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EXCITATION OF NEBULAR SPECTRUM LINES

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ABSTRACT

Evidence is presented which supports Seaton's suggestion that permitted lines in gaseous nebulae, other than those of hydrogen and helium, can be produced by direct photoexcitation by the central star rather than by recombination. Observations are presented for three nebulae in which O III lines are found but for which the He II lines are not present. This finding contradicts recombination theory which predicts that the O III lines would not appear unless the He II lines are present, as O^{2+} and He^{+} have nearly identical ionization potentials. Next, a strong correlation between the ratio of the strengths of the λ3918 + λ3920 and the λ4267 doublets of C II and the temperature of the exciting star is established. Since this ratio shows a range of values of a factor of 13, it seems unlikely that recombination would be the dominant mechanism acting to produce all the lines. This correlation appears to be a sensitive spectroscopic indicator of the temperature of the central star An equivalent Harmon-Seaton temperature for the central star of IC 4997, which is not directly observable, is found to be 54,000° K. The observations indicate that the λ3918 + λ3920 doublet is produced dominantly by direct excitation of C^{+} whereas the λ4267 line may indeed be principally a recombination line from C^{2+}.

I. INTRODUCTION

Seaton (1968) has demonstrated that in a nebula permitted spectra of ions other than hydrogen and helium are excited by direct photoexcitation by the exciting star. He found that when the oxygen-to-hydrogen ratio is measured from the O III and O IV lines, the ratio is higher for high-excitation nebulae than for low-excitation nebulae, which implies a correlation with central-star temperature, a situation which should not exist. Seaton further suggested that the approximate relative abundances of O^{3+} and O^{4+} can be computed from the simple relation

$$\frac{N(O^{3+}) + N(O^{4+}) + \ldots}{N(O^{+}) + N(O^{2+})} = \frac{N(He^{2+})}{N(He^{+})}, \tag{1}$$

since the ionization potential of O^{2+} (54.71 eV) is almost exactly the same as He^{+} (54.17 eV).

Kaler (1970) made extensive use of this relationship in the calculation of the oxygen-to-hydrogen ratios of a large number of nebulae, where the results support Seaton's idea and the method of equation (1).

Since the measurement of abundance ratios is so important in the evaluation of the fundamental processes which produce nebular spectra, it is important to search for observational data which will test his contention. Two supporting arguments are given in the next two sections. The second argument leads to a method for the empirical determination of the temperatures of exciting stars.

II. THE O III LINES

Let us first assume that the O III and He II lines are produced by recombination, for which oxygen and helium must be in the O^{3+} and He^{2+} ionization stages, respectively. As noted above, the ionization potentials of O^{2+} and He^{+} are nearly identical. The photoionization cross-sections for O^{2+} and for He^{+} are within about a factor of 2 of one another (Seaton 1958, 1960), and Burgess and Seaton (1960) show that the effective recombination coefficients of the $3s\ ^3P$–$3p\ ^3D$ and the $3s\ ^3P$–$3p\ ^3S$ multiplets of O III are comparable to that of He II λ4686 within a factor of 4. Since helium is over 100 times as abundant as oxygen, we would expect that λ4686 would always be stronger than these O III lines by at least a factor of 10 and, for the central-star temperatures where the helium is marginally doubly ionized, that the He II lines would appear and not the O III lines. When the O III lines are enhanced by fluorescence (Bowen 1935), the multiplets named above are still weaker than He II λ4686.

For three planetary nebulae we find just the opposite. NGC 6572, NGC 6543, and IC 4997 exhibit lines of O III, but no trace of the nebular λ4686 line is present. Table 1 shows the O III lines observed in these nebulae, arranged by multiplet. The sources of the observations are given in column (2) of table 3. The identifications were made with the aid of Moore's (1945) Revised Multiplet Table (RMT). In table 1, column (1) gives the laboratory wavelength from the RMT; column (2), the last four significant figures of the observed wavelengths; column (3), the observed intensities on the basis of I(HB) = 100. No correction is made for interstellar reddening since all the lines in a multiplet are so close in wavelength. The RMT multiplet numbers are given in parentheses before the transition.

