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SOME SPECTROSCOPIC CHARACTERISTICS OF THE OB STARS: AN INVESTIGATION OF THE SPACE DISTRIBUTION OF CERTAIN OB STARS AND THE REFERENCE FRAME OF THE CLASSIFICATION

NOLAN R. WALBORN*
Yerkes Observatory, University of Chicago
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ABSTRACT

The distances of stars from the Victoria list of revised Hγ spectrophotometric absolute magnitudes have been reinvestigated, by means of MK spectral classification at 63 Å mm^{-1}. The two systems are compared with respect to spectral types, absolute magnitudes, and space distribution of the individual stars; marked systematic and accidental differences are found to exist for the O9–B2 stars. The Victoria spectral types are systematically later, and the revised Victoria magnitudes consistently fainter, than those derived here. In terms of the present classifications, it is found that the OB stars investigated delineate the Local and Perseus spiral arms.

Because the work was done with a dispersion approximately twice that of the MK atlas, the spectral classification has been investigated in detail. With a reference frame based upon ratios of the lines of helium and silicon, an increased classification resolution in the range O9–B1 is obtained. Also, a luminosity classification for the earlier O stars is proposed.

I. INTRODUCTION

The concept of a spectroscopic natural group was introduced in order to permit, with a given spectral resolution, the identification of members in relatively homogeneous spectral-type and/or luminosity categories, by means of easily recognized spectral characteristics (Morgan 1951; Nassau and Morgan 1951). The desired properties of a natural group were compactness, that is, as small an extent as possible in one or both coordinates of the H-R diagram; distinctness, or well-defined boundaries; and uniqueness, a minimal possibility of including spectra from an unconnected domain of the diagram. The boundary of the OB natural group was defined in terms of the two-dimensional spectral types of the MK system, as a step function passing through progressively brighter luminosity classes with advancing spectral type.

In general, one may hope to improve the compactness, distinctness, and uniqueness of the natural groups by going to a higher spectral resolution. Correspondingly, the dispersions in temperature and luminosity within groups can be expected to be smaller, and conclusions concerning their space distributions more definite (Morgan, Keenan, and Kellman 1943). The highly desirable limit to this process is a system in which one may place some degree of confidence in the results for individual stars. The MK system of two-dimensional spectral classification may be regarded as a system of very compact natural groups; each group is defined by one or more standard stars, and the classification consists of placing a nonpeculiar star in a group such that its spectrum bears a closer similarity to that of the corresponding standard than to that of the standard for any surrounding group. The sizes of the groups and their relations to the physical parameters are of course arbitrary, and will depend upon the criteria available in the region of the H-R diagram and at the spectral resolution in question.

Numerous observational systems have been devised to obtain two-dimensional classifications of the OB stars. Recent reviews have been given by Strömgren (1963, 1966)

* Visiting student, Kitt Peak National Observatory, which is operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation. Present address: David Dunlap Observatory, University of Toronto.

and Underhill (1966). It is sometimes stated that the MK system has been rendered obsolete by other approaches to stellar classification, due to the quantitative nature of the latter and to the greater speed of photoelectric techniques. This view can be questioned with respect to two important points. One is that in general the quantitative systems involve operational processes in addition to, or very different from, those involved in the MK system—for example, the determination of equivalent widths of absorption lines. In this regard, the MK system serves as a well-established, independent reference system with which to investigate the existence or otherwise of systematic or accidental differences with other systems; the more highly refined the MK system becomes, the greater will be its value for this purpose. Another important point is that because of the larger amount of information obtained, as well as because of the emphasis on line visibility and careful inspection of the entire spectrum, the MK system is probably the most effective for the detection of peculiar stars, whether as individuals or groups, which might remain undetected or introduce spurious results in the systems based upon fewer information elements (Abt 1963; Strömgren 1966; Morgan 1966).

Because the MK system is defined essentially in terms of standard stars, the effects of transferring it to different spectral resolutions are amenable to investigation. The present work is concerned with an investigation of the characteristics of the OB group, by means of classification spectrograms of dispersion 63 Å mm^{-1} and widening 1.2 mm. They were obtained during the summer and fall of 1969 with the Meinel spectrograph and the No. 1 36-inch reflector at Kitt Peak National Observatory; the processing was as described in Abt *et al.* (1968). The stars observed fall into two categories. (1) Stars from the list of Petrie and Lee (1966) were observed for the purposes of a comparison with the revised Victoria system of spectroscopic absolute magnitudes (Petrie 1965), and of a reinvestigation of the space distribution of OB field stars within about 3 kpc of the Sun (Petrie and Petrie 1968); the results will be discussed in § II. (2) A considerable number of stars from the various published lists of MK standards were observed, in order to investigate the spectral classification with the higher dispersion; they will be considered in § III.

While this investigation is oriented primarily toward questions of galactic structure, some incidental results of possible relevance to the interpretation of early O atmospheres and to the study of stellar evolution have been obtained (§ III*c* below and Walborn 1970, respectively). This circumstance is believed due to a favorable compromise between the opposing requirements of spectral resolution and sample size obtained for the OB stars with the present observational system.

II. THE VICTORIA SYSTEM

a) *Historical Background*

As part of an extensive program in optical galactic structure at the Dominion Astrophysical Observatory (Victoria), a system for the determination of stellar luminosities from spectrophotometric measures of the equivalent width of Hγ has been developed there. Various authors criticized the original Victoria calibrations (Petrie and Maunsell 1949; Petrie 1952) because of the systematic and accidental effects present with respect to other systems (Kopylov 1958; Johnson and Iriarte 1958; Sinnerstad 1961). A revised calibration of Hγ equivalent width in terms of absolute magnitude was performed by Petrie (1965). Distances of several clusters in the revised system were found to agree with photometric values (except for NGC 2244, which was the only one studied whose distance depends strongly on early O stars). However, Petrie found the revised calibration to be 0.6 mag fainter than the present MK calibration (Blaauw 1963) for the class V stars in the range O9–B2.

Recent work has provided indications that systematic differences may remain between the revised Victoria system of absolute magnitudes and other systems (Hoag and

Applequist 1965; Hutchings 1966; Racine 1968; Walker and Hodge 1968; Murphy 1969). In particular, Walker and Hodge from their study of several associations found it necessary to revise Petrie's (1965) spectral-type corrections in the range O6–O8 by an average of −1.3 mag.

A list of 571 stars of the northern hemisphere with absolute magnitudes determined on the revised Victoria system has been given by Petrie and Lee (1966). From this list, 183 stars have been observed at 63 Å mm^{-1} on Kitt Peak. The stars selected for observation in general have (Victoria) spectral types of B2.5 or earlier, and most of them have *B*-magnitudes between 7.0 and 9.0. These stars are listed in the Catalog (Table 1). The columns give (1) HD, HDE, or BD number, (2) present spectral classification, (3) absolute magnitude from the calibration of Blaauw (1963), and (4) remarks. Some new notations used in the 63 Å mm^{-1} classification will be explained in § III.

b) The Spectral Types

The dispersion used for the most part in the program at Victoria for observing faint B stars was 51 Å mm^{-1} at Hγ (Petrie and Pearce 1962). The Victoria spectral types are from a visual examination of the spectrograms, the visibility of certain lines and certain line ratios being used as criteria. In a comparison of discrepant Victoria and MK spectral types, Underhill (1966) stated that the former were likely to be more nearly correct, due to the higher dispersion employed. However, an increased dispersion is not a sufficient condition for an improved spectral classification; the widening and other factors are equally important, as will be discussed in § III. In their study of He I $\lambda\lambda$4387 and 4471 and of the interstellar band at λ4430, from the same Victoria plate material, Walker and Hodge (1966) state (referring to the microphotometer tracings) that these are "the only absorption features which are strong enough not to be confused by the plate grain on these narrow spectra, and which are present in the spectra of most O and B stars."

A comparison of the present spectral types (Table 1) with the Victoria types (Petrie and Lee 1966) is given in Figure 1. In Figure 1*a*, the comparison is made for 116 sharp-lined stars. The solid line corresponds to coincidence (as do the short segments on either side of it between O9.7 and B0.7), while the dashed lines indicate a difference of one-tenth of a spectral type. Two conclusions are apparent: (1) for the great majority of stars, the agreement is to a tenth of a spectral type or better; (2) there is a systematic effect, in the sense that at B0 and later many more stars are given later types by Petrie. This effect was also noted by Underhill (1966). In Figure 1*a* approximately 11 percent of the classifications differ by more than a tenth of a class. (Stars for which the present classification may be uncertain due to a peculiarity are indicated by large filled circles.)

In Figure 1*b*, the analogous comparison is made for sixty-three stars with broadened lines, the degree of broadening as indicated by the n-parameter (§ III*b*) being denoted by different symbols. Again, stars for which an uncertainty is attached to the present classification either because of a peculiarity or because of the broadened lines are indicated by large filled circles. While the systematic effect noted in Figure 1*a* is still present, the degree of scatter is greater, 32 percent of the classifications differing by more than a tenth of a spectral class. Here there are two O9 stars classified B1 and B3, and an O6.5 star classified B1, in the Victoria list.

The smooth systematic difference between the Victoria and present spectral types would not be expected in the first approximation to have an effect upon the absolute magnitudes obtained for the stars, since standard stars classified in the same system were used in the Victoria calibration. On the other hand, many broader-lined stars selectively have even later types in the Victoria classification (Figs. 1*a* and 1*b*); since the numerical spectral-type corrections given by Petrie (1965) decrease monotonically from O6 to B5, the effect would be to obtain absolute magnitudes for these stars that are too bright (with respect to Petrie's system).

