

SPECTROPHOTOMETRIC STUDIES OF GASEOUS NEBULAE. XXII. THE IRREGULAR RING NEBULA NGC 6445

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ABSTRACT

The relatively high excitation, low surface brightness nebula NGC 6445 ($8 + 3^{\circ}1$) shows a spectrum characterized by strong lines of He II $\lambda 4685$, [Ne III] as well as [O II] $\lambda 3727$, [S II], [O I], and probably [N II]. The observational data strongly suggest an excess of helium.

Subject headings: abundances, nebular — planetary nebulae

I. INTRODUCTION

Even though characterized by strong lines of [O II] $\lambda 3727$, NGC 6445 must be included among high-excitation planetary nebulae which presumably represent an advanced stage of evolution. Curtis (1918) remarked that the "brighter portion is a very irregular square-shouldered ring, $38'' \times 29''$, beyond which extended very faint ansae." The nebula has also been compared to a coral atoll and enclosed lagoon. The spectrum has been studied by Minkowski (1942), by Page (1942), who noted the great strength of $\lambda 3727$ [O II], and by Aller (1951), who noted that the monochromatic images resemble one another closely.

Low surface brightness and southerly declination render NGC 6445 a difficult object for study from northern latitudes. We have combined photoelectric measurements secured with the 60-inch (152-cm) Mount Wilson telescope, with photometrically calibrated spectrograms obtained there, and with the nebular spectrograph at the 120-inch (305-cm) Lick telescope. The photometric procedures are described in previous papers of the series.

Table 1 lists the observed lines and their adopted intensities as they appear integrated over the nebula. It was not practical to measure weak lines photoelectrically, and for these we have to rely on the photographic measurements. The measured intensities of most of the lines (which depend on photographic photometry) suffer from the fact that this nebula has a surface brightness about 100 times lower than an object such as NGC 6543. On the scale $I(H\beta) = 100$, the weakest lines have intensities of the order of 1.5 to 3 and are subject to accidental errors of about 50 percent. The photographic observations were confined to the region $\lambda 3700$ – $\lambda 5010$; lines measured photoelectrically are indicated by an asterisk. We observed lines of H, He I, He II, [N II] (blended with $H\beta$), [O I], [O II], [O III], [Ne III], [S II], [S III] (?), [Ar III], [Ar V].

Especially noteworthy is the high excitation of the nebula (excitation class 8) as illustrated by the strength of the He II $\lambda 4686$ line ($I = 65$), while at the same time, low-excitation lines of [O I], [O II], and [S II] are very prominent. The most striking feature

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TABLE 1
SPECTRUM OF NGC 6445

| λ | Identification | <i>I</i> |
|------------------|-------------------|----------|
| 3722..... | H14 [S III] | 1.7 |
| 3726*..... | [O II] | 110 |
| 3729*..... | [O II] | 101 |
| 3734..... | H13 | 1.4 |
| 3750..... | H12 | 1.9 |
| 3755..... | O III | 0.7 |
| 3759..... | O III | 1.5 |
| 3770..... | H11 | 2.4 |
| 3797..... | H10 | 3.2 |
| 3835..... | H9 | 5.2 |
| 3868*..... | [Ne III] | 129 |
| 3889*..... | H8 + He | 14 |
| 3967..... | [Ne III] | 41 |
| 3970..... | H7 | 10 |
| 4026..... | He I | 2.2 |
| 4068..... | [S II] | 5.4 |
| 4076..... | [S II] | 2.7 |
| 4101..... | H δ | 22 |
| 4340..... | H γ | 42 |
| 4363..... | [O III] | 18 |
| 4471..... | He I | 6.0 |
| 4686*..... | He II | 65 |
| 4711..... | [Ar IV] } | 5.1 |
| 4713..... | He I } | |
| 4740..... | [Ar IV] | 3.8 |
| 4861*..... | H β | 100 |
| 4959*..... | [O III] | 490 |
| 5007*..... | [O III] | 1510 |
| 5876*..... | He I | 30 |
| 6300, 6312*..... | [O I] + [S III] | 24 |
| 6563, etc.*..... | H α [N II] | 1820 |
| 6716, 6730*..... | [S II] | 125 |
| 7065*..... | He I | 6.8 |
| 7135*..... | [Ar III] | 27 |
| 7320, 7330*..... | [O II] | 65 |

* Lines measured photoelectrically.

of the nebular spectrum, however, is the great strength of the helium lines which appear to admit no other interpretation than a high He/H ratio, perhaps twice that found in normal nebulae.

