

STUDIES OF JUPITER'S EQUATORIAL THERMAL LIMB DARKENING DURING THE 1965 APPARITION

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ABSTRACT

Refinements in the statistical analysis underlying a determination of the average observational limb darkening, and in the quasi-static model-atmospheric determination of theoretical limb darkening, are presented for Jupiter. Photometric scans totaling forty-one are used. An account of the effects of ammonia on temperature structure (back warming), as well as the integrated equation of transfer determining specific intensity (blocking), is incorporated into the theory. The theoretical monochromatic brightness is weighted by the 8-14- μ spectral response of the photometer (which affords observational comparison) and by atmospheric transmission. The theoretical limb darkening has then been convolved with smear functions due to the astronomical seeing and finite instrumental aperture. The resulting effective temperature is between 135° and 140° K, values which agree satisfactorily with the direct bolometric measurement and which confirm the existence of an internal Jovian heat source larger than the absorbed solar radiation. The helium-to-hydrogen ratio is not a strong influence on the comparison.

I. INTRODUCTION

The temperature in Jupiter's stratosphere lies below the freezing point of NH_3 so that NH_3 vapor is likely to be in equilibrium with the solid ammonia. In this circumstance, the partial pressure of NH_3 , and consequently the NH_3 absorption, varies exponentially with the local temperature. Since the stratospheric temperature is proportional to the planet's effective temperature, Jupiter's limb darkening in the spectral region of the 10- μ band of NH_3 provides a sensitive observational indication of the planet's effective temperature (Trafton 1967). Further support for this simple view of the ammonia concentration in the Jovian upper atmosphere arises from the fact that thermal mapping of Jupiter in the 10- μ band of NH_3 on many separate occasions reveals picture contrasts highly subdued, sometimes undetectable beyond simple limb darkening, compared with the picture presented in reflected sunlight (Willey 1968). Profound anomalies appear to be infrequent. The year-to-year variation in 10- μ brightness temperature as measured during the last decade has been less than 2° K.

A preliminary derivation of Jupiter's equatorial effective temperature for the 1965 opposition was carried out by Trafton and Willey (1969, 1970) by comparing the observed limb darkening in the 8-14- μ region with theoretical limb-darkening curves. We have since improved the models by taking into better account the effect of the NH_3 rotational band on the Jovian temperature structure. In this paper we present the results of this analysis.

II. THE THEORETICAL LIMB-DARKENING CURVES

The model atmospheres employed are modifications of nongray models in which hydrostatic and local thermodynamic equilibrium prevail, together with radiative equilibrium above the ammonia-lifting condensation level and convective equilibrium below (Trafton 1967). The models originally take into account the influence of the continuous opacity of He- H_2 mixture on temperature structure. In the present study the rotational NH_3 band has been added to the equation imposing flux constancy. A scheme for incorporating the 10- and 16- μ vibration-rotation bands of NH_3 into the equation of radiative transfer has also been introduced. We assume that NH_3 is saturated down to a level

where its vapor pressure equals 6.7×10^{-4} times the total pressure and that the NH_3 mixing ratio is constant below this level. The exception is the strong $10\text{-}\mu$ band of NH_3 , which we treat, with little error, as being saturated at all levels. We also assume that no particle scattering contributes significantly to the extinction either in the $8\text{--}14\text{-}\mu$ spectral region or in other parts of the thermal spectrum. Furthermore, we neglect the absorption in the tail of the $7.5\text{-}\mu$ CH_4 band in the observed bandpass.

The continuous thermal opacity of the hydrogen-helium mixture is due to transitions in the field interactions of the molecules, both in translation relative to mutual potential wells and in changes of rotational orientation of pressure-induced dipoles with respect to the average intermolecular potential. These are pure absorption processes compared with which Rayleigh scattering and Raman scattering are negligible at thermal wavelengths. This opacity is sufficient to cause the emergent infrared thermal radiation to originate in a depth range all of which is some tens of kilometers above the visible cloud structure, as would be demanded by the much lower brightness temperatures at thermal wavelengths compared with spectroscopically derived temperatures characteristic of the cloud species absorbing sunlight (Trafton 1967; Wildey 1968). The rationale of employing constant-flux atmospheric models rests on the reasonability of the assumption that sunlight is negligibly absorbed until it is in the vicinity of the clouds. Although solar ultraviolet is scattered at higher levels, there is not yet convincing evidence that it is absorbed. So high above the clouds the possibility seems relatively remote that a Mie-scattering coefficient may be significant at thermal wavelengths compared with absorption. However, studies of multicolor imagery and polarization reveal the presence of at least a mild haze in this general region. If ammonia vapor is in equilibrium with the solid, one might expect such a haze on theoretical grounds alone. Whether such a haze significantly affects Jupiter's thermal limb darkening is not presently known, and this must be regarded as a fundamental limitation of the present study.