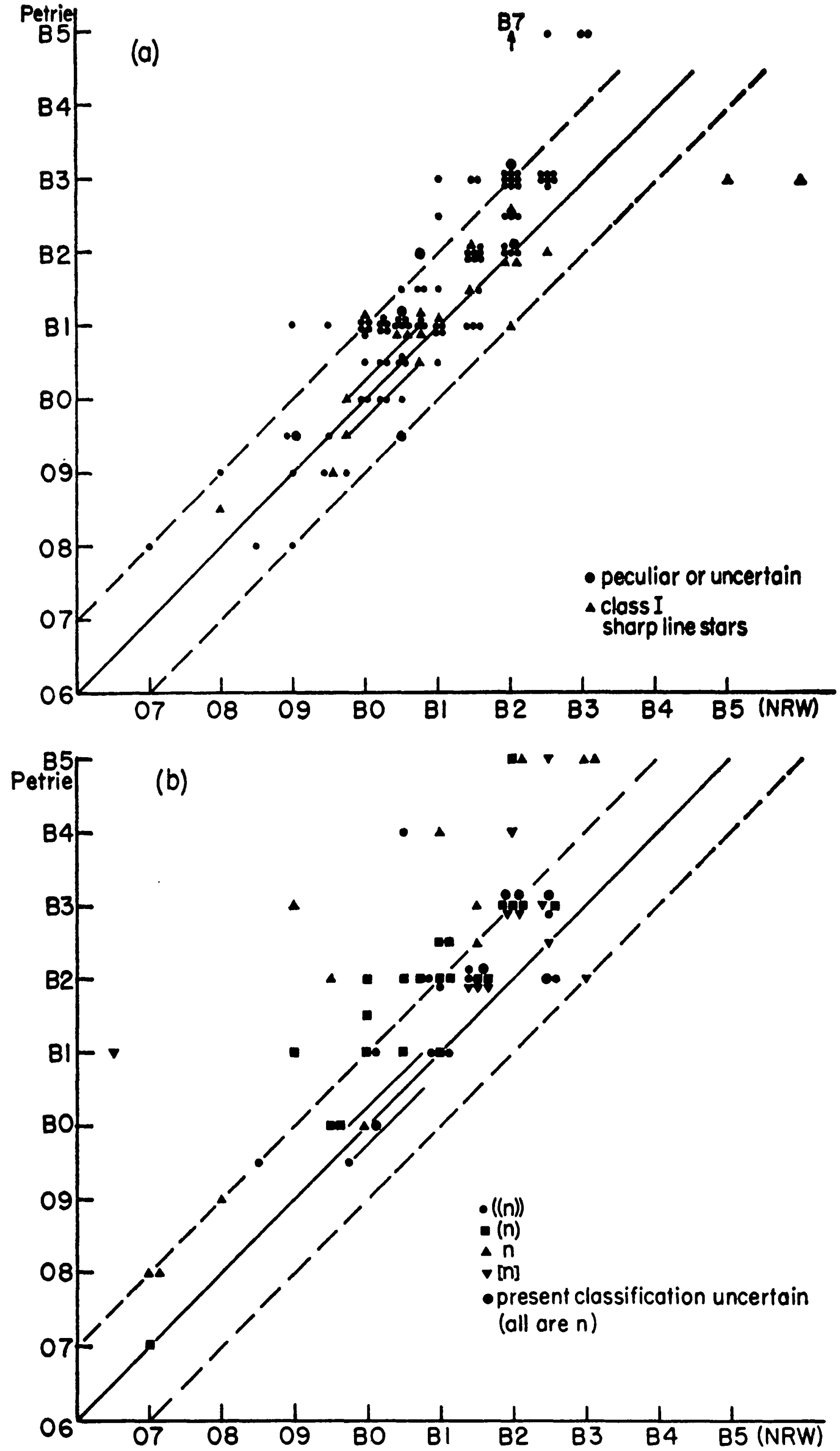

FIG. 1.—Comparison of Victoria with present spectral types: (*a*) Sharp-lined stars, (*b*) stars with broadened lines.

TABLE 1

CATALOG OF SPECTRAL CLASSIFICATIONS FOR STARS FROM PETRIE AND LEE (1966)

HD (1)	Walborn (2)	M_V (3)	Notes (4)
73	B1.5 IV	−3.7	1
1334	B2.5 V	−2.1	2
1544	B0.5 III(n)	−4.7	3
1743	B0.2 IV	−4.7	3
3827	B0.7 V((n))	−3.8	
4460	B1.5 V((n))	−3.1	
4768	B5 Ia	−7.0	3
5551	B1.5 Ia	−6.7	3
5553	B1.5 III–IV(n)	−3.9	4
5882	B2.5:V:n	...	5
6182	B0.5 IIp	−5.2	3
6675	B0.2 III	−4.9	1
7636	B2 III:[n]e+	...	6
10125	O9.7 II	−5.6	
10898	B2.5 Iab	−6.3	
232552	B0:IV:ne+	...	7
12509	B1.5 II	−4.9	
12740	B1.5 II	−4.9	
12882	B2.5 III:[n]e+	...	7
13590	B3 III:np	...	8
13621	B0.5 III–IV((n))	−4.6	3
13716	B0.5 III	−4.7	
13745	O9.7 II((n))	−5.6	3
13831	O+B1:III:n	...	3
13866	B2 Ib–II:p	−5.3	3
14443	B2 Ibp	−5.7	3
15137	O9.5 II–III(n)	−5.6	
15642	O9.5 III:n	−5.4	3
15690	B2 Iab	−6.3	3
18326	O7 V(n)	−5.4	
18409	O9.7 Ib	−5.9	
20134	B2.5 IV–V	−2.5	3
20508	B1.5 IV	−3.7	
20798	B2 III–IV	−3.5	
20898	B1 IV((n))	−4.1	
21212	B1.5 IV:[n]e+	...	6
21806	B1 V(n)	−3.6	5
23060	B2 IV–V	−2.9	3
24190	B2 V(n)	−2.5	3
24560	B1.5 V:[n]e	...	7
29441	B2.5: V:ne	...	
30677	B1 II–III((n))	−4.7	4
31894	B2 IV–V	−2.9	
32018	B2 IV	−3.3	
32672	B2 IV	−3.3	
32989	B2 IV	−3.3	
34626	B1.5 IV:np	...	
36013	B3 V:n	−1.7:	3
36212	B2.5 II	−4.7	
36441	B1.5 IV–V((n))	−3.4	
36483	O9 V(n)	−4.8	
36895	B2 IV–V	−2.9	
37032	B0 IV–V	−4.6	
37366	O9.5 V	−4.6	
37737	O9.5 III(n)	−5.4	

NOTES:

1. Classification standard.
2. Weak shell characteristics.
3. Association member.
4. Line-breadth standard.
5. Spectroscopic binary from line appearance.
6. α Cyg emission sharp.
7. α Cyg emission broad.
8. ζ Tau-type shell star.
9. Variable radial velocity (Petrie and Pearce 1962).
10. Emission variable (Petrie and Pearce 1962).

TABLE 1—*Continued*

HD (1)	Walborn (2)	M_V (3)	Notes (4)
39680	O6.5 V[n]e	−5.5	
39746	B1 II((n))	−5.0	3
40894	B2 IV–V	−2.9	
41689	B1 V(n)	−3.6	
41690	B1.5 II–III	−4.5	
42379	B1 Ib	−5.7	3
42401	B2 V	−2.5	1
43753	B0.5 II–III	−5.0	3
44637	B3 III:[n]e	...	3
45789	B2.5 IV–V	−2.5	
45911	B2 IV–V	−2.9	
46056	O8 Vn	−5.2	3, 5
46106	B0 V	...	3
46149	O8.5 V	−5.0	1, 3
46202	O9 V	−4.8	3
46485	O7 V:n(e)	...	3
46573	O7 III((f))	...	3
46867	B0.5 III–IV	−4.6	
47382	B0 III–IV	−4.9	
48279	O8 V	−5.2	
65079	B2 V(n)(e?)	−2.5	
66665	B0.7 IV–V	−4.1	
154445	B1.5 V	−3.1	1
162094	B3 V	−1.7	
162365	B2 IV	−3.3	
163472	B2 IV–V	−2.9	
166331	B1.5 III	−4.0	
169798	B2.5 IV–V	−2.5	2
171871	B2 II:p	...	
174298	B1.5 IV	−3.7	
174571	B2.5 V:[n]e		
176304	B2.5 III	−3.3	
180642	B1.5 II–III	−4.5	
181653	B1 II–III	−4.7	
181858	B3 V	−1.7	
184279	B1 IV	−4.1	
+28°3434	B0.5 III:	...	
186618	B0.7 IV	−4.3	1
+23°3759	B0 III:((n))	...	3
186994	B0.2 IV	−4.7	
189957	O9.5 III	−5.4	1
190336	B0.7 II–III	−4.9	
227586	B0.5 IVp	−4.5	5, 9
190919	B0.7 Ib	−5.8	1, 3
190944	B1.5 V:[n]e+	...	7, 10
227634	B0 II	−5.4	3
+35°3955	B0.7 Iab	−6.2	3
191139	B0.5 II	−5.2	
191381	B2 IV	−3.3	
191395	B0.5 V	−4.0	
191396	B0.2 III	−4.9	
191423	O9 III:n	...	3
191456	B0.5 II–III	−5.0	
191473	B1 III	−4.4	
191495	B0 IV–V(n)	−4.6	
191531	B0.5 III–IV	−4.6	
191611	B0.5 Ia	−6.4	
191917	B1 III	−4.4	
191980	B5–6 III:p	...	
192001	O9.5 IV	−5.1	
192035	B0 III–IV(n)	−4.9	
192039	B0 IV	−4.8	1
192660	B0 Ib	−5.8	3
192968	B1 Vn	−3.6	

TABLE 1—*Continued*

HD (1)	Walborn (2)	M_V (3)	Notes (4)
193007........	B0.2 III	−4.9	
193032........	B0.2 III	−4.9	
193076........	B0.7 II	−5.1	
193183........	B1.5 Ib	−5.7	3
193443........	O9 III	−5.7	3
193444........	B0.5 III	−4.7	
193516........	B0.7 IVp	−4.3	
193794........	B0 III–IVn	−4.9	
194057........	B0.7 Iab:	...	
194092........	B0.5 V	−4.0	3
194194........	B1.5 IV	−3.7	
194279........	B2 Ia	−6.8	3
194280........	O9.7 Iab	−6.2	
194739........	B2.5 V	−2.1	
194779........	B2 II	−4.8	
194839........	B0.5 Ia+	...	
195229........	B0.2 III	−4.9	
196025........	B2 IV–V	−2.9	
196421........	B2 IV	−3.3	1
197460........	B1 II(n)	−5.0	
197702........	B1 III(n)	−4.4	
199216........	B1 II	−5.0	3
199308........	B2 IV–V	−2.9	
201345........	O9 Vp	−4.8	
201666........	B2 V(n)	−2.5	
201795........	B0.7 V	−3.8	1, 3
202124........	O9.5 Iab	−6.2	
202253........	B1.5 IV	−3.7	
202347........	B1.5 V	−3.1	
203664........	B0.5 III(n)	−4.7	
203699........	B2.5 IV:[n]e	...	
204116........	B1.5 IV(n)	−3.7	3
204722........	B1.5 IV:np	...	3, 4, 5, 9
239729........	B0 V	−4.4	3
207308........	B0.7 III–IV(n)	−4.5	3
207329........	B2.5 Ia((n))	−6.8	
207563........	B2.5 IV–V	−2.5	
208106........	B2: V:np	...	5, 9
208185 AB....	B2: V:p	...	3, 5
210386........	B1.5 II–III	−4.5	
235795........	B2 V(n)	−2.5	
212455........	B6 Ib	−5.7	3
213405........	B1 III(n)	−4.4	
214432........	B2.5 V((n))	−2.1	1, 3
215733........	B1 II	−5.0	
216044........	B0 III–IV	−4.9	
216092........	B2 V:n	...	
216438........	B1 II–III	−4.7	
216532........	O8.5 V((n))	−5.0	3
216898........	O9 IV	−5.3	3
217035 AB.....	B0 IV:(n)	...	3, 5, 9
217086........	O7 Vn	−5.4	3
217463........	B1.5 V:n	...	3
217490........	B0 II	−5.4	
218195........	O9 III	−5.7	
218323........	B0.2 IV	−4.7	3
218325........	B2.5 III:(n)p	...	8
219188........	B0.5 II–III(n)	−5.0	
223501........	B2 V:[n](e?)	...	
223924........	B1.5 III:n	...	
223987........	B1 II–III	−4.7	
224257........	B0.2 IV	−4.7	
225095........	B2 IV[n]e+	−3.3	7
225160........	O8 Ib(f)	...	1

c) The Absolute Magnitudes

A comparison of the revised Victoria absolute magnitudes (Petrie and Lee 1966) with those resulting from the present spectral classification (Table 1) and Blaauw's (1963) calibration of the MK system is presented in Figure 2 for five groups of (present) spectral types. In the figures, the present luminosity classification is given as the abscissa and an absolute-magnitude scale as the ordinate. Blaauw's calibration of each luminosity class (for the first-mentioned spectral type of each figure) is plotted as a square, and the Victoria absolute magnitude for each star is also plotted, the broader-lined stars and peculiar stars (for a few of which there is an uncertainty in the present luminosity classification) being indicated by smaller symbols. Stars for which there was a difference of more than one-tenth in the spectral type between the Victoria and present classifications have been eliminated from these figures.

Two conclusions are evident. (1) There is a marked systematic effect, the Blaauw calibration forming essentially an upper envelope to the Victoria magnitudes, with the center of gravity of the latter lying on the average between 0.5 and 1.0 mag fainter. Therefore, the revised Victoria absolute magnitudes continue to be consistently fainter than those obtained from the MK calibration. This result confirms and extends one by Murphy (1969); there is also an intimation of the result in the work of Racine (1968). (2) There is a scatter in the individual magnitudes on the order of ± 0.5 mag about the center of gravity.

