DOES RÖMER'S METHOD YIELD A UNIDIRECTIONAL SPEED OF LIGHT?

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Abstract

It is shown that any finite number of clocks at rest in an inertial reference system can be synchronized in an infinity of epistemologically permissible ways, the Reichenbach parameters ϵ_{ij} for pairs of the clocks taking values generally different from one-half. They are determined from a system of linear equations for which the Einsteinian case, where each $\epsilon_{ij} = \frac{1}{2}$, constitutes the trivial solution. Römer's method of finding the speed of light is then examined from the viewpoint of a reference system in which the coordinate clocks are in a general (non-Einsteinian) state of synchronization. It is shown that the quantity c which the method yields is *expressly not the one-way speed of light*, but the average speed over a closed path.

I. Introduction

In his opening paper on relativity theory, Einstein (1905) established *as a matter of definition* the equality of the speeds of light in the two opposite directions along a straight segment AB in an inertial reference system. Combined with the accepted physical fact that the average vacuum speed of light over the closed path ABA (and over any closed contour) always turns out to have the particular value c, namely, about 3×10^{10} cm sec^{-1}, this definition then confers upon c the secondary role of a *conventional unidirectional speed.* In the above statement, the "average" speed of course refers to that calculated from the measured path of the light signal and from the lapse of time between dispatch and return of the signal as determined by a single clock stationed at the source.

The conventionality of the one-way speed of light has, in recent times, been emphasized by Grünbaum (1955, 1964). In his words, "since no statement concerning a one-way transit time or one-way velocity derives its meaning from mere facts but also requires a prior *stipulation* of the criterion of clock synchronization, a choice . . . which renders the transit times (velocities) of light in opposite directions *un*equal, cannot possibly conflict with such physical isotropies and symmetries as prevail *independently* of our descriptive conventions". On the other hand, there exist observational methods of determining the speed of light, notable among them being the historically celebrated method of Römer, which yield the value c for what *seems* to be the unidirectional speed apparently independently of synchronization conventions. It has been claimed in a recent article (Brown 1967) that Römer's method in fact "nullifies Einstein's contention, repeated by Eddington and others, that we only know the out-and-return velocity, not the one-way velocity, so that the time of arrival of a signal at a distant point is never known from observation but can only be a convention".

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The object of the present study is to discover and elucidate this conventional element in the customary interpretation of Römer's observations. While in progress towards this goal, we shall see in some detail how any finite set of clocks at rest in an inertial reference system is theoretically capable of being synchronized in an infinity of epistemologically acceptable ways. Evaluated in a reference system in which the synchronization of coordinate clocks is not restricted to Einstein's convention, "the speed of light" resulting from the method of Römer is expressly *not* the one-way speed, but the average speed over a closed path.

II. A Pair of Synchronized Clocks

Let C_1 and C_2 be a pair of similar clocks at rest at points A_1 and A_2 respectively of an inertial reference system I, where the distance $A_1A_2 = a$. Similarity of clocks in this context, as usually in relativity, means that the clocks would go at the same rate if kept together so as to permit direct comparison. At an event E_1 at A_1, a light signal is dispatched towards A_2, is instantaneously reflected there at the event E_2 back towards A_1, and subsequently returns to A_1 at the event E_3. Let t_1 and t_3 be the readings of C_1 at the first and third events, with t_2 the reading of C_2 at the second event. We say that *C_2 is synchronized with C_1 according to the parameter* ϵ_{12} if its reading at E_2 satisfies the Reichenbach (1928) formula

$$t_2 = t_1 + \epsilon_{12}(t_3 - t_1)\,, \tag{1}$$

where ϵ_{12} is a fraction between 0 and 1. The particular choice $\epsilon_{12} = \frac{1}{2}$ is a constituent element in Einstein's principle of the constancy of the speed of light and hence of the special theory of relativity, but other choices of ϵ_{12} from within the interval $(0,1)$ are physically just as permissible, although will lead to a more complex descriptive picture of light propagation. As this matter has been dealt with by Grünbaum with great lucidity, there is no need to labour the point.