We employ Gille and Lee's (1969) random exponential band model for NH_3 to describe the rotational NH_3 band absorption in the Jovian model atmospheres. The effective mean rotational NH_3 absorption coefficient in the limit of small line-width to line-spacing ratio α/δ is (Trafton 1970)

$$K'(j) = \frac{2\alpha}{N_j\delta} \left\{ \sum_i^{N_j} [\langle k_\nu(i) \rangle]^{1/2} \right\}^2 ,$$

where $k_\nu(i)$ is the monochromatic absorption coefficient for the i th line. This mean is a local average over a frequency interval just small enough so that B_ν does not vary significantly over this interval but large enough to smooth the rapid frequency variation of the NH_3 band. It varies slowly with frequency interval j . Its use greatly reduces the calculations required in constructing a nongray model and is justified by the *a posteriori* observation that $dB_\nu/d\tau_s$ is linear in this absorption coefficient over the rotational band, where τ_s is the standard optical depth.

To test the effect of the rotational NH_3 band on the limb darkening, we computed profiles from model atmospheres alternately containing and not containing this band. Figure 1 illustrates the comparison for the $T_e = 140^\circ\text{K}$ and $\text{He}/\text{H}_2 = 0$ model. The difference is even less for the $\text{He}/\text{H}_2 = 1$ models because the stronger translational absorption in this case more effectively masks the rotational NH_3 band. These comparisons indicate that the approximations in our treatment of the rotational NH_3 band absorption do not significantly affect the results.

The effect of the 10- and $16\text{-}\mu$ vibration-rotation bands on the $8\text{--}14\text{-}\mu$ emergent specific intensity was incorporated into the theory as follows.

The high-resolution transmission data from Garing, Nielson, and Rao (1959) and Mould, Price, and Wilkinson (1959) have been used to construct, line by line (where possible), a model of the 10- and $16\text{-}\mu$ bands of NH_3 . A microscope-projector was used to

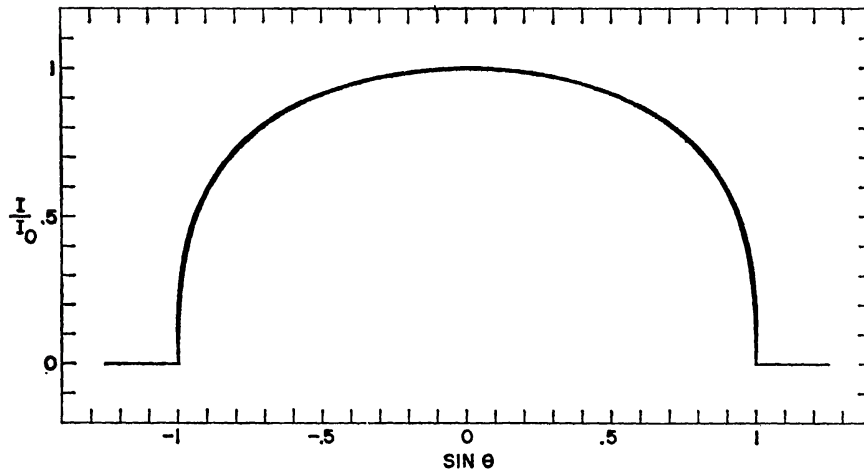


FIG. 1.—The influence of NH_3 on $10\text{-}\mu$ limb darkening with respect only to its modification of the temperature structure upon being incorporated into the equations of radiative equilibrium. Given the temperature structure in each case, the NH_3 monochromatic absorption is included in both integrations of the equation of transfer producing the two limb-darkening curves. Model parameters are $T_e = 140^\circ \text{K}$ and $\text{He}/\text{H}_2 = 0$. Resolution is infinite.

enlarge the published spectra and measure the equivalent width and wavenumber of each line. Line symmetry was used to decide how to partition the equivalent widths in blends. Next we derived the line strengths for each line assuming Lorentz profiles and the Ladenburg-Reiche (1913) theory of the curve of growth (see also the review by Goody 1964). The two sets of data helped to resolve any ambiguities as to whether a line was saturated or on the linear part of the curve of growth. Most of the data on the $10\text{-}\mu$ band were saturated.

Realizing that most of the radiation would pass through the far wings of the line and not near the line centers which are fairly Lorentzian, we adopted a line whose core was Lorentzian but whose wings varied as an arbitrary power of the frequency displacement from the line center.

A band model for room temperature was constructed for more than 505 lines. The equivalent width of this band between 660 and 1300 cm^{-1} for a number of pressures, path lengths, and line-shape parameters was computed. We assumed that all the lines had the same half-width. This is not true, but the value of the half-widths does not sensitively affect the computed equivalent widths. Spectral resolution is insufficient to permit direct determination of the half-widths, and therefore some value must be assumed. The value is related to the normalization of the line strengths since they are derived simultaneously from the laboratory data.