One photographic plate was taken which allows us to measure line intensities at different points in the nebula. In table 2 we present results for the central region of the nebula, and at two points in the ring, one of high excitation and one of low excitation. Note in particular the great variation of the He I, He II and [O II] lines between high- and low-excitation portions of the ring. The high value for $\lambda 4340$ H γ is probably due to the line being overexposed on the plate.

II. SPACE ABSORPTION

Let us define $C = \log F(\text{H}\beta)_{\text{true}}/F(\text{H}\beta)_{\text{obs}}$ where $F(\text{H}\beta)_{\text{obs}}$ is the flux in H β actually received at the Earth and $F(\text{H}\beta)_{\text{true}}$ is the flux that would be received if there were no space absorption. Various estimates of C are available, mostly based on comparison of radiofrequency and optical fluxes, although sometimes based on data obtained in the optical region of the spectrum. Utilizing only optical data, Minkowski (1965)

TABLE 2
STRATIFICATION EFFECTS IN NGC 6445

| λ | RING | | CENTER |
|-------------------------------|-------------------|-------------------|-------------------|
| | High | Low | |
| 4959..... | 515 | 475 | 575 |
| 4861..... | 100 | 100 | 100 |
| 4686..... | 55 | 5 | 76 |
| 4471..... | 4 | 6 | ... |
| 4363..... | 18 | 16 | 19 |
| 4340..... | 47 | 47 | 46 |
| 4101..... | 24 | 22 | 21 |
| 3889..... | 14 | 15 | 15 |
| 3868..... | 124 | 124 | 121 |
| 3729..... | 111 | 172 | 118 |
| 3726..... | 136 | 158 | 122 |
| Relative H β flux..... | 1.00 | 0.70 | 0.27 |
| T_e ($^{\circ}$ K)..... | 13,700 | 13,300 | 13,400 |
| N_e per cm^3 | 1.4×10^3 | 7.3×10^2 | 4.8×10^2 |
| He/H..... | 0.17 | 0.17 | > 0.07 |
| O/H..... | 4.20(-4) | 3.0(-4) | ... |

obtained $C = 1.49$, which seems to agree fairly well with the estimate by Aller and Milne (1972), who obtained $C = 1.41$, from a comparison of 11-cm and H β fluxes. Cahn and Kaler (1970) adopt a value of 1.21, although in their table 3 they quote values of C ranging from 1.04 to 1.52, which depend upon the value of the H β flux chosen.

A comparison of the observed Balmer decrement with theoretical predictions of Clarke (1965) and of Brocklehurst (1971), for electron densities of the order of 10^3 and electron temperatures of the order of $15,000^{\circ}$ K, together with a comparison of observed and predicted (Brocklehurst 1972) helium line intensities, suggest a value of $C = 0.8$. With this value of C , the measured values of the line intensities in the spectrum of NGC 6445 admit of an internally consistent interpretation. A larger value of C , such as 1.2 or 1.4, gives ionic concentrations (abundances) depending on wavelength for a given species such as S II or He I. Accordingly, we have adopted $C = 0.8$.

The values of C derived from the two methods are reconcilable. Cahn and Kaler (1970) used a value of electron temperature of 5000° K as appropriate to hydrogen emission. If we assume that the hydrogen radiates at a temperature appropriate to that derived from the forbidden lines, and if we assume that the ratio of total to selective absorption is of the order of 4 rather than 3, the value of C derived from the radio observations comes down to meet that derived only from the optical observations.

III. SUGGESTED INTERPRETATION OF THE OBSERVATIONS

The measurements of the fluxes received at the Earth in $\lambda 5007$ of [O III] as made by O'Dell (1963) and by Aller and Liller (1968) are in excellent accord, $\log F(N_1) = -9.97$. Adopting an angular radius of $16.6''$ (O'Dell 1963), and $\log F(N_1)/F(\text{H}\beta) = 1.18$ from our present photometry, we get $\log F(\text{H}\beta) = -11.15$, which compares well with the value of $\log F(\text{H}\beta) = 11.20$ found by O'Dell (1963). The corresponding flux through the nebular surface is given by $\log F(\text{H}\beta) = -2.96 + 0.8 = -2.16$, to allow for space absorption. We compute the density from the surface brightness by equation (125) of Aller and Liller (1968), assuming the nebula to be a shell of ratio (shell thickness)/(shell radius) = 0.36 (Curtis 1918). Curtis remarks that NGC 6445 shows severe

departures from a simple shell hypothesis, so that the filling factor, ϵ , cannot be deduced easily from the photographs of the nebula.

