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THE DYNAMICS OF M STARS IN THE SOUTHERN GALACTIC CAP

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ABSTRACT

The radial velocities of HD M stars in the Southern Galactic Cap have been measured with a photoelectric speedometer. Their velocity dispersion is 7 km s^{-1} in the plane, rises to 28 km s^{-1}, and thereafter remains steady. Using conventional dynamical theory, the density variation of these stars with distance below the plane implies a density of gravitating matter of 0.21 $M_{\odot}$ pc^{-3} in the solar neighborhood. This is not significantly higher than the values obtained by similar methods for other groups of stars. The distribution of total energies of motion perpendicular to the plane accords with the collapse picture of the Galaxy.

I. INTRODUCTION

It was demonstrated by Oort (1932) that a comparison of the density and velocity dispersion of a group of stars at different heights above the galactic plane provided a measure of the acceleration in the same direction. He showed that this acceleration was accordant with the gravitational acceleration of known matter. Oort's method was followed by Hill (1960) and Oort (1960), who found a density of gravitating matter of 0.15 $\mathfrak{M}_{\odot}$ pc^{-3}.

Oort's method is essentially a differential one; explicit solutions for the distribution of density and velocity dispersion in one-dimensional self-gravitating stellar systems have been discussed by Camm (1950), Woolley (1957), Jones (1962), and Woolley and Stewart (1967). Extensive radial-velocity measures of A0 stars have substantially confirmed the predictions of these papers (Wayman 1961; Woolley *et al.* 1969; Harding, Fahim, and Haslam 1971).

It is generally believed that the stellar content of the disk of the Galaxy corresponds in kinematic behavior, age, and chemical composition with the range of these variables found in open clusters. Similarly, the stellar content of the galactic halo corresponds to the range found in globular clusters. Thus the A0 stars near the Sun are typical main-sequence members of the metal-rich young disk; in fact they define its kinematic boundaries (Eggen 1963). Moving from the disk to the halo, the typical HD A0 stars show an increasing percentage of post–red-giant blue-horizontal-branch stars. These stars have different masses from the young disk stars. Their luminosities probably also differ, but the exact amount is uncertain. Their velocity dispersions differ by a factor of 10 (Rodgers 1971). The two groups of stars are not distinguished in the *Henry Draper Catalogue*, although they can be separated on objective-prism plates at higher dispersion (e.g., MacConnell *et al.* 1971).

M stars are rare in globular clusters both because the weakness in metals inhibits the formation of TiO bands and because the tip of the giant branch does not always reach to sufficiently low temperatures. Analogously they may also be expected to be rare in the halo, and consequently more homogeneous in their kinematic behavior than the A0 stars. *UBVRI* measures have recently been published by Eggen (1970*a*) for all HD M stars in the South Galactic Cap (SGC) together with distances based on the (M_I, $R - I$) plot. As all the other data are available for these stars, a knowledge of their radial velocities provides a new determination of the Galactic gravitation field perpendicular to the plane.

II. THE RADIAL VELOCITIES

a) *Image-Tube Plates*

A series of plates was taken at 10 Å mm^{-1} with the coudé spectrograph of the Mount Stromlo 1.88-m reflector. A magnetically focused image tube with S20 cathode was used. The image suffered from "S distortion," but the wavelength interval 6410–6610 Å was sensibly straight. The star lines measured were selected from Davis (1947). The plates were measured on the Gollnow setting device (Gollnow, Rudge, and Thomas 1967). The velocity system was checked against the K-type standards of Evans (1966) and a correction of $+1.2$ km s^{-1} determined. The external errors were assumed equal to the internal errors based on the interagreement of the lines. They ranged from 0.7 to 4.5 km s^{-1} (s.e.).

b) *Stellar Speedometer*

A further series of radial-velocity measurements was made with a stellar speedometer similar to the radial-velocity spectrometer described by Griffin (1967). The term "speedometer" was adopted for brevity and also because it measures a scalar speed and not a vector velocity. This instrument was situated in the coudé spectrograph of the 1.88-m reflector. The mask was designed to match the spectrum of the M giant β Peg as described by Davis (1947) over the wavelength interval 5338–5449 Å. The same standards were used as in § II*a*, and the standard error of one night's observation found to be ± 1.6 km s^{-1} (s.e.). The speedometer observations are combined with the image-tube observations in table 1.