Next, we fitted these band models to the low-resolution data of France and Williams (1966), who made extensive measurements of the equivalent width of the band from 660 to 1300 cm^{-1} over a very large range of both pressure and path length. Unfortunately, the measurements were all at room temperature and not at Jovian temperature. It was found impossible to match the observed and calculated bands when Lorentz profiles were assumed. On the other hand, a very good match was obtained over the entire domain of pressure and path length when the wings of the line profiles varied according to a different exponent (1.78). This line shape for the band model was adopted.

In the Jovian atmosphere, NH_3 self-broadening is negligible. The pressure-broadening coefficients for NH_3 in a mixture of H_2 and He are unknown for the $10\text{-}\mu$ band of NH_3 , so we assume that they are essentially the same as those for the microwave lines of NH_3 (Legan *et al.* 1965). These room-temperature data are known to within 5 percent accuracy. That we obtain a good fit for our semiempirical model of the 10- and $16\text{-}\mu$ bands

of NH_3 to both the high-resolution and the low-resolution laboratory data using a single line width indicates that the distribution of line widths in the band does not effect the mean transmission visibly. When α/δ is small, the mean transmission deviates significantly from unity only after the individual lines have become saturated, so the limb darkening does not depend sensitively on the line shape or width in the core.

The line shape in the wings was derived from pure NH_3 or $\text{NH}_3\text{-N}_2$ mixtures and may be different for mixtures of NH_3 with H_2 and He. We assume that the error arising from this uncertainty has only a secondary effect on the determination of the effective temperature.

The band model was made to represent Jovian temperatures by altering the Boltzmann factors and partition function of the bands. A temperature of 130°K was assumed, except near the short-wavelength wing of the band where a mean temperature of 180°K was used. This rendered the $16\text{-}\mu$ band nearly insignificant, as well as many of the lines in the $10\text{-}\mu$ band.

Assumptions after Townes and Schawlow (1955) were made regarding the theoretical dependence of collisional half-width on temperature. The local temperature was then used to evaluate this half-width.

Using a frequency grid fine enough to delineate the frequency dependence of the band model, even at low pressures, we calculated mean transmissions over intervals of 10 cm^{-1} throughout the $8\text{--}14\text{-}\mu$ region for isothermal and isobaric paths. Tables of these mean transmissions were generated for a grid of effective pressures and path lengths. The effective pressure depended on the local temperature as well as on the partial pressures of the broadening gasses.

The Curtis-Godson approximation (Curtis 1952; Godson 1953) was generalized to apply to the new line shape. This, of course, required that the mean transmission along a homogeneous path agree with that along a path for which both the pressure and temperature varied, both in the strong-band and weak-band limits. "Effective" line parameters described the band model for the equivalent homogeneous path. The expressions for the equivalent pressure P_e and equivalent absorber A_e corresponding to a homogeneous atmosphere are

$$(P_e)^{0.78} = (\int dA_e P^{0.66}) / (\int dA_e P^{-0.12}), \quad A_e = \int dA,$$

where P is the local " NH_3 equivalent pressure" and where we have neglected the variation of the mean line strength with depth and the line-to-line variation of the half-width.

Because the 10- and $16\text{-}\mu$ bands of NH_3 lie on the tail of the thermal spectrum, they do not significantly affect Jupiter's temperature structure. Consequently, we added these bands to the models only after the structure had been determined by using the pressure-induced He- H_2 opacities and the rotational NH_3 band.

At each level of the model, the modified Curtis-Godson approximation was used to find what homogeneous path was equivalent to the actual path between a given depth in a specific model and the surface in a specific direction. These effective pressures and path lengths were calculated for each depth of each model. The appropriate value for the particular $\cos \theta$ (angle of specific intensity with local vertical) under consideration was interpolated in the table of mean transmission. Accordingly, the following modified integrated equation of transfer was used:

$$I_\nu(\mu, 0) = \int_0^\infty B_\nu(T) \exp \left[\frac{-\tau_\nu(\text{H}_2 + \text{He})}{\mu} \right] T_\nu(\text{NH}_3) \frac{d\tau_\nu}{\mu},$$

where the frequency-dependent quantities are averages over intervals of 10 cm^{-1} , the integration is over all the layers of the model shallower than total $\tau_\nu = 12$, $\tau_\nu(\text{H}_2 + \text{He})$ is the monochromatic optical depth of the helium and hydrogen opacities, and $T_\nu(\text{NH}_3)$ is the mean transmission of NH_3 at wavenumber ν and along path $\mu = \cos \theta$.