The existence of O III in these nebulae appears to be well established. Nearly complete multiplets are present, particularly multiplets (2) and (14). IC 4997 presents the weakest case, but since most of multiplet (2) is present, the identification appears good.

An examination of the RMT shows many possible blends at the observed wavelengths of the O III lines. In nearly all cases, blends with lines of other ions can be ruled out, as stronger lines of the blending multiplet are not present. The only exception is a possible blend of N III, multiplet (4) at λ3754.62, with the λ3754.67 line of O III, multiplet (2). This blend can also be eliminated as the λ3754/λ3791 intensity ratio is about what is expected from the theoretical relative line strength given by Aller (1963). These theoretical line ratios provide a test for the O III identification. We compare lines with common upper levels. Table 2 gives intensity ratios for NGC 6572 and NGC 6543 together with the theoretical ratios (Aller 1963) where we assume *LS* coupling. The lines of IC 4997 are so weak and the errors so large that this comparison would not be meaningful. The λ3811 line [multiplet (2)] identified in NGC 6572 and IC 4997 is probably spurious.

The shorter-wavelength lines ($\lambda < 3600$ Å) do not appear in the spectrum of NGC 6543, as the efficiency of the instruments used for the observations is rather low in this region.

Figure 1 shows the tracings of the longest exposure plates (see the original references) of these three nebulae in the region of λ4686 of He II. No trace of that nebular line (which should be as narrow as the other lines) is visible, although a broad line which is almost certainly of stellar origin is present. In all three of these spectra the central star contaminates the nebular spectrum. The O III lines are certainly of nebular origin because of their narrowness.

We assume that the hydrogen and helium lines are produced by recombination, a process that seems quite certain (Seaton 1968). Since He^{2+} is not present in the nebula, we would not expect O^{3+} to be present. The O III spectrum must then be produced by O^{2+} from some mechanism other than recombination, presumably by direct photoexcitation from the ground state.

III. THE C II LINES

Two multiplets of C II are widely observed in the spectra of gaseous nebulae: the λ4267 ($3\ ^2D$–$4\ ^2F^o$) doublet, which always appears blended, and the λ31918.98–λ3920.68 ($3\ ^2P^o$–$4\ ^2S$) doublet, which under moderate dispersions is resolved. The ratio of the intensities of these two doublets is highly variable from one nebula to another, showing a variation of a factor of 13. This result is quite remarkable, especially if we assume that

TABLE 1

IDENTIFICATION OF O III

A.

	NGC 6572		NGC 6543		IC 4997	
λ_R (1)	λ (2)	I_0 (3)	λ (4)	I_0 (5)	λ (6)	I_0 (7)
(2) $3s\,^3P^o$–$3p\,^3D$:						
3754.67..............	54.69	0.20	54.90	0.41	54.55	0.04
3757.21..............	57.33	0.07	57.30	0.26	57.64	0.04
3759.87..............	59.84	0.14	59.96	0.35	73.97	0.03
3774.00..............	73.94	0.08	74.22	0.14	91.58	0.03
3791.26..............	91.14	0.06	91.32	0.12	...	...
3810.96..............	11.2	0.07	...	...	10.59	0.03
(3) $3s\,^3P^o$–$3p\,^3S$:						
3340.74..............	40.74	0.13	...	...	...	...
3299.36..............	99.36	0.09	...	...	...	...
(8) $3p\,^3D$–$3d\,^3F$:						
3260.98..............	60.7	0.17	...	...	60.7	0.02
(12) $3p\,^3S$–$3d\,^3P^o$:						
3121.71..............	21.80	0.42	...	...	...	...
3132.86..............	31.98	0.59	...	...	...	...
(14) $3p\,^3P$–$3d\,^3D^o$:						
3702.75..............	02.69	0.21	02.84	0.10	...	...
3707.24..............	07.19	0.22	07.05	0.38	07.32	0.04
3714.03..............	13.97	0.16	13.72	0.15	14.92	0.05
3715.06..............	15.08	0.21	15.08	0.20	...	...
(15) $3p\,^3P$–$3d\,^3P^o$:						
3444.10..............	44.10	0.11	...	...	43.20	0.03
(17) $3p\,^1D$–$3d\,^1F^o$:						
3961.59..............	...	...	...	...	61.39	0.05

B.