Minkowski's (1965) distance estimate of 1220 pc is in good accord with Cahn and Kaler's (1970) estimate of 1400 pc, which we have adopted to calculate the electron density. With this model, space absorption parameter $C = 0.8$, surface brightness in $H\beta = 6.9 \times 10^{-3}$ ergs $\text{cm}^{-2} \text{s}^{-1}$ and a distance of 1400 pc, we find $N_e = 1100 \text{ cm}^{-3}$.

We assume that the electron temperature derived from the [O III] $\lambda\lambda 5007, 4959$, and 4363 lines is appropriate also for the Balmer lines (Miller and Mathews 1972). From the intensities of these [O III] lines, interpreted with the aid of new cross-sections by Eissner *et al.* (1969), an electron temperature of $15,000^\circ \text{K}$ is derived.

We have calculated the electron temperature at the three points in the nebula discussed above. The values are given in table 2, and show no significant variation. The electron density may also be obtained from the $\lambda 3729/\lambda 3726$ intensity ratio and from the [S II] $\lambda 6716/\lambda 6730$ ratio. Unfortunately, the latter is not observed, although we have measured the $I(\lambda 4068)/[I(\lambda 6716) + I(\lambda 6730)]$ ratio, which does serve to establish a relation between temperature and density in the [S II] zone (Krueger *et al.* 1970). From the observed $\lambda 3729/\lambda 3726$ intensity ratio of 0.92 and the tables by Eissner *et al.*, we get $x = 0.074$, which corresponds to a density of 910 electrons cm^{-3} at $T_e = 15,000^\circ \text{K}$. The ratio of $\lambda 4068$ to the red [S II] lines gives $x = 0.1$ for temperatures near $15,000^\circ \text{K}$. On the other hand, a temperature as low as 8000°K in the [S II] zone would require $x \sim 0.9$. The [O II] auroral lines at $\lambda\lambda 7319, 7330$ may also be compared with the nebular $\lambda 3727$ pair. The intensity ratios are consistent with $x = 0.074$ and $T = 15,000^\circ \text{K}$. Thus, it appears that the [O II] and [O III] lines are indeed produced in essentially the same volumes in the shell of NGC 6445, as suggested by the monochromatic images on the slitless spectrograms. For the remaining calculations, we adopt $x = 0.074$ and $T_e = 15,000^\circ \text{K}$, although certain lines, such as those of [O I], may originate in dense condensations.

We have also calculated the electron densities from the [O II] ratios observed in the three different parts of the nebula. We present these values in table 2, together with the relative $H\beta$ flux observed at these points. Note that the central density is lowest, as is consistent with the appearance of direct photographs, and that the electron density is in qualitative accord with the relative $H\beta$ flux, as expected.

Recent calculations by Brocklehurst (1972) make it practical to use several lines of helium to calculate the concentration of He^+ ions (see also Peimbert and Costero 1969). For ionized helium, $\lambda 4686$, one may use the data of Clarke (1965) and Brocklehurst (1971). Giving the greatest weight to $\lambda\lambda 5876$ and 7065 which are observed photoelectrically, we derive $N(\text{He}^+)/N(\text{H}^+) = 0.150$. From $\lambda 4686$, we find $N(\text{He}^+)/N(\text{H}^+) = 0.074$. Hence, the total helium/hydrogen ratio is 0.22. There appears to be no escape from the conclusion that the helium/hydrogen ratio is greater than in normal planetaries.

The He/H ratios derived from the individual points of table 2 are lower than the value given above, although still quite high in absolute terms. The difference may be due to the difficulty of measuring $\lambda 4471$ on this one plate. The significant point is that He/H is the same in both the high- and low-excitation portions of the ring. The variations in $\lambda 4471$ He I and $\lambda 4686$ He II compensate for one another just as expected.