As has been stated earlier, the NH_3 in Jupiter's atmosphere has a negligible effect on the planet's temperature structure, except for the higher-temperature models where the effect is small and arises from the rotational band rather than the 8–14- μ vibration-rotation bands. Nevertheless, the observational bandpass encompasses a small fraction of Jupiter's thermal spectrum, and therein NH_3 dominates the average absorption coefficient in comparison with H_2 -He. It has been said that the 8–14- μ bands are saturated in Jupiter's spectrum. Ordinarily, for a constant-flux LTE atmosphere, such a statement implies that the monochromatic brightness temperature over the wavelength interval observed is equivalent to the boundary temperature of the model atmosphere. Such a temperature (approximately 100° K) is certainly lower than that observed (129° K). This is due to the fact that the H_2 -He opacity is effective at much higher atmospheric levels due to the strong concentration of NH_3 near the cloud tops imposed by its vapor-pressure equilibrium with the solid phase. The meaning of saturation is thus that within the bands the flux between line cores rises very little. If we make the usual approximation that the mean optical depth of continuum formation is 0.6, crudely corresponding to a depth of 45–50 km below the top of the Jovian photosphere (or roughly 20 km above cloud-temperature level) according to model selections we will make in the next section of this paper, then the average level from which the 8–14- μ flux emerges is approximately 2 km higher than this. At this level the H_2 -He absorption coefficient is below that of NH_3 in the same wavelength range by almost an order of magnitude. The depth-integrated contribution functions of the two molecular groups become more nearly equal, however, with NH_3 retaining dominance. It is easily appreciated that the confinements of NH_3 to a narrower depth range than H_2 -He thus causes it to flatten the limb darkening of Jupiter over what the planet would exhibit with an atmosphere of only H_2 and He. It is also of interest that the confinement of the effects of NH_3 to regions blanketed by absorbing H_2 -He, in which negligible NH_3 is present, promotes the adequacy of the approximation methods used to determine the effect of NH_3 on temperature structure. The smallness of the effect is nevertheless the best guarantor of this adequacy.

III. THE OBSERVATIONAL LIMB DARKENING

The observational limb darkening of Jupiter was obtained by making infrared scans across an approximate Jovian diameter. The 200-inch Hale telescope of Palomar Observatory was used. The scans (totaling 41) were collected on various dates throughout the 1965 apparition, as shown in Table 1. As equatorial-diametric scans, they are estimated to have passed within about 5°, or possibly 10°, of Jovidetic arc of Jupiter's sub-Earth point, and the angle of the scan line with the projected Jovian pole varied from about 73° to 77°. The scan direction was actually in right ascension at the rate of +2500 arc sec hour⁻¹. A mercury-doped germanium photoconductor cooled with liquid hydrogen was used. Aspects of the technique and photometric system have been previously reported (Willey 1966*a*, 1968).

TABLE 1
THE OBSERVATIONS

Julian Day (2438000+)	Number of Scans	Jovian Equatorial Diameter	Axial Position Angle
789.58	14	41"13	343°
812.60	5	38"19	343°
805.59	8	39"04	343°
867.63	4	33"60	347°
834.61	3	35"91	345°
824.61	7	36"87	344°

Each scan was first standardized as to the $\sin \theta$ scale (*abscissa*). This was not done in an absolute sense since an observational limb darkening does not go to zero at $\sin \theta = \pm 1$, and no astrometric calibration of scan rate was available with which a physical ephemeris (semidiameter) might have provided such a calibration. Forty points were chosen evenly along the abscissa where the ordinate (infrared specific intensity) was evaluated. This division satisfied the Nyquist criterion fairly well. Points 0 and 40 were placed at the extrapolated intersection of the limb signal with the sky and background, and the ordinate was defined zero at 0 and 40. Next, in order to weigh all points of a scan evenly in the intercomparison of scans, and to avoid undue biasing of the atmospheric-extinction analysis by the random error and atmospheric-extinction fluctuation of the center point of each scan, the ordinate was scaled by normalizing the area under each scan to unit value. Thus the atmospheric extinction correction to first order in $\sec z$ (zenith angle) was automatically incorporated in this normalization.

From this point on in the analysis, the measured scan ordinates at each value of the scan abscissa were treated as statistically independent and were analyzed separately.

At each point of the abscissa, then, the following analysis was performed. The ordinate points were replaced by their logarithms as a newly defined and superseding ordinate. The mean value of the ordinate was computed. The difference between individual ordinate values and the mean ordinate was fitted to a second-degree polynomial in $\sec z$. A perfectly gray extinction across the 8–14- μ band, or a lack of dependence of the relative Jovian infrared spectrum on limb distance, would result in zero coefficients for both first- and second-order terms in $\sec z$ (in fact, rigorous independence of scan shape on $\sec z$). The residuals between the individual ordinates as reduced observations and the ordinates as predicted by the polynomial were determined. Those observations were thrown out for which the residual was greater than 2.5 times the average residual. The analysis was performed anew on the revised data set. This process was repeated until none of the observations were rejected. Thus it was possible to select different sets of scans from which the final ordinates at different abscissae would be determined. The results are summarized in Table 2. In the extremes producing the final result all forty-one observa-