	NGC 6572		IC 4997	
λ_R (1)	λ (2)	I_0 (3)	λ (4)	I_0 (5)
(21) $3s\,^5P$–$3p\,^5D^o$:				
3695.37.............	95.34	0.02	...	...
3709.54.............	09.24	0.04	...	...
(23) $3s\,^3P$–$3p\,^3D^o$:				
4073.90.............	...	...	73.9	0.01
4081.10.............	...	...	81.40	0.30
(25) $3p\,^5D^o$–$3d\,^5F$:				
3466.90.............	...	...	67.30	0.05
(27) $3p\,^5P^o$–$3d\,^5D$:				
3384.45.............	85.8	0.10	...	...
(28) $3p\,^5P^o$–$3d\,^5P$:				
3348.05.............	47.94	0.08	...	...
3355.92.............	56.0	0.15	...	...
(31) $3p\,^3D^o$–$3d\,^3D$:				
3200.95.............	01.2	0.15	...	...
(32) $3p\,^5S^o$–$3d\,^5D$:				
4529.7..............	29.05	0.03	...	...

TABLE 2

LINE RATIOS FOR O III, MULTIPLET (2)

Nebula	$I(\lambda 3754)/$ $I(\lambda 3791)$	$I(\lambda 3757)/$ $I(\lambda 3774)$	$I(\lambda 3757)/$ $I(\lambda 3811)$
Theoretical.........	3.0	1.35	20
NGC 6572..........	3.3	0.88	1.0
NGC 6543..........	3.4	1.87	...

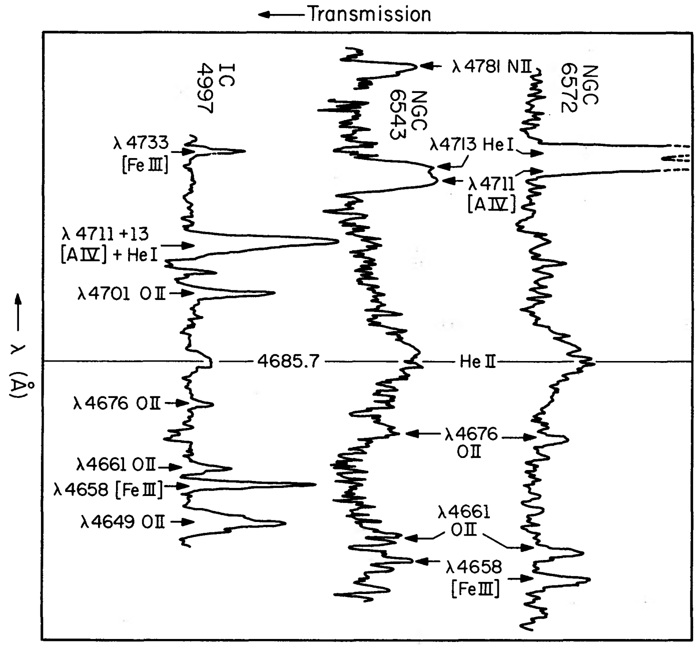

FIG. 1.—Transmission tracings of the longest-exposure plates of NGC 6572, NGC 6543, and IC 4997 in the neighborhood of the λ4686 He II line.

the lines are the products of recombination. The hydrogen and helium line ratios, for example, show only slow dependences (less than a factor of 3 variation) upon nebular electron temperature and density.