We now complement equation (1) with the physical fact, mentioned in the Introduction, that the average speed of the signal over the closed path $A_1A_2A_1$ as determined with the aid of the single clock C_1 must be equal to c. This is to say, $2a/(t_3 - t_1) = c$. It then follows that the unidirectional speeds of light over the paths A_1A_2 and A_2A_1 are respectively

$$c_{12} = a/(t_2 - t_1) = c/\{2\epsilon_{12}\}\,, \tag{2a}$$

$$c_{21} = a/(t_3 - t_2) = c/\{2(1-\epsilon_{12})\}\,. \tag{2b}$$

It is to be observed that precisely the same unidirectional speeds of light would result if, instead of C_2 having been synchronized with C_1 according to ϵ_{12}, the clock C_1 were synchronized with C_2 according to the different but related parameter $\epsilon_{21} = 1 - \epsilon_{12}$. We may say that in both cases the clocks acquire the same relative setting. A given *state of synchronization* as a mutual relationship between the two clocks is thus determined either by ϵ_{12} (referring to the *second* clock having been synchronized with the *first*) or equivalently by $1 - \epsilon_{12}$ (referring to the *first* clock having been synchronized with the *second*). In a symbolic notation, a given state of synchronization of the two clocks may then be represented either by $C_1\,\epsilon_{12}\,C_2$ or equivalently by $C_2(1-\epsilon_{12})C_1$.

While the one-way speed of light in a given direction is thus to a degree arbitrary, it is a function of the Reichenbach parameter ϵ_{12} defining the state of synchronization of the two clocks in terms of whose readings this speed is observed, and hence can only be altered if one or the other of the clocks is reset.

III. Three Synchronized Clocks

Let C_1, C_2, and C_3 be three similar clocks at rest at points A_1, A_2, and A_3 respectively of an inertial reference system I, where the distances $A_i A_j = a_{ij}$ $(i,j = 1,2,3)$. Assume that the pair C_1, C_2 has been synchronized to the state $C_1\,\epsilon_{12}\,C_2$, and the pair C_2, C_3 to the state $C_2\,\epsilon_{23}\,C_3$. We ask: does this automatically guarantee the synchronization of C_3 and C_1 to some state $C_3\,\epsilon_{31}\,C_1$, and what is the value of the parameter ϵ_{31}? The answer is that clock synchronization is not automatically transitive for all choices of ϵ_{12} and ϵ_{23}, as there is a condition to be satisfied, but assuming the condition to hold, the parameter ϵ_{31} is determined from the requirement that the average speed of light over the closed path $A_1 A_2 A_3 A_1$ (with reflections at A_2 and A_3) must be equal to c.

Now the total time required by a light signal, according to the single clock C_1, to describe the above triangular contour may be expressed, if the three clocks are in a state of synchronization, as the sum of the times taken (as determined by the appropriate pairs of clocks) to cover the three sides of the triangle, and is given by $a_{12}/c_{12} + a_{23}/c_{23} + a_{31}/c_{31}$, which by virtue of equation (2a) is

$$2(a_{12}\,\epsilon_{12} + a_{23}\,\epsilon_{23} + a_{31}\,\epsilon_{31})/c\,.$$

Since the total length $a_{12}+a_{23}+a_{31}$ of the contour when divided by the above expression must yield the value c, we obtain the equation

$$a_{12}(1-2\epsilon_{12})+a_{23}(1-2\epsilon_{23})+a_{31}(1-2\epsilon_{31}) = 0\,, \tag{3a}$$

which gives ϵ_{31} in terms of the chosen values for ϵ_{12} and ϵ_{23}. In order to be meaningful, however, the resulting ϵ_{31} must fall within the interval $(0,1)$, a condition which somewhat restricts the selection of ϵ_{12} and ϵ_{23} yet still leaves an infinity of possibilities.