TABLE 2
SYNTHESIS OF OBSERVED LIMB DARKENING

Abscissa Point	Scans Used	Number of Iterations to Stabilize	Abscissa Point	Scans Used	Number of Iterations to Stabilize
1.....	38	1	21	36	2
2.....	34	4	22	35	3
3.....	34	3	23	36	3
4.....	30	5	24	37	2
5.....	34	4	25	37	2
6.....	41	0	26	37	2
7.....	38	2	27	38	2
8.....	30	5	28	38	1
9.....	35	2	29	40	1
10.....	29	6	30	40	1
11.....	39	1	31	41	0
12.....	38	3	32	40	1
13.....	36	3	33	37	2
14.....	36	3	34	32	6
15.....	35	4	35	34	4
16.....	32	5	36	41	0
17.....	32	5	37	38	2
18.....	34	4	38	33	3
19.....	36	3	39	36	2
20.....	33	4			

tions were retained at three points of the abscissa while at one point only twenty-nine observations were retained. With the final set of thirty-eight polynomials, the limb darkening was computed at $\sec z = 1$. This result differed negligibly from the limb darkening associated with the average at each abscissa point of ordinates of the corresponding data set. Thus no real dependence of scan shape on $\sec z$ was found. The ordinate was finally converted back to an antilogarithmic scale.

In the course of evaluating the data, the high-spatial-frequency features of individual scans (except those obviously much higher than the optical transfer function of the photometer extends) have been honored without adjudication of physical reality in the individual case. The signal-to-noise ratio of a single independent scan is not generally sufficient for this unless such sharp anomalies are of high amplitude. As a consequence of the statistical analysis employed, features present on only a few scans are rejected, whether they are real or only artifacts of noise. In view of the large number of scans involved, any sharp feature or limb-darkening asymmetry which emerges in the final result must be reasonably considered an aspect of Jovian "steady state" insofar as such a condition can be given meaningful existence. The formal probable error of every point on the final limb-darkening curve is considerably under 1 percent, and definitely smaller than the degree of limb-darkening asymmetry.

In Figure 2 may be seen all of the individual scans incorporated into the present study. Although there are many scans where response to band structure is obvious (this aspect of the Jovian infrared emission has been previously discussed [Willey 1968]), there is no obvious bias leading to the emphasis of a particular feature in the overall synthesis. However, it may well be noted that in the statistical analysis as shown in Table 2, the

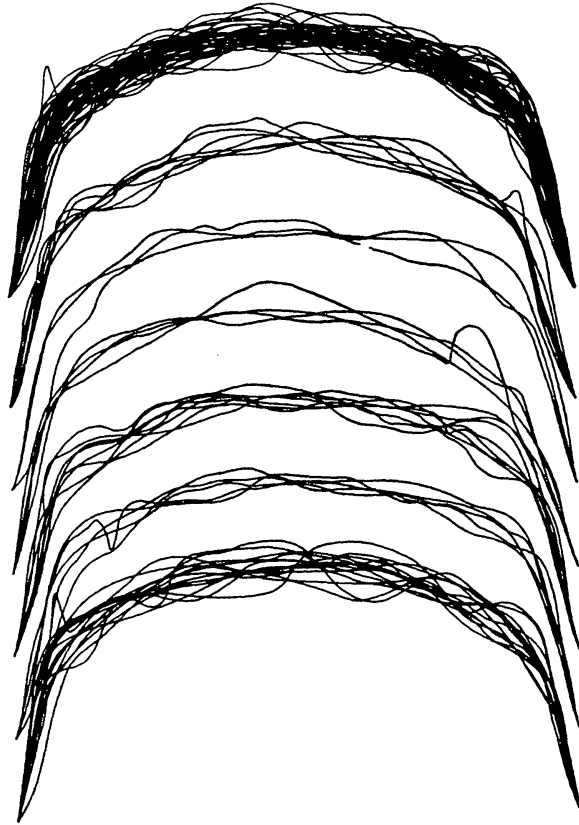


FIG. 2.—All normalized observational limb-darkening curves prior to statistical analysis. They are shown separated by observing run; at the top may be seen a plot of all of the scans against the same ordinate. The ascending order of the rest of the curve sets corresponds to the ordering in Table 1.

number of individual observations rejected on the morning side is sixty-seven whereas there were only thirty-nine rejections on the afternoon side.

IV. COMPARISON OF THEORY AND OBSERVATION

In order to obtain a theoretical comparison with the observational limb darkening it was first necessary to weigh the emergent monochromatic specific intensity by the spectral responsivity of the photometer and the atmospheric transmission and then to map it as a function of $\sin \theta$. The results were the same whether the atmospheric transmission used was that of Sinton and Strong (1960) or that of Goetz (1967). The equation of transfer was integrated over depth at ten values of $\cos \theta$. Linear interpolation was found to be quite adequate in transforming I_ν from μ to $\sin \theta$ between all points of the model output except between $\mu = 0$ and $\mu = 0.02$ ($\mu = \cos \theta$). In this region it was considered that the interpolation formula should fit both the value and the slope of the $I(\mu)$ output curve at $\mu = 0.02$, and furthermore the value of the interpolation formula should be zero at $\mu = 0$ whereas its first derivative should approach infinity as $\mu \rightarrow 0$. A convenient formula found to possess these properties with no points of inflection in the region of use, was

$$f(x) = AX^{-1/B},$$

where

$$B = 4I(\mu = 0.02)/[I(\mu = 0.1) - I(\mu = 0.0)] \quad \text{and} \quad A = I(\mu = 0.02)(0.02)^{-1/B}.$$

The efficacy of the formula was verified by integrating the transfer equation for $\mu = 0.01$.

