	12.12	11.22	10.36	-0.28
N194	117	-60	0.0177	3.725	III	14	12.89	11.99	11.13	12.75	11.85	10.99	+0.06
N741	151	-54	0.0188	3.751	II	10	13.04	12.01	11.24	12.87	11.84	11.07	-0.08
N1600	200	-33	0.0160	3.681	I	10	12.74	11.82	10.96	12.57	11.65	10.79	+0.08
N2563	203	+29	0.0159	3.677	III	17	12.83	11.88	11.03	12.71	11.76	10.91	+0.20
N2832	191	+44	0.0200	3.778	I-II	23	13.27	12.29	11.45	13.18	12.20	11.36	+0.14
N3158	183	+55	0.0234	3.846	I-II	18	13.32	12.36	11.50	13.20	12.24	11.38	-0.16
N5044	311	+46	0.0087	3.415	III	29	11.69	10.70	9.89	11.62	10.63	9.82	+0.38
N5077	314	+50	0.0084	3.401	III	6	11.72	10.71	9.87	11.50	10.49	9.65	+0.32
N5353	83	+71	0.0076	3.359	III	5	11.31	10.29	9.44	11.08	10.06	9.21	+0.10
N5846	0	+49	0.0060	3.257	III	10	9.90	10.94	9.09	9.73	10.77	8.92	+0.28
N7242	92	-16	0.0204	3.787	II-III	15	13.41	12.41	11.55	13.28	12.28	11.42	+0.18
N7385	82	-41	0.0258	3.889	III	15	13.56	12.56	11.68	13.43	12.43	11.55	-0.18

*$(\text{mag})_C^T \equiv (\text{mag})_C - \langle \Delta M \rangle_{\text{Table 3}}^{BM} - \delta V_1(\text{richness})$.

*The δV_C^T calculated from $V_C^T = 5 \log cz - 6.83$.

Formal analysis of the errors in table 7 was not made by the method of steepness of descent to $\sigma(q_0, C)_{\min}$ as in Paper II, but comparison here with the equivalent of figure 10 of that paper shows that the errors in table 7 are larger here by -30 percent. Especially inaccurate is q_0 from R_c because of the scarcity of many clusters with large z.

The principal conclusion from table 7 is that the corrections for BM effect and richness do not appreciably affect q_0 in our particular sample. For example, the V-magnitude data give $q_0(V_c) = +0.94 \pm 1$ ($2\,\sigma$); $q_0(V_c^{BM}) = +0.95 \pm 1$ ($2\,\sigma$); and $q_0(V_c^T) = +0.80 \pm 1$ ($2\,\sigma$), which are the same within the errors.

But obviously, q_0 is not determined by these data. The two problems that presently block an adequate solution are (1) the unknown correction due to evolutionary changes in absolute luminosity in the look-back time, and (2) the lack of data for the clusters with $z \gtrsim 0.4$ that are required in great numbers.

We must emphasize that none of the data or the analysis in any paper of this series has solved these problems. The determination of q_0 is clearly a problem for the future. Only the grossest alternative solutions such as $q_0 = -1$ or $q_0 > 3$ can be discarded at this moment. However, the prediction of steady-state cosmology, where no mean evolutionary correction is needed, is clearly at variance with even the present data.

VI. SUMMARY OF CLUSTER PROPERTIES

Despite the failure to yet determine q_0, a number of properties of groups and clusters are suggested by the data here and in Paper VI that may bear on questions of