It is seen from the symmetry of (3a) that it makes no difference at which of the clocks the signal starts its triangular journey. Remembering that $\epsilon_{ji} = 1-\epsilon_{ij}$, we quickly verify that the sense in which the contour is described is likewise immaterial. The state of synchronization of a set of three clocks is thus described mathematically by *a single relation* connecting the Reichenbach parameters for each of the three pairs.

Equation (3a) may be simplified by setting

$$1-2\epsilon_{23} = x_1\,, \qquad 1-2\epsilon_{31} = x_2\,, \qquad 1-2\epsilon_{12} = x_3\,,$$

for then it becomes

$$a_{23}\,x_1 + a_{31}\,x_2 + a_{12}\,x_3 = 0\,. \tag{3b}$$

The condition that each ϵ_{ij} must be within the interval $(0,1)$ now appears as the condition that each $|x_i| < 1$. Einstein's choice $\epsilon_{23} = \epsilon_{31} = \epsilon_{12} = \frac{1}{2}$ is seen to correspond to the trivial solution $x_1 = x_2 = x_3 = 0$.

In what follows, it is convenient to write the equation in an even simpler notation, namely

$$B_{12}+B_{23}+B_{31} = 0\,, \tag{3c}$$

where

$$B_{ij} = a_{ij}(1-2\epsilon_{ij})\,. \tag{4}$$

We note that the quantities B_{ij} are antisymmetric, i.e.

$$B_{ji} = -B_{ij}\,. \tag{5}$$

IV. Any Number of Synchronized Clocks

We now generalize the discussion to a set of s similar clocks C_i situated at rest at points A_i, where s is any finite integer and $i = 1, 2, \ldots, s$. The number of Reichenbach parameters ϵ_{ij} involved (leaving out the transpositions $\epsilon_{ji} = 1-\epsilon_{ij}$) is in this case

$$n(s) = \binom{s}{2} = \tfrac{1}{2}s(s-1)\,.$$

Again it turns out that the whole set can be synchronized in infinitely many ways, with $s-1$ of the parameters chosen freely but sufficiently close to the value $\frac{1}{2}$ in order to confine the remaining ones (solutions of $m(s) = \frac{1}{2}(s-2)(s-1)$ linear equations) within the interval $(0, 1)$ as required.

In order to verify this statement, let us first consider r of the clocks, namely $C_1, C_2, \ldots, C_r$, where $r < s$. If this is a synchronized set, then the average speed of light around *every* closed contour, or polygon, determined by some or all of the points $A_1, A_2, \ldots, A_r$ must be equal to c, a condition which is embodied for each contour by an equation of the type (3). For example, for the contour $A_1 A_2 A_3 \ldots A_r A_1$ we have

$$B_{12}+B_{23}+B_{3.}+\ldots+B_{.r}+B_{r1} = 0\,, \tag{6}$$

with analogous equations for all other contours consisting of the same r points taken in a different sequence, as well as for all smaller contours down to triangles. Among all these equations let the maximum number of *linearly independent* ones be denoted by $m(r)$. We now ask: if the number of clocks is increased by one, by introducing say the clock C_{r+1} at A_{r+1}, how many more *linearly independent* equations must be added to the previous $m(r)$ ones in order to embody a state of synchronization of the now augmented set of $r+1$ clocks? The additional equations will clearly be independent of the previous ones, as they all relate to contours involving the added point A_{r+1}.

Consider the system of $r-1$ obviously linearly independent equations

$$\left.\begin{array}{l} B_{r+1,1}\ \ +B_{1r}\ \ +B_{r,r+1} = 0\,, \\ B_{r+1,2}\ \ +B_{2r}\ \ +B_{r,r+1} = 0\,, \\ \ldots\quad\ldots\quad\ldots\quad\ldots \\ B_{r+1,r-1}+B_{r-1,r}+B_{r,r+1} = 0\,. \end{array}\right\} \tag{7}$$

