PREFERENTIAL PERIHELION AND APHELION DISTANCES IN THE PLANETARY SYSTEM

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Abstract

In this paper a series of precise relationships between planetary perihelion and aphelion distances are described, leading to the definition of preferential perihelion and aphelion distances, designated as *meeting distances*. The probability that the observed relationships could be accidental is low enough to make the relationship significant.

MAIN RELATIONSHIPS BETWEEN PLANETARY PERIHELIA AND APHELIA

The investigation presented in this paper provides an empirical rule for the distribution of heliocentric distances of planetary perihelia and aphelia. A similar kind of relationships is also found in several satellite systems (Barricelli 1972).

The simplest way of presenting the relationships is perhaps to describe step by step the observations which led to their discovery. In Table 1 the planetary revolution periods (P), the semi-major axes (R), eccentricities (E), and the perihelion (Pr) and aphelion distances (Ap) are given. We notice that the ratio between the revolution periods of Neptune and Pluto is much lower than in any other pair of consecutive planets and almost exactly 2/3, Pluto's revolution period being 248.4 years and Neptune's 164.8 years, with a difference of less than 0.5% from 2/3 of Pluto's revolution period. Another peculiarity of these two planets is that they have a very similar perihelion distance, Pluto's perihelion distance being 29.692 A.U. and Neptune's 29.814 A.U.

The discovery of other relationships between planetary perihelion and aphelion distances was then made possible by the following observation:

Knowing Pluto's revolution period and perihelion distance, and selecting another orbit with revolution period 2/3 of Pluto's and the same perihelion distance, we are able to calculate the aphelion distance of this new orbit, which is 30.623. This distance differs by less than 1% from Neptune's aphelion distance which is 30.327. Suppose now that, for the sake of curiosity or as an attempt to generalize this method, instead of selecting the ratio 2/3 we had selected another simple rational number, for example 1/2, and had carried out the calculation of a new connecting orbit with a similar procedure The new orbit would have a semi-major axis (derived from Pluto's by applying Kepler's 3rd law) equal to 24.895 A.U. which is less than Pluto's perihelion distance of 29.692 A.U. The new connecting orbit could be ascribed an aphelion distance coinciding with Pluto's perihelion distance, in which case its perihelion distance would be 20.097 A.U. If we look at Table 1 we notice that this distance is almost identical with the aphelion distance of Uranus, which is 20.094 A.U. and differs from the preceding figure by less than 0.018 per cent.

At this point one may stop and ask whether this coincidence could again be the result of chance or whether the investigation of this kind of relationships could be worth pursuing further. The *a priori* probability that this kind of coincidences relating perihelion and aphelion distances of three planets could occur by chance is obviously low, and we decided therefore to pursue the investigation.

The next step would have to be the determination of a series of connecting orbits with rational period relationships to Pluto's orbit and with common perihelion and/or aphelion distances. To start with, it may seem sensible to construct a series of orbits with periods expressed by negative powers of 2 where Pluto's revolution period P_1 is taken as unit (to be called "primary connecting orbits" or "primary *links*"). This is done in the first part of Table 2 where each primary orbit has its aphelion at the same heliocentric distance (designated as primary meeting distance or *PM* in the table) where the preceding primary orbit has its perihelion. The first orbit in the table has of course the perihelion and aphelion distances coinciding with Pluto's.

Some already described coincidences between primary meeting distances (*PM*) and planetary perihelion and aphelion distances are displayed in Table 2. With the exception of the already noted relationships, this first step did not give any new coincidences. We are however prepared for the possibility that rational numbers other than powers of 2, such as for example the ratio 2/3 used to identify the aphelion of Neptune, might be involved. The question is which ratios, and how they should be applied? A possible procedure which seemed appealing both because of its simplicity and because it had already given the successful coincidences described above, was that the primary meeting distances constructed in Table 2 should be either directly compared with planetary perihelia and aphelia or used as a basis for the construction of

secondary orbits and secondary meeting distances obtained by other rational numbers. This is done in Table 2 on three occasions in order to fit the perihelion distances of Jupiter and Saturn and the aphelion distance of Neptune. The Saturn perihelion distance of 9.0041 A.U. presents another case of an extremely good fit with a meeting distance at 9.0065 defined as the aphelion of the orbit with the rational period $(5/3).2^{-4}$ (designated as 1.67×2^{-4}) and perihelion at the closest *PM* distance. The probability that all these coincidences could be accidental is once more significantly reduced. Whatever the cause of these phenomena might be, attempts to find an explanation may have a better chance of success if postponed.