The data are presented in table 3, where column (1) gives the name of the nebula and column (2) gives the sources of the observations. Columns (3) and (4) give the observed intensities of the λ3918 and λ3920 lines, respectively. The intensity ratio of these two lines is given in column (5). Column (6) gives the observed intensity of the blend of the two, I_0(blend). When the observations were made with low dispersion, this is all that is observed. Column (7) gives the observed intensity of the λ4267 line $I_0(\lambda 4267)$. All these intensities are on the basis of $I_0(\mathrm{H}\beta) = 100$.

Column (8) gives the adopted values for the interstellar extinction constant. Most of these are derived from comparisons of radio flux densities with Hβ fluxes, except for IC 4997, IC 2149, and the Orion Nebula, for which Paschen-Balmer ratios were used. They are essentially taken from Cahn and Kaler (1971) (which contains references to the observations) except that they were modified by new data from Higgs (1971). In addition, new iterative solutions were calculated in conjunction with the hydrogen

TABLE 3

C II LINE INTENSITIES

Nebula (1)	Reference (2)	$I_0(\lambda3918)$ (3)	$I_0(\lambda3920)$ (4)	$I_0(\lambda3920)/I_0(\lambda3918)$ (5)	I_0(blend) (6)	$I_0(\lambda4267)$ (7)	c (8)	I_c(blend) (9)	$I_c(\lambda4267)$ (10)	log R_c (11)	$10^{-3}\,T_*$ ° K (12)
Orion........	1	0.13	0.26	2.0	0.39	0.35	0.36	0.48	0.40	+0.08	40
NGC 2440....	2	...	...	...	<0.05	0.38	0.73	<0.08	0.49	<−0.81	>100
NGC 3242....	3	...	...	...	0.08:	0.74	0.32	0.10	0.83	−0.93	93
NGC 6543....	4	0.09	0.27	3.0	0.36	0.95	0.25	0.42	1.03	−0.40	66
NGC 6572....	5, 6	0.05	0.08	1.6	0.13	0.63	0.51	0.17	0.75	−0.64	62
NGC 6826*...	7	...	0.2	...	0.3	0.9	0.07	0.3	0.9	−0.47	69
NGC 6891....	8	...	...	...	0.52	1.04	0.44	0.67	1.21	−0.26	55
NGC 7009....	9, 6	0.04	0.05	1.25	0.09	1.04	0.09	0.09	1.07	−1.05	81
NGC 7027....	10	0.01	0.02	2.0	0.03	0.44	1.36	0.07	0.70	−1.03	>100
NGC 7662....	11	0.04	0.05	1.25	0.09	0.71	0.38	0.11	0.81	−0.86	100
IC 418.......	12	0.16	0.29	1.8	0.45	0.61	0.49	0.60	0.72	−0.08	43
IC 2149......	13	...	...	...	<0.20	0.46	0.38	<0.25	0.52	<−0.32	49
IC 2165......	14	...	...	...	<0.06	0.41	0.87	<0.10	0.55	<−0.75	>100
IC 4997......	5	0.02	0.05	2.5	0.07	0.17	0.57	0.10	0.21	−0.33	53

* $I_0(\lambda3918)$ not observed; assume I(blend) = 1.5 $I_0(\lambda3920)$.

REFERENCES:

1. Kaler *et al.* (1965).
2. Aller *et al.* (1968).
3. Czyzak *et al.* (1966).
4. Czyzak *et al.* (1968).
5. Aller and Kaler (1964*c*).
6. Kaler and Aller (1969).
7. Czyzak *et al.* (1971).
8. Aller, Czyzak, and Kaler (unpublished).
9. Aller and Kaler (1964*a*).
10. Aller, Kaler, and Czyzak (unpublished).
11. Aller *et al.* (1966).
12. Aller and Kaler (1964*b*).
13. Kaler, Lee, Aller, Czyzak, and Yoss (unpublished).
14. Kaler *et al.* (1968).

temperatures found in Kaler (1970). Where no hydrogen temperature is available, a value of 7000° K is adopted (Peimbert 1971) as typical.