We will establish that these equations, together with the earlier $m(r)$ equations, imply a condition of the type (6) for each contour determined by any or all of the $r+1$ points $A_1, A_2, \ldots, A_{r+1}$. Actually it suffices to prove the result for those contours which involve the added point A_{r+1}, since the earlier equations take care of all the contours from which A_{r+1} is absent. In other words, it is sufficient to prove the vanishing of all expressions of the type

$$V = B_{r+1,p}+B_{p.}+\ldots+B_{.q}+B_{q,r+1}\,.$$

Now by virtue of the earlier $m(r)$ equations and by (5) we must have

$$B_{p.}+\ldots+B_{.q} = -B_{qp} = B_{pq},$$

so that our expression at once reduces to

$$V = B_{r+1,p}+B_{pq}+B_{q,r+1}\,. \tag{8}$$

Here the indices p and q belong to the set $(1,2,\ldots,r)$.

If $p = q$, then $B_{pq} = B_{pp} = 0$ (for $a_{pp} = 0$) and so $V = 0$ by (5). Therefore let $p \neq q$. There now remain two possibilities: either (i) $p = r$ or (ii) p is one of $1,2,\ldots,r-1$. In case (i), expression (8) is

$$V = B_{r+1,r}+B_{rq}+B_{q,r+1} = -(B_{r+1,q}+B_{qr}+B_{r,r+1})\,.$$

As $q \neq r$, it must be one of $1,2,\ldots,r-1$, and the expression vanishes by virtue of one of the equations (7). In case (ii), expression (8) becomes, with (7),

$$V = -B_{pr}-B_{r,r+1}+B_{pq}+B_{q,r+1} = -(B_{r+1,q}+B_{qp}+B_{pr}+B_{r,r+1})$$

$$= -(B_{r+1,q}+B_{qr}+B_{r,r+1})\,,$$

which vanishes trivially when $q = r$ and again by virtue of one of the equations (7) when $q \neq r$. Therefore in all cases $V = 0$.

We have thus established that the number of linearly independent equations determining the Reichenbach parameters for a set of $r+1$ synchronized clocks exceeds by $r-1$ the number for r synchronized clocks, i.e.

$$m(r+1) = m(r)+r-1\,.$$

Moreover, we have actually identified a possible set of the additional $r-1$ equations, namely the set (7), which is clearly the simplest possible choice.

Since we already know from Section III that $m(3) = 1$, it follows from the above difference equation that

$$m(s) = \tfrac{1}{2}(s-2)(s-1)\,.$$

Each of these $m(s)$ equations is a linear homogeneous equation of the type (3b), the total number of unknowns x_i being, as mentioned earlier, $n(s) = \frac{1}{2}s(s-1)$. By the usual algebraic theory, every solution $\xi = (x_1, x_2, \ldots, x_n)$ is expressible as a linear combination of $n(s)-m(s) = s-1$ particular linearly independent solutions

$\xi_1, \xi_2, \ldots, \xi_{s-1}$, and involves $s-1$ unknowns $x_{m+1}, x_{m+2}, \ldots, x_n$ which may be selected arbitrarily. Provided that the arbitrary x_i are sufficiently close to zero (which means that the corresponding ϵ_{ij} are sufficiently close to the value $\frac{1}{2}$), the remaining x_i, numbering m, will have absolute values less than unity (which means that the corresponding ϵ_{ij} are between 0 and 1), as will be clear from the numerical example in Section V.

We again note that Einstein's synchronization convention corresponds to the trivial solution $x_1 = x_2 = \ldots = x_n = 0$.

V. Numerical Example

Consider a set of four clocks at the corners of a square $A_1A_2A_3A_4$ of side a (Fig. 1). A state of synchronization is defined by the six parameters ϵ_{12}, ϵ_{13}, ϵ_{14}, ϵ_{23}, ϵ_{24}, ϵ_{34}, which correspond to a solution of a system of $m(4) = 3$ equations. In the case of only three clocks we had the one equation (3c), to which we now add two more equations of the type (7). The complete system of equations is therefore