c) *R Sculptoris*

The carbon star R Scl (HD 8879) lies in the SGC, and a plate was secured under the same conditions as described in § II*a*. The spectrum is a confused combination of emission and absorption lines. Fifteen lines of the (5, 2) band of CN (Davis and Phillips 1963) could be identified in emission with radial velocity -54 km s^{-1} $\pm$ 4 (s.e.). Seven absorption lines common to other carbon stars could be identified with radial velocity -4 km s^{-1} $\pm$ 3 (s.e.).

III. SPECTRAL TYPES

Spectral classifications were made on three series of plates: (i) IIaO plates taken with the Meinel spectrograph on the 50-inch (127-cm) reflector at Mount Stromlo; (ii) IIaF plates taken with the Meinel spectrograph on the 40-inch (102-cm) reflector at Siding Spring; (iii) image-tube plates described in § II*a*. Sufficient MK standards were observed to give spectral subtypes with the usual accuracy and to distinguish luminosity classes III and V. Few supergiant standards were observed, and there is a distinct possibility that some stars classified as III may in fact be II. The three series are in good accord and are combined in table 1.

IV. DISTANCE SCALE

The luminosities M_v taken from Eggen (1970*a*) for constant stars are brighter than the calibration of the MK system given by Keenan (1963) by 0.6 $\pm$ 0.2 mag (s.e. per star). Photometric luminosities on essentially the same system have been determined for many bright M giants by Eggen (1967) and Eggen and Stokes (1970). On the basis

TABLE 1A

NEWLY DETERMINED RADIAL VELOCITIES AND SPECTRAL TYPES

HD No.	No. of Nights Speedo.	Image Tube	Radial Velocity km/s	-z pc	Spectral Type	Δ(U-B)	e Galactic Eccentricity
218655	2	-	- 7.5	*	M 0 III		
223737	1	1	+69.4	*	M 7 III+		
223783	2	-	-18.5	633	M 4 III	-.08	.41
225016	1	1	+53.2	1346	M 6 III	.05	
178	3	1	+17.7	747	M 6 III	.30	.52
180	1	1	- 4.6	1082	M 5 III	.05	.64
393	1	-	+ 3.3	1015	M 6 III	.10	.13
402	2	-	-17.1	264	M 1 III	-.06	.16
437	2	-	- 0.5	450	M 2 III	-.03	.10
672	3	-	-12.9	647	M 5 III	.20	.27
1628	2	2	+ 2.7	1540	M 6 III	.25	
1813	1	1	+25.2	1111	M 5 III	.02	.25
1923	2	-	+ 3.9	421	M 3 III	.06	.29
2225	1	1	-20.6	1760	M 4 III	.11	.50
2268	2	-	+ 0.5	308	M 2 III	.02	.16
2367	1	-	+38.9	1173	M 5 III	.12	.38
2489	1	1	+49.5	1015	M 5 III++	.40	.54
2960	1	1	+ 8.2	1156	M 2 III	-.04	.54
3101	2	-	+ 2.7	731	M 5 III	-.20	.24
3287	2	-	-40.5	728	M 5 III	.00	.26
3373	1	1	- 3.7	1005	M 5 III	.2:	.22
3514	1	1	-36.7	1266	M 6 III	.20	>1.
4053	2	-	+14.5	404	M 2 III	-.05	.26
4226	1	-	+12.0	1005	M 5 III	-.10	.44
5445	2	-	+21.9	240	M 1 III	-.03	.02
5473	2	-	+76.3	692	M 4 III	.05	.38
5735	2	-	+28.6	402	M 3 III	-.02	.31
6290	2	1	+ 4.1	410	M 3 III	-.01	.23
6816	1	1	-12.9	1010	M 5 III	.00	.20
7122	2	1	+ 8.7	374	M 5 III	.10	.05
7235	1	1	+62.3	974	M 5 III	.13	.20
7674	1	-	+25.1	862	M 3 III	.03	.06
8447	2	-	+ 0.7	460	M 3 III	.01	.16
8680	1	1	+ 9.8	1099	M 6 III	-.03	.31
9085	2	-	+ 7.5	469	M 4 III	-.03	.04
9184	2	-	+17.5	*	M 3 III		
9497	1	-	- 2.2	889	M 6 III	.12	.23
9642	1	1	- 3.9	1122	M 5 III	.14	.36
9692	1	-	+18.3	386	-	-.07	.18
9875	2	-	+10.4	688	M 2 III	.00	.62
9894	2	-	+43.1	618	M 2 III	.03	.29
10254	1	-	- 7.7	466	M 2 III	-.05	.14
11695	2	1	+ 5.6	*	M 4 III		
12066	1	-	+33.2	1140	-	-.10	.49
12890	1	-	+52.1	888	-	.13	>1.
16554	1	-	+25.3	*	-		