Weaver and Ebert (1964) carried out a recalibration of the MK system based upon the photometric cluster moduli of Johnson *et al.* (1961), in which the upper main sequence is considerably fainter than in the Blaauw calibration. If the Weaver-Ebert calibration were used, the systematic effect between the MK and Victoria magnitudes at class V would essentially disappear. However, Fitzgerald (1969) repeated the Weaver-Ebert procedure, using cluster moduli redetermined by Hoag and Applequist (1965) from the same *UBV* data used by Johnson *et al.* (1961), and he found absolute magnitudes differing only slightly in general from those of Blaauw, in the same range. A comparison of

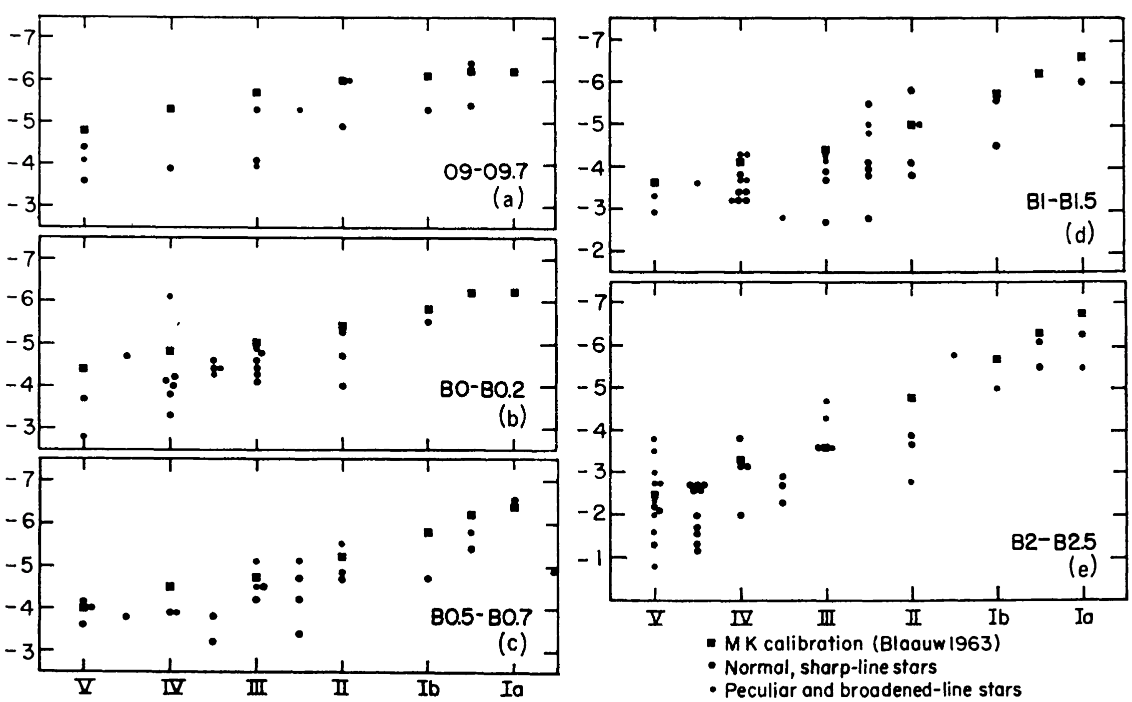

FIG. 2.—Comparison of Victoria with present absolute magnitudes, for several ranges of spectral type: (*a*) O9–O9.7, (*b*) B0–B0.2, (*c*) B0.5–B0.7, (*d*) B1–B1.5, (*e*) B2–B2.5.

the moduli obtained by Hoag and Applequist with those by Johnson *et al.* shows no systematic differences.

The final Hγ moduli derived for the calibrating clusters by Petrie (1965) agree almost exactly with the photometric values tabulated by Johnson *et al.* (1961), except for NGC 2244 which results with an Hγ modulus 1.4 mag smaller. On the other hand, Becker (1963) obtains a photometric modulus 0.6 mag greater for the α Per cluster than the Petrie Hγ and Johnson *et al.* (1961) value. The comparisons made by Hoag and Applequist show that the various determinations of the photometric modulus have random differences on the order of 0.5 mag for many clusters. It may be that among the calibrating clusters used by Petrie one or two of the photometric moduli were too small.

The accidental effect noted above is, of course, the combined result of errors in the present classifications and in the Victoria absolute magnitudes. According to Petrie and Lee (1966), the accidental errors in the measurements of the Hγ equivalent widths were found to be nearly constant at about 10–15 percent in the range 1.8–8.5 Å, corresponding to an average mean error in the absolute magnitudes of about ± 0.2 mag. There is evidence, however, that larger accidental errors may occur in the equivalent widths, and hence in the magnitudes. One reason for this conclusion is provided by the star HD 217490 = HD 217507 = BD+58°2521. Because of an error in the HD catalog, this star has two numbers there. The star is included under both HD numbers in the catalogs of Petrie and Pearce (1962) and Petrie and Lee (1966). The radial-velocity measurements (Petrie and Pearce 1962) show no evidence of variability, and similar mean values are found from the two series, which overlap in time. The respective entries for HD 217490 and 217507 in the second catalog are (from two measures in each case) 2.2 and 3.4 Å for the equivalent widths of Hγ, and -5.3 and -4.0 for the absolute magnitudes.

d) The Space Distribution

A review of work in optical galactic structure has been given by Sharpless (1965). Evidence for optical spiral structure in the solar neighborhood was found by Morgan, Sharpless, and Osterbrock (1952*a*, *b*) and Morgan, Whitford, and Code (1953). The study of the distributions of H II regions and young stellar aggregates was further developed by Becker and his associates (Becker and Fenkhart 1963; Becker 1963, 1964). In the latter determinations of distance for H II regions, absolute magnitudes determined by various authors from Hγ measures of the exciting stars were used; the Victoria results were reduced to Kopylov's (1958) system (Becker and Fenkhart 1963).

The space distribution and motions of 688 individual stars with distances determined by means of the Victoria absolute-magnitude system were discussed by Petrie and Petrie (1968). (About 30 percent of the stars in the Petrie and Lee 1966 list with spectral type earlier than B3 are members of associations, but they are treated as individuals instead of being grouped into average points.) An essentially uniform distribution projected onto the galactic plane was found, both for the stars of types O8–B2 and for the B3–B9 stars studied. Since it was felt that the errors in the absolute magnitudes were not sufficient to smooth the distribution had the stars originally been concentrated in spiral arms, and in view of the relatively small peculiar motions and evolutionary lifetimes of O and early B stars, it was concluded that "the [O8–B2] stars . . . were not born exclusively in thin spiral arms unless the time scale of stellar ages is grossly in error." If these conclusions are correct, important questions arise in the field of galactic astronomy or of stellar evolution. After Roberts (1957), it is generally believed that all O and B stars in the Galaxy probably originated in associations, and the evidence that these young aggregates appear selectively in rather well-defined spiral features in the solar neighborhood is strong.

Before the space distributions derived here for the Victoria program stars of Table 1 are discussed, the still troublesome questions of the MK absolute-magnitude calibration and of the ratio of total to selective extinction will be considered briefly. In this work, the MK calibration of Blaauw (1963) and the classical ratio of visual extinction to reddening $R = 3.0$ (Blanco 1956; Hiltner and Johnson 1956) have been adopted.

The absolute-magnitude calibration of the MK system by Blaauw (1963) in the spectral range of interest here incorporates the results of Johnson and Iriarte (1958) and Schmidt-Kaler (1963), who used cluster-sequence fittings starting from the geometric distance for the Hyades. The independent results from the distance of the Scorpius-Centaurus moving cluster determined by Bertiau (1958) are shown to yield the same result for the distance modulus of h and χ Persei, and they agree well with the Hyades-based results at types B1–3 V and B1 III. On the other hand, Bertiau's values at the two additional common points B0 V and B2 IV are about a half-magnitude fainter than those listed by Blaauw. The substantially different recalibration by Weaver and Ebert (1964), and its criticism by Fitzgerald (1969), have been mentioned above; the most outstanding difference between Blaauw's and Fitzgerald's calibrations in the range of interest is at types O9 III and B0 III, which are brighter by 0.7 and 0.5 mag, respectively, in Fitzgerald's list. Lesh (1968) has performed a calibration in the range B1–B5 classes V through III, based upon the narrow-band photometry of Borgman and Blaauw (1964); her magnitudes are consistently fainter than those of Blaauw (1963) by a few tenths. The reason for this result is unclear, since the Borgman and Blaauw calibration is derived from Bertiau's (1958) Scorpius-Centaurus distance. In summary, the various calibrations (except that by Weaver and Ebert) deviate from Blaauw's (1963) in either sense, usually by relatively small amounts except for some isolated types.

The important question of the appropriate value for the ratio of total to selective extinction continues to be a matter of debate. (1) Schmidt-Kaler (1967, 1969) has suggested a value $R = 3.2$. For the present purpose, however, the difference between that value and $R = 3.0$ will not be critical. (2) There is evidence that a value near $R = 3.8$ may be more appropriate in some very young regions associated with ionized hydrogen (Hiltner and Morgan 1969; Ishida 1969; Morgan 1970; Dufour and Lee 1970). In Table 1, however, there are very few stars with types earlier than O9, and they will be eliminated from the final space-distribution diagrams due to the greater uncertainty of their absolute-magnitude calibration at the present time. (3) Johnson (1965, 1968) has proposed that the value of R shows strong variations with galactic longitude along the plane, reaching values as large as 6.3 in some regions. On the other hand, these results can be questioned for several reasons (Walker 1962; Becker 1966; Schmidt-Kaler 1967; Grubissich 1968). Moreover, comparisons of optical and radio fluxes from H II regions do not confirm the high values for R found by Johnson (Gebel 1968; Dufour and Lee 1970).

The space distributions projected onto the galactic plane resulting from the present work are shown in Figures 3 and 4. Whenever possible, the photometry was taken from Blanco *et al.* (1968), with preference for observations by Hiltner, Johnson, and their associates; otherwise, the magnitude is from Petrie and Lee (1966), with 0.1 mag subtracted from the values given there; and the color is inferred from the color excesses given by Walker and Hodge (1966), together with the intrinsic colors of Johnson (1958). The intrinsic colors used for the MK spectral types are those given by Johnson (1963). In the case of the new intermediate spectral types introduced here, the intrinsic colors and absolute magnitudes used for the present are linear interpolations between adjacent values in the tables of Johnson (1963) and Blaauw (1963), respectively. Clearly, it is desirable eventually to calibrate the present spectral classification directly, by means of extensive observations in clusters with independently known moduli; however, since only one primary MK standard was changed significantly with respect to its luminosity classification, and in view of the results of § III*d* below, it is not expected that the results will differ too greatly from the previous calibrations. The uncertainty in the absolute magnitudes is difficult to determine from the present data; it is believed to be smaller

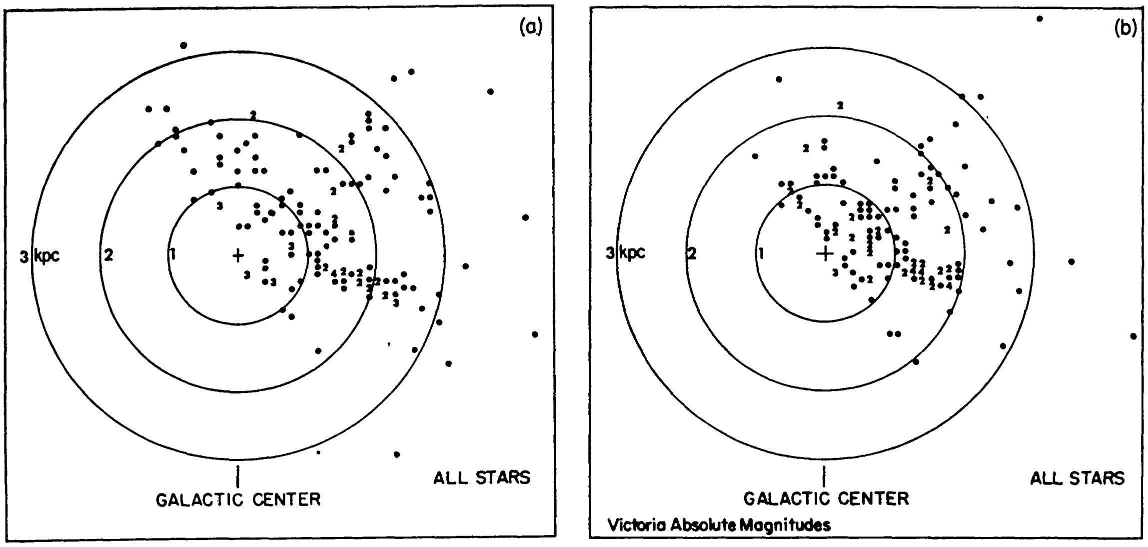

FIG. 3.—Space distribution of all stars for which present absolute magnitudes are given in the catalog. (*a*) Present absolute magnitudes, (*b*) Victoria absolute magnitudes.