$$B_{12}+B_{23}+B_{31} = 0\,,$$

$$B_{41}+B_{13}+B_{34} = 0\,,$$

$$B_{42}+B_{23}+B_{34} = 0\,,$$

where, of course, $B_{31} = -B_{13}$, $B_{41} = -B_{14}$, $B_{42} = -B_{24}$. Using the relation (4) with $a_{12} = a_{23} = a_{34} = a_{14} = a$, $a_{13} = a_{24} = \surd 2\,a$, and setting

$$\left.\begin{aligned} x_1 &= 1-2\epsilon_{23}\,, & x_4 &= 1-2\epsilon_{12}\,, \\ x_2 &= 1-2\epsilon_{24}\,, & x_5 &= 1-2\epsilon_{13}\,, \\ x_3 &= 1-2\epsilon_{34}\,, & x_6 &= 1-2\epsilon_{14}\,, \end{aligned}\right\} \tag{9}$$

the equations may be written (after cancellation of a) in the form

$$x_1 + \;0\; + 0 + x_4 - \surd 2\,x_5 + 0 \;= 0\,,$$

$$0 + \;0\; + x_3 + 0 + \surd 2\,x_5 - x_6 = 0\,,$$

$$x_1 - \surd 2\,x_2 + x_3 + 0 + \;0\; + 0 \;= 0\,.$$

The matrix of this system is of rank 3, with the leading 3×3 minor nonzero. The last three unknowns x_4, x_5, and x_6 may therefore be assigned arbitrary values. Taking these to be respectively 1, 0, and 0, we obtain the particular solution

$$\xi_1 = (-1, -1/\surd 2, 0, 1, 0, 0)\,.$$

Two other linearly independent solutions are

$$\xi_2 = (\surd 2, 0, -\surd 2, 0, 1, 0)$$

and

$$\xi_3 = (0, 1/\surd 2, 1, 0, 0, 1)\,.$$

The general solution is therefore

$$\xi = (x_1, \ldots, x_6) = k_1 \xi_1 + k_2 \xi_2 + k_3 \xi_3$$
$$= \{-k_1 + \surd 2\, k_2,\ (1/\surd 2)(-k_1 + k_3),\ -\surd 2\, k_2 + k_3,\ k_1,\ k_2,\ k_3\}, \quad (10)$$

where k_1, k_2, and k_3 are arbitrary constants.

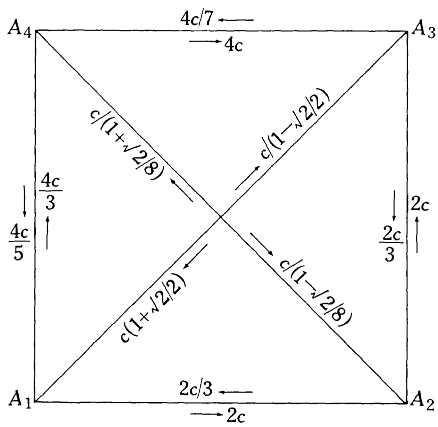

Fig. 1.—Diagram illustrating the numerical example of four clocks at the corners of a square. From the one-way speeds of light indicated the average speed over every closed contour is c.

From among the infinitely many possibilities which this result presents, let us take, for example,

$$k_1 = 1/2, \qquad k_2 = 1/\surd 2, \qquad k_3 = 1/4,$$

for which

$$\xi = (1/2,\ -1/4\surd 2,\ -3/4,\ 1/2,\ 1/\surd 2,\ 1/4). \quad (11)$$

This amounts to our having selected x_4, x_5, and x_6 arbitrarily, for which choice the expression (10) then quickly yields the remaining unknowns x_1, x_2, and x_3. With the present choice the absolute magnitude of each of the latter happens to be less than unity, in conformity with our physical requirement. Had this not been the case, we should have chosen absolutely smaller x_4, x_5, and x_6 (that is, k_1, k_2, and k_3), for in accordance with the general expression (10) the resulting x_1, x_2, and x_3 would thereby, likewise, have smaller absolute magnitudes.

The use of result (11) together with relations (9) and (2) now leads to one-way speeds of light as indicated in Figure 1. It may be verified by direct inspection that the average speed over every closed contour is c, as required.