Columns (9) and (10) present values of I_0(blend) and $I_0(\lambda 4267)$ corrected for interstellar extinction, designated I_c(blend) and $I_c(\lambda 4267)$, respectively. The reddening function used is that suggested by Seaton (1960).

The log of the intensity ratio of these doublets is presented in column (11) as log R_c where

$$\log R_c = \log I_0(\text{blend}) - \log I_0(\lambda 4267) + \Delta f_\lambda c\,, \qquad (2)$$

where Δf_λ is the difference in the interstellar extinction function between λ3920 and λ4267 which equals 0.10. Since this value is so low, log R_c has very little dependence upon interstellar extinction.

The last column gives the temperatures of the exciting stars, T_*. Most of these are Zanstra temperatures computed by Harmon and Seaton (1966). Values for IC 4997 and for the Orion Nebula are from Kaler (1967) as computed by a modified version of Stoy's (1933) method. The Orion Nebula is excited primarily by the O6 star θ^1 Ori C. The value of T_* given in table 4 is identical to that derived by Morton (1969) for an O6 star. The central stars of IC 2165, NGC 2440, and NGC 7027 cannot be observed. Values of T_* are simply given as $> 10^5$ ° K because of the high degree of nebular excitation present.

Before the ratio R_c can be used, care must be taken that the lines are not affected by blends. Examination of the RMT shows no likely blends in the neighborhood of λ4267. Table 4 shows the possible identifications of the λλ3918, 3920 lines.

Each of the nebulae which do not simply involve upper limits to R_c has been checked for blends with N II, O II, or S III. The criteria used in checking for blends were as follows:

N II: presence of other singlet lines, particularly $3p\ {}^1P_1$–$3d\ {}^1D^o{}_2$ (λ4447.03) and $3s\ {}^1P^o{}_1$–$3p\ {}^1D_2$ (λ3995.00).

S III: presence of $3d\ {}^3D^o{}_3$–$4p\ {}^3P_2$ (λ3928.62) line. From *LS*-coupling considerations (Aller 1963) $I(3928.62)/I(3920.37) = 5.8$.

O II: presence of $3s'^2D_{5/2}$–$3p'^2P^o{}_{3/2}$ (λ3911.96) or $3s'^2D_{3/2}$–$3p'^2P^o{}_{3/2}$ (λ3912.09), which will appear blended. This O II multiplet has never been observed. Since it involves a doubly excited electronic state, the O II blend can probably be completely ruled out.

As an additional aid in checking for blends, the ratio $I(3920.68)/I(3918.98)$ should be 2.0, again assuming *LS* coupling, which agrees with the observed ratios presented in column (4) of table 3.

It is found from the above considerations that the C II lines are probably not blended with other lines above the 10 percent level in any of the nebulae, except possibly NGC 7009. For this object, if the λ3918 line is assumed to contain N II, log R_c could be as low as −1.22. It is unlikely that photometric errors could result in an error in R_c of more

TABLE 4

POSSIBLE IDENTIFICATIONS OF λλ3918 AND 3920

λ (Å)	Identification	RMT Multiplet	Transition
3918.98	C II	4	$3\ {}^2P^o{}_{1/2}$–$4{}^2S_{1/2}$
3919.00	N II	17	$3p\ {}^1P_1$–$3d{}^1P^o{}_2$
3919.29	O II	17	$3s'^2D_{3/2}$–$3p'^2P^o{}_{1/2}$
3920.37	S III	8	$3d\ {}^3D^o{}_2$–$3p\ {}^3P_2$
3920.68	C II	4	$3\ {}^2P^o{}_{3/2}$–$4\ {}^2S_{1/2}$

than 50 percent for even the most poorly observed C II lines. The variation in R_c is so large, though, that even a factor of 2 error introduced by blending or by some other cause does not change the conclusions of this paper.