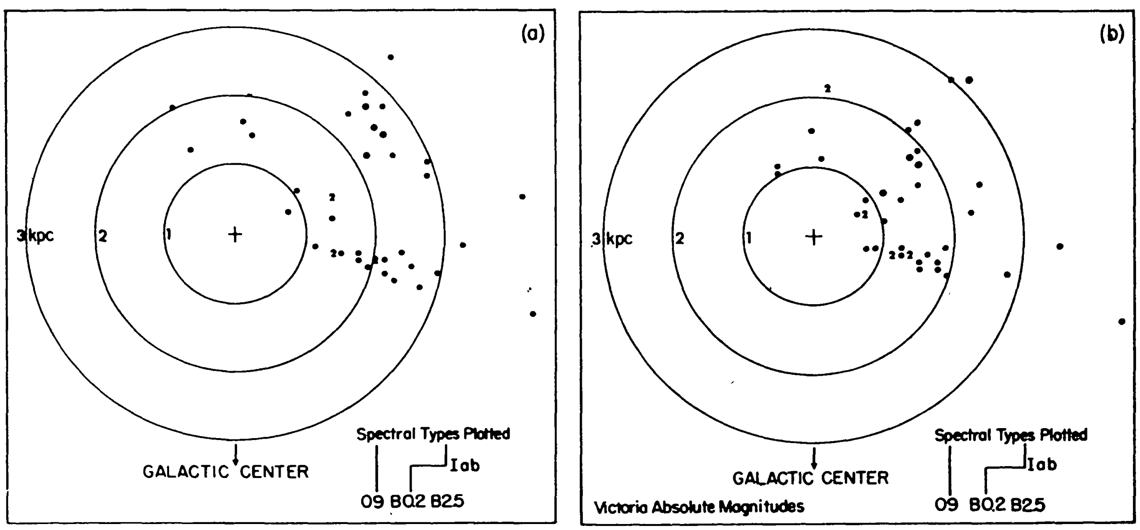

FIG. 4.—Space distribution of normal stars of all luminosity classes from O9 to B0.2, classes Iab and brighter from B0.5 to B2.5. (*a*) Present absolute magnitudes, (*b*) Victoria absolute magnitudes. Four stars studied by Münch (1957) are indicated by larger symbols.

than 0.5 mag on the average. Future observations in clusters should provide a more definite value.

All stars in Table 1 with absolute magnitudes determined in the present work are plotted in Figure 3*a*. The Sun is located at the center of the space-distribution diagrams, and the concentric circles correspond to distances of 1, 2, and 3 kpc. There are essentially no stars corresponding to the Sagittarius Arm, due to the declination limits on the Victoria observations. For comparison, the same stars are plotted in Figure 3*b*, with the revised Victoria absolute magnitudes from Petrie and Lee (1966) substituted for the present ones. Only the absolute magnitudes have been changed; that is, the present spectral types (intrinsic colors) apply to Figure 3*b* also. The effect of the systematic difference between the two luminosity systems is seen in the general contraction of the distance scale in Figure 3*b* with respect to that in Figure 3*a*.

In Figure 4 the following two-dimensional spectroscopic limits have been placed on the stars plotted: all luminosity classes from O9 to B0.2, together with classes Iab and brighter to type B2.5; there are forty-one stars within this boundary. (All peculiar stars and stars for which there was any uncertainty connected with the present luminosity classification have also been eliminated.) In Figure 4*a* the present absolute magnitudes are used; here the Perseus Arm is clearly delineated, well separated from the Local Arm and at an average distance of 2.5 kpc from the Sun in the direction $l^{II} = 130°$. Again for comparison, the Victoria absolute magnitudes are substituted and the same stars replotted in Figure 4*b;* here there is no evidence for the spiral structure, and the uniform distribution found by Petrie and Petrie (1968) has been recovered. In essence, what has happened in Figure 4*b* is that most of the stars in the Perseus Arm have been scattered into the interarm region of Figure 4*a*, less than about 2 kpc from the Sun, due to the systematically fainter Victoria absolute magnitudes.

With respect to this last group of stars, it is of interest to note that some of them (HD 1743, 5551, 10125, 13745) were included in the study of optical interstellar absorption lines by Münch (1957). These stars are indicated by larger symbols in Figure 4. Münch's work provides a striking demonstration that the interstellar gas in the directions of the Perseus Arm ($100° < l^{II} < 165°$) is concentrated into two regions at different distances from the Sun, with a region of much lower density between. He found that the interstellar lines in every star more distant than approximately 2 kpc (in terms of the MK system) in these directions have two strong, well-separated components. Interpreted as due to velocity shifts from galactic rotation, the two components were found to be due to material located on the average at 0.5 and 2.5 kpc from the Sun. For comparison, 21-cm studies in these directions which made use of the Schmidt (1965) model of the Galaxy place the maximum density of the H I features in the Perseus Arm at 3–4 kpc (Lindblad 1967; Kerr 1969); the true distance is probably somewhat smaller, due to the effect of noncircular motions in this region (Rickard 1968). These latter distance determinations are independent of the MK calibration; it seems that the abovementioned stars are unlikely to be less than 2 kpc from the Sun, as is indicated for three of them by the Victoria absolute magnitudes. Therefore, these stars provide independent support for the present luminosity system.

III. THE REFERENCE FRAME

a) General Considerations

A higher dispersion is not in itself a sufficient condition for an improved accuracy of spectral classification. The results are a question to be investigated for the particular region of the H-R diagram under consideration, and a variety of factors must be taken into account: (1) Increased widening of the spectrograms is an important requirement for the early-type stars (Morgan *et al.* 1943). (2) The availability of suitable criteria with a smooth variation in the classification coordinates must be investigated, particularly if blends or diffuse features such as Balmer-line wings were used at lower dispersion (Morgan *et al.* 1943; Morgan, in Abt 1963; Keenan 1963). (3) A higher dispersion will probably entail a loss of accuracy for the broadest-lined stars. In the determination of individual spectroscopic distances it is best to eliminate them in any event. (4) In addition, there is the question whether, at the higher dispersion, random effects due to variables other than temperature and luminosity have become dominant, that is, whether the "cosmic noise" level has exceeded that of the observational noise. If that is the case in general, then nothing will have been gained with respect to a two-dimensional classification for the stars in question.

On the other hand, if these considerations are favorably resolved, then the following results may be expected with the higher dispersion: (1) a systematically increased accuracy in the classification, that is, a smaller range of uncertainty on the average in the classification coordinates; and (2) the possibility of discovering new categories of stars that are peculiar with respect to the classification frame employed.

Some operational concepts and procedures should be mentioned briefly. The evaluation of the two-dimensional system and choice of classification criteria are made by means of an initial study of the MK standards throughout and surrounding the region of interest in the H-R diagram, obtained with the equipment to be used for the investigation. An arbitrary empirical reference frame is defined in this way. It is not necessarily unique; in principle, different primary criteria might be chosen, and possibly systematically or randomly different spectral types result.

In the classification, one applies a given criterion primarily to the coordinate in which it varies most rapidly. Since all criteria actually vary with both coordinates, however, a given one may appear the same at two or more points in the reference frame (for instance, the effect of a slightly lower temperature on a certain line ratio may be compensated by that of a slightly higher luminosity, giving rise to the "diagonal effect"); therefore, the classification is a process of logical elimination by successive consideration of all criteria in the spectrum. If the spectrum cannot be uniquely placed at a defined position of the reference frame, one next tries to do so at a new position slightly displaced in one or both coordinates; if that likewise proves impossible, and a contradiction arises in each attempted classification, then the spectrum must be labeled peculiar. In the case of a higher spectral resolution, it may become convenient to define new intermediate or interpolated spectral types, if a significant number of stars can be described most concisely by that means.

Finally, the calibration of the empirical classification in terms of physical parameters is a separate problem to be undertaken after the reference frame has been defined. In this way effects of errors in the calibration upon the classification are avoided (Morgan 1958; Keenan 1963). Of course, it is the calibration and the subsequent application to problems of galactic astronomy and astrophysics which will eventually determine the degree of validity and usefulness of the empirical reference frame and the classification relative to it.

b) Spectral Classification O9–B2.5

From the preliminary study of the MK standards at 63 Å mm^{-1} it was decided that the primary criteria in the present reference frame should be ratios of the lines of neutral and ionized helium, and of the first three ions of silicon. These elements have the advantage of a considerable number of fairly strong and unblended lines in the classification region λλ3900–4900, and in addition the presence of two or more ionization states from the same element provides a sensitive criterion of spectral type. In the case of helium, this effect is extremely useful from O9 to B0, for Si III and Si IV from B0 to B1.5, and for Si II and Si III at B2 and later. Also a number of silicon/helium line ratios are luminosity-sensitive. The primary classification line ratios are listed in Table 2, together with the coordinate of most rapid variation. The variation of three of the most important is illustrated schematically in Figure 5. It was found that a smooth and consistent two-dimensional variation could be obtained in terms of these criteria; the reference frame so derived will be referred to as the helium-silicon grid.

Most of the remaining lines and blends visible in O and early B spectra at this dispersion, other than the Balmer series of hydrogen, are due to C III, N II, N III, and O II. Most of them show a pronounced positive luminosity effect. Although they display a smooth variation among the majority of spectra, it has been found that in some stars, particularly in the case of nitrogen, their strengths are anomalous with respect to the helium-silicon grid (Walborn 1970). As a result, these lines are used as secondary criteria only.