VI. The Time-dilation Formula

Before we are in a position to evaluate Römer's method of determining the speed of light, it is necessary to attend to one more preliminary.

In the *special theory of relativity*—that is to say, in an inertial reference system I in which the coordinate clocks have been synchronized according to the *Einstein convention* $\epsilon_{ij} = \frac{1}{2}$—the increment $d\tau$ of the proper time of a moving clock C is related to the corresponding increment $\mathrm{d}t$ of the coordinate time of the reference system through the time-dilation formula

$$\mathrm{d}\tau = (1-v^2/c^2)^{\frac{1}{2}}\,\mathrm{d}t\,, \tag{12}$$

where v is the instantaneous speed of C in I. In particular, suppose that v is constant over some path, *not necessarily straight*, connecting two points A_1 and A_2 at which are situated synchronized clocks C_1 and C_2. If the clock C leaves A_1 at instant t_1 (according to C_1) and arrives at A_2 at instant $t_2 = t_1+\delta t_{12}$ (according to C_2), then by virtue of the constant speed the increment of the proper time of C for the entire trip is related to δt_{12} once again by

$$\delta\tau_{12} = (1-v^2/c^2)^{\frac{1}{2}}\,\delta t_{12}\,. \tag{13}$$

Now suppose, on the other hand, that the clocks C_1 and C_2 are in a state of synchronization $C_1\,\epsilon_{12}\,C_2$, where generally $\epsilon_{12} \neq \frac{1}{2}$. In what way should formula (13) now be altered? To begin with, the condition that the coordinate speed v of C should be constant over the whole path would seem to lose its meaning under the circumstances in which there are no other coordinate clocks save the two clocks C_1 and C_2 at the termini of the path. Nevertheless it is possible to preserve the idea of uniform motion if we define it in terms of what may be called the *self-measured speed* q of the clock C in the inertial reference system I. This concept has been used before by Ives (1948); it is defined as $\mathrm{d}\sigma/\mathrm{d}\tau$, where $\mathrm{d}\tau$ is the increment of the proper time of C, that is to say, the increment of its own reading, corresponding to which the clock covers a distance element $\mathrm{d}\sigma$ in the reference system I. As Ives has remarked, this is one of the most common ways of measuring speed in everyday practice; it is used by the mariner with his chronometer, by the automobilist in traversing the "measured mile", etc. Also, most important for the present purpose, the self-measured speed is independent of the question of clock synchronization in I.

The connection between q and the special-relativistic coordinate speed v results from equation (12) on division by $\mathrm{d}\sigma$:

$$1/q = (1-v^2/c^2)^{\frac{1}{2}}(1/v)\,.$$

As a consequence, relation (13) now becomes

$$\delta\tau_{12} = (1+q^2/c^2)^{-\frac{1}{2}}\,\delta t_{12}\,, \tag{14}$$

and the condition of constancy of the special-relativistic speed v along the given curve between A_1 and A_2 now becomes the condition of uniformity of motion as defined by the statement that $q =$ constant over the curve in question.

At zero instant according to C_1, let a light signal be dispatched from A_1 towards A_2. This signal of course follows the *straight* path A_1A_2, of length a, say. If the clocks C_1 and C_2 are in the state of synchronization $C_1\,\epsilon_{12}\,C_2$, then the time of arrival of the signal is, according to C_2,

$$t' = a/c_{12} = 2a\epsilon_{12}/c\,.$$

For Einsteinian synchronization, on the other hand, the clock C_2 would display at the event of the signal's arrival the reading

$$t = a/c\,.$$

The difference of these expressions, namely

$$t-t' = a(1-2\epsilon_{12})/c\,,$$

gives the amount by which the reading of C_2 must be adjusted in order to reset C_2 from Einsteinian synchronization to the state of synchronization $C_1\,\epsilon_{12}\,C_2$ with respect to the clock C_1. It is now evident that, on such a resynchronization, the formula (14) must be altered to read

$$\delta\tau_{12} = (1+q^2/c^2)^{-\frac{1}{2}}\{\delta t_{12}+a(1-2\epsilon_{12})/c\}\,, \tag{15}$$

where δt_{12} is the duration of the journey of the clock C (at *uniform* self-measured speed q along the given curve) from A_1 to A_2 as determined by the clocks C_1 and C_2 in the state of synchronization $C_1\,\epsilon_{12}\,C_2$.