Figure 2 shows log R_c plotted against the temperature of the central star, T_*. As T_* increases, log R_c decreases very markedly. The very existence of this strong correlation argues for a direct influence of the central star. If the lines were formed by recombination, a dependence of R_c on electron temperature would be present. A plot of log R_c against the electron temperatures given by Kaler (1970) shows no such correlation except as reflected by the fact that the nebulae with high central-star temperatures tend as a group to have higher electron temperatures, and vice versa. If we look at log R_c versus T_e for each group, absolutely no correlation exists.

It is apparent that the C II $I(\lambda\lambda 3918, 3920)/I(\lambda 4267)$ doublet ratio is a sensitive empirical indicator of the temperature of the nebula's exciting star. We confine ourselves to nebulae with known values of R_c which have Zanstra temperatures computed by Harmon and Seaton (1966). In addition, 10^5 ° K is adopted for NGC 7027. There is no apparent deviation of these points from a straight line. If we fit a line through the points by the method of least squares, we find that the (log R_c, T_*)-relation can be represented by the equation

$$\log R_c = 0.51 - 1.55 \times 10^{-5} T_* \, . \tag{3}$$

We may now use this relation to determine an equivalent Zanstra temperature for the central star of IC 4997. This nebula appears as a nearly stellar object in which the star cannot be separated from the nebula, so that a Zanstra temperature cannot be computed. The value of R_c from table 4, when introduced into equation (3), yields a value of $T_* = 54{,}000°$ K, nearly the same as that found by Kaler (1967) from Stoy's method. From the scatter of figure 2 the error would be about ± 7000° K.

In the same way we find $T_* = 28{,}000°$ K for the exciting star of the Orion Nebula, lower than expected. Possibly the relation becomes nonlinear at low temperatures. It seems more likely, however, that we cannot compare simply the C II emission from a massive H II region with that from planetary nebulae.

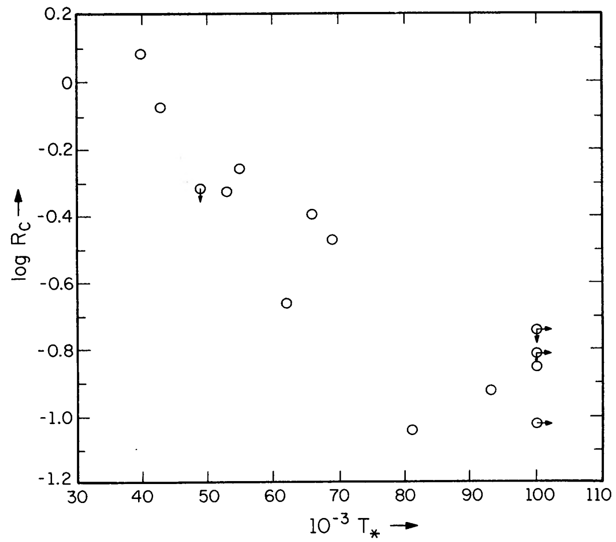

FIG. 2.—The log of the ratio of the intensity of the λ3918 + λ3920 doublet of C II to that of λ4267, log R_c, plotted against the temperature of the exciting star, T_*.

Inspection of table 3 shows some other interesting properties of the C II lines. The variation in the intensity of the λλ3918, 3920 doublet is more than twice that of the λ4267 doublet. The variations in λ4267 do not appear to have much to do with T_*. It is quite obvious, however, that the nebulae with high T_* have weaker λλ3918, 3920 lines than do those with low T_*. Divide the nebulae into high- and low-temperature groups at 70,000° K. There are about an equal number per group. The mean corrected λ4267 intensities are 0.74 and 0.71 for the two groups respectively, whereas the mean blended λλ3918, 3920 intensity is less than 0.09 for the high-temperature group and 0.39 for the low-temperature group. The upper limits placed on λλ3918, 3920 for NGC 2440 and IC 2165 were used in this calculation. Some of the variation in line strengths must be due to the real variation of ionic abundance and of the carbon-to-hydrogen ratio. The low intensities of both doublets for IC 4997 may well be related to the low oxygen-to-hydrogen ratio for this nebula (Kaler 1970).