1971ApJS...23..257W

TABLE 2

PRIMARY CLASSIFICATION CRITERIA

Ratio	Coordinates	Remarks
	A. Spectral Types O9–B1	
He I λ4009/He I λ4026	Spectral type	
Si IV λ4089/Hδ or He I λ4121	Spectral type and luminosity class	
Si IV λ4116/He I λ4121	Spectral type and luminosity class	Generally blended at MK atlas dispersion
He II λ4200/He I λ4144	Spectral type	Defines O9, O9.5
He II λ4541/He I λ4471	Spectral type	Defines O9, O9.5
He II λ4541/He I λ4387	Luminosity class	Important at O8
Si III λ4552/He II λ4541	Spectral type	Defines O9.7
Si III λ4552/He I λ4387	Luminosity class	
Si III λ4552/Si IV λ4089	Spectral type	
He II λ4686/He I λ4713	Luminosity class	Negative luminosity effect
	B. Spectral Types B1.5–B2.5	
He I λ4009 appearance	Luminosity class	Diffuse at class V
He I λ4009/He I λ4026	Spectral type	Maximum at B2
He I λ4121/He I λ4144	Luminosity class	
Si II λ4128–30/He I λ4121	Spectral type	Defines B2.5
Si III λ4552/He I λ4387	Luminosity class	
(C III λλ4647–4650–4651/He I λ4713)	Spectral type	Assists in resolving diagonal ambiguity B1 V–B2 III and in defining B1.5; used with caution

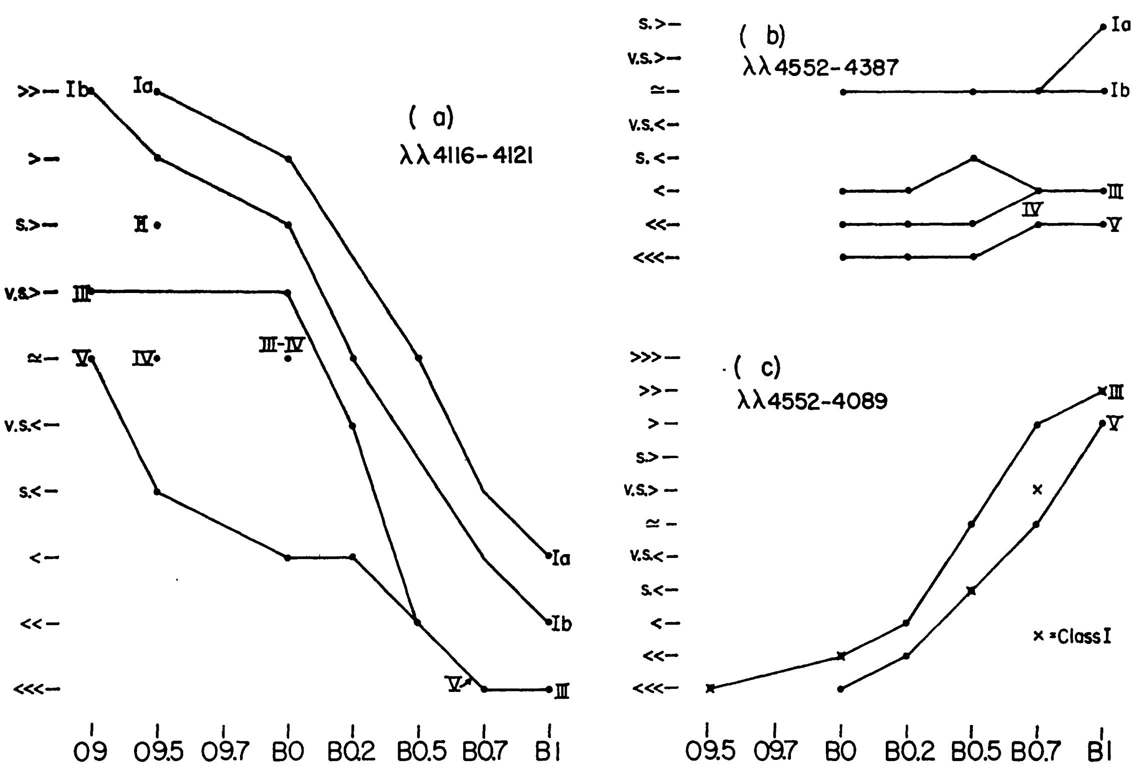

FIG. 5.—Schematic representation of the variation of three important classification criteria. The appearance of the criterion at each point is obtained by replacing the dash between the wavelengths with the corresponding symbol on the ordinate scale; "s." means "slightly," and "v.s." means "very slightly." The values of the line ratios at each point are defined by the spectra of the standard stars.

Helium-weak stars of early type are known (Sharpless 1952; Sargent and Searle 1968), as are B stars with helium enhanced and/or hydrogen deficient (Greenstein and Wallerstein 1958; Morgan and Lodén 1966; Hack 1967). Also, a characteristic of one well-known class of Ap stars is that the lines of Si II are enhanced. In view of these observations, the significance of a two-dimensional classification frame based upon the lines of helium and silicon might be questioned. The basic question, however, is the following: Can a significant number of the members of the spectroscopic group under consideration be smoothly and consistently described in terms of a well-defined two-dimensional reference frame? The affirmative answer in the case of the OB stars at 63 Å mm^{-1} and the helium-silicon grid allows confidence that the assigned classifications are basically determined by two parameters. The assumption that these are the stellar temperatures and luminosities is tested by the calibration and the results of applications to particular problems.

Some new interpolated spectral types have proved useful in the classification. The new types are one between O9.5 and B0 (O9.7) and one on either side of B0.5 (B0.2 and B0.7, respectively). It should be emphasized that despite the notation, the new types do not represent classification to a hundredth of a spectral class, but are essentially interpolations between previous MK positions, for use with the higher dispersion.

The new types are illustrated by a sequence of spectra at luminosity class III (giants) in Figure 6 (Plate 1). The discussion here will also illustrate the classification procedure itself as described earlier in general terms, that is, as a process of logical elimination by contradiction. In the case of HD 6675 (B0.2 III): if the spectrum is B0, then the weaker Si IV lines require a luminosity class below HD 48434 (B0 III); however, the weakness of He II λ4686 and the strength of the Si III and O II lines would indicate one equal to or above the B0 III star, and therefore the spectrum cannot be B0. A slightly later type would satisfy all of these criteria, and the absence of He II λλ4200 and 4541, clearly visible at B0, supports this interpretation. That the star is not as late as B0.5 can be seen by an analogous process relative to 1 Cas (B0.5 III), and so HD 6675 is placed at the interpolated type.

The case of ε Per (B0.7 III) illustrates the usefulness of interpolated types in sharpening the separation of temperature and luminosity variation in the criteria. It was classified B0.5 V in the MK list (Johnson and Morgan 1953); Garrison (1967) used it at B0.5 IV, and Lesh (1968) classified it B0.5 III. It is clear that if the star is of type B0.5, then the weaker Si IV relative to 1 Cas would require a lower luminosity class (Fig. 6); on the other hand, the weakness or absence of He II λ4686 (clearly visible on the 63 Å mm^{-1} plates at B0.5 IV and V), as well as the strength of Si III and O II, implies a giant luminosity class. Such a class between B0.5 and B1 represents all criteria in a satisfactory way.

For the new types among the supergiant spectra, the most important line is Si III λ4552 (actually the strongest line of a triplet whose other members are at λλ4568 and 4575; Struve and Elvey 1930). In the O9.5 Ia standard α Cam, this line is much weaker than He II λ4541, while in ε Ori (B0 Ia) it is much stronger. Since λ4541 is falling off very rapidly with advancing type here while λ4552 is just beginning to strengthen, the ratio of these two lines should provide a sensitive indication of spectral type. Therefore, supergiants in which the ratio Si III λ4552/He II λ4541 has a value close to unity are classified O9.7. It does not appear that this subdivision will be useful at the lower luminosities (class III and below), where the Si III lines, which have a positive luminosity effect, are too weak to serve as a reliable criterion earlier than B0. Some O9.7 spectra are illustrated in Walborn (1971).

In the classification of the supergiants from B0 to B1, the ratio Si III λ4552/Si IV λ4089 is most important. In some standard stars previously classified B1, its value is considerably smaller than in others at the same type (although in no case is it as small as at B0.5). Supergiants in whose spectra this ratio is only slightly greater than unity are here classified B0.7. The bright standard κ Cas (B0.7 Ia) merits special mention

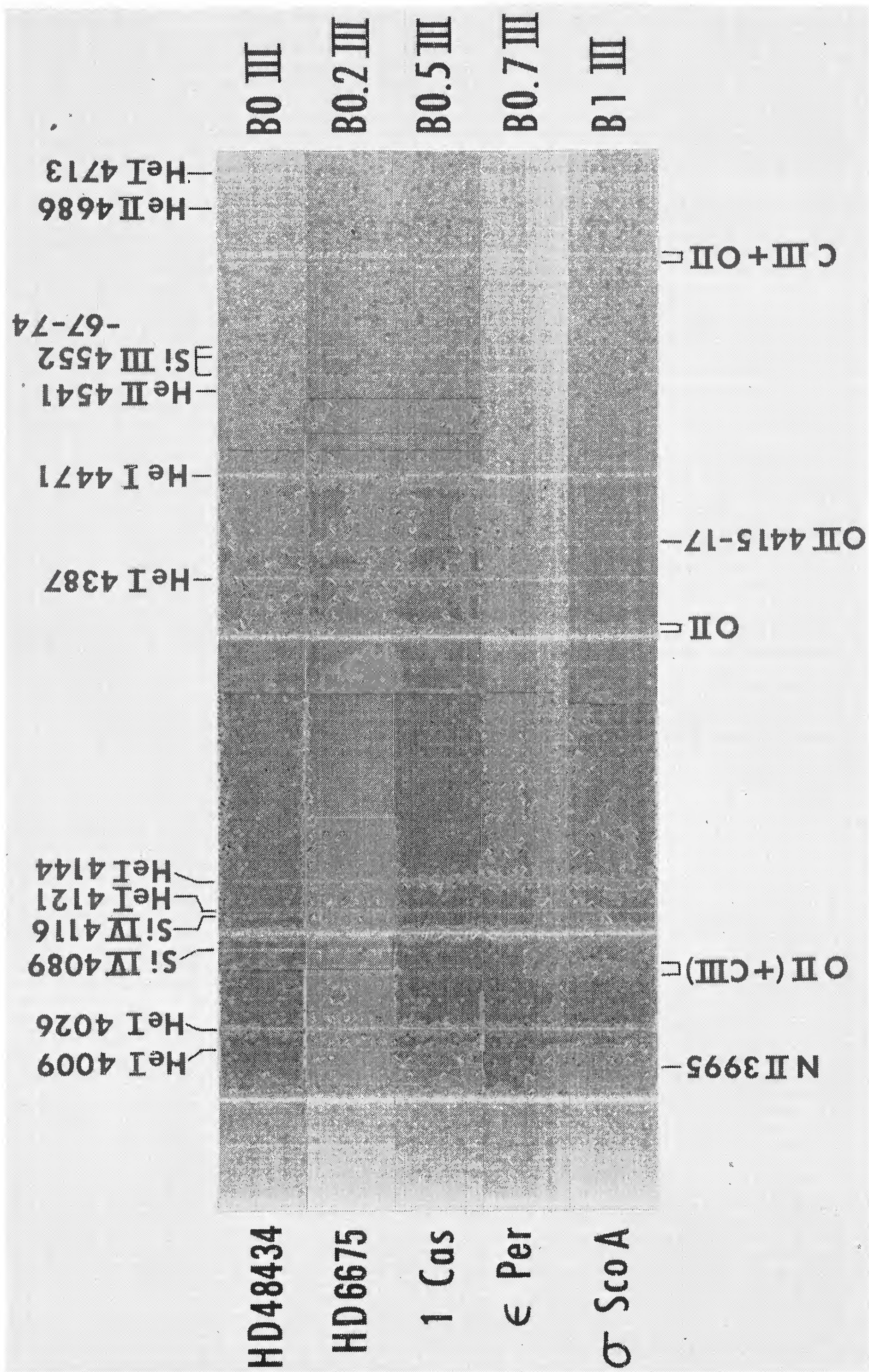

FIG. 6.—A spectral-type sequence from B0 to B1 at luminosity class III

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at this point. In a high-dispersion spectrophotometric study, Wilson (1956) found evidence that κ Cas is of earlier type than other B1 supergiants, but nearer B1 than to B0 (ϵ Ori). Also, κ Cas has been found to be deficient in the strength of the nitrogen lines; nevertheless, it is included as a standard because in terms of the helium and silicon lines, upon which the classification is based, it provides a good definition of its type.