Since the Pluto series (or chain) of connecting orbits (links) and meeting distances shows several coincidences with planetary perihelia and aphelia (namely Neptune *Ap*, Neptune *Pr*, Uranus *Ap*, Saturn *Pr* and Jupiter *Pr*), it was tempting to try whether a similar chain could be constructed on some other planet. Indeed it was found that the orbit of Mercury could be used in a similar way to construct a series of connecting orbits and meeting distances fitting perihelion or aphelion distances of the planets Venus, Earth, Mars, Jupiter, Saturn, Neptune and Pluto. In the Mercury chain, which is given in the second part of Table 2, all the primary links have revolution periods expressed by positive powers of 2 in terms of Mercury's revolution period P_2. Several secondary links and meeting distances which fit the perihelia or aphelia of various planets are given in the same table. Again we observe several good fits which once more confirm the validity of this method of identifying preferential perihelion and/or aphelion distances.

The reader will have noticed that there are no free parameters available once the Pluto chain is precisely linked to Pluto and the Mercury chain is precisely linked to Mercury.

Four types of rational numbers are used in the secondary links of Table 2. The ratio of revolution periods between a secondary link and the primary link immediately below it can be either $4/3 = 1.33$ (Neptune's *Ap* link in the Pluto chain, Neptune's and Pluto's *Pr* link in the Mercury chain and Venus's *Ap* link), or the harmonic reciprocal (meaning reciprocal when we disregard powers of 2) ratio $3/2 = 1.5$ (Jupiter's *Ap* and Mars's *Pr* links), or $5/3 = 1.67$ (Saturn's *Pr* link), or its harmonic reciprocal $6/5 = 1.2$ (Jupiter's *Pr*, Saturn's *Ap* and Earth's *Ap* links). These are the only rational numbers besides 2 which are needed to express the period ratios between secondary and primary links in a chain.

Another coincidence to which we would like to attract the reader's attention is the fact that the ratio between the revolution periods of Pluto and Mercury is close to a power of 2, namely 1031 (differing only 0.7% from $1024 = 2^{10}$). This is an essential requirement for the possibility of connecting the Pluto and Mercury chains into a single chain system.

The probability that the coincidences described could be the result of chance can be calculated and is found to be quite low enough to make the relationships significant.

THE PROBLEM OF SECULAR PERTURBATIONS

The existence of precise relationships between perihelion as well as aphelion distances of many planets is indeed a surprising observation. Secular perturbations by major planets which in less than hundred thousand years would produce significant alterations of several planetary perihelion and aphelion distances have been calculated (Brouwer and Clemence 1961). This raises major problems in connection with the coincidences presented above, which are indeed curious phenomena apparently too precise to be accidental, but also too unlikely to be real.

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Table 1

Planetary elements*

P = Revolution period, R = Semi major axis, E = *eccentricity*, Pr = Perihelion distance, Ap = Aphelion distance

Planet	P YEARS	R A.U.	E	Pr A.U.	Ap A.U.
Pluto	248.42	39.5177	0.24864	29.69190	49.34358
Neptune	164.78	30.0707	0.00853	29.81408	30.32727
Uranus	84.001	19.19098	0.04704	18.28816	20.09380
Saturn	29.456	9.53884	0.05606	9.00410	10.07359
Jupiter	11.862	5.20280	0.04825	4.95175	5.45386
Mars	1.8808	1.52369	0.09331	1.38151	1.66587
Earth	1.0000	1.00000	0.01675	0.98325	1.01675
Venus	0.61518	0.72333	0.00682	0.71840	0.72827
Mercury	0.24091	0.38710	0.20561	0.30751	0.46669

* The data are from *Astronomitsjeski Yeshegodnik*, SSSR 1970.