We now analyze the data using formulae and tables as follows: [O II], [O III] (Eissner *et al.* 1969); [O I], [N II], [S III], [Ne III] (Aller and Czyzak 1968, 1972); [S II], [Ar IV] (Krueger *et al.* 1970; Czyzak *et al.* 1970). The choice of the basic parameters for the latter differ slightly from those of Saraph and Seaton (1970). The first four columns of table 3 give the ion, the lines chosen, the adopted intensity corrected for space absorption, and the ratio of the ionic concentration with respect to hydrogen. The numbers in parentheses give the power of 10 by which the entry is to be multiplied, e.g., 3.7 (-5) means 3.7×10^{-5} . The lines of [N II] $\lambda\lambda 6548, 6584$ and of [S III] $\lambda 3722$ are

TABLE 3
IONIC CONCENTRATIONS

| Ion | Adopted I_c | (ion/H) | Suggested Abundance | |
|-------------|---------------|---------|---------------------|---------------|
| He I..... | 4026 | 3.4 | 0.155 | 0.150 0.22 |
| | 4471 | 7.2 | 0.154 | |
| | 5876 | 19.2 | 0.145 | |
| | 7065 | 2.9 | 0.154 | |
| He II..... | 4686 | 70 | 0.074 | |
| N II..... | 6584, 6548 | (640) | 3.2 (-5): | 1.4 (-4): |
| O I..... | 6300 | (11) | 0.5 (-5) | |
| O II..... | 3726 | 391 | { 3.7 (-5) ... } | 2.5 (-4) |
| | 3729 | | | [2.7 (-4)] |
| O III..... | 5007 | 1865 | 1.26 (-4) | |
| | 4959 | | | |
| Ne III..... | 3868 | 220 | 5.5 (-5) | 1.1 (-4): |
| S II..... | 4068 | 8.0 | 1.5 (-6) | » 1.0 (-5) |
| | 6716, 6730 | 60 | 1.3 (-6) | |
| S III..... | 3722 | (1.2) | 1.0 (-6): | |
| Ar III..... | 4740 | 11.2 | 4.2 (-7) | 1.5 (-6) |
| Ar IV..... | 7135 | 4.0 | 5.8 (-7) | |

blended with hydrogen lines, so the contribution of hydrogen has to be estimated and subtracted. The estimate for $\lambda 3722$ is influenced by large accidental errors; that for $\lambda\lambda 6584, 6548$ depends on the validity of our choice for space absorption. The resultant abundance estimates are thus uncertain and are denoted by a colon. Unfortunately, the [N II] $\lambda 5755$ auroral line falls in a region of the spectrum badly affected by sky background in low-resolution scans.

Both [O II] and [O III] radiations seem to originate in strata with the same electron temperature, $T_e \sim 15,000^\circ \text{K}$. The neutral oxygen radiation must come from some region of lower ionization. If it is produced in a cool, dense zone of electron temperature 7500°K , its computed concentration is increased by about a factor of 5.

Neon is represented by strong lines of [Ne III]; in fact, the ratio of [Ne III]/[O III] intensities is unusually high for this nebula. It is unfortunate that the nebula is so far south that the region of the ultraviolet [Ne V] lines cannot be observed effectively.

Both $\lambda 4068$ and the combined red nebular transitions of [S II] give accordant ionic concentrations, but the [S III] estimate is uncertain because it involves an estimate of a contribution to a blend. To resolve this problem, high-resolution spectra in the red spectral region are needed. The argon ionic concentrations are estimated from the $\lambda\lambda 7135$ and 4740 lines, for which the intensity measurements should be reasonably reliable.

IV. ESTIMATES OF TOTAL ELEMENTAL ABUNDANCES

To derive the total abundances of various elements, one has to allow for the distribution of atoms among different stages of ionization. One can use an empirical or semiempirical procedure, or one can construct a model for the nebula, adjusting the parameters—nebular radius and density, temperature and radius of central star, and nebular chemical composition—until the spectral line intensities are correctly represented. The latter approach is justified only for those nebulae for which reasonably complete observational data are available.

For helium, one simply adds the contributions of $N(\text{He}^+)$ and $N(\text{He}^{++})$, but for other elements one can follow semiempirical recipes such as those proposed by Seaton (1968) or by Peimbert and Costero (1969).

Since ionization stages of oxygen higher than O^{++} can be present only in nebular domains where helium is twice ionized, Seaton suggested that the concentration of

these ions could be evaluated from

$$\frac{N(\text{He}^{++}) + N(\text{He}^+)}{N(\text{He}^+)} = \frac{N(\text{O})}{N(\text{O}^+) + N(\text{O}^{++})}. \quad (1)$$

The O/H ratios calculated from the observed lines and the above relation for the two parts of the ring do not agree too well with one another (see table 2). The disparity of about 40 percent may simply reflect the difficulty of allowing for unobserved ionization stages.

Estimation of the high ionization stages of nitrogen in a high-excitation nebula is very difficult. The expression proposed by Peimbert and Costero (1969) for low to moderate excitation nebulae may give a lower limit, since it does not allow for N^{+3} and higher stages of ionization. To estimate the contribution of these stages it would be necessary to calculate the radiation field of the nebula.