* b > -72.5 not discussed in §IV.

\+ Hα absent or slight emission.

++ Classified as an emission line star in Figure 4 on the basis of the (U-B) variation reported by Eggen; no Hα emission on one red plate in the present study.

TABLE 1B

RADIAL VELOCITIES FROM EGGEN (1970*a*)

HD No.	Radial Velocity (km s^{-1})	$-z$ (pc)	$\Delta(U - B)$	Galactic Eccentricity e
151	+34.0	535	+0.25:*	0.19
1038	−22.5	140	−0.04	0.15
1115	+35.0	406	+0.80*	0.24
1721	+95.1	425	+0.05	0.32
1760	+29.1	161	+0.00*	0.08
1879	−22.0	294	+0.06	0.29
2429	+12.2	195	+0.01	0.12
12255	−15.0	169	+0.01	0.10
12274	+18.0	158	+0.04	0.13

* Emission-line star.

TABLE 1C

STARS WITH UNKNOWN RADIAL VELOCITY

HD No.	$-z$ (pc)
222024	1100
222159	1224
224225	874
2438	329
2585	916
5920	596
6198	955
10778	380
10831	501
12106	871
12551	1150

of the photometric luminosities the 25 closest stars were selected. Their trigonometric parallaxes (Jenkins 1952, 1963) indicate a correction to the photometric luminosities of 0.1 ± 0.5 mag (s.e. of mean). Thus the trigonometric parallaxes offer no independent check on the luminosity scale. The photometric luminosities are used in this paper because the spectroscopic luminosities would require an average reduction of about 20 percent in the V velocities of the moving groups used to set up the photometric calibration, and this is felt to be untenable. Stars not observed by Eggen are given $M_v = -1.0$ to accord with the photometric system.

V. DYNAMICAL DISCUSSION

All HD stars of type M in the Southern Galactic Cap were included in the density measurements. The radius of the Cap was taken as 17°5 to cover the present observations and the cone truncated at 500, 1000, and 1500 pc. The local density of HD M stars was found from Eggen (1967), and Eggen and Stokes (1970). Fifty-one percent of M stars brighter than $V = 5.5$ were found in the two lists. The density was determined in a sphere of radius 150 pc centered in the Sun, which was assumed to lie in the Galactic plane.

Following table 1 all stars were assumed to be giants. If Luyten's (1968) luminosity function is assumed to consist solely of M dwarfs when M_{pg} is fainter than 10, then five M dwarfs brighter than $V = 10$ are expected in this area. Some fraction of Luyten's faint stars must be white dwarfs, so there is no serious contradiction with the fact that no M dwarfs were found. The distant group is the one most susceptible to contamination by dwarfs; such contamination will be qualitatively compensated by any incompleteness in the HD catalog at its faint limit.