The question whether the type B0.2 will be useful at the higher luminosities (class II and above) must be left to the future, when a more extensive sample in this spectral range at 63 Å mm^{-1} has become available.

To summarize, classifications B0.2 and B0.7 V to III are interpolations which resolve conflicts between criteria involving Si IV on the one hand and Si III, He II λ4686, and O II on the other. Types O9.7 and B0.7 at classes II to I are interpolations based on the line ratios Si III λ4552/He II λ4541 and Si III λ4552/Si IV λ4089, respectively. The dwarf, giant, and supergiant standard stars which fundamentally define the helium-silicon classification grid are given in Table 3.

Two brief additional remarks about the present spectral types are necessary. The first concerns the n-parameter, which describes the degree of broadening of the spectral lines. At 63 Å mm^{-1} the different degrees of broadening are quite noticeable, and they have been denoted in the spectral types. The symbol ((n)) indicates that the lines $\lambda\lambda$4116 and 4121 are just merged, and (n) represents a degree of broadening intermediate between that and the broadening in the n spectra of the 125 Å mm^{-1} types. It is believed that the higher spectral resolution at 63 Å mm^{-1} is maintained for the ((n)) and probably the (n) stars; the accuracy should be comparable to that of the lower dispersion in the case of the n stars, and for the nn or most extreme broad-lined stars it is probably inferior. The only nn star in the present program is ζ Oph. The additional notation [n] is used to denote those stars in which the degree of broadening of the hydrogen lines is much greater than that of the helium lines. These stars often show Balmer cores and the α Cyg (primarily Fe II) lines from a shell, either in absorption or emission, the latter sometimes broadened and sometimes not. For the stars discussed in § II (Table 1), projected rotational velocities have been estimated by Walker and Hodge (1966); the mean values for each n-parameter are as follows: ((n)) 105 km sec^{-1}, (n) 224 km sec^{-1} (HD 18326 and HD 217035 omitted), n 437 km sec^{-1} (HD 208106 omitted), and [n] 199 km sec^{-1}. Determinations by Slettebak (1949, 1956) and Slettebak and Howard (1955) indicate a more compressed scale, with average values near 180, 250, and 320 km sec^{-1} for the ((n)), (n), and n stars, respectively.

TABLE 3

STANDARD STARS O9–B3

Sp	Dwarfs (V)	Giants (III)	Supergiants
O9.	10 Lac	ι Ori	HD 210809 (Ib)
O9.5.	AE Aur	HD 189957	19 Cep (Ib), α Cam (Ia)
O9.7.	. . .	. . .	ζ Ori (Ib), HD 195592 (Ia)
B0.	υ Ori	HD 48434	ϵ Ori (Ia)
B0.2.	τ Sco	HD 6675	. . .
B0.5.	HD 36960	1 Cas	κ Ori (Ia)
B0.7.	HD 201795	ϵ Per	HD 190919 (Ib), κ Cas (Ia)*
B1.	ω^1 Sco	σ Sco	ζ Per (Ib), HD 13854 (Iab)
B1.5.	HD 154445	12 Lac	HD 190603 (Ia$^+$)
B2.	HD 42401	γ Ori	9 Cep (Ib), χ^2 Ori (Ia)
B2.5.	HD 214432	π^2 Cyg	3 Gem (Ib), 55 Cyg (Ia)
B3.	η Aur, η UMa	HD 21483	o^2 CMa (Ia)

* Nitrogen-deficient star.

The second remark concerns the e-parameter, used here as usual to signify Balmer cores in emission. In addition, the forms (e) and e^+ will be used to denote probable emission at Hβ and/or Hα, and α Cyg shell lines in emission, respectively. The more extended form introduced by Lesh (1968) will not be used, because (1) the higher resolution would probably introduce systematic differences and (2) Hβ is not visible on the 63 Å mm^{-1} plates.

c) A Proposed Luminosity Classification Earlier than O9

i) *Historical Background*

From the empirical spectroscopic point of view, a fundamental difficulty in the classification of the O-type stars is the very small number of reasonably strong features in their photographic spectra. No luminosity classification for the stars earlier than O9 was given in the MKK atlas (Morgan *et al.* 1943). On the other hand, evidence of an external nature indicated that the Of stars are more luminous than the absorption O stars (Roman 1951).

There is evidence for a small but observable luminosity effect in the Balmer lines of the O stars. (i) Underhill (1955) found no relation between the absolute magnitude and equivalent widths of ultraviolet hydrogen lines. However, it has been pointed out (Botto and Hack 1962) that no account was taken by Underhill of the temperature effect in the Balmer lines; at the higher gravities it is expected to be of the same order as the luminosity effect for the O stars (Mihalas 1970). (This point also provides a possible explanation for the results of Walker and Hodge 1968, who found no correlation between V_0 and the equivalent width of Hγ for O stars in associations, except in NGC 2244; both IC 1805 and Cyg OB2 may contain evolved stars, whereas in NGC 2244 all of the O stars are probably on the main sequence.) Also, the absolute magnitudes obtained for the O stars by Underhill are based on cluster moduli which are systematically smaller than those given by other authors (Hoag and Applequist 1965; Becker 1963; Morgan *et al.* 1953); several moduli are from the earlier Hγ–absolute-magnitude calibration for B stars (Petrie 1952). This circumstance results in rather faint absolute magnitudes for two members of NGC 6871 classified Of in her study (HD 190429N and HD 190864; the latter spectrum is "intermediate"—see below). A third star classified Of, HD 46223, is a main-sequence star, according to the definitions introduced below. (ii) In other investigations based upon the Balmer lines, a detectable luminosity effect among the early O stars was found (Wilson 1957; Kopylov 1958; Botto and Hack 1962; Hack 1963). (iii) On the theoretical side, a considerable gravity effect in the hydrogen lines had been predicted on the basis of LTE calculations (Kopylov 1956; Mihalas 1965); recent non-LTE models predict a smaller, but still measurable, effect (Mihalas 1970).

An important and related problem is whether the Of phenomenon is due to a shell mechanism, or whether it arises from an atmospheric process which may be related to the luminosities of the stars. Underhill (1960, 1966, 1970) has proposed that the shell interpretation is to be preferred, a possibility which it is well to consider, since shells are known to simulate luminosity effects in various regions of the H-R diagram. On the other hand, considerable observational evidence supports the interpretation that the Of emission lines actually are formed in the upper atmospheres of the stars, and that they are related to the stellar luminosities (Oke 1954; Wilson 1955, 1957; Slettebak 1956). Recently, additional external evidence has become available which suggests that the Of stars are evolved objects more luminous than the main sequence at their spectral types (van den Bergh 1968; Schild, Hiltner, and Sanduleak 1969; Hiltner and Morgan 1969; Morgan 1970).

It appears that one factor which may have hindered the detection of luminosity differences between the Of and absorption-line O stars is a problem of definition of the former. As introduced by Plaskett and Pearce (1931), this was that N III λλ4634–4640–4642 and He II λ4686 appeared in emission. As is well known, with sufficient spectral resolution, some emission in the N III lines is visible in the spectra of nearly all early O stars, and in those of highest luminosity at least as late as O9.5. Some authors have labeled such stars "f" also; in some lists more than 50 percent of the O stars appear as "Of," while Underhill (1966) found that in the lists of Morgan *et al.* (1955) and Hiltner (1956) the number is 13 percent. In this work the following extension of the f-parameter will be used to describe the different emission-absorption combinations shown in the classification region by the O stars earlier than O9 (examples are given in parentheses): ((f)), weak N III λλ4634–4640–4642 emission and strong He II λ4686 absorption (HD 46223, λ Ori); (f), N III emission, with λ4686 absorption weak or "neutralized" (HD 15558, 225160); f, N III and λ4686 emission (λ Cep); and f^+, Si IV λλ4089 and 4116 as well as N III and λ4686 emission (HD 15570).

ii) *Transferred Luminosity Criteria*

One of the well-established luminosity indicators in the spectral type range O9–B0.5 is the remarkable negative luminosity effect in the line He II λ4686. Strongest in the dwarfs, it declines with increasing luminosity and becomes extremely weak or absent in the Ia supergiants, while the other He II lines in the classification region remain strong. On the 63 Å mm^{-1} plates, it can be seen that the N III lines λλ4634–4640–4642 behave in a similar way, relative to the other N III lines; while the latter show a marked positive luminosity effect, λλ4634–4640–4642 are usually stronger in the dwarfs. This effect has been pointed out and illustrated in connection with the nitrogen-anomalous spectrum of HD 201345 (Walborn 1970). Since the *Ngf*-value for λ4640 is twice that of λ4097 (Oke 1954), it appears that some process is inhibiting the former absorption in high-luminosity stars.

It seems possible that there is a connection between the selective weakening of these absorption lines with increasing luminosity and the selective appearance of the same lines in emission in Of stars. Therefore, the hypothesis is proposed that their negative luminosity effect at types O9–B0.5 is due to the same emission process as the Of phenomenon, and that the latter may therefore also be interpreted as a luminosity effect. (I am indebted to Dr. L. Auer for an early suggestion that the λ4686 absorption negative luminosity effect might be due to emission filling in the line.)

Wilson (1955, 1957) demonstrated an absorption-to-emission behavior with luminosity at spectral types O9–B0 in the line C III λ5696, which is also selectively in emission in Of stars. He also suggested an interpretation of his data on the early O stars similar to that which is proposed here.

In order to provide support for the present interpretation of the Of stars, one may seek luminosity criteria in the late O stars, where luminosity effects are well established, that are capable of being transferred to spectra of slightly earlier type. It appears that such a transferrable luminosity criterion is provided by the line ratio He II λ4541/He I λ4387. Referring to the illustration in Walborn (1970), one can see an increase in its value between the O9 V star 10 Lac and the O9 Ib supergiant HD 210809. In the O9 III star ι Ori its value is intermediate between these two, being very close to unity. Therefore, it is considered to be defined as a luminosity criterion by the O9 stars.

The spectral types within class O are determined from the line ratios He II λ4541/He I λ4471, and He II λ4200/He I + II λ4026. The zero-point and scale-defining values are that the first ratio has a value of unity at type O7, and that He I is almost invisible on the present plates at O4 (so that a temperature spread may exist at this last type). For a given value of the two spectral-type ratios, the additional ratio He II λ4541/He I λ4387 can have different values, which are interpreted as being determined by the luminosity of the star. There are in fact theoretical reasons for expecting such a behavior of this ratio, for a given value of λ4541/λ4471 (Mihalas 1970).