The greater sensitivity of the λλ3918, 3920 doublet to central-star temperature can perhaps be understood in that the ^2S upper term can be excited directly from the ^2P ground state of C^+ whereas the ^2F upper term of λ4267 cannot. I suggest that the lines are produced by both direct excitation of C^+ and recombination from C^{2+} where the λλ3918, 3920 doublet is produced dominantly by the former and λ4267 by the latter. As the temperature of the central star increases and the carbon becomes more highly ionized, the amount of C^+ in the nebula, and thus the strength of the λλ3918, 3920 doublet, decreases. Singly ionized carbon has nearly the same ionization potential as neutral helium. The data presented by Kaler (1970) show that the amount of neutral helium in a nebula rapidly decreases with increasing central-star temperature, which supports the above interpretation. The C^{2+}, however, will probably exist in some abundance over a much greater range of central-star temperatures, and will show a maximum relative abundance somewhere in the middle temperature range. The ionization potential of C^{2+} (47.87 eV) is reasonably comparable to that of He^+ (54.50 eV). From Kaler (1970) it is seen that He^+ is quite abundant as compared with both neutral helium and He^{2+} at nearly all central-star temperatures. Quite possibly, the C II λ4267 doublet can, at least in approximation, be used for abundance determination with the application of only recombination theory. A complete explanation of the behavior of the C II lines (and other permited lines) will require a detailed application of transfer theory. Until this is done, carbon abundances derived from nebulae will be suspect.

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REFERENCES

Aller, L. H. 1963, *The Atmospheres of the Sun and Stars* (2d ed.; New York: Ronald Press Company).
Aller, L. H., Czyzak, S. J., and Kaler, J. B. 1968, *Ap. J.*, **151**, 187.
Aller, L. H., and Kaler, J. B. 1964*a*, *Ap. J.*, **139**, 1074.
———. 1964*b*, *ibid.*, **140**, 936.
———. 1964*c*, *ibid.*, p. 621.
Aller, L. H., Kaler, J. B., and Bowen, I. S. 1966, *Ap. J.*, **144**, 291.
Bowen, I. S. 1935, *Ap. J.*, **81**, 1.
Burgess, A., and Seaton, M. J. 1960, *M.N.R.A.S.*, **121**, 76.
Cahn, J. H., and Kaler, J. B. 1971, *Ap. J. Suppl.*, No. 189, **22**, 319.
Czyzak, S. J., Aller, L. H., and Kaler, J. B. 1968, *Ap. J.*, **154**, 543.
———. 1971, *ibid.*, **168**, 405.
Czyzak, S. J., Aller, L. H., Kaler, J. B., and Faulkner, D. J. 1966, *Ap. J.*, **143**, 327.

Harmon, R. J., and Seaton, M. J. 1966, *M.N.R.A.S.*, **132**, 15.
Higgs, L. A. 1971, *M.N.R.A.S.*, **153**, 315.
Kaler, J. B. 1967, *A.J.*, **72**, 305.
———. 1970, *Ap. J.*, **160**, 887.
Kaler, J. B., and Aller, L. H. 1969, *Ap. J.*, **157**, 1231.
Kaler, J. B., Aller, L. H., and Bowen, I. S. 1965, *Ap. J.*, **141**, 912.
Kaler, J. B., Czyzak, S. J., and Aller, L. H. 1968, *Ap. J.*, **153**, 43.
Moore, C. S. 1945, *Revised Multiplet Table* (Princeton Contr. No. 22).
Morton, D. C. 1969, *Ap. J.*, **128**, 629.
Peimbert, M. 1971, *Bol. Obs. Tonantzintla y Tacubaya*, **6**, 29.
Seaton, M. J. 1958, *Rev. Mod. Phys.*, **30**, 979.
———. 1960, *Rept. Progr. Phys.*, **23**, 313.
———. 1968, *M.N.R.A.S.*, **139**, 129.
Stoy, R. H. 1933, *M.N.R.A.S.*, **93**, 508.