The raw data on the stellar density distribution are given in the first part of table 2. Because the density varies substantially within each z division, the mean density is not the density at mean height in that division. A preliminary analysis (cf. fig. 1) indicated that the number density, Δ, varied approximately as $\exp(-|z|/200)$ (z in pc). This formula was used to compute the difference between $z(\bar{\Delta})$, the value of z where the local density equals the mean density, and $\bar{z}$, the mean value of z. This correction is applied in the second part of table 2 for each of the three truncated cones. Again with this formula, the density at $z = 0$ is 1.32 times the mean density in the sphere. This factor and the incompleteness factor of the lists have been applied to the density in the second part of table 2. The densities from table 2 are plotted in figure 1 as a function of z, together with their random sampling errors.

The velocity components perpendicular to the galactic plane W, discussed in the third part of table 2, were taken to be the reverse of the radial velocities in table 1A, supplemented by nine values from Eggen (1970*a*) in table 1B. The sample number, n, is slightly smaller than in the first part of table 2 because a few stars were not measured for radial velocity. It may readily be shown that for a cap of this size the dispersion of the radial velocities is indistinguishable from that of the W motions. The errors of observation are too small to affect the dispersions. The W velocities in the sphere were taken from Eggen (1967) and Eggen and Stokes (1970), assuming the stars in their lists to be a random sample of the stars in the sphere.

The densities and velocity dispersions were discussed by the method of Woolley and Stewart (1967), using their preferred potential C. The necessary extrapolation could be made with confidence because the density of gravitating matter at their limit is effectively zero. It was verified that this potential is in substantial agreement with that of Schmidt (1956) when reduced to a nondimensional form.

The best fit to the densities in figure 1 and the velocity dispersions in figure 2 is given by the sum of two groups: the first has a velocity dispersion of 7 km s^{-1} and provides 72 percent of the stars in the plane, and the second has a velocity dispersion of 28 km s^{-1} and provides 28 percent of the stars in the plane. The characteristic height l of Woolley and Stewart's model C is then 87 pc, corresponding to a density of 0.21 $\mathfrak{M}_\odot$ pc^{-3} in the solar neighborhood.

Discussion of the errors is simplified by the fact that the low-velocity population contributes significantly to the density only in the plane. The characteristic height l is determined from the three points out of the plane whose velocity dispersion is essentially that of the high-velocity population. This dispersion was found from the whole group of stars in the Cap and has a standard deviation of ± 10 percent. Because the point at -805 pc is almost centrally placed between those at -366 pc and -1231 pc, it is the two latter which determine the slope of the $\log \Delta(z)$ curve, and hence l. It is readily shown that the error bars in figure 1 correspond to a standard error of ± 8 percent in l. The zero point of the (M_I, $R - I$) relation is estimated to have a standard error of ± 0.2 mag, corresponding to ± 10 percent in l. If Keenan's (1963) luminosities were used, then l would be reduced by 32 percent and the density would be increased to the high value of 0.37 $\mathfrak{M}_\odot$ pc^{-3}. The range of the four model potentials tabulated by Woolley and Stewart (1967) indicates an uncertainty of ± 5 percent in l arising from uncertainty in the dimensionless potential. All these uncertainties are squared in computing the density which accordingly has an uncertainty of ± 30 percent (s.e.).

TABLE 2

VARIATION OF DENSITY AND VELOCITY DISPERSION WITH DISTANCE

Shape	z Limits (pc)		Raw Data on Star Numbers			Adjusted as Described in § V		W Motion Data		
			n	$\bar{z}$ (pc)	Volume 10^6 pc^3	$\overline{\Delta}$ 10^{-8} pc^{-3}	$z(\overline{\Delta})$ (pc)	n	$\overline{W}$ (km s^{-1})	σ_w (km s^{-1})
Sphere..................	+ 150	− 150	53	0	14	958.	0	53	− 6	15
Truncated cone.........	0	− 500	23	− 335	13	177.	− 366	21	−10	26
Truncated cone.........	− 500	−1000	19	− 755	91	20.9	− 805	13	−19	34
Truncated cone.........	−1000	−1500	17	−1118	260	6.53	−1231	14	−12	25
Whole cone..........								48	−13	28