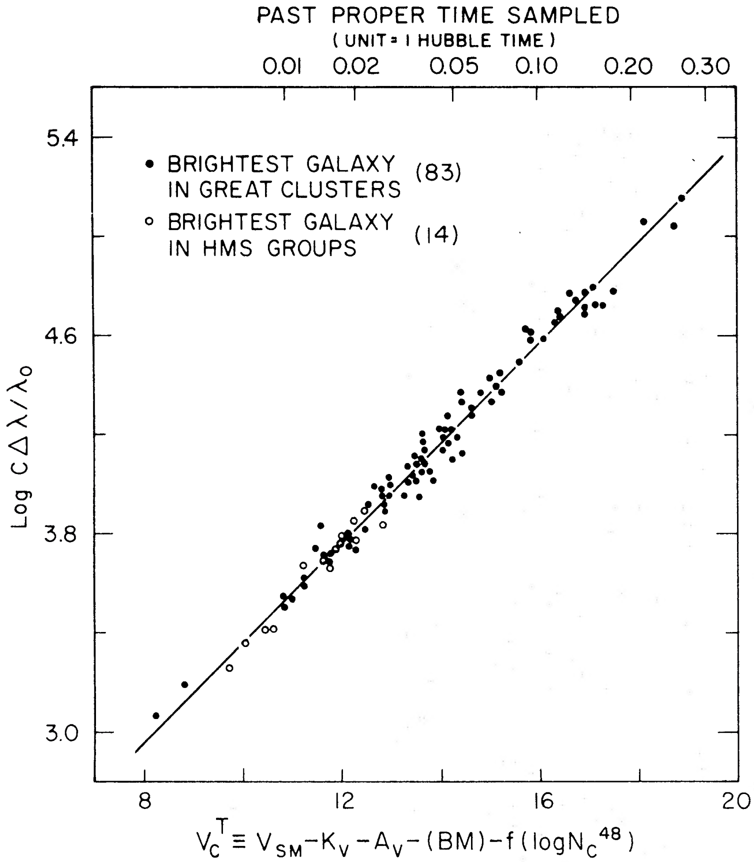

FIG. 7.—Hubble diagram for totally corrected V magnitudes (aperture, K-term, galactic absorption, Bautz-Morgan, and richness) from data in columns (5) and (12) of table 6. *Open circles*, HMS groups; *closed circles*, all others. The look-back time for $q_0 = +1$ (Sandage 1961) is shown along the top border as it applies to the redshift read from the line at the given value of the abscissa.

formation and evolution. Many of these must be tested by more complete observations, but it may be useful to list those properties that seemed most suggestive to us.

1. The absolute magnitudes of first-ranked cluster E and S0 galaxies are nearly independent of cluster richness (Paper II, fig. 8; figs. 4 and 5 here), providing that the group is not *compact*, as defined below. Absolute magnitudes of the second- and third-ranked galaxies vary significantly with richness (fig. 4 here).

2. First-ranked galaxies in *compact* groups (defined here to be those where the ratio of projected separation to apparent diameter of the brightest galaxies is less than 2) are absolutely *faint* as judged by their position in the Hubble diagram (Paper VI, figs. 2 and 4). Examples are G68 and G6027. Is this an initial condition, or the result of later interactions?

3. The Bautz-Morgan groups and the total population (richness) are not correlated. This may suggest that the central dominant cD galaxy in BM class I clusters does not grow by accretion of matter obtained by tidal stripping (Gallagher and Ostriker 1972) of other members, but rather that some initial condition is involved. Otherwise, the *brightest* first-ranked galaxy in BM class I clusters should occur in the most populous cluster, contrary to observation [there is only a weak correlation, if any, between $\Delta M_v(1)$ and N_c^{48} for BM class I clusters from the data of table 1].