Considering now the sequence of O8 and O8.5 spectra illustrated in Figure 7 (Plate 2),

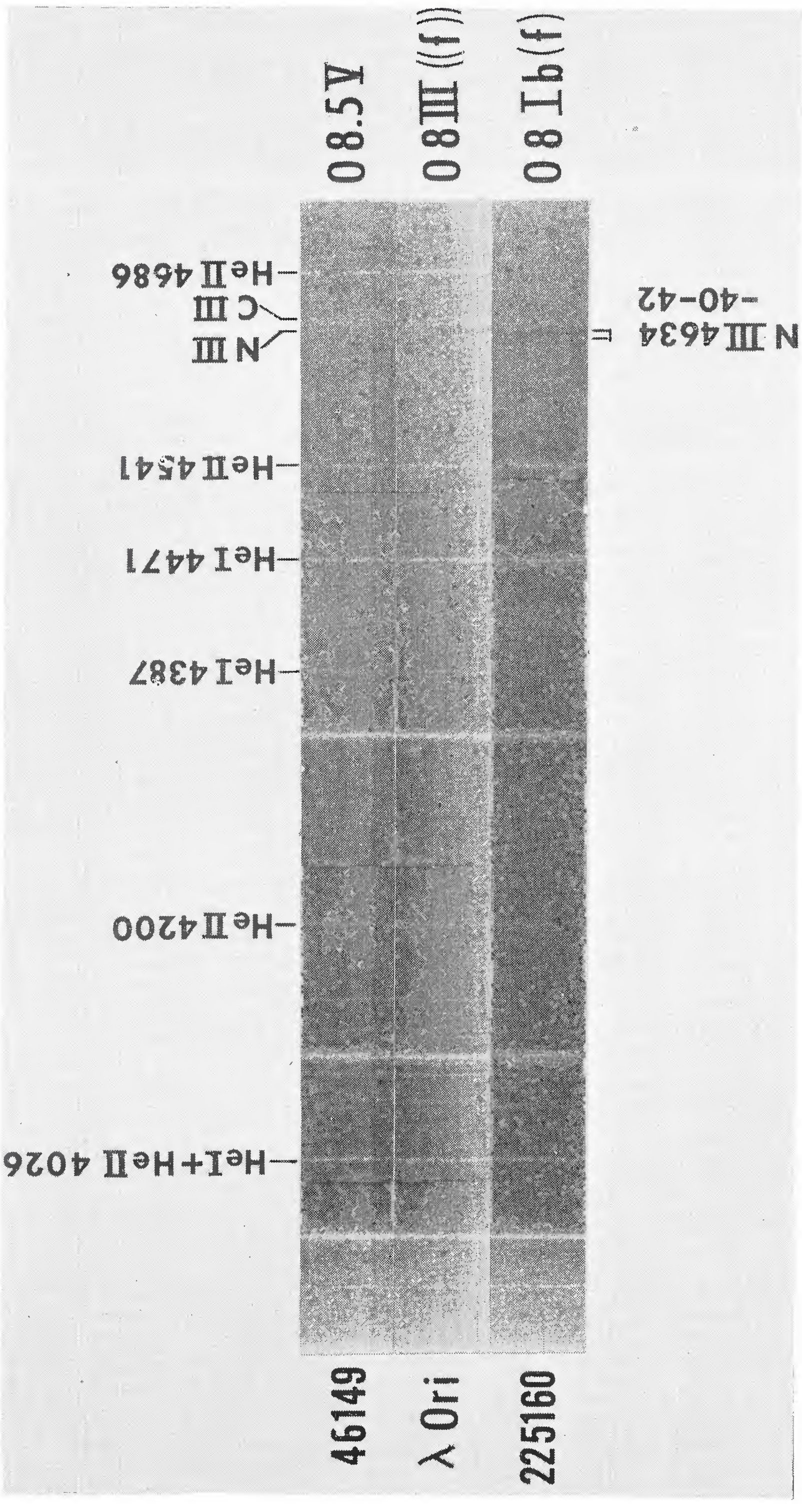

FIG. 7.—Luminosity effects at spectral types O8–O8.5

WALBORN (*see* page 274)

none of which is here classified as Of, one can see an increase in the value of the ratio λ4541/λ4387 in passing from HD 46149 through λ Ori to HD 225160. Of course, λ4541/λ4387 will also increase as one goes to earlier spectral types at a given luminosity; in the present case, however, the greater strength of the Si IV lines λλ4089 and 4116 in λ Ori shows that it is not an earlier-type dwarf than HD 46149, since both stars are on the high-temperature side of the Si IV maximum; in fact, they provide additional evidence that λ Ori is of higher luminosity than HD 46149. In the O8f stars HD 151804 and 152408, λ4387 is nearly invisible and the Si IV lines are extremely strong. (I am indebted to Dr. W. A. Hiltner for permission to refer to spectrograms of these two stars; see Hiltner, Garrison, and Schild 1969.) From these considerations, the sequence of O8 and O8.5 spectra in Figure 7 is interpreted as a luminosity sequence.

On further considering Figure 7, one finds that in the direction of increasing λ4541/λ4387, the emission strength at λλ4634–4640–4642 also increases, and the λ4686 absorption declines (i.e., the λ4686 emission also increases, on the above interpretation). These latter effects are then taken to be defined as further luminosity criteria by the O8 and O8.5 stars, and they are in turn transferred to the earlier O stars to provide a luminosity classification there. In this way the proposed luminosity classification for the O-type stars is derived, from purely empirical spectroscopic considerations. It is given in terms of the classification criteria in Table 4, together with some standard stars which define it.

The behavior of N III λλ4634–4640–4642 and He II λ4686 in O-type spectra may now be described in the following way: (i) Along the main sequence (luminosity class V), one finds strong λ4686 absorption, with N III emission becoming faintly visible at about O7 V and increasing gradually in strength toward earlier types. (ii) In addition, the N III emission strength increases with luminosity, becoming visible as late as types O8, O9, and O9.5 at classes III, Ib, and Ia, respectively, that is, along a diagonal line in the two-dimensional grid. (iii) The λ4686 absorption weakens with increasing luminosity, becoming essentially absent at O9–O9.5 Ia. (iv) The most luminous stars at O8 and earlier have both λ4686 and the N III lines in emission (Of). The λ4686 emission may increase in strength as one proceeds from O8 to earlier types among the most luminous stars, if certain peculiar Of stars showing strong P Cygni features in the classification region are excluded (Brucato 1970). (v) Intermediate between the Of and class V stars, one finds spectra with weakened or "neutralized" λ4686 absorption and intermediate N III emission strength.

TABLE 4

PROPOSED LUMINOSITY CLASSIFICATION FOR THE EARLY O STARS

Luminosity Class	Criteria	Standard Stars
	A. Spectral Types O7–O8.5	
Ia	Of; Si IV very strong	HD 151804, O8 Iaf
Ib	He II λ4686 absorption weak or neutralized; Si IV strong	HD 225160, O8 Ib(f)
III	He II λ4686 absorption and Si IV strong; He II λ4541/He I λ4387 greater than at V	ξ Per, O7.5 III((f)) λ Ori, O8 III((f))
V	He II λ4686 absorption very strong	15 S Mon, O7 V((f)) HD 46149, O8.5 V
	B. Spectral Types O4–O6	
I	Of and f^+	HD 15570, O4 If^+
III	He II λ4686 absorption weak or neutralized	HD 15558, O5 III(f)
V	He II λ4686 absorption strong	HD 46223, O4 V((f)) HD 46150, O5 V((f))

It will be noted that the f-parameter becomes essentially redundant, since the spectral type + luminosity class determine it. However, it is felt worthwhile to retain it in the classification for the present, for two reasons: (i) the correspondence of the f-parameter and luminosity class is not one-to-one, constant values of the former occuring along diagonal lines in the two-dimensional reference frame at certain places; and (ii) the f-parameter is purely descriptive of the appearance of the spectrum, while the proposed luminosity classification remains subject to confirmation by further calibration work and by theoretical computations in progress (Mihalas 1970). The actual range in absolute visual magnitude among the early O stars is not expected to exceed about 2.0 mag. Also, it will be of importance to acquire further information about the time scale and magnitude of variations in the emission lines of these stars (Brucato 1970).

iii) *Discussion*

The relationship between the spectroscopic phenomena at type O and that among the early B stars is an important question. It has been suggested (Underhill 1960, 1966, 1970) that the Of stars are the higher-temperature analogue of the Be stars. The evidence discussed here, however, suggests definite differences between these two kinds of objects. (i) The Be stars are well differentiated spectroscopically from the nonemission B stars, and it seems reasonably well established that the underlying cause is the rapid rotation of the former. The Of-type emission phenomenon, on the other hand, is an essentially continuous one, affecting nearly all early O stars in (smoothly) varying degrees. (ii) From the empirical spectroscopic point of view, in terms of the behavior of the lines N III $\lambda\lambda$4634–4640–4642, He II λ4686, and C III λ5696 (Wilson 1955), which are selectively in emission in Of stars and have a selective negative luminosity effect at late O and early B types, the Of stars seem to be related to the absorption-line O stars as the O9–B0.5 supergiants are to their lower-luminosity counterparts. (iii) Spectroscopic analogues of the Be stars at spectral type O do in fact exist; examples are HD 39680, O6.5 V[n]e, and HD 46485, O7 V:n(e).

Botto and Hack (1962) and Hack (1963) have given equivalent widths of Hγ for many O and early B stars, which they have used as part of a luminosity classification for the O stars. In addition to the Merate observations, some of the measures are from Williams (1936), Underhill (1955), and Slettebak (1956); no corrections for any systematic or random differences among the sources have been made. A diagram of these Hγ measures versus present spectral types and luminosity classes is given in Figure 8. A clean separation among the class V and IV, III, II, and I regions is possible throughout the entire diagram. (The lines drawn on the diagram are intended only to emphasize this fact and have no physical significance.) All stars of types O4–B0.2 classified to date on the present system for which Hγ measures are listed by Hack have been plotted, with two exceptions (which if plotted would transgress a dividing line between regions): HD 46056 (O8 Vn, $W(\mathrm{H}\gamma) = 2.08$), which is probably a spectroscopic binary showing double lines on the present plate (I am indebted to Dr. P. J. Treanor for pointing this out), with variable velocity reported by Petrie and Pearce (1962) and possible emission at Hβ by Merrill and Burwell (1949); and 68 Cyg (O7.5 V:n, $W(\mathrm{H}\gamma) = 2.02$), for which there is some uncertainty in the present luminosity classification due to the broad lines. Altogether, thirty-eight stars are plotted; they will be listed together with classifications for additional O stars in a later paper. If one assumes the validity of the luminosity classification proposed here, Figure 8 provides strong evidence for a luminosity effect in the Balmer lines as early as type O4, as was reported by Hack.

d) Comparison with Previous MK Classifications

The present spectral types are compared with previous MK classifications in Figure 9. There are 118 stars, all of which are from Table 1. The previous MK types have been