The overall atmospheric-instrumental convolution function was determined as a tabulated function by numerically folding the Gaussian profile of atmospheric seeing with the semielliptical profile of the circular photometer aperture as follows. The relationship between the observed limb darkening and the true limb darkening, of infinite resolution, may be represented as two convolutions of the true profile performed in sequence by the seeing smear function and the photometer smear function. The actual diffraction of the 200-inch telescope, which is of course not of Gaussian profile, and which is $0''.7$ at 10μ on the Rayleigh criterion, is assumed incorporable into seeing with little error. Thus we have

$$\Phi(\nu'') = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(\nu) \Lambda_s(\nu - \nu') \Lambda_p(\nu' - \nu'') d\nu d\nu',$$

where

$$\Lambda_s(\chi) = \exp(-\frac{1}{2}\chi^2/\sigma^2);$$

$$\Lambda_p(\chi) = (a^2 - \chi^2)^{1/2}, \quad -a < \chi < a;$$

$$\Lambda_p(\chi) = 0, \quad |\chi| > a,$$

which is reducible to

$$\Phi(\nu') = \int_{-\infty}^{\infty} \phi(\nu' - \nu) \Lambda_r(\nu) d\nu,$$

where

$$\Lambda_r(\nu) = \int_{-a}^a (a^2 - \chi^2)^{1/2} \exp[-(\nu - \chi)^2/2\sigma^2] d\chi.$$

Units are $\sin \theta$. The function $\Phi(x)$ is observed limb darkening and $\phi(x)$ is true limb darkening. If one uses a mean Jovian equatorial semidiameter, weighted by number of scans, of $19''.26$, the value of a is 0.07915 . The value of σ is taken to be implied by a visual estimate of the seeing on the Mount Wilson scale and a size of seeing-tremor-disk corresponding to a Gaussian threshold of 10 percent of peak value. Inasmuch as the seeing smear is somewhat inexact, each model has been treated for several values of seeing.

The smeared theoretical limb darkening was then computed at fifty points in $\sin \theta$ from -1.25 to $+1.25$ by numerically folding the (infinitely resolved) theoretical limb-darkening curve with the overall atmospheric-instrumental profile corresponding to the

"seeing" used in the individual computation. The curve was then normalized to $I(0) = 1$.

For comparison between theory and observation the analyzed observational limb-darkening curve was now normalized to a central value of unity, and its abscissa was calibrated in terms of $\sin \theta$ by requiring the areas under the observational curve and the smeared theoretical curve to be equal. The complete results of this study are tabulated in Table 3.

V. DISCUSSION

The tabulation shows that the effect of varying the He/H_2 ratio on the theoretical limb-darkening curves is small compared with the effect of varying the effective temperature. In principle, we could derive He/H_2 from the magnitude of the specific intensity once we know T_e from the normalized limb darkening. We could not accomplish this because the neglect of the $7.5\text{-}\mu$ bands of CH_4 alters the flux in the $8\text{--}14\text{-}\mu$ range by nearly 20 percent. Whereas this neglect precludes a determination of the He/H_2 ratio, it affects the brightness temperature by less than 2° K and the limb darkening is scarcely affected at all so that it does not interfere significantly with our T_e determination.

The best fits imply an effective temperature for Jupiter's equatorial region between 135° and 140° K , with perhaps slight preference for the latter figure. This judgment ignores the disturbance (*sharp shoulder*) near the eastern limb which, if taken into account, would raise the value of the effective temperature closer to that found in our preliminary report but would result in a worse overall fit. In Figure 3 the observational-theoretical comparison of limb darkening is shown for the case $T_e = 135^\circ\text{ K}$, $\text{He}/\text{H}_2 = 1$, and seeing = 3. Figures 4–6 show the comparison for variation of model parameters in the neighborhood of best fit. In Figure 7, the comparisons that are shown are for the same model parameters as Figure 3; however, all values of seeing (quite in excess of seeing uncertainty) plus perfect resolution are incorporated. The seeing effects are surprisingly small except immediately near the limb. The general disagreement between theory and observation almost exactly at the limb surely arises from the inadequacy of a Gaussian representation of seeing in the wings of the resulting smear. That the fit for other values of the effective temperature is significantly worse may be readily verified by plotting values from Table 3. There is slight indication that the fits may be better for seeing 4 than seeing 3. That the seeing at $10\text{ }\mu$ is better than that in the visual range is reasonable, but by what amount is difficult to ascertain. The use of the seeing 4 comparisons may thus be justified even though seeing 3 was selected as nominal even with a subjective allowance for the wavelength difference. The comparisons seem uniformly improved, so that the same T_e is redeemed. It may be mentioned in passing that inas-