VII. Analysis of Römer's Method

Let us now consider the circumstances underlying the *principle* of the method of Römer for the determination of the speed of light.

C is a clock in motion with a constant self-measured speed q in a circle of diameter d which is at rest in an inertial reference system I, while at an external point A_0, also at rest in I, a beacon is emitting flashes of light at what are judged to be equal intervals T by a local clock C_0. The flashes are received by C at varying intervals, as measured by C itself, depending on the direction of its instantaneous velocity. This, of course, is an example of the Doppler effect.

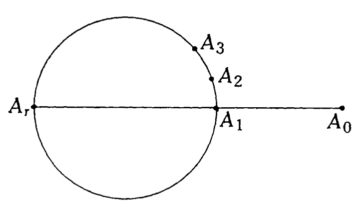

Fig. 2.—Diagram illustrating a clock in circular motion receiving light signals from A_0 at the points $A_1, A_2, \ldots, A_r$.

Suppose that the first signal reaches the clock C when the latter is at the point A_1 of the circle which is nearest A_0 (Fig. 2). At this event, let C display a reading τ_1. The second signal arrives when C is at a point A_2 and reads τ_2, the third when C is at A_3 and reads τ_3, etc. Finally, let the event at which C is furthest from A_0 be the event of arrival of the rth signal, let the point in question be labelled A_r, and let τ_r be the corresponding reading of C. The instants of emission of the signals according to C_0 will be taken as $0, T, 2T, \ldots, (r-1)T$ respectively.

Now if we wish to relate the received periods to the emitted ones and then to proceed to draw any physical conclusions which the example is capable of yielding, it is necessary to think in terms of *a set of coordinate clocks* C_i at rest at the points

A_i ($i = 1, 2, \ldots, r$), in addition to the clock C_0 at A_0, all in some state of synchronization as defined by a set of mutually compatible Reichenbach parameters. For if the right to utilize a "coordinate time" in the inertial reference system I should be denied us, then there would remain no theoretical structure at all to carry our subsequent argumentation. Römer's observations would then remain beyond interpretation, at least until the advent of radically new theoretical concepts.

In the standard interpretation of the observations, the speed of light is tacitly assumed to have the same value c over each of the paths $A_0 A_1, A_0 A_2, A_0 A_3, \ldots, A_0 A_r$, which is of course equivalent to assuming the value $\frac{1}{2}$ for all the parameters ϵ_{ij} connecting pairs C_i, C_j ($i, j = 0, 1, 2, \ldots, r$) of the coordinate clocks. There is no physical or epistemological reason, however, that would preclude the use of any other set of parameter values. We will therefore make no particular numerical choice; it is sufficient merely to assume, quite generally, that the ϵ_{ij} constitute *any* mutually compatible set. This means, mathematically, that an equation of the type (6) must be satisfied for every polygon defined by three or more of the clocks.

We are now in a position to relate the times of emission and receipt of the signals *in the reference system I*. Using, as before, the symbol a_{ij} for the distance $A_i A_j$, we may say that signals 1, 2, 3, ..., r are received at the instants

$$
\begin{aligned}
t_1 &= a_{01}/c_{01} = 2\epsilon_{01}\, a_{01}/c\,,\\
t_2 &= T + a_{02}/c_{02} = T + 2\epsilon_{02}\, a_{02}/c\,,\\
t_3 &= 2T + a_{03}/c_{03} = 2T + 2\epsilon_{03}\, a_{03}/c\,,\\
&\cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot\\
t_r &= (r-1)T + a_{0r}/c_{0r} = (r-1)T + 2\epsilon_{0r}\, a_{0r}/c\,.
\end{aligned}
$$