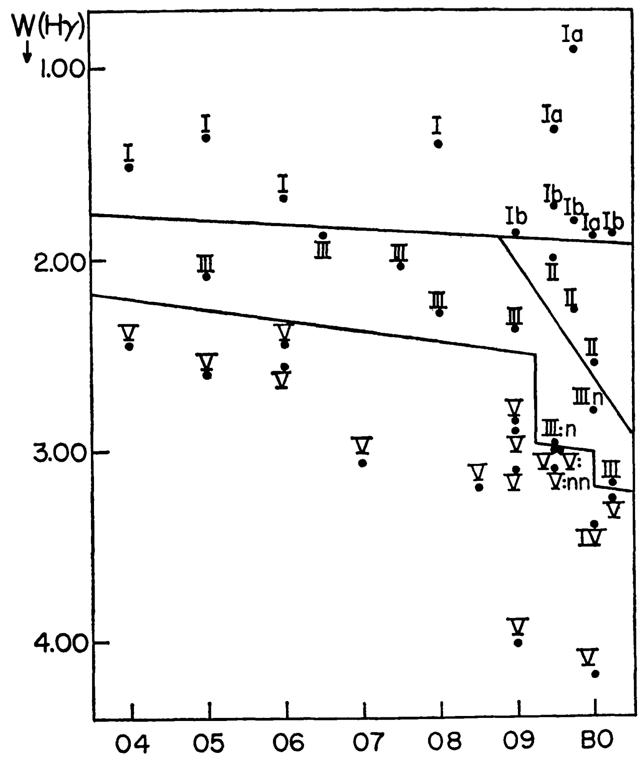

FIG. 8.—The luminosity effect in Hγ for the O stars

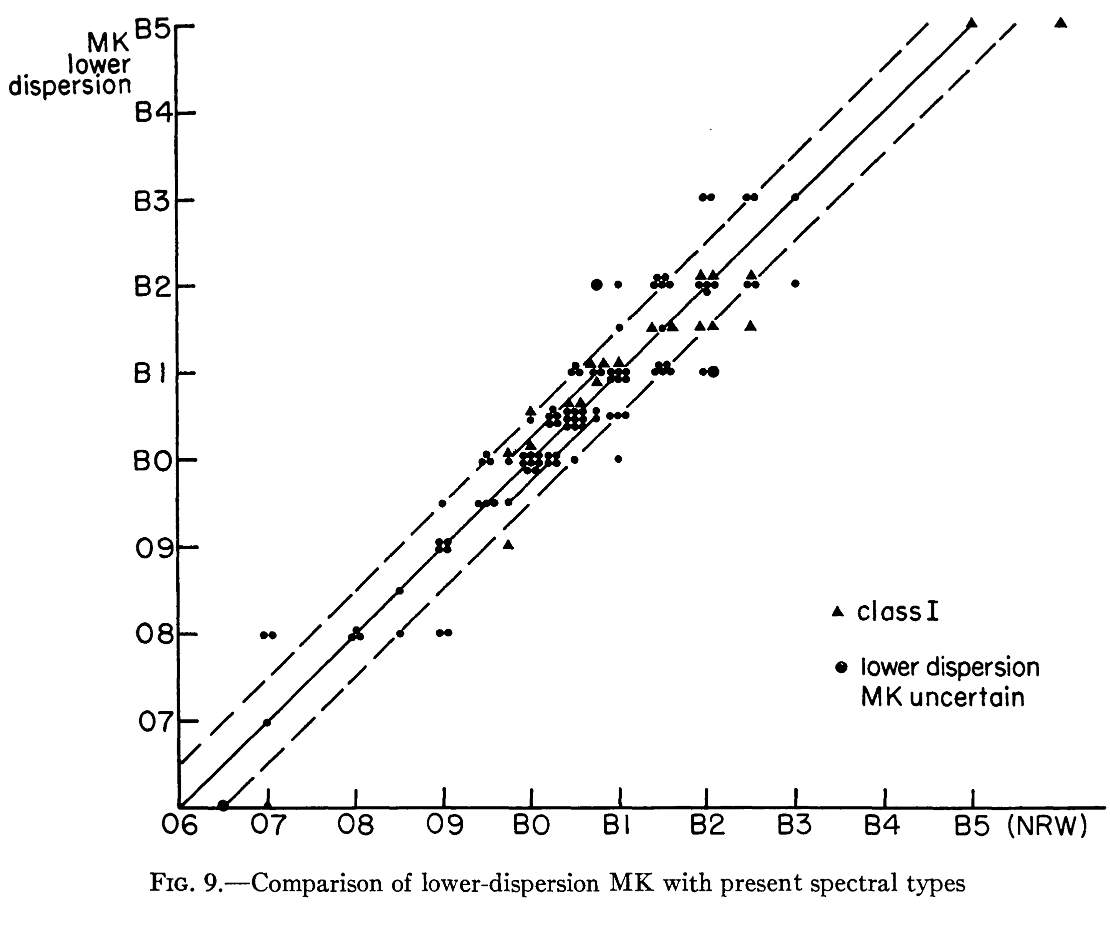

FIG. 9.—Comparison of lower-dispersion MK with present spectral types

taken only from Morgan *et al.* (1955), Hiltner (1956), and (for a few brighter stars) Lesh (1968). In this case the dashed lines represent a difference of one-twentieth of a spectral class. There is no systematic effect; there are only seventeen stars for which the disagreement is larger than one-twentieth, and only two for which it is larger than a tenth, of a spectral class. Of the latter two stars, one is the peculiar nitrogen-enhanced star HD 193516 (Walborn 1970), which will be discussed in detail at a later time; the other is discussed in the next paragraph. The 63 Å mm^{-1} work confirms the lower-dispersion classifications, as well as an earlier result that MK spectral types of normal early B stars can be reproduced to a twentieth of a class on the average (Garrison 1967).

The only case of serious disagreement between a present spectral type and a previous MK classification is HD 12882, here classified B2.5 III: [n]e+, but B6 Ia by Morgan *et al.* (1955). An identification problem is of course possible; on the other hand, in view of the present classification, there is the interesting possibility that the disagreement is due to a drastic change in the shell spectrum. In that case, the star might be similar to Pleione (28 Tau = HD 23862, B8pe), which in 1943–1947 displayed a shell simulating the appearance of an A5 supergiant spectrum. It is of interest to note the radial-velocity measurements listed by Petrie and Pearce (1962) for HD 12882; while they note "central emission at Hβ; absorption lines diffuse and velocity unreliable," the individual measurements seem consistent with the possibility of a declining expanding shell.

The comparison of present with previous luminosity classes is made in Figure 10, for

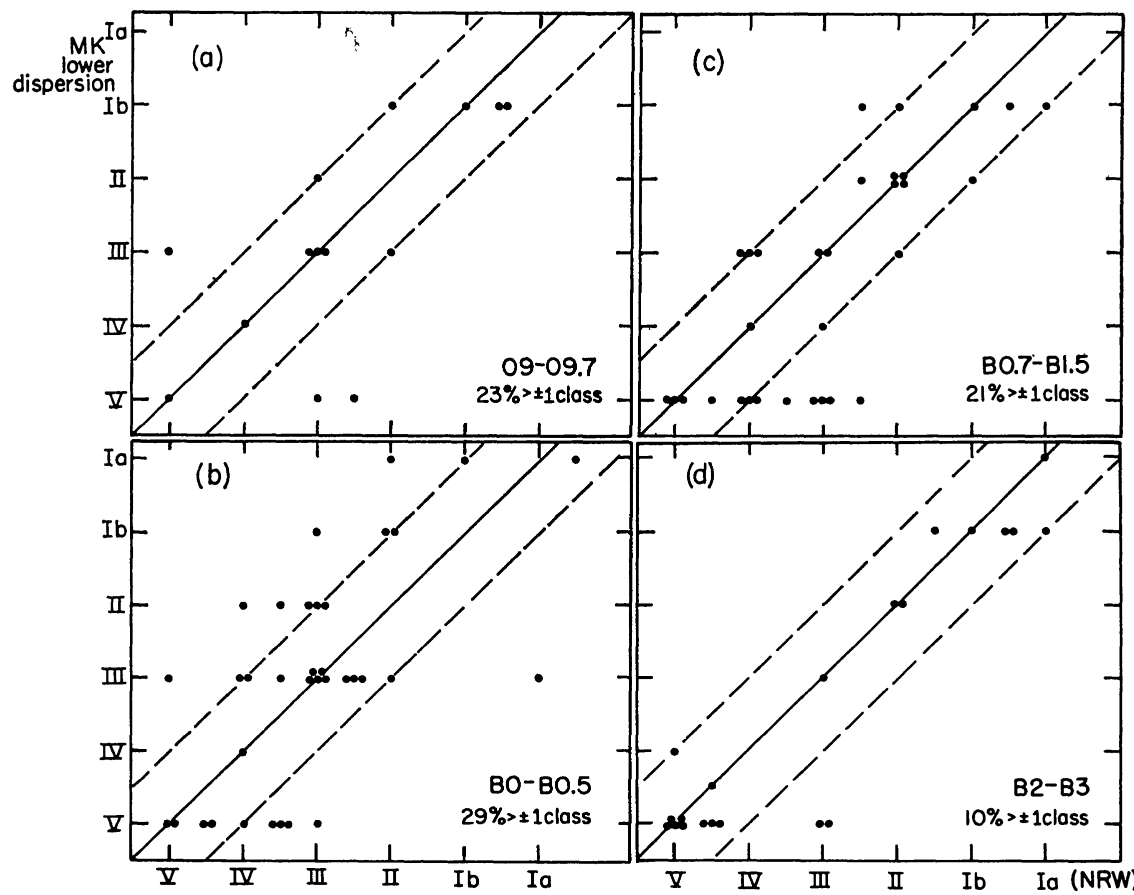

FIG. 10.—Comparison of lower-dispersion MK with present luminosity classes, for several ranges of spectral type: (*a*) O9–O9.7, (*b*) B0–B0.5, (*c*) B0.7–B1.5, (*d*) B2–B3.

four ranges of (present) spectral type. Here the agreement is fairly good. There seems to be a tendency of the present classification to assign brighter classes for some stars in the range V to III. A tendency for previous classes to be brighter than the present ones for the B0–B0.5 II–Ia stars may also be real. The majority of the classifications agree to within one luminosity class, but there are a number of stars for which the differences are larger. The percentage of such cases remains nearly constant with an average value of 24 percent from type O9 to B1; the decline to 10 percent in Figure 9*d* is probably real despite the small number of stars, and due to the strong maximum of the He I spectrum (which shows marked luminosity effects) at type B2 (there are very few B3 stars in the present sample). In many cases the disagreement in classification is actually along a (short) diagonal of positive slope in the two-dimensional frame, so that the difference in the deduced absolute magnitudes is smaller than might be inferred from the difference in luminosity class alone. On the other hand, a difference of one luminosity class is quite significant in some parts of the calibration. It is hoped that the present luminosity classification might be of somewhat higher accuracy systematically than that in the lower-dispersion work; that expectation was one of the primary motivations for this investigation.

As a final comparison with previous two-dimensional MK spectral types, reference is made to recent classifications in h and χ Persei by Slettebak (1968), based upon spectrograms with dispersion 40 Å mm^{-1}. The comparison is made in Table 5, where types from the lower-dispersion MK sources cited above are also given. In every case in which displacements from the latter have been made in both higher-dispersion studies, they are in the same sense in one or both coordinates.

IV. CONCLUSIONS

The primary conclusions of this investigation may be summarized as follows:

1. It has been found that a dispersion of 63 Å mm^{-1}, with attention to the other relevant factors discussed in § III, is well suited for spectroscopic study and MK spectral classification of the OB group. It is believed that the following results are due to the improved spectral resolution obtained: (*a*) an increased classification resolution in the range O9–B1, (*b*) an empirical luminosity classification for the early O stars, and (*c*) the detection of an anomalous behavior of the nitrogen and carbon absorption spectra in some OB stars (Walborn 1970, 1971).

2. With respect to the present classification system (and its calibration), both the spectral types and the absolute magnitudes of the Victoria system show systematic and accidental differences. The revised Victoria absolute magnitudes are consistently fainter by a significant amount than those obtained in this work. On the other hand, the present spectral types agree with those of the lower-dispersion MK classification to within one-twentieth of a spectral class.

TABLE 5

SPECTRAL TYPES IN h AND χ PERSEI

Star (HD)	Walborn	Slettebak 1968	MK Lower-Dispersion
13621.....	B0.5 III–IV((n))	B0 V	B1 V
13745.....	O9.7 II((n))	B0 II–III	B0 III
13866.....	B2 Ib–II:p	B2 II	B2 Ib
14443.....	B2 Ibp	B2 Ibp	B2 Ib
15642.....	O9.5 III:n	O9.5 III	B0 III:
15690.....	B2 Iab	B2 Ib	B1.5 Ib

3. The present MK classification seems sufficiently reliable to allow study of space distributions from spectroscopic distances for individual OB stars, in terms of a comparison with the results for stellar aggregates. Evidence is found that the OB field stars (and association members treated as individuals) in the northern hemisphere in the apparent magnitude range 7–9 can be shown to follow the space distribution of the H II regions, OB associations, and youngest clusters in the solar neighborhood, that is, a concentration into the Local and Perseus Arms. The different results obtained by Petrie and Petrie (1968) are understood as due primarily to the systematically fainter Victoria absolute magnitudes.

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