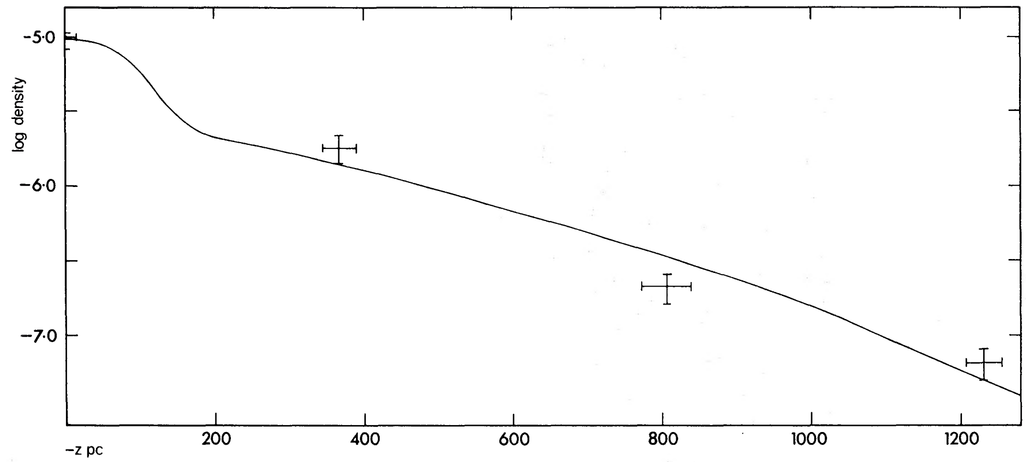

FIG. 1.—Log density (expressed in number of stars a cubic parsec) as a function of distance from the plane. The error bars are 2 standard errors in length. The continuous line is the adopted solution.

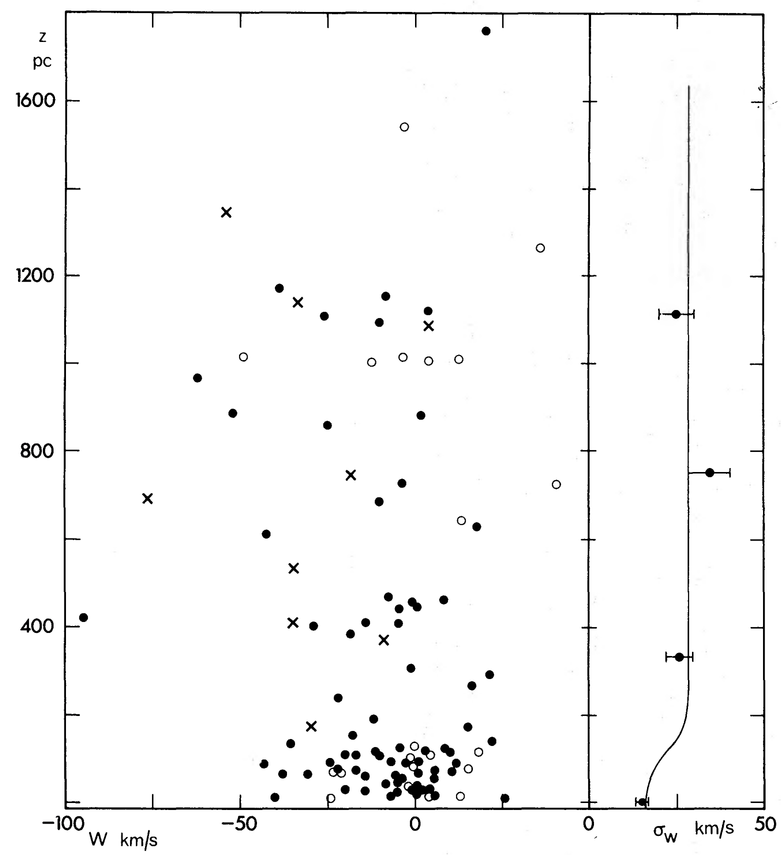

FIG. 2.—(*a*) $|z|$ pc plotted as a function of W in km s^{-1}. For stars in the Cap, W is taken as the reverse of the radial velocity. *Filled circles*, constant stars; *open circles*, erratic variables; *crosses*, quasi-periodic variables. (*b*) Velocity dispersion of points in fig. 2*a* divided into four groups as described in table 2. The error bars are 2 standard errors in length, computed from the formula $\sigma/\sqrt{(2n)}$ which assumes that the parent population is Gaussian. The continuous line is the adopted solution.