The period between the first and second signals *according to the moving clock C* is therefore, with the aid of equation (15), where for the sake of brevity we set $(1+q^2/c^2)^{-\frac{1}{2}} = Q$,

$$
\begin{aligned}
\tau_2 - \tau_1 &= Q[t_2 - t_1 + a_{12}(1-2\epsilon_{12})/c]\\
&= Q[T + 2\epsilon_{02}\, a_{02}/c - 2\epsilon_{01}\, a_{01}/c + a_{12}(1-2\epsilon_{12})/c]\\
&= Q[T + (a_{20} - a_{10})/c + \{a_{01}(1-2\epsilon_{01}) + a_{12}(1-2\epsilon_{12}) + a_{20}(1-2\epsilon_{20})\}/c]\\
&= Q[T + (a_{20} - a_{10})/c]\,,
\end{aligned}
$$

because of the synchronization of the coordinate clocks in I. In the same way we obtain

$$
\begin{aligned}
\tau_3 - \tau_2 &= Q[T + (a_{30} - a_{20})/c]\,,\\
&\cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot \quad \cdot\cdot\\
\tau_r - \tau_{r-1} &= Q[T + (a_{r0} - a_{r-1,0})/c]\,.
\end{aligned}
$$

These $r-1$ equations give on summation

$$\tau_r - \tau_1 = Q[(r-1)T + (a_{r0} - a_{10})/c],$$

or

$$\tau_r = \tau_1 + Q[(r-1)T + a_{r1}/c]. \tag{16}$$

Suppose, next, that the flashes of the beacon are sufficiently rapid in order that the distances a_{20} and a_{10} be very nearly equal. The period according to C between the first and second signals is then approximately

$$\tau_2 - \tau_1 = QT.$$

If an observer moving with C expects the periods between successive signals to remain constant, then accordingly he will expect the arrival of the rth signal at the time

$$\tau_r' = \tau_1 + (r-1)QT,$$

which is an earlier time than that given by expression (16). The cumulative "delay" of the signals during the motion of C from A_0 to A_r is therefore

$$\tau_r - \tau_r' = Qa_{r1}/c = Qd/c \approx d/c, \tag{17}$$

if $Q \approx 1$.

What we have arrived at is the well-known result of Römer. According to its customary interpretation, the accumulated retardation of the signals during half a revolution of C represents the time taken by light (which apparently had only travelled one way) to traverse the diameter of the circle. Identifying C with an observatory carried by the Earth in its orbit around the Sun, and the beacon with the periodic occurrence of eclipses of one of the moons of the planet Jupiter, the retardation $\tau_r - \tau_r'$ may be determined by direct observations over a period of six months and the supposedly one-way speed of light calculated from the formula (17). Our analysis, however, has revealed this interpretation as profoundly erroneous. The quantity c which appears in (17) is traced back to relation (2a), where it expressly represents the average two-way, or round-trip, speed of light. On the contrary, unidirectional speeds of light have been explicitly different from c unless the particular Reichenbach parameter had been chosen to have the value one-half. We therefore conclude that the method of Römer (as indeed *all* observational methods of finding the speed of light) can only supply an *objective average speed in a closed path.*

VIII. References

Brown, G. B. (1967).—*Bull. Inst. Phys., Lond.* **18**, 71.

Einstein, A. (1905).—*Annln. Phys.* **17**, 891. English translation (1922) in "The Principle of Relativity". (Dover: New York.)

Grünbaum, A. (1955).—*Am. J. Phys.* **23**, 450.

Grünbaum, A. (1964).—"Philosophical Problems of Space and Time." Ch. 12. (Routledge & Kegan Paul: London.)

Ives, H. E. (1948).—*J. opt. Soc. Am.* **38**, 879.

Reichenbach, H. (1928).—"Philosophie der Raum-Zeit-Lehre." Sec. 19. (de Gruyter: Berlin.) English translation (1958): "The Philosophy of Space and Time". (Dover: New York.)