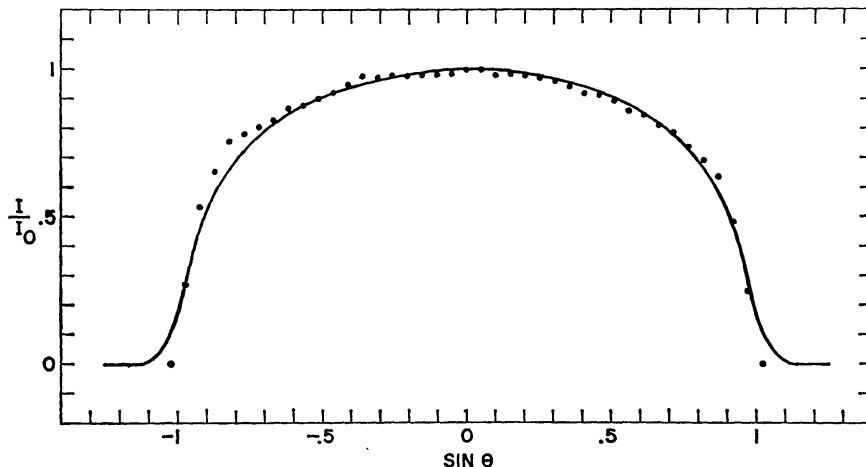


FIG. 3.—Observational-theoretical comparison of Jovian $10\text{-}\mu$ limb darkening for $T_e = 135^\circ$, $\text{He}/\text{H}_2 = 1$, and nominal infrared seeing. Filled circles, observations.