Other recent determinations of the density of gravitating matter are (in $\mathfrak{M}_\odot$ pc^{-3}) Oort (1960) 0.15; Jones (1962) 0.14; and Woolley and Stewart (1967) 0.11. Accuracies of 10–20 percent are claimed, so that the present value is not significantly discordant from any of them. The true value must be about 0.14 $\mathfrak{M}_\odot$ pc^{-3} $\pm$ 0.01 (s.e.). Luyten's (1968) luminosity function implies a stellar density of 0.064 $\mathfrak{M}_\odot$ pc^{-3}, and gas and dust probably contribute another 0.024 $\mathfrak{M}_\odot$ pc^{-3} (Woolley and Stewart 1967). Interstellar H_2 may contribute another 0.018 $\mathfrak{M}_\odot$ pc^{-3} (Carruthers 1970). There remains a significant amount of matter to be accounted for.

The interagreement of four different investigations based on groups of stars or observational material which are almost completely independent is evidence that the fundamental assumptions of the method are correct. The first is that the stars are "well-mixed" or alternatively that they obey Jeans's relation. The second is that it is permissible to treat the motions in the W direction as completely free from any interlocking with the U and V motions. Harding *et al.* (1971) found that the mean motion $\bar{W}$ of A0 stars increased with distance in the Southern Galactic Cap, implying a breach of the "well-mixed" principle. The present observations reveal a reverse effect but of doubtful statistical significance. At their face value the observations indicate a net flux of M stars away from the plane and of A0 stars toward it. If stars are formed only in the plane, they evolve from type M to type A on a timescale of 10^8 years. This suggestion need not be taken seriously, but it does underline an implicit assumption in the method: that stellar evolution for the stars in question is so slow relative to the dynamical cycle time that the "well-mixed" principle is not violated.

VI. GALACTIC HISTORY

While it is not the purpose of the present paper to prove the collapse picture (Eggen, Lynden-Bell, and Sandage 1962) of galactic history, it is of some interest to test the accordance of the present observations with that theory. The parameters z and W vary continually for any given star and therefore shed little light on the history of the Galaxy. It is more germane to study the total energy of the W motion,

$$\tfrac{1}{2}W^2 - \Phi(R_\odot, z)\,,$$

which varies only between fixed limits for a steady galactic potential (Ollongren 1962). It is therefore a useful indicator of a star's history, back to the end of the collapse phase of the Galaxy. It may readily be shown not to be an adiabatic invariant, unlike the eccentricity of the galactic orbit (Eggen *et al.* 1962). Accepting that the density of gravitating matter in the Solar vicinity is 0.14 $\mathfrak{M}_\odot$ pc^{-3}, then $l = 107$ pc and the potential can be computed at any z. The total energy was computed for all stars in the sphere and the cone, and used to derive both the motion W_0 when passing through the plane, and the greatest excursion $z_{\max}$.

The eccentricity of the galactic orbit, e, under the Eggen *et al.* (1962) potential is plotted against W_0 and $z_{\max}$ in figure 3. It was computed from the U, V velocity components of Eggen (1970*a*) and is listed in tables 1A and 1B. Because the eccentricity is adiabatically invariant, stars of high eccentricity and low angular momentum are identified with those formed during the collapse phase. Similarly, stars of low eccentricity and high angular momentum are supposedly formed in the disk. Because the collapse phase preceded the formation of the disk, high-eccentricity stars can be called old and low-eccentricity stars young. Returning to figure 3, the correlation between $z_{\max}$ and e indicates a correlation between $z_{\max}$ and age—evidence that during collapse the Galaxy had a greater extent in z than at present. Perfect agreement between e and $z_{\max}$ is prevented by their failure to conserve, especially through the collapse phase, and by errors of observation. In particular, the two stars with positive energy are not well-substantiated Galactic escapers because their eccentricities depend on poorly determined proper motions.