An equally difficult situation is obtained for sulfur. If we can get the concentration of $N(\text{S}^+)$ and $N(\text{S}^{++})$, we must remember that the ionization potential of S^{++} is actually less than that of O^+ , 34.7 versus 35.1 eV. In moderate-excitation nebulae where oxygen does not exist in ionization stages above the second, Peimbert and Costero's recipe for sulfur is probably valid, i.e.,

$$\frac{N(\text{S})}{N(\text{H}^+)} \sim \frac{N(\text{S}^+) + N(\text{S}^{++}) + N(\text{S}^{+3})}{N(\text{H}^+)} = \frac{N(\text{O}^+) + N(\text{O}^{++})}{N(\text{O}^+)} \frac{N(\text{S}^+) + N(\text{S}^{++})}{N(\text{H}^+)}; \quad (2)$$

but what about a high-excitation nebula such as NGC 6445? Clearly the estimate is much too low. Note that S^{+3} has an ionization potential of 47.3 eV, which is considerably less than that of 54.4 eV for He II , so that overwhelmingly the largest proportion of sulfur could exist as S^{+3} or S^{+4} . The Seaton-type correction cannot be expected to apply to sulfur in a high-excitation nebula, so we have to fall back on the older empirical methods of Bowen and Wyse (1939) or, better yet, calculate the nebular radiation field.

Neon presents another problem. Here we are discomfited by the circumstance that we do not have observations in the ultraviolet to evaluate the intensity of the [Ne V] lines, if they exist. The only lines of neon observed are those of [Ne III]. To allow for the lower stages of ionization we may employ the Seaton-Peimbert recipe

$$\frac{N(\text{Ne}^+) + N(\text{Ne}^{++})}{N(\text{H}^+)} \sim \frac{N(\text{O}^+) + N(\text{O}^{++})}{N(\text{O}^{++})} \frac{N(\text{Ne}^{++})}{N(\text{H}^+)}, \quad (3)$$

while for the higher stages

$$\frac{N(\text{Ne}^+ + \text{Ne}^{++} + \text{Ne}^{+3} + \text{Ne}^{+4})}{N(\text{Ne}^+ + \text{Ne}^{++})} \sim \frac{N(\text{He}^+ + \text{He}^{++})}{N(\text{He}^+)}. \quad (4)$$

Hence, finally

$$\frac{N(\text{Ne})}{N(\text{H}^+)} \sim \left[\frac{N(\text{O}^+ + \text{O}^{++})}{N(\text{O}^{++})} \right] \left[\frac{N(\text{He}^+ + \text{He}^{++})}{N(\text{He}^+)} \right] \frac{N(\text{Ne}^{++})}{N(\text{H}^+)}. \quad (5)$$

For Ar^+ , Ar^{++} , and Ar^{+3} , we note that the ionization potentials are respectively 27.6, 40.9, and 59.8 eV, so that to estimate the contribution of the higher stages of ionization we apply the same recipe as for oxygen, since the radiation field is reduced in much the same manner as the He ionization,

$$\frac{N(\text{Ar})}{N(\text{H}^+)} = \frac{N(\text{He}^{++}) + N(\text{He}^+)}{N(\text{He}^+)} \frac{N(\text{Ar}^{++}) + N(\text{Ar}^{+3})}{N(\text{H}^+)}. \quad (6)$$

These approximations are all more or less unsatisfactory for a nebula such as NGC 6445 which shows a wide range in excitation. Except for helium, the derived abundances may all be lower limits.

Clearly, we can improve on these results only by: (a) securing additional observations in the southern hemisphere so that reliable intensities can be established for some of the weaker lines, especially the lines of Ne v and other ions that fall in the ultraviolet; (b) Securing higher-resolution data from the visual region, using a red-sensitive image tube (one must have accurate internal calibrations checks for the image-tube procedures, to supplement the photoelectric); (c) constructing a model for the nebula, calculating the radiation field, and deriving ionic concentrations, as has been done for NGC 7662 and a few other planetaries.

The low surface brightness of the nebula (two orders of magnitude below that of a typical well-studied planetary such as NGC 6210, 7009, or 7662) imposes very severe restrictions on the kind and amount of observational data one can obtain. The effort seems to be justified, since NGC 6445 seems to have a high abundance of helium. This result would suggest that in some planetaries the ejected shell may contain a contribution from regions within the evolving star where hydrogen has already been converted into helium.

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