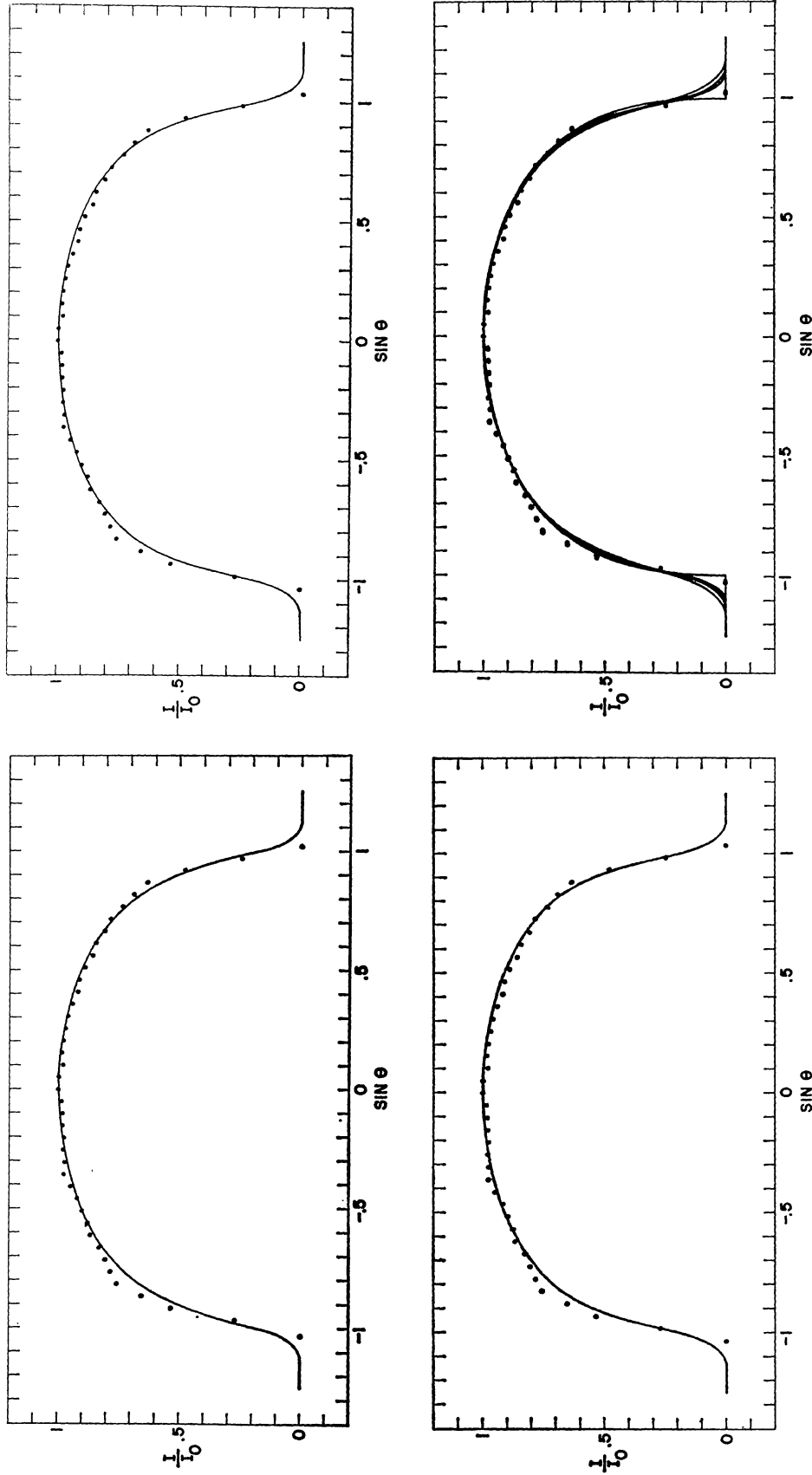


FIG. 4 (*top left*).—Observational-theoretical comparison of Jupiter's 8-14- μ limb darkening for $T_e = 135^\circ$, $\text{He}/\text{H}_2 = 0$, and nominal seeing.

FIG. 5 (*top right*).—Same as Figs. 3 and 4 except for $T_e = 140^\circ$ and $\text{He}/\text{H}_2 = 1$.

FIG. 6 (*bottom left*).—Same as Fig. 5 but for $T_e = 140^\circ$ and $\text{He}/\text{H}_2 = 0$.

FIG. 7 (*bottom right*).—Observational-theoretical comparison of Jupiter's thermal limb darkening paired for instrumental smear plus different values of astronomical seeing (1-2, 3, 4, 5) and one case of infinite resolution. The authors' calibration of the seeing scale may be somewhat subjective. The actual values of σ assumed in the Gaussian function in units of the Jovian radius are 0.0036, 0.012, 0.036, and 0.060. The radius of the photometer aperture is 0.07915.

much as the limb-darkening asymmetry is real, the scaling procedure ought perhaps to be applied to the area between the western limb and the center of the disk, rather than to the entire limb-darkening curve. This would make the fit between theory and observation even better than shown in Figures 3-6. In this connection it is also worth emphasizing that any method which scales the observations independently for each model comparison will minimize rather than maximize distinctions in goodness of fit between the various models. The general agreement of our value of T_e with that of Aumann, Gillespie, and Low (1969) of $134^\circ \pm 4^\circ$ K as determined by direct bolometry of the whole disk is support for the assumptions in the model atmosphere. These temperatures are well above those expected on the basis of solar heating alone, and the near equality of the equatorial and whole-disk values is additional support for the existence of a significant internal source of heat.

According to Gierasch and Goody (1969) the radiative time constants deep in Jupiter's atmosphere are long compared with probable dynamical ones. Nevertheless, the upper atmosphere must be in local radiative equilibrium (Trafton and Münch 1969). Observational evidence for a significant depth in Jupiter's equatorial zone to which radiative equilibrium holds is contained in the figures, which show that the radiative models fit the observations very well except for an isolated zone near the eastern limb. Because the mean depth of formation of the observed 8-14- μ radiation lies at a Planck mean optical depth of unity in the thermal spectrum, radiative equilibrium is the rule rather than the exception, at least to depths of $\tau \sim 1$.

The disturbance near the eastern limb may represent the penetration of a deeper dynamical disturbance to the layers of the 8-14- μ emission. If dynamical, however, it is apparently coupled to the diurnal insolation because the observational limb darkening derived in the present study represents the averages of many scans over several months in which transient phenomena were rejected. The flux in this bulge is otherwise difficult to explain.

Although we offer no confident explanation for the sharp shoulder on the Eastern limb, we note that it may represent a solar induced non-LTE transient phenomenon close to Jovian sunrise and that the decay of this effect justifies our model fitting on the afternoon-sunset limb while ignoring the discrepancy on the morning-sunrise limb. The very much larger number of observations rejected on the morning half of the disk compared to the afternoon half is undoubtedly significant. The disturbance may bear some physical relationship to the phenomenon of increased thermal radiation from regions of Jupiter's atmosphere shadowed by the Galilean satellites, although that phenomenon was unobservable during the apparition under investigation (Willey 1966b).

The effective temperature favored here is approximately 10° K lower than that favored in our preliminary report. This is because (a) we earlier treated only the eastern limb of Jupiter in our comparison and (b) the original method of allowing for the back-warming effect of the rotational NH_3 band (at layers in the model where the temperature was high enough to cause significant rotational NH_3 absorption above the layer, the τ_ν was multiplied by a factor of 20 for all frequencies lying in the rotational band and the model was then run to constant flux) was a crude overestimate.

In summary, these 1965 opposition observations imply that, over Jupiter's equatorial region, departures from radiative equilibrium are events isolated in space and time. Dynamics apparently plays a secondary role to radiative equilibrium in determining Jupiter's atmospheric structure above the $\tau = 1$ level. Furthermore, the effective temperature implied by the limb darkening at 10 μ is in good agreement with the direct bolometric measurement.

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