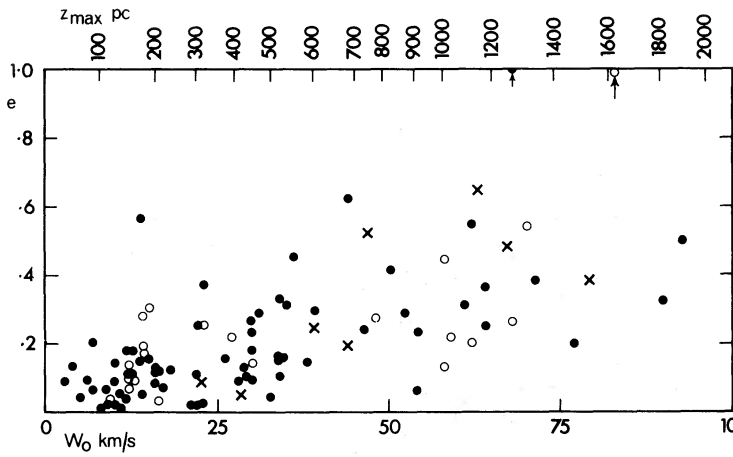

FIG. 3.—Eccentricity of the Galactic orbit under the Eggen, Lynden-Bell, and Sandage potential plotted as a function of z_{max} (pc) (*upper margin*) or W_0 (km s^{-1}) (*lower margin*). Symbols as in fig. 2.

To discuss the chemical history of the Galaxy, a good criterion of stellar chemical composition is required. The best criterion for such red stars is $\Delta(U - B)$, the ultraviolet excess at fixed $(R - I)$ (Eggen 1969). In figure 4 $\Delta(U - B)$ is plotted as a function of z_{max} and W_0. The correlation is poor and quite unlike figure 20 of Eggen (1970*b*), where $\delta(U - B)$ shows a discontinuous increase at 800 pc. Some of the deviations in $\Delta(U - B)$ may be associated with hydrogen emission, but HD 178 [at $\Delta(U - B) = 0.3$, $W_0 = 47$ km s^{-1}] has no Hα emission on the two red plates obtained. Either the chemical history of the Galaxy bears little relation to its kinematic history as typified by z_{max}, or $\Delta(U - B)$ is a poor measure of chemical composition. The first possibility would imply that the enrichment of the interstellar medium proceeded at

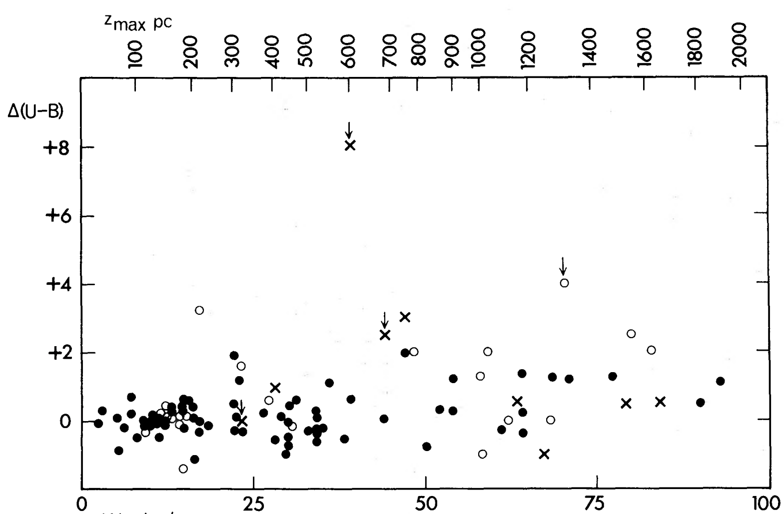

FIG. 4.—Ultraviolet excess $\Delta(U - B)$ plotted as a function of z_{max} or W_0. Symbols as in fig. 2. Arrowed stars have hydrogen emission.

different rates at different parts of the Galaxy. Certainly there is no hope of distinguishing between the two kinematic populations discussed in § V by their $\Delta(U - B)$ values.

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