Table 2

Links and meeting distances in the Pluto and Mercury chains*

Chain	P_0 P_1 = Pluto rev. period, P_2 = Mercury rev. period	E_0	R_0 A.U.	Meeting distances A.U.		Planetary perihelion (Pr) or aphelion (Ap) coincidences	Planet
Pluto chain				PM	49.3436	(49.3436 Ap)	Pluto (base)
	P_1	0.2486	39.5177				
	$1.33\times2^{-1}P_1$	0.0154	30.1577	$1.33Ap$	30.6235	30.3273 Ap	Neptune
				PM	29.6919	(29.6919 Pr)	Pluto (base)
						29.8141 Pr	Neptune
	$2^{-1}P_1$	0.1927	24.8946				
				PM	20.0973	20.0938 Ap	Uranus
	$2^{-2}P_1$	0.2815	15.6826				
				PM	11.2679	—	—
	$2^{-3}P_1$	0.1405	9.8794				
	$1.67\times2^{-4}P^1$	0.0295	8.7487	$1.67Ap$	9.0065	9.0041 Pr.	Saturn
				PM	8.4910	—	—
	$2^{-4}P_1$	0.3643	6.2237				
	$1.2\times2^{-5}P_1$	0.1064	4.4274	$1.2Ap$	4.8984	4.9517 Pr	Jupiter
				PM	3.9563	—	—
	$2^{-5}P_1$	0.0091	3.9207				
				PM	3.8850	—	—
	$2^{-6}P_1$	0.5730	2.4699				
				PM	1.0548	—	—

* P_0, E_0, R_0 represent the period, the eccentricity and the semi-major axis of a link, respectively.

Table 2. Continued (see opposite page)

Preferential Distances

Mercury chain							
				PM	30.4071	30.3273 *Ap*	Neptune
	$1.33\times2^{9}\ P_2$	0.0132	30.0120	1.33 *Pr*	29.6170	29.6919 *Pr*	Pluto
						29.8141 *Pr*	Neptune
	$2^{9}\ P_2$	0.2274	24.7744				
				PM	19.1417	—	—
	$2^{8}\ P_2$	0.2265	15.6069				
				PM	12.0720	—	—
	$1.2\times2^{7}\ P_2$	0.0873	11.1024	1.2 *Pr*	10.1328	10.0736 *Ap*	Saturn
	$2^{7}\ P_2$	0.2279	9.8317				
				PM	7.5914	—	—
	$2^{6}\ P_2$	0.2257	6.1936				
	$1.5\times2^{5}\ P_2$	0.0620	5.1127	1.5 *Ap*	5.4296	5.4539 *Ap*	Jupiter
				PM	4.7958	—	—
	$2^{5}\ P_2$	0.2291	3.9017				
				PM	3.0077	(2.9776 *Ap*)	(Ceres)
	$1.2\times2^{4}\ P_2$	0.0836	2.7756	1.2 *Pr*	2.5435	(2.5574 *Pr*)	(Ceres)
	$2^{4}\ P_2$	0.2237	2.4579				
				PM	1.9082	—	—
	$2^{3}\ P_2$	0.2324	1.5484				
	$1.5\times2^{2}\ P_2$	0.0701	1.2782	1.5 *Ap*	1.3676	1.3815 *Pr*	Mars
				PM	1.1886	—	—
	$1.2\times2^{2}\ P_2$	0.0791	1.1015	1.2 *Pr*	1.0144	1.01675 *Ap*	Earth
	$2^{2}\ P_2$	0.2185	0.9754				
				PM	0.7623	—	—
	$1.33\times2\ P_2$	0.0240	0.7444	1.33 *Pr*	0.7265	0.72827 *Ap*	Venus
	$2\ P_2$	0.2405	0.6145				
				PM	0.4667	(0.46668 *Ap*)	Mercury (base)
	P_2	0.2056	0.3875				
				PM	0.3075	(0.3075 *Pr*)	Mercury (base)