1973ApJ...183..743S

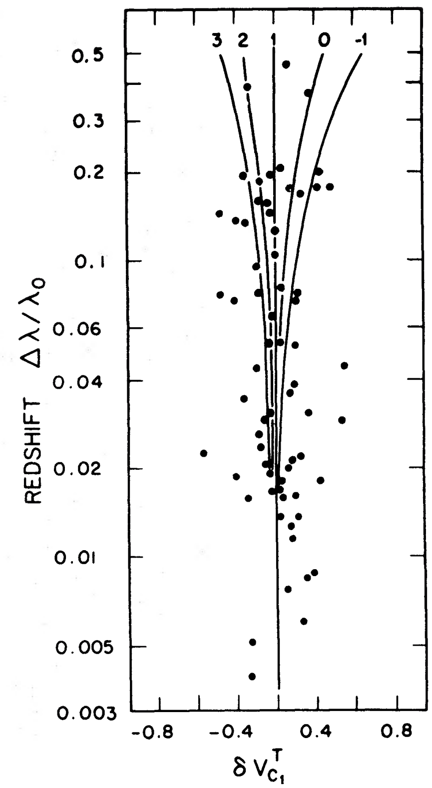

FIG. 8.—Magnitude residuals from fig. 7 read horizontally from the line correlated with redshift. Theoretical curves for various apparent q_0 values are shown. Negative magnitude residuals are in the sense of brighter luminosities. The true q_0 value depends on unknown evolutionary corrections not discussed here.

TABLE 7

TEN SOLUTIONS FOR q_0 USING MAGNITUDES IN THREE COLORS WITH AND WITHOUT CORRECTIONS FOR BAUTZ-MORGAN AND RICHNESS EFFECTS

Magnitude Type	Sample	Source	N	q_0	σ(M) mag
B_C	First cluster group plus radio clusters	Table 4 of Paper VI	47	0.84	0.381
V_C			47	0.94	0.365
R_C			32	1.55	0.358
B_C^{BM}	First cluster group plus radio clusters	Table 6 here Cols 8-10	47	0.84	0.340
V_C^{BM}			47	0.95	0.317
R_C^{BM}			32	1.15	0.316
B_C^{T}	First cluster group plus radio clusters	Table 6 here Cols 11-13	47	0.65	0.340
V_C^{T}			47	0.80	0.317
R_C^{T}			32	1.10	0.307
V_C^{T}	Everything	Table 6, Col 12	97	1.13	0.282

4. Item 3 makes even more puzzling the *reversal* of *sense* in the correlations of absolute magnitude and BM type between first-ranked and fainter members (fig. 2 here). The brighter the dominant galaxy becomes, the absolutely fainter will be the second- and third-ranked members. "The rich are rich at the expense of the poor, progressively." If it were not that the luminosity of the dominant members depends only weakly, if at all, on cluster richness, we would take this point (dominance at the expense of fainter galaxies) as favorable to the view of Gallagher and Ostriker. However, the near independence of properties on population is puzzling.

5. But a *compound* effect of cluster richness is involved for the fainter galaxies, as seen by appropriately combining figures 2 and 4. In small groups of Bautz-Morgan class I, the *second-* and *third*-ranked members are very faint absolutely. In *equally small groups* of BM class III, the second and third are not nearly so faint. Examples are the HMS groups G1600, 2832, 3158. Does the formation of a large dominant galaxy in a small group leave too little matter to form large secondary members? In this regard, it is interesting that the *sum* of the luminosities of the first- and second-ranked members is nearly constant, and independent of Bautz-Morgan type (first two panels of fig. 2).

We expect that some or all of these five statements will be modified as better data become available. It is too early to tell if these or their modified versions will be helpful in eventually formulating a theory of cluster formation, with later evolution.

It is a pleasure to thank Drs. Bautz and Abell for sending us a preprint of their paper where the problem of absolute magnitudes of galaxies as a function of Bautz-Morgan class was first discussed. We also thank M. Riley for the computer programming required to calculate table 7. One of us (E. H.) wishes to thank the Carnegie Institution for a fellowship to work in Pasadena for two summer periods.

Again it is a particular pleasure to thank W. L. W. Sargent for permitting us to use and to quote his new redshift values for the Abell clusters 98, 274, 655, 2029, 2224, and 2670. These clusters proved to be of special importance because several of them are of Bautz-Morgan class I, and their inclusion appreciably strengthens